Dynamic Harmonic Estimation Using a Novel Robust Filtering Strategy: Iterated Sliding Innovation Cubature Filter

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***Abstract*— Active filters (AFs) are effective tools for mitigating the detrimental effects of harmonic components on the power systems. The performance of AFs is significantly dependent on designing an accurate and robust estimator which is responsible for providing reference harmonic values. In this paper, a novel technique, called sliding innovation cubature filter, is proposed to estimate the harmonic parameters, i.e., magnitude and phase, in various operating conditions. The proposed method exploits the concept of sliding mode control in the formulation of the measurement update step in the Bayesian filtering framework to enhance the robustness of the estimator. Furthermore, the iterated version of the proposed algorithm, called iterative sliding innovation cubature filter, is presented to enhance the accuracy of the estimator. The proposed method keeps its robustness and accuracy in the noisy conditions under the fault occurrence as well as power system transients. The obtained results from both simulation and experimental setup confirm that the proposed estimator is more accurate and robust with a higher convergence speed compared to the well-known discrete Fourier transform (DFT), cubature Kalman filter (CKF), iterated extended Kalman filter (IEKF), and particle filter (PF).**

***Index Terms*—Harmonic estimation, Cubature Kalman Filter, Sliding Innovation Filter, sliding boundary, transient signal.**

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I. Introduction

POWER electronics science has brought about significant advances in modern power systems. The main reason behind this is the developments of semiconductors in reliability, switching speed, and thermal features, which have accelerated the application of power electronic devices in flexible power system and control. Nevertheless, power-switching devices have nonlinear and time-varying dynamics leading to non-sinusoidal currents flowing through the power lines [1]. In such a situation, the system stability and power quality may be at risk of high harmonic distortions. Hence, the active power filters are used to compensate for the undesired harmonics in the power system. Harmonic estimation algorithms, being the core of active filters, are applied to extract harmonics parameters of a distorted power signal in online mode [2]. As requirements for high power quality grows rapidly, researchers have conducted many studies to develop harmonic estimation techniques in power systems.

Discrete Fourier Transform (DFT) in its various forms is a simple and time-efficient algorithm that is repeatedly applied for harmonic parameters estimation [3-7]. However, this method yields reliable and accurate results only when the sampling frequency is an integer multiple of the frequency of harmonic components existing in the power system. Due to generation-demand imbalance, frequency fluctuation in a power system is inevitable. So, Coherent sampling is not guaranteed in DFT-based methods leading to so-called leakage and picket fence undesirable effects [8]. While the DFT is appropriate for just spectral analysis, Discrete Wavelet Transform (DWT) is a signal processing tool for the time-frequency analysis which is capable of extracting the non-stationary and noisy signal components. However, DWT-based algorithms with fine tuning become computationally intensive and they also take some energy to invest to select the proper wavelets for a specific signal processing application [9]. Several different methods for the mitigation of DWT drawbacks have been proposed [10-12]. Nevertheless, some processing time issues are still in effect for DWT-based algorithms due to computational limitations in the frequency domain, and their sensitivity to sampling parameters. Hence, time-domain processing algorithms have attracted more interest in extracting signal parameters.

Phase-locked loop (PLL) is also a well-known tool for signal processing applications, especially in telecommunication industries [13, 14]. In spite of precise results in steady states, the PLL-based methods do not perform efficiently in oscillatory conditions where the signal parameters have high dynamics. Besides, these methods require complex hardware resources to produce the desired reference signals [15, 16]. Artificial Neural Networks are capable of approximating the mapping relationship between states and measurements in a nonlinear process. Hence, they are frequently utilized for estimating the harmonic components of the power signals [17-19]. Nonetheless, the unavailability of adequate training data and the presence of undesirable noises in the signal can lead to convergence issues or heavy iterative computation [20]. Evolutionary algorithms often provide well-approximating solutions to some types of estimation problems [21, 22]. As an advantage, they do not need any assumptions about the underlying nonlinear system. However, in high dimensional estimation applications, e.g., parameter estimation of a signal with multiple harmonic components, the computational complexity of EAs may be a prohibiting factor. Hence, this category of methods is applied to harmonic estimation problems in combination with the analytical estimators to control the computational burden [23-25].

Least Squares Estimator (LSE) in its batch format has been repeatedly implemented by researchers for harmonic parameters estimation [26-28]. Nevertheless, the batch LSE requires the information from the previous step in its estimation process which leads to structure matrix augmentation. In other words, as the number of iterations increases, its computational burden outgrows our resources which is a limiting factor in online applications. To overcome this issue, the recursive and nonlinear versions of LS-based methods have been applied [29]. However, these modified versions show poor results for the systems with a high degree of nonlinearity [30]. Kalman Filter (KF), as a filtering approach, has been recognized as a high-performance estimation tool. Because of its strong theory, a wide area of research has been conducted to propose appropriate versions of KFs to find optimal and robust solutions for various estimation problems. Also, a significant trend has been shaped to establish the state-space model of harmonic estimation problems based on KF formulations [31-36]. Despite obtaining acceptable simulation results, the robustness of KF-based methods in the presence of power system disturbances (transients, faults, time-varying parameters, etc.) has not been practically addressed [37].

This paper presents a novel Bayesian filtering paradigm to estimate harmonic parameters of a distorted power signal. The advantages of the proposed approach are its precise and robust solution in the presence of faults, transients, and noise polluted signals. The time update estimates of the proposed filter are obtained by applying the efficient and simple numerical integration using cubature points generated by the third-order cubature rule. Because of time-varying dynamics, transients, and fault conditions in the power systems, modeling errors in the state-space representation of the distorted waveforms are inevitable. The main contribution of the paper is proposing a Sliding Innovation (SI) term in the Bayesian filtering framework which guarantees the robustness of the estimates in the measurement update phase. Furthermore, in the measurement update stage, the SI term is recalculated in an iterative process to compensate for the modeling errors caused by different operating conditions. Such an iterative process enhances the estimation accuracy of the proposed SI-based strategy. Different conditions of a distorted power signal are simulated and experimentally tested to demonstrate the capabilities of the proposed filter, namely Iterated Sliding Innovation Cubature Filter (ISICF), in harmonic estimation problems. Furthermore, to highlight the performance of the proposed technique the accuracy and convergence properties of ISICF are compared to those of conventional DFT, CKF, IEKF and PF methods. It should be mentioned that the proposed filtering paradigm can be used for a variety of estimation problems in which the systems cannot be modeled precisely.

II. Proposed Algorithm

The discrete form of a distorted voltage or current waveform in a power system can be represented by a series of Harmonics whose frequencies are integer multiples of the power system frequency represented by:

$$Z_k = \sum_{n=1}^{N} A_n sin(2\pi f_n k\tau_s + \varphi_n) + v_k \quad (1)$$

where f_n is the frequency of n-th order harmonic of the power signal, and N is the total number of the harmonics. The measurement signal Z_k at discrete time k with the interval of τ_s is corrupted with additive noise v_k. The amplitudes A_n and phases φ_n are states to be estimated. As can be observed, the general formulation of the measurement Z is a nonlinear function of the φ_n. Then, the target parameters should be estimated by applying a nonlinear paradigm. In presence of a high number of harmonics, nonlinear solving of harmonic estimation problems would be time-consuming. Hence, a computationally efficient Bayesian filtering method with simple numerical integration suiting for high dimensional state estimation problems is used in this paper [38]. The KF in its various forms is established as a two-stage estimation approach [39, 40]. The first stage includes time update equations. The following state vector is given for the harmonic estimation problem:

$$\mathbf{X} = [\varphi_1, \varphi_2, \dots, \varphi_n, A_1, A_2, \dots A_n]^T \quad (2)$$

Cholesky factorization on the covariance matrix $\boldsymbol{P}$ is performed to obtain square roots of the state estimation errors:

$$\boldsymbol{P}_{k-1|k-1} = \boldsymbol{S}_{k-1|k-1}\boldsymbol{S}_{k-1|k-1}^T \quad (3)$$

Using square root matrix $\boldsymbol{S}$, the propagated cubature points are extracted:

$$\boldsymbol{X}_{i,k-1|k-1} = \boldsymbol{S}_{k-1|k-1}\boldsymbol{\xi}_i + \hat{\boldsymbol{x}}_{k-1|k-1} \quad (4)$$

Where $i = 1, 2, \dots, 2m$, and m is the number of states to be estimated. $\boldsymbol{\xi}_i$ is a vector with equal elements as follows:

$$\begin{cases} \boldsymbol{\xi}_i = \sqrt{m} & i \leq m \\ \boldsymbol{\xi}_i = -\sqrt{m} & i > m \end{cases} \tag{5}$$

The propagated cubature points are evaluated using the state transition equation of the system. Then, a simple integration is implemented to obtain the time update estimation of the states:

$$\boldsymbol{X}^*_{i,k|k-1} = f\left(\boldsymbol{X}_{i,k-1|k-1}\right) \tag{6}$$

$$\hat{\boldsymbol{x}}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{2m} \boldsymbol{X}^*_{i,k|k-1} \tag{7}$$

In the harmonic estimation problem, the state transition matrix f is m dimensional identity matrix. The time update process is accomplished by updating the error covariance matrix:

$$\boldsymbol{P}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{2m} \boldsymbol{X}^*_{i,k|k-1}\boldsymbol{X}^{*T}_{i,k|k-1} - \hat{\boldsymbol{x}}_{k|k-1}\hat{\boldsymbol{x}}^T_{k|k-1} + \boldsymbol{Q} \tag{8}$$

where $\boldsymbol{Q}$ is the model noise diagonal matrix whose non-zero values are tuned by the designer. The second stage is to derive the measurement update equations for error and state matrices. Once a new measurement is received, the error covariance matrix is factorized just the same as the time update stage:

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{S}_{k|k-1}\boldsymbol{S}^T_{k|k-1} \tag{9}$$

The cubature points are attained based on factorized matrix **S**:

$$\boldsymbol{X}_{i,k|k-1} = \boldsymbol{S}_{k|k-1}\boldsymbol{\xi}_i + \hat{\boldsymbol{x}}_{k|k-1} \tag{10}$$

Applying the measurement model of the system cubature points are evaluated then, using simple numerical integration of the cubature points a deterministic value is estimated for the measurement in the measurement update stage:

$$\boldsymbol{Z}_{i,k|k-1} = h\left(\boldsymbol{X}_{i,k|k-1}, \boldsymbol{u}_k\right) \tag{11}$$

$$\hat{\boldsymbol{z}}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{2m} \boldsymbol{Z}_{i,k|k-1} \tag{12}$$

Unlike the common filtering strategies in which the filter gain is derived as a function of the state error covariance, the proposed filter applies the Sliding Innovation (SI) term (inspired by the concept of sliding mode control) in its gain formulation [41, 42]. The Innovation $\tilde{\boldsymbol{z}}$ and gain matrix $\boldsymbol{G}$ are calculated as follows:

$$\tilde{\boldsymbol{z}}_{k|k-1} = Z_k - \hat{\boldsymbol{z}}_{k|k-1} \tag{13}$$

$$\boldsymbol{G}_k = \boldsymbol{H}^{-1}\overline{sat}\left(\frac{|\tilde{\boldsymbol{z}}_{k|k-1}|}{\delta}\right) \tag{14}$$

As seen, the saturation function $\overline{sat}$ and a control parameter δ are utilized to keep the gain value in the targeted sliding boundary layer. $\overline{sat}$ yields an output sliding between +1 and -1, and δ is the sliding boundary layer width. Based on the level of uncertainties in the estimation process, the sliding boundary width can be determined (further details on computing the δ values are explained in the next section). Note that the measurement equation is nonlinear in the harmonic estimation problem. Hence, the measurement matrix h is linearized around the time-update estimates using first-order Taylor series expansion:

$$\boldsymbol{H}_k = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}_{k|k-1}} \tag{15}$$

The estimates for states and covariance matrices are then updated using the gain matrix:

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{G}_k\tilde{\boldsymbol{z}}_{k|k-1} \tag{16}$$

$$\boldsymbol{P}_{k|k} = (\boldsymbol{I} - \boldsymbol{G}_k\boldsymbol{H}_k)\boldsymbol{P}_{k|k-1}(\boldsymbol{I} - \boldsymbol{G}_k\boldsymbol{H}_k)^T + \boldsymbol{G}_k\sigma_v\boldsymbol{G}^T_k \tag{17}$$

The current measurement information is utilized to estimate $\hat{\boldsymbol{x}}_{k|k}$; then, it is expected that a better estimation is obtained for the updated state vector $\hat{\boldsymbol{x}}_{k|k}$ compared to $\hat{\boldsymbol{x}}_{k|k-1}$. This leads to the idea of reconstructing the linearized measurement matrix $\boldsymbol{H}_k$ around the new estimate $\hat{\boldsymbol{x}}_{k|k}$ to reduce the linearization error. Then, the process given in the (14) through (17) is used to recalculate the state vector and error covariance matrix in the measurement update stage. Re-linearizing the measurement matrix can be repeated as many times as desired, although for most problems the majority of the possible improvement is obtained by only one-time re-linearization [39, 43].

The proposed filtering procedure makes use of the sliding innovation concept in the gain matrix formulation to obtain robust estimates for a typical nonlinear system whose measurement and system equations are evaluated using cubature points. Hence, the filter is named Sliding Innovation Cubature Filter (SICF). When applying the re-linearized measurement matrix in an iterative process, the filter is called as Iterative Sliding Innovation Cubature Filter (ISICF). The formulation of the proposed filtering paradigm in the context of the harmonic estimation problem was presented in this section. The theory can be expanded to be used in a wide area of nonlinear estimation research. The flowchart of the proposed algorithm is shown in Fig. 1.

A. *Determination of sliding boundary width:*

The proposed method applies sliding boundary width δ to keep the estimation uncertainty in a bounded range. The δ parameter regulates the filtering gain such that state estimates stay within a region of the true state trajectory in the whole estimation process. For this purpose, the sliding boundary layer width is defined as a function of the modeling uncertainty and noise present in the estimation process [44, 45]. To keep the uncertainties in a constraint boundary, the Lyapunov stability equation is applied. Firstly, the state estimation error $\tilde{\boldsymbol{x}}$ in terms of process uncertainties is obtained. The gain formulation in (14) is applied to the measurement update (16) which results in:

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{H}_k^{-1}\overline{sat}\left(\frac{|\tilde{\boldsymbol{z}}_{k|k-1}|}{\delta}\right)\tilde{\boldsymbol{z}}_{k|k-1} \quad (18)$$

Then, the state estimation error equation is exploited according to (19):

$$\tilde{\boldsymbol{x}}_{k|k} = \tilde{\boldsymbol{x}}_{k|k-1} - \boldsymbol{H}_k^{-1}\overline{sat}\left(\frac{|\tilde{\boldsymbol{z}}_{k|k-1}|}{\delta}\right)\tilde{\boldsymbol{z}}_{k|k-1} \quad (19)$$

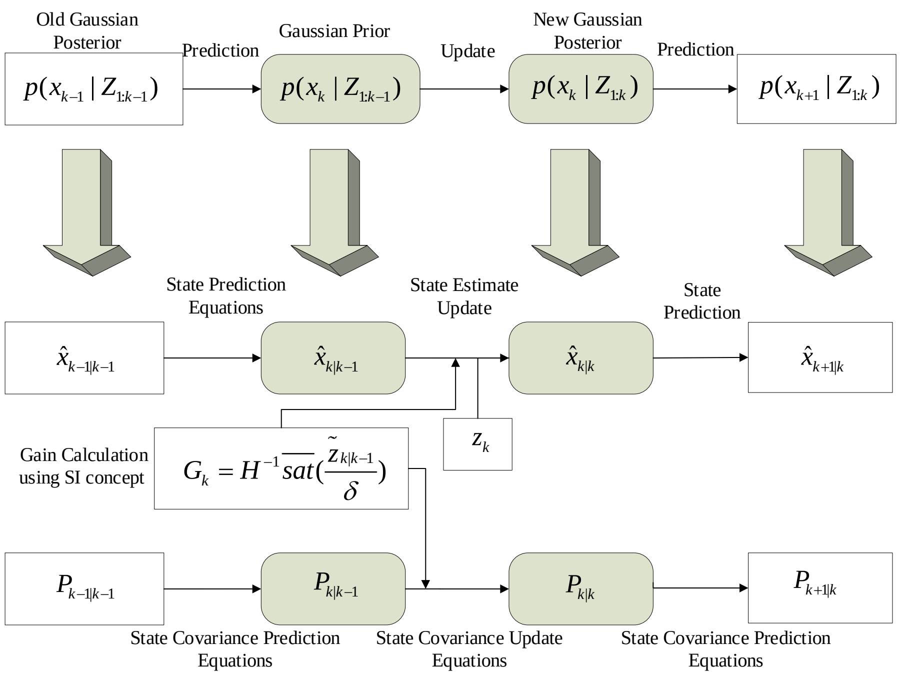


Fig. 1. Flowchart of the proposed ISICF algorithm

Since the state transition matrix f is the identity matrix, then the (19) can be written as follows:

$$\tilde{\boldsymbol{x}}_{k|k} = \tilde{\boldsymbol{x}}_{k-1|k-1} - \boldsymbol{H}_k^{-1}\overline{sat}\left(\frac{|\tilde{\boldsymbol{z}}_{k|k-1}|}{\delta}\right)\tilde{\boldsymbol{z}}_{k|k-1} \quad (20)$$

Then, the following is the recursive way of estimating the state error in terms of the initial condition:

$$\tilde{\boldsymbol{x}}_{k|k} = \tilde{\boldsymbol{x}}_{0|0} - \sum_{k=1}^{k} \boldsymbol{H}_k^{-1}\overline{sat}\left(\frac{|\tilde{\boldsymbol{z}}_{k|k-1}|}{\delta}\right)\tilde{\boldsymbol{z}}_{k|k-1} \quad (21)$$

The above Lyapunov stability equation is marginally stable if the state estimation error $\tilde{x}$ is bounded for all iterations k and all bounded initial conditions $\tilde{x}_{0|0}$. Then, the sliding boundary width δ should be assigned to satisfy the following equation:

$$\sum_{k=1}^{k} \boldsymbol{H}_k^{-1}\overline{sat}\left(\frac{|\tilde{\boldsymbol{z}}_{k|k-1}|}{\delta}\right)\tilde{\boldsymbol{z}}_{k|k-1} < \xi \quad (22)$$

where ξ is a limit which is determined by the filter designer. The above equation extracts a theoretical bound for the estimation error. Based on our empirical studies, the ξ is selected to be 0.02 for the harmonic estimation problem.

III. Simulation and Experimental Results:

To demonstrate the performance of the ISICF in the estimation of power signals, different scenarios are taken into consideration. Two categories of experiments including software simulations and experimental tests are considered in this respect. To drive the software simulations, a typical distorted waveform containing a chain of common power harmonics is defined as a reference. The harmonic content of the selected waveform is in accordance with the electrical current of high-intensity discharge electronic devices and arc furnaces [37]. Furthermore, a dynamic signal, which is constructed based on the reference waveform, is defined to show the strength of the proposed method in tracking the time-varying harmonic amplitudes. Simulation results of ISICF in each step are numerically compared with those of the well-known DFT, CKF, IEKF, and PF methods.

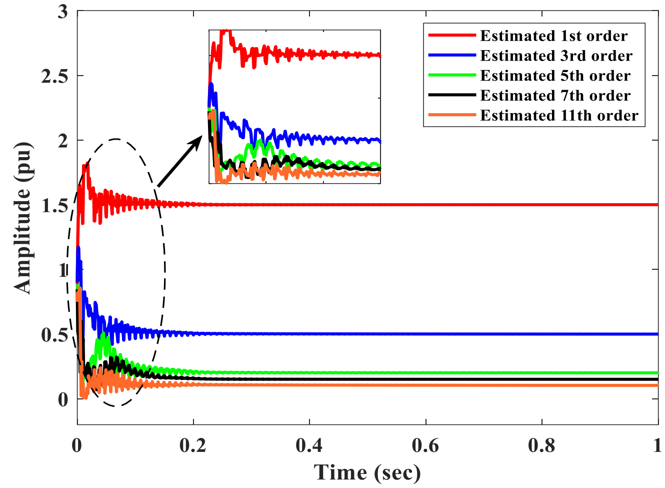
In order to verify the practical application of the proposed method, experimental setups are developed to generate the practical measurement data with abrupt changes. Then, the transient measured data are processed by applying ISICF to estimate their harmonic contents. Using the obtained parameters, we construct the estimated waveform, whose result is compared with the true waveform. Moreover, a real-time test setup is established to evaluate the convergence speed and quality of the proposed method in practical cases. Note that the initial conditions for all simulations and experiments are selected as follows: $\hat{x}_{0|0}$ is a $m \times 1$ vector with zero elements; $\boldsymbol{P}_{0|0}$ is $10^3 \times Identity(m)$; $\mathbf{Q}$ is $10^{-10} \times Identity(m)$; σ_v is 0.01; $\delta \in (0.02,\ 0.2)$. The δ value is selected based on the theoretical bound obtained by (22). Moreover, the sampling rate for all simulations is considered to be 1200 Hz (24 samples per cycle).

A. Static signal estimation

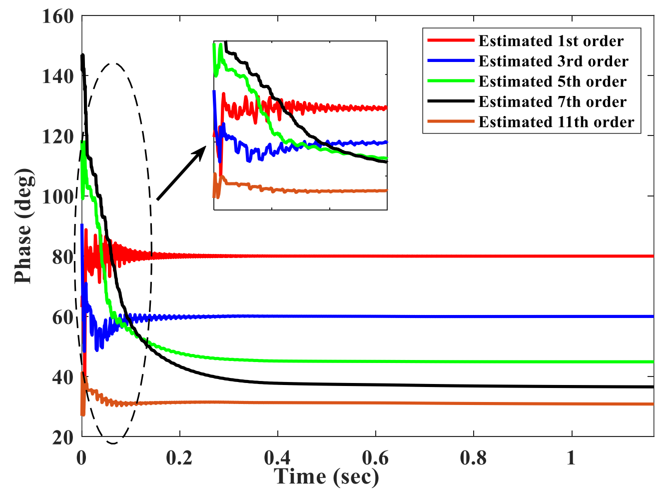
As per the first stage of simulations, the static signal model defined in (23) is applied to investigate the capability of the ISICF in harmonic parameter estimation.

$$Z_k = 1.5sin(\omega t + 80°) + 0.5sin(3\omega t + 60°) + 0.2sin(5\omega t + 45°) + 0.15sin(7\omega t + 36°) + 0.1sin(11\omega t + 30°) + v_k \tag{23}$$

where ω is the angular frequency of the power signals with the fundamental frequency of 50Hz. Furthermore, the signal is corrupted with different noise levels to explore the noise rejection performance of the proposed algorithm. In this regard, simulations under different noisy conditions, including 15dB and 20dB signal-to-noise ratios (SNRs). Initial parameters of the ISICF are kept constant at all noise levels. The estimation problem is also implemented by the well-known DFT, CKF, IEKF, and PF approaches. Since the graphical representation of the estimation algorithms for both noise levels are visually the same, only those obtained by the proposed ISICF method for the noise level (SNR = 20dB) are shown in Fig 2. It should be mentioned that the standard deviation of the injected Gaussian noise i.e., v_k is considered to be 0.022 for this case. Although the convergence rate of the parameter estimation of higher-order harmonics is lower than that of low orders, overall waveform tracking quality using the proposed ISICF, depicted in Fig. 3, demonstrates that the algorithm converges to the true value in around half a cycle. Also, all estimated parameters are stable after converging to the final values. The mean square error (MSE) and variance indices of the waveform estimation error are employed as a measure of accuracy and robustness, respectively. The results corresponding to the last 200ms of the simulation time (between 0.8s and 1s) at different noise levels are selected for the computation of these indices. Results given in Table I show that ISICF estimates the true waveform precisely at different noise levels. In spite of the excellent results of DFT in low noise levels, this method loses its accuracy and robustness in the presence of intensive noise. The IEKF and PF algorithms have almost the same tracking behavior as ISICF for static signal estimation. However, numerical indices (mean and standard deviation) reveal that this method is not as precise as ISICF.



(a)



(b)

Fig. 2. Estimation results obtained by proposed ISICF algorithm for static signal: (a) Amplitudes (b) Phases.

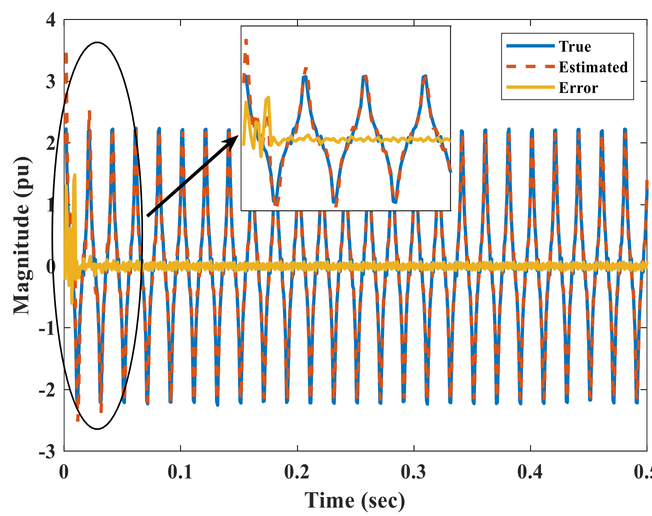


Fig. 3. Estimated waveform and corresponding error using proposed ISICF algorithm for static signal.

TABLE I
NUMERICAL COMPARISON OF ESTIMATORS IN STATIC WAVEFORM ESTIMATION

	ISICF		DFT		CKF		IEKF		PF	
SNR	MSE	Variance	MSE	Variance	MSE	Variance	MSE	Variance	MSE	Variance
20 dB	8.4464 ×10-6	6.7595 ×10-6	1.3006 ×10-8	1.2159 ×10-8	4.7551 ×10-5	6.9462 ×10-5	2.3682 ×10-5	2.4419 ×10-5	5.8256 ×10-5	7.9437 ×10-5
15 dB	4.0056 ×10-5	5.3092 ×10-5	7.6182 ×10-2	1.8437 ×10-1	2.3331 ×10-3	4.4318 ×10-2	3.0113 ×10-4	2.7008 ×10-3	2.8016 ×10-3	5.7552 ×10-2

B. *Dynamic signal estimation*

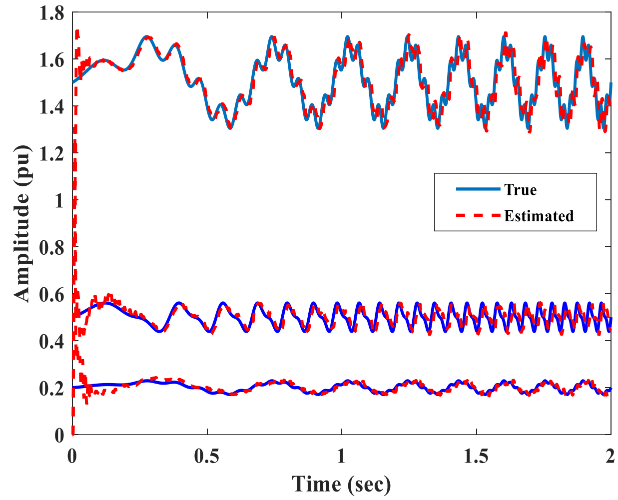
Due to the existence of time-varying conditions in the power systems, the electrical current has dynamic nature. The severity of dynamics depends on generation topology and load configuration. To explore the estimation capability of the proposed method in the presence of the dynamic parameters, time-variant terms are inserted into the structure of the signal, defined in the previous sub-section, as follows:

$$Z_k = (1.5 + \mathrm{a}_1(t))sin(\omega t + 80°) + (0.5 + \mathrm{a}_3(t))sin(3\omega t + 60°) + (0.2 + \mathrm{a}_5(t))sin(5\omega t + 45°) + 0.15sin(7\omega t + 36°) + 0.1sin(11\omega t + 30°) + v_k \quad (24)$$

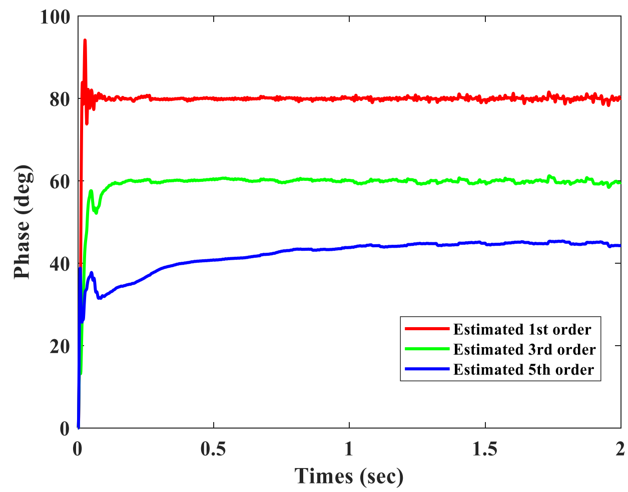
where

$$\mathrm{a}_1(t) = 0.15sin(2\pi \mathrm{f}_1 t) + 0.05sin(2\pi \mathrm{f}_3 t),$$
$$\mathrm{a}_3(t) = 0.05sin(2\pi \mathrm{f}_2 t) + 0.02sin(2\pi \mathrm{f}_3 t)$$
$$\mathrm{a}_5(t) = 0.025sin(2\pi \mathrm{f}_1 t) + 0.005sin(2\pi \mathrm{f}_3 t)$$
$$\mathrm{f}_1 = 0.25 + 1.875\mathrm{t\ Hz}$$
$$\mathrm{f}_2 = 0.75 + 5.625\mathrm{t\ Hz}$$
$$\mathrm{f}_3 = 1.5 + 11.25\mathrm{t\ Hz}$$

As can be observed in the presented model, the 1st, 3rd, and 5th harmonics have time-varying amplitudes whose frequencies are also set to be variable to make a more severe condition on the estimation process of the proposed method. Also, a zero-mean Gaussian noise with a standard deviation of 0.022 is added to the signal to make the model closer to real conditions. Estimation results of the proposed algorithm for estimating dynamic signal parameters and overall waveform are presented in Figs. 4 and 5, respectively. Although the process model used in the structure of the estimator is kept the same as one defined in the static signal test, the ISICF still tracks the dynamic parameters accurately. Nevertheless, the required time for the convergence of parameter estimation is slightly more than that of a static test. Furthermore, CKF, IEKF and PF estimators have been applied to the same dynamic signal, and comparison results for the error of waveform estimation during the last 200ms of simulation (reported in Table II) show the superiority of the ISICF in tracking dynamic signals. The main reason behind this is the sliding innovation strategy applied in ISICF that places more emphasis on the measurements than the process model when encountering model uncertainties. However, such model errors can significantly affect the CKF, IEKF and PF estimation results. In the investigated dynamic situation, the estimation errors of the DFT method are higher than minimum requirements and are not reported consequently.



(a)



(b)

Fig. 4. Estimation results obtained by proposed ISICF algorithm for dynamic signal: (a) Dynamic amplitudes (b) Corresponding phases.

TABLE II
NUMERICAL COMPARISON OF ESTIMATORS IN DYNAMIC WAVEFORM ESTIMATION
(SNR=20 DB)

ISICF		DFT		CKF		IEKF		PF	
MSE	Variance	MSE	Variance	MSE	Variance	MSE	Variance	MSE	Variance
2.3049 ×10-4	5.9457 ×10-4	1.0008 ×10-1	9.4554 ×10-2	8.8105 ×10-3	1.3501 ×10-2	8.2441 ×10-4	6.9406 ×10-3	9.3063 ×10-3	1.4069 ×10-2

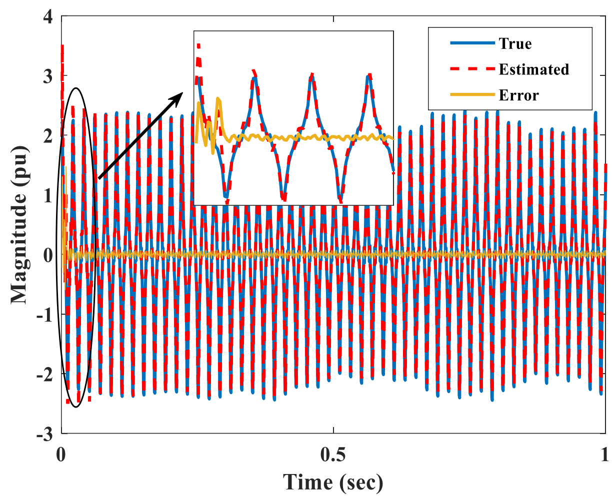


Fig. 5. Estimated waveform and corresponding error using proposed ISICF algorithm for static signal.

C. Abrupt changes estimation

The abrupt changes in electrical signals caused by transient phenomena are inevitable in power systems. Since these short-time events last from a few microseconds, up to a few milliseconds, the estimation of their abrupt changes is usually challenging. In this section, a laboratory setup is designed to extract real data from a load switching circuit. Then, the proposed method is applied to estimate the abrupt changes in voltage magnitudes logged from the dedicated setup. The setup includes a 0.1H inductor in series with two parallel transformers whose power ratings are 100VA and 200VA, respectively. The no-load 100VA transformer is directly fed by an AC source while the 200VA transformer with a 48W LED driver load is powered by a switch. The analog voltage of the inductor is measured by a fast response voltage transducer (LV 25-P). The measured data is then logged using an analog to digital NI USB-6009 data acquisition (DAQ) card at a 1200 Hz sampling rate. The experimental setup is demonstrated in Fig. 6. To generate an abrupt change in the voltage of the series inductor, the 200VA transformer with LED load is switched on at 4.725 seconds. The transient behavior of inductor voltage is also shown in Fig. 6. The logged data is analyzed in off-line mode by the proposed algorithm in MATLAB and the graphical results of the estimated signal are presented in Fig. 7. As observed, ISICF provides close estimation results to the real measured signal. On the other hand, CKF, IEKF and PF present poor estimations of abrupt changes. Since significant changes in the signal are not modeled in the state space representation of estimators, the efficiency of estimation in the time update phase is considerably degraded. Using CKF, IEKF and PF algorithms, this error is propagated to the measurement phase which results in low-quality final estimates. In contrast, ISICF exploits the use of sliding innovation term in its measurement update phase to compensate for the errors of the time update phase. Hence, final solutions obtained by the proposed method are more reliable when encountering the transients in the signal. A numerical comparison of the results obtained by both estimators is presented in Table III. In the investigated situation, the estimation errors of the DFT method are higher than minimum requirements and are not reported consequently.

D. Real-time implementation

To show the performance of the proposed algorithm in the practical application a Hardware-In-the-Loop (HIL) setup is designed. The main objective of the HIL setup is to validate the real-time operation of the algorithm. Because of the inherent processing delays of MATLAB codes particularly when reading data from DAQ cards, it is not considered a real-time tool in most cases.

To cope with this issue, the implemented codes are compiled to the C++ programming language which strongly gains traction in real-time systems. The code is implemented in an embedded hardware system based on PC/104 micro-computer set that has wide applications in real-time processing. A VDX-6354 processor card (Vortex86DX 800MHz CPU module) and a PCM-5114 DAQ card are dedicated hardware to PC/104 to process and acquire the actual input. Also, a fast response current sensor is employed to collect the current of an AC power line feeding a total of 6KW LED luminaires. The setup diagram is shown in Fig. 8. The sampling rate of DAQ card is set to 1200 Hz for reading the current measured by sensor. The measured current contains a large amount of 3rd, 5^{th,} and 7th harmonics. Once a measurement is collected by the DAQ card, a 5V digital activation pulse is applied to the General-Purpose Input Output (GPIO) port of the processor card and one iteration of ISICF is simultaneously run to provide the estimates associated with the current iteration. The digital pulse is then deactivated (set to 0V) at the end of processing time. The chain of pulses is detected by a digital scope to analyze the computational performance of the proposed ISICF. As the sampling frequency is set to be 1200Hz, the processing time (duration of digital activation pulse) should be less than $(1/1200)s$ to guarantee the real-time performance of the proposed method in practical applications. Although the processing time of all iterations is not exactly the same, as shown in Fig. 9, it is always less than $(1/3000)s$ which confirms the performance of the ISICF in real-time applications. This originates in the fact that the proposed ISICF algorithm approximates nonlinear multivariate integrals using a simple integration method that leads to a relatively low

TABLE IV
NUMERICAL RESULTS OF ISICF IN THE REAL-TIME ESTIMATION

Filter	ISICF	
	MSE	Variance
Statistical Indices	4.3049	6.9460 ×10-4

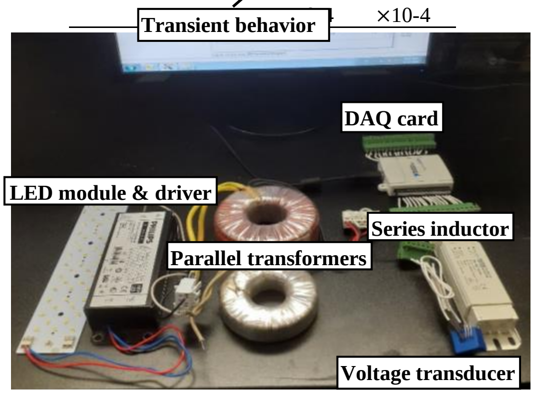


Fig. 6. Experimental setup for testing abrupt changes in the signal.

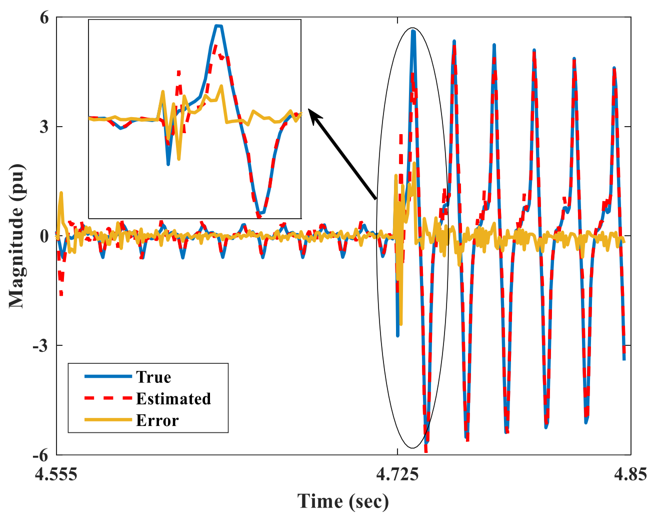


Fig. 7. Estimated waveform and corresponding error using proposed ISICF

TABLE III
NUMERICAL COMPARISON OF ISICF WITH CKF AND IEKF IN ABRUPT CHANGE ESTIMATION

ISICF		CKF		IEKF		PF	
MSE	Variance	MSE	Variance	MSE	Variance	MSE	Variance
3.8802 ×10-4	6.0130 ×10-4	1.3813 ×10-2	1.6582 ×10-2	4.7529 ×10-3	3.5866 ×10-3	1.4277 ×10-2	1.8092 ×10-2

computational burden. DAQ card outputs the estimated waveform at its analog output channel. Then, using the scope, the measured and estimated waveforms are shown in Fig. 9. The MSE and variance of estimation error for the last 200ms of the process are computed and results are given in Table IV. Results confirm that ISICF has acceptable performance even in practical test cases.

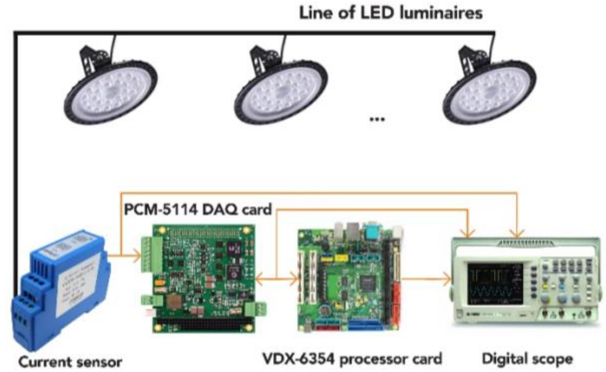
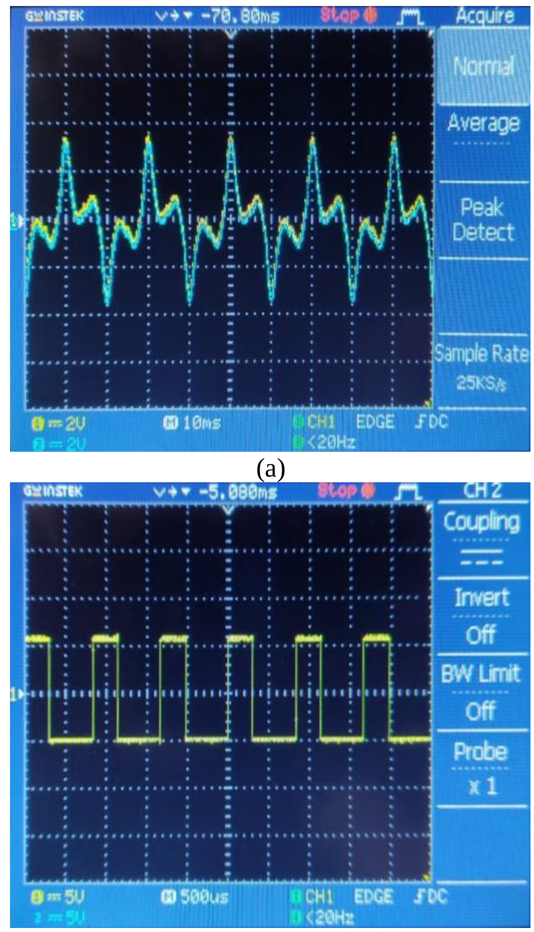


Fig. 8. Real-time experimental setup.



(a)

(b)

Fig. 9. Results obtained by proposed ISICF algorithm for real-time setup: (a) Measured and estimated waveforms (b) Processing time.

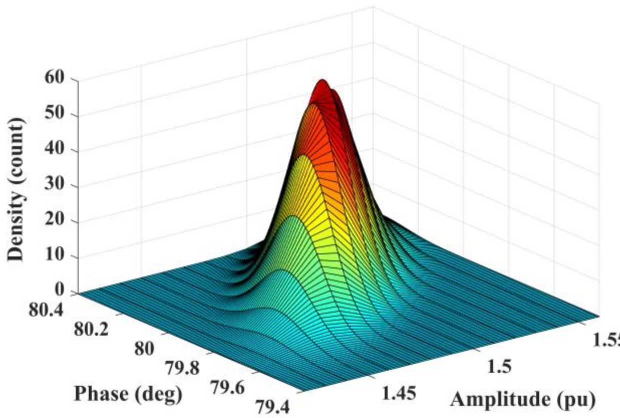


Fig. 10. Statistical distribution of the fundamental harmonic components obtained by MC runs.

IV. Stability Analysis of the Proposed Method Using Monte Carlo Method

The stability proof of the ISICF algorithm has been explored in section II. In this section, a statistical analysis based on Monte Carlo (MC) method is conducted to show the stability of the results obtained by the proposed method. For this purpose, in each MC run, the sliding boundary width δ is randomly selected from the range defined by equation (22); then, the output parameters including estimated amplitudes and phases are statistically analyzed to assess the stability of the estimation process. The MC campaign consists of 250 runs in which the noise level with SNR=20dB is set. The δ value for each run is selected from the Gaussian distribution $N(0.11,0.03)$ which covers the range obtained in the equation (22). The statistical distribution of the fundamental harmonic components obtained by MC runs is shown in Fig. 10. The MC analysis outputs for other harmonic components follow almost the same statistical characteristics. The results obtained by MC analysis support the theoretical stability analysis in section II confirming that the estimation results of ISICF are stable in the selected range of the sliding boundary width.

V. Conclusion

A novel robust estimator, called ISICF, exploiting the concept of sliding mode control in the formulation of the measurement update step in the Bayesian filtering framework was proposed for the harmonic estimation problem in this paper. The condition for stability of ISICF was obtained using the Lyapunov stability equation. The algorithm uses iterated structure to maintain the accuracy of estimation results in an acceptable boundary. Different simulation case studies have been carried out in MATLAB to show the performance of the ISICF in the presence of the noise and system unmodeled dynamics. Furthermore, an HIL setup based on real-time coding and an embedded hardware system was implemented to validate the real-time application of the proposed algorithm in practical conditions. Eventually, the theoretically extracted boundedness was supported by applying the MC campaign. The proposed ISICF can be used for harmonic estimation problems in various applications, including power quality assessment, monitoring, fault detection, etc.

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