Synchrophasor Phase Angle Data Unwrapping Using an Unscented Kalman Filter

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***Abstract*—The wrapping operation causes discontinuities in phase angle measurements from synchrophasor devices. While various unwrapping algorithms traditionally used for synchrophasor data have proven effective under typical operating conditions, they are likely to fail when faced with atypical data because of equipment malfunctions. For example, the underestimation of the phase angle jump when the data originates from a device with an internal clock or other timing mechanism can introduce this kind of error. This letter proposes using an unscented Kalman filter (UKF) for unwrapping one dimensional synchrophasor data. This technique is especially suitable for analyzing low-quality data. The effectiveness of this method is demonstrated on real phase angle measurements from devices with documented timing issues in the Dominion Energy power system.**

***Index Terms*--Kalman filter, Power system monitoring, Synchrophasors.**

I. Introduction

IN compliance with the IEEE C37.118 standard [1], synchrophasor devices report angles strictly between the $\pm\pi$ radian. In reality, the frequency is rarely 60 Hz; therefore, the angles can drift and often undergo jumps when crossing the $\pm\pi$ limit. This phenomenon is called wrapping.

One dimensional phase angle unwrapping is a well-studied problem with regards to synchrophasor phase angle data. Python and MATLAB have standard functions for unwrapping data [2] that are popularly used for offline applications. These, however, suffer from memory management issues and are not ideal for computation environments that have limited memory resources or for online applications. A workaround for this issue is proposed in [3]. Noise in the phase angle measurements can corrupt these approaches, especially if the gradient of the wrapped phase angle curve is underestimated. While low pass filtering could certainly help, it is not straightforward. This is because the interaction of noise with wrapping can significantly distort the spectral content. Even when filtering is possible, it comes with the disadvantage of introducing an artificial delay in the signal that translates into a phase shift.

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Phase angle unwrapping (PU) is a mature area of research in remote-sensing applications; for example, in processing 2D data obtained through synthetic aperture radar interferometry (InSAR) [4]. The existing PU methods are divided into the following categories: (1) path-following-based techniques, (2) optimization-based techniques, and (3) techniques based on integrated denoising and unwrapping. One of the most popular approaches belonging to the first category proposed by Goldstein et al. [5] identifies closed loops along which the directional gradients are nonzero and unwraps the data by avoiding those loops. In the 2nd category of approaches, optimization methods operate by defining an objective function to be minimized. This is a function of the gradient of the unwrapped phase angle values [6]. While being fairly robust, usually the time and space complexities of both of these approaches are high, which brings us to the third category of techniques. One of the earliest works [7] in this category analyzed the potential advantages of using Kalman Filtering for multi-dimensional phase angle unwrapping. In [8], an Extended Kalman Filter (EKF) was applied to real and synthetic data and showed promise. However, it failed when traversing portions of the signal with low coherence (high noise), alongwith other issues that were attributed to the linear approximation inside the EKF. In this regard, [9] proposed using an Unscented Kalman Filter (UKF) [11]. [10] extended the approach to multi-baseline InSAR, which involves processing multiple SAR images as opposed to a single one.

In traditional synchrophasor applications, unwrapping is done before denoising. However, unwrapping can significantly benefit from the results of denoising. This makes the third category of techniques fairly attractive for synchrophasor data, because they can benefit from both denoising and unwrapping simultaneously. This work explores the use of the Kalman filter to address atypical cases observed in field data. None of the currently used techniques for phasor measurement unit (PMU) data analysis provided satisfactory results.

This letter is organized as follows. Section II presents the problem formulation of the phase angle unwrapping problem in one dimension. Section III presents the proposed approach using the UKF. Section IV analyzes results obtained on signals from Dominion Energy substations suffering from clock synchronization issues. These are compared against the standard Python unwrapping function to demonstrate the effectiveness.

II. Problem Formulation

The relationship between the measured, wrapped phase angle and the estimated, unwrapped phase angle is as follows:

$$\phi_u(k) = \phi_w(k) + 2\pi\gamma(k), \quad (1)$$

where $\phi_u(k)$ and $\phi_w(k) \in [-\pi, +\pi]$ are unwrapped and wrapped phase angles respectively at k^{th} time step and $\gamma(k) \in Z$ is an integer correction factor. The conventional approach to obtain $\gamma(k)$, referred to as the 1D Itoh algorithm [2], proceeds by estimating the gradient $\hat{\Delta}_u(k) = \hat{\phi}_u(k+1) - \hat{\phi}_u(k)$ between neighboring points as follows:

$$\hat{\Delta}_u(k) = \begin{cases} \phi_w(k+1) - \phi_w(k), |\phi_w(k+1) - \phi_w(k)| < \pi \\ \phi_w(k+1) - \phi_w(k) - 2\pi, \phi_w(k+1) - \phi_w(k) > \pi \\ \phi_w(k+1) - \phi_w(k) + 2\pi, \phi_w(k+1) - \phi_w(k) < -\pi. \end{cases} \quad (2)$$

This is followed by integrating $\hat{\Delta}_u(k)$ starting from a reference value for $\hat{\phi}_u(0)$, as follows:

$$\hat{\phi}_u(k+1) = \hat{\phi}_u(k) + \hat{\Delta}_u(k). \quad (3)$$

However, this approach fails because you need to calculate the phase slope $\hat{\Delta}_u$. An error in estimation towards the beginning of the signal accumulates in successive points, especially when using a two-point derivative calculation.

III. Phase Angle Unwrapping Using a UKF

A. State Space Equation

Wrapping results in a discontinuity in the phase angle, and consequently, nondifferentiability. Now, we cannot justify using a zero-order state-space model for the phase angle because the power system is never at 60 Hz and the ambient perturbations constantly excite the phase angle dynamics. Therefore, relaxing the continuity condition to differentiability, from (3), the following state-space model is used for estimation:

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + w = f(x(k)) + w, \quad (4)$$

where $x = [\phi_u, \Delta_u]^T \in R^n$ and $w \sim N(0, Q)$ is the model noise modeled as a zero-mean Gaussian process with covariance matrix as Q.

B. Observation Equation

For phase angle unwrapping, only ϕ_w is measured. We know that the effect of wrapping shifts $\phi_u(k)$ by integer multiples of 2π; and thus, cos and sin of ϕ_w are the same as that for ϕ_u. Therefore, we do not use ϕ_w directly in the observation equation even though it is available. Instead we use sin and cos as follows:

$$z(k) = \begin{bmatrix} \cos(x_1(k)) \\ \sin(x_1(k)) \end{bmatrix} + v = h(x(k)) + v, \quad (5)$$

where $z(k) = [\cos(\phi_w(k)), \sin(\phi_w(k))]^T$ and $v(k) \sim N(0, R)$ are the measurement noise modeled as a zero-mean Gaussian process with a covariance matrix, R. The nonlinearity of $h(x)$ makes UKF optimal when compared to the extended Kalman filter.

C. Overall Estimator using UKF

The unscented transform [11] at the core of UKF operates by finding a minimal number of sigma points that capture the mean and variance of a state passing through a nonlinear transform. Let $\widehat{x(0)}^+$ and $P_{xx}(0)^+$ denote the posterior estimate and posterior covariance of the state value at time $k = 0$. The algorithm iterates through the following three steps for $k = 1$ to ∞.

Algorithm: UKF

Step 1: Calculation of sigma points χ

$$\chi_0 = \widehat{x(k)}^+$$

$$\chi_i = \chi_0 + \left(\left(\sqrt{(n+\lambda)P_{xx}(k)^+}\right)_i\right)^T, 1 \le i \le n$$

$$\chi_i = \chi_0 - \left(\left(\sqrt{(n+\lambda)P_{xx}(k)^+}\right)_i\right)^T, n+1 \le i \le 2n$$

$$W_0^{(m)} = \frac{\lambda}{n+\lambda}$$

$$W_0^{(c)} = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta)$$

$$W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n+\lambda)}$$

$$\lambda = \alpha^2(n+\kappa) - n.$$

Step 2: Prior estimates of states and covariance at $k+1$ are obtained by propagating through (4)

$$\widehat{x(k+1)}^- = \sum_i W_i^{(m)} f(\chi_i)$$

$$P_{xx}(k+1)^- = \sum_i W_i^{(c)} \left(f(\chi_i) - \widehat{x(k+1)}^-\right)\left(f(\chi_i) - \widehat{x(k+1)}^-\right)^T + Q$$

$$\widehat{z(k+1)}^- = \sum_{i=1:2n} W_i^{(m)} h(f(\chi_i))$$

$$P_{zz}(k+1)^- = \sum_i W_i^{(c)} \left(h(f(\chi_i)) - \widehat{z(k+1)}^-\right)\left(h(f(\chi_i)) - \widehat{z(k+1)}^-\right)^T + R$$

$$P_{xz}(k+1)^- = \sum_i W_i^{(c)} \left(f(\chi_i) - \widehat{x(k+1)}^-\right)\left(h(f(\chi_i)) - \widehat{z(k+1)}^-\right)^T.$$

Step 3: Posterior estimates (correction step)

$$K = P_{xz}(k+1)^- \times [P_{zz}(k+1)^-]^{-1}$$

$$\widehat{x(k+1)}^+ = \widehat{x(k+1)}^- + K\left(z(k+1) - \widehat{z(k+1)}^-\right)$$

$$P_{xx}(k+1)^+ = P_{xx}(k+1)^- - KP_{zz}(k+1)^- K^T.$$

While the UKF inherently performs denoising, it is difficult to gain insight into how the signal is modified in the frequency domain. Thus, for applications such as small-signal analysis [12], we want to start with unwrapped data with no denoising effect from the UKF, i.e. estimate of $\gamma(k)\forall k$ in (1) should be an integer. The following post processing can be performed on the UKF results:

$$\hat{\phi}_u(k) = \hat{\phi}_u^{UKF} + 2\pi \text{ round}\left(\frac{(\hat{\phi}_u^{UKF}(k) - \phi_w(k))}{2\pi}\right), \quad (6)$$

where $\hat{\phi}_u^{UKF}$ is the estimated, unwrapped phase angle obtained from UKF, and $\hat{\phi}_u$ is the final estimate minus the denoising estimate of UKF.

IV. Results

To illustrate the virtues of the proposed approach, studies were conducted using synchrophasor data obtained from a PMU in the Dominion system with a history of device timing issues. These measurements were collected on the high side of two generator step-up units (GSU) at a 60 Hz reporting rate. The main goal of the unwrapping is to provide data for PMU applications; thereby, the unwrapped signals undergo frequency domain analysis through Welch's periodogram [13] to illustrate the impact of existing and proposed unwrapping approaches. The spectra are computed using a two-minute FFT window and 50% overlap between successive windows. The window function used is Hanning's. The model error and measurement noise covariance matrices for the UKF are set to $Q = diag([0.001, 0.01])$ and $R = diag([0.1, 0.1])$. The remainder of the parameters are set to default values with $\alpha = 1, \beta = 0,$ and $\kappa = 3 - n = 1$. These can be optimized to improve the filter performance.

A. Regular Phase Angle Data (Low Noise)

For this study, a five-minute window of data was collected from the first GSU. Fig. 1, illustrates that the wrapped data measurements have a high signal to noise ratio (SNR) making the unwrapping problem trivial. The unwrapping performed by MATLAB's algorithm looks almost identical to the proposed UKF approach.

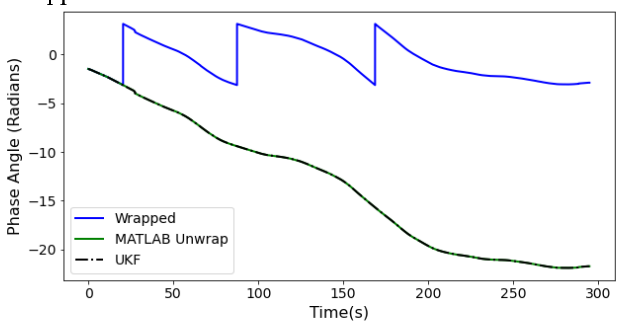

Fig. 1. First GSU's phase angle plot

To gain better insight into the signal content, the unwrapped signals were analyzed in the frequency domain. Fig. 2, reveals that removing the denoising effect of the UKF gives similar results to MATLAB's algorithm. Moreover, not applying denoising reduces the energy at higher frequencies (> 4 Hz), i.e. suppressing the noise. This can be made more aggressive by reducing the model noise covariance matrix, i.e. Q term corresponding to the second equation in (4).

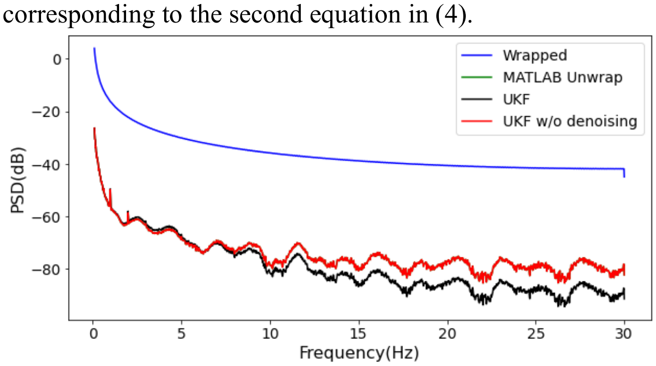

Fig. 2. PSD of first GSU's phase angle

B. Noisy Data

In this study, the same five-minute window was used for the second GSU with a history of digital fault recorder alarm triggers owing to the device's timing issues. What makes this data set challenging is the interaction of noise with the wrapping operation, which was evident from the high variance regions in the signal between 30-60s, 90-150s, and 170-190s. Fig. 3, illustrates these results. Notice that MATLAB's unwrapping algorithm fails in the presence of noisy phase angle data, resulting in an evident error accumulation. Notably, the proposed approach can easily unwrap the data. This can be confirmed by comparing the unwrapped angle with that of the other coherent generator in the same substation as seen in Fig. 1.

Next, the unwrapped angles are plotted in the frequency domain in Fig. 4. Observe that poor unwrapping results in a significant distortion to the low-frequency spectrum in the 0-5 Hz range. This is a major issue because electromechanical modes monitored using PMU data lie within this range. The effect of UKF's noise suppression can also be seen where the energy of >5 Hz frequency terms has been reduced when compared to the formulation in (6).

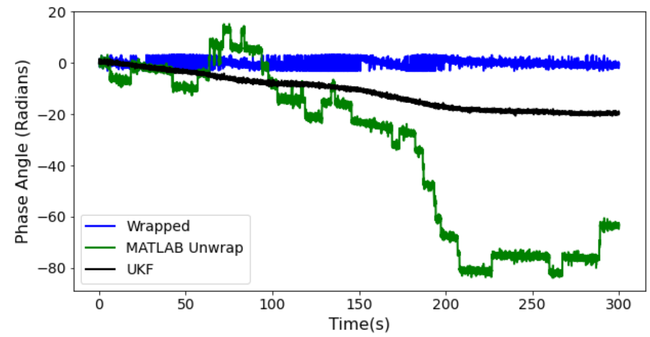

Fig. 3. Second GSU's phase angle plot

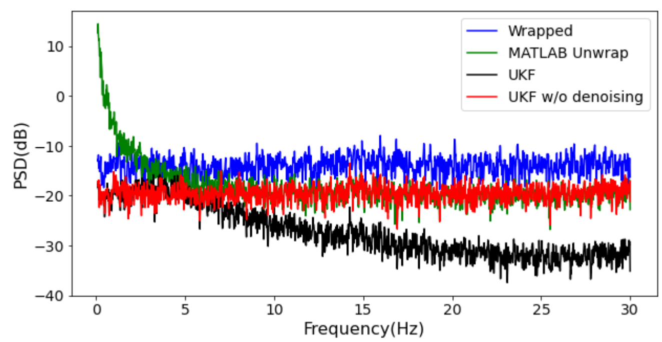

Fig. 4. PSD of second GSU's phase angle

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