Coherency-Independent Structured Model Reduction of Power Systems

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***Abstract*—This paper proposes a new model reduction algorithm for power systems based on an extension of balanced truncation. The algorithm is applicable to power systems which are divided into a study area which requires a high-fidelity model and an external area, making up most of the power system, which is to be reduced. The division of the power system can be made arbitrarily and does not rely on the identification of coherent generators. The proposed algorithm yields a reduced order system with a full nonlinear description of the study area and a reduced linear model of the external area.**

***Index Terms*—Dynamic equivalents, internal systems, model reduction of power systems, structured model reduction.**

I. Introduction

POWER system dynamic model reduction, typically known as *power system dynamic equivalencing* [1], has the main aim of providing an equivalent system model able to reproduce the aggregated steady-state [2] and dynamic characteristics of the full-order network [1], while at the same time being compatible with the available computation tools for power system analysis [3]. In this equivalent model, the *study area* is a portion of the network which is preserved with full detail, i.e., all the mathematical description of the power apparatus involved are untouched, while the *external area*, consisting of the remaining part of the network, is replaced with a simpler mathematical description, i.e., a reduced-order model.

Coherency-based power system model reduction [4]–[9] considers two important stages: the first stage is the identification of coherency in the generators of the power system [10], and the second stage is the dynamic reduction of the system. The dynamic reduction process itself is carried out by aggregating the network [11] and aggregating the generators [8], [12]. Some approaches that are capable of retaining a part of the network have been proposed in [13], [14]. These methods have been proven in practical industrial applications [1], and offer benefits such as [15], [16]: 1) providing homogeneous reduced models consisting of typical power system elements, 2) providing a reduced model that partially retains the coordinates of the stable and unstable equilibrium points, potential and kinetic energy of the full model, and 3) being able to preserve eigenvalues and eigenvectors important for disturbances of the study system.

The nature of coherency properties is to cluster generator groups which impose the areas in which the network can be divided. In addition, it has been recently recognized that the boundaries between the study area and the external system need to be rebuffered to properly consider system operating changes [17]. This is due to the fact that changes in operating conditions may raise variations in generator coherency behavior [18] resulting in shifting the boundary of the study area to include generators that are strongly coupled to the external area. Therefore, it is challenging to properly utilize coherency-based model reduction when a very specific and arbitrarily part of the power network needs to be reduced.

For applications such as small-signal power system security assessment of very large networks [19] with arbitrary network boundaries, as well as for the design of power plant controllers for system-wide phenomena such as inter-area oscillations [20], model reduction methods capable of arbitrarily demarcating a boundary separating the study area from the external area without having to comply with coherency properties might be of practical value.

Recently, it has been shown how recent model reduction algorithms popular in automatic control can be applicable to power systems [21], [22]. These algorithms typically have a strong theoretical foundation and they are also very general in the sense that they are not targeted to a particular application. This makes them a good candidate for the reduction of power systems composed not only of synchronous machines but also, for instance, renewable energy sources [17].

Model reduction where various structural constraints are taken into account ("structured model reduction") has been considered in several papers. For example, in [23] frequency-weighted model reduction problems are considered, and in [24] controller reduction is considered. More general interconnection structures have been considered, for example, in [25]–[27].

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This paper presents a structured model order reduction algorithm based on an extension balanced truncation. The idea is to reduce the external system while trying to minimize the effect it will have on the study area. This is the main objective of structured model reduction, namely to reduce models locally while ensuring a small global model error. The remainder of this paper is organized as follows. In Section II we formulate the model reduction problem. Section III summarizes the theory of structured model order reduction and in Section IV it is shown how this theory can be applied to power systems. Section V presents the application of the proposed algorithm to two power system models on which the merits of the proposed model reduction algorithm are evaluated.

II. Problem Formulation

In this study the power systems are divided into a study area, which has variables of interest to us, and an external area, with variables we are not interested in apart from their effect on the study area (Fig. 1). The problem we try to solve is how the model order of the external area can be reduced while having as little detrimental impact on the dynamics of the study area as possible. We will require that the study area is described by its original nonlinear equations to allow for a physical interpretation of it. The interface between the study area and the external area is defined by their n tie-lines and the corresponding buses. Each bus in the network satisfies [28]

$$\begin{aligned} P_i =& V_i^2 G_{ii} + \sum_{j=1;j\neq i}^{N} V_i V_j B_{ij} \sin(\theta_i - \theta_j) \\ &+ \sum_{j=1;j\neq i}^{N} V_i V_j G_{ij} \cos(\theta_i - \theta_j) \\ Q_i =& - V_i^2 B_{ii} + \sum_{j=1;j\neq i}^{N} V_i V_j G_{ij} \sin(\theta_i - \theta_j) \\ &- \sum_{j=1;j\neq i}^{N} V_i V_j B_{ij} \cos(\theta_i - \theta_j) \end{aligned} \tag{1}$$

where P and Q are the active and reactive power, G and B are the real and imaginary part of the admittance, and V and θ are the voltage magnitudes and phases of the buses. The interface between the study area and the external area is chosen so that (1) is well-defined for the buses at the tie-lines. This means that for the external area we will define the voltage magnitudes V_i^{study} and phases θ_i^{study} adjacent to it as input signals of the external area and as outputs of the study area (Fig. 1). Conversely, we define V_i^{ext} and phases θ_i^{ext} to be inputs of the study area and outputs of the external area.

III. Structured Model Reduction

Next we will fit the model reduction of power systems into the general framework of structured model reduction. This model reduction refers to the reduction of states of subsystems connected in a network, while preserving the network topology. This implies a local model reduction of the subsystems while, at the same time, the objective is to capture the global dynamics of

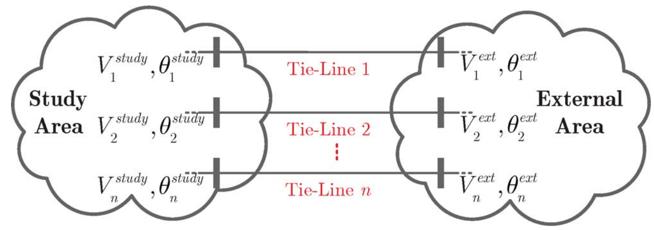

Fig. 1. Power system is divided into a study area and an external area, which is to be reduced.

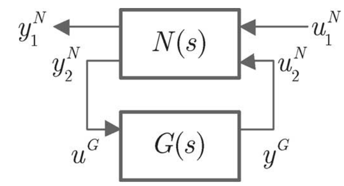

Fig. 2. Interconnected system. G is the system that should be reduced and N is the interconnecting network. u_1^N and y_1^N are the external input and output, respectively. u^G and y^G are the input and output of the system that should be reduced.

the interconnection. Fig. 2 shows the general setup with q subsystems $G_k(s)$ collected in $G(s) = \operatorname{diag}\{G_1(s), \ldots, G_q(s)\}$ and interconnected by the network $N(s)$, which contains information about the topology of the full system. It can be noted that the network itself may have its own dynamics. To relate this to the model reduction of power systems consider a set of q separate external systems interconnected with a study area. The external systems would then correspond to $G(s)$ and the study area to $N(s)$, which in this case would contain both the network topology defining the interconnection of the external areas and dynamics of its own. The interconnecting signal u^G will be the voltages V_i^{study} and phases θ_i^{study} and, similarly, y^G will be V_i^{ext} and θ_i^{ext}.

We will now introduce the notation $\mathcal{F}_l(N, G)$ for the lower linear fractional transformation and use it to denote the transfer function from u_1^N to y_1^N when N and G are connected (Fig. 2). Given that subsystem G_k has m_k input and p_k output signals and with $\|\cdot\|_\infty$ being defined as the induced L_2-norm [29], the objective is to find the reduced order system $\hat{G}$ composed of its q subsystems such that

$$\left\| \mathcal{F}_l(N, G) - \mathcal{F}_l(N, \hat{G}) \right\|_\infty \tag{2}$$

is made as small as possible and

$$\hat{G} \in \{F(s) : F(s) = \operatorname{diag}\{F_1(s), \ldots, F_q(s)\}\}$$

where $F_k(s) \in \mathbb{C}^{p_k \times m_k}$, $k = 1, \ldots, q$.

Structured model reduction can be contrasted with the method used in [30], where the model reduction of the external area G is made independently of the study area N. The reason that structured model reduction is preferable to this approach is that it emphasizes the importance of having an accurate model of G around the frequencies that are of primary importance when the external area is connected to the study area. Thus, if there is an inter-area mode between the study area and the external area, the structured model reduction algorithm will achieve good accuracy around the inter-area frequency, while this can be lost if the external area is reduced independently of the study area.

The minimum of (2) is very difficult to find since it is a nonconvex optimization problem. We will therefore have to resort to suboptimal methods, which yield solutions satisfying the constraints while trying to minimize the norm of the model error. In this paper we will use a model reduction algorithm that is inspired by balanced truncation, see for example in [23] and [31]. To enforce the structural constraints we use a generalization of balanced truncation as described in [26], [27], [32], and [33]. The notation used here closely follows the one used in [26] and [27].

The algorithm uses the reachability and observability Gramians P and Q which can be found as the solutions to the Lyapunov equations

$$AP + PA^T + BB^T = 0, \qquad A^TQ + QA + C^TC = 0 \quad (3)$$

where the matrices A, B, C realize the interconnected system $\mathcal{F}_l(N, G)$. Assuming that the state-vector is partitioned as

$$x = \begin{bmatrix} x_N^T \; x_1^T \; \dots \; x_q^T \end{bmatrix}^T \quad (4)$$

we can introduce a partition of P and Q in (5) and (6), respectively, with the blocks P_N, Q_N for the interconnecting network N that is not reduced and with the blocks P_k, Q_k for subsystem G_k that should be reduced separately, but in a way so that the closed-loop dynamics is retained:

$$P = \begin{bmatrix} P_N & P_{NG} \\ P_{NG}^T & P_G \end{bmatrix}, \quad P_G = \begin{bmatrix} P_1 & \dots & P_{1q} \\ \vdots & \ddots & \vdots \\ P_{1q}^T & \cdots & P_q \end{bmatrix} \quad (5)$$

$$Q = \begin{bmatrix} Q_N & Q_{NG} \\ Q_{NG}^T & Q_G \end{bmatrix}, \quad Q_G = \begin{bmatrix} Q_1 & \dots & Q_{1q} \\ \vdots & \ddots & \vdots \\ Q_{1q}^T & \cdots & Q_q \end{bmatrix}. \quad (6)$$

The method then balances subsystem $G_k(s)$ by the coordinate transformation $x_k = T_k\bar{x}_k$ that makes the transformed sub-Gramians $\bar{P}_k = T_k^{-1}P_kT_k^{-T}$ and $\bar{Q}_k = T_k^TQ_kT_k$ subsystem balanced, which means that

$$\begin{aligned} &\bar{P}_k = \bar{Q}_k = \Sigma_k = \operatorname{diag}\{\sigma_{k,1}, \dots, \sigma_{k,n_k}\}, \\ &\sigma_{k,1} \geq \dots \geq \sigma_{k,n_k}, \\ &\sigma_{k,j} = \sqrt{\lambda_j(P_kQ_k)} = \sqrt{\lambda_j(\bar{P}_k\bar{Q}_k)} \end{aligned} \quad (7)$$

where λ_j denotes the jth eigenvalue. Thus if the original state vector has the structure (4), then the transformed system will have the states $\bar{x}$ defined by

$$T\bar{x} = x$$

where

$$T = \operatorname{diag}(T_N, T_1, \dots, T_q), \; T_N \in \mathbb{R}^{n_N \times n_N}, \; T_k \in \mathbb{R}^{n_k \times n_k}$$

and n_N and n_k are the order of system N and G_k, respectively.

After the change of basis, either truncation or singular perturbation can be carried out to reduce the model order of the subsystems [34]. To decide which states to remove, the structured Hankel singular values in (7) can be of aid. The reasoning is that the structured Hankel singular values indicate how reachable and observable the states of the subsystems are when we control the global input signal u_1^N and observe the global output signal y_1^N (Fig. 2).

IV. Structured Model Reduction Applied to Power Systems

We will now apply the theory in Section III to power systems divided into a study area and an external area. Although the theory is applicable to power systems with several external areas, we will restrict ourselves to a single one, i.e., $q = 1$ using the notation in Section III. We now propose a four-step algorithm for the reduction of the external area.

A. Four-Step Algorithm

1) Defining the Model: A general power system can be modeled with differential algebraic equations (DAE) of the form

$$\begin{aligned} \dot{x} &= f(x, x_{\text{alg}}, u) \\ 0 &= g(x, x_{\text{alg}}, u). \end{aligned} \quad (8)$$

Generators, controllers, etc. require both states x and algebraic variables x_{alg} for their modeling. Algebraic variables are also needed to model bus voltages. The signal u is used to describe exogenous inputs to the power system, which could be for instance time-varying loads or a reference signal to a controller.

If we divide the power system (8) into a study area N and an external area G, we get

$$\begin{aligned} \dot{x}^G &= f^G\left(x^G, x_{\text{alg}}^G, u^G\right) \\ 0 &= g^G\left(x^G, x_{\text{alg}}^G, u^G\right) \end{aligned} \quad (9)$$

and

$$\begin{aligned} \dot{x}^N &= f^N\left(x^N, x_{\text{alg}}^N, u_1^N, u_2^N\right) \\ 0 &= g^N\left(x^N, x_{\text{alg}}^N, u_1^N, u_2^N\right). \end{aligned} \quad (10)$$

The variables u^G and u_2^N are the voltage magnitudes and phases of the buses at the tie-line as described in Section II and u_1^N is the same exogenous input as in (8); see Fig. 2.

2) Linearizing: In order to apply the structured model reduction algorithm described in Section III it is first necessary to linearize both the study area and the external area. By solving the power flow problem, the steady-state of the power system is acquired around which the linearization is done. The linearization of ((9), (10)) will take the form

$$\begin{aligned} \begin{pmatrix} \dot{x}^G \\ 0 \end{pmatrix} &= \begin{pmatrix} A_{11}^G & A_{12}^G \\ A_{21}^G & A_{22}^G \end{pmatrix} \begin{pmatrix} x^G \\ x_{\text{alg}}^G \end{pmatrix} + \begin{pmatrix} B_1^G \\ B_2^G \end{pmatrix} u^G \\ \begin{pmatrix} \dot{x}^N \\ 0 \end{pmatrix} &= \begin{pmatrix} A_{11}^N & A_{12}^N \\ A_{21}^N & A_{22}^N \end{pmatrix} \begin{pmatrix} x^N \\ x_{\text{alg}}^N \end{pmatrix} \\ &\quad + \begin{pmatrix} B_{11}^N & B_{12}^N \\ B_{21}^N & B_{22}^N \end{pmatrix} \begin{pmatrix} u_1^N \\ u_2^N \end{pmatrix}. \end{aligned}$$

The algebraic variables x_{alg}^G and x_{alg}^N can be solved for

$$\begin{aligned} x_{\text{alg}}^G &= -A_{22}^{G^{-1}}\left(A_{21}^Gx^G + B_2^Gu^G\right) \\ x_{\text{alg}}^N &= -A_{22}^{N^{-1}}\left(A_{21}^Nx^N + B_{21}^Nu_1^N + B_{22}^Nu_2^N\right) \end{aligned}$$

and if the matrices M^G and M^N select which algebraic variables the two subsystems output, i.e., the tie-line voltage magnitudes and phases, the DAEs can be recast into the following ordinary differential equations:

$$\begin{aligned} \dot{x}^G &= \left(A_{11}^G - A_{12}^G A_{22}^{G^{-1}} A_{21}^G\right) x^G + \left(B_1^G - A_{12}^G A_{22}^{G^{-1}} B_2^G\right) u^G \\ y^G &= M^G \left(-A_{22}^{G^{-1}} \left(A_{21}^G x^G + B_2^G u^G\right)\right) \end{aligned} \tag{11}$$

and

$$\begin{aligned} \dot{x}^N &= A^N x^N + B^N \begin{pmatrix} u_1^N \\ u_2^N \end{pmatrix} \\ \begin{pmatrix} y_1^N \\ y_2^N \end{pmatrix} &= C^N x^N + D^N \begin{pmatrix} u_1^N \\ u_2^N \end{pmatrix} \end{aligned} \tag{12}$$

where

$$\begin{aligned} A^N &= A_{11}^N - A_{12}^N A_{22}^{N^{-1}} A_{21}^N \\ B^N &= \begin{pmatrix} B_{11}^N - A_{12}^N A_{22}^{N^{-1}} B_{21}^N & B_{12}^N - A_{12}^N A_{22}^{N^{-1}} B_{22}^N \end{pmatrix} \\ C^N &= \begin{pmatrix} 0 \\ -M^N A_{22}^{N^{-1}} A_{21}^N \end{pmatrix} \\ D^N &= \begin{pmatrix} 0 & I \\ -M^N A_{22}^{N^{-1}} B_{21}^N & -M^N A_{22}^{N^{-1}} B_{22}^N \end{pmatrix}. \end{aligned}$$

We can note that the system N has one input signal u_1^N and one output signal y_1^N apart from the input-output pair that defines its interconnection with the external area G. These are the exogenous inputs and the global outputs and they should preferably be selected so that they have a high participation in the modes that are of most importance. Participation factors can be used as a guide towards this end [35].

3) Model Reduction: With G and N on the form (11) and (12), the state space equations for the interconnected system $\mathcal{F}_l(N, G)$ can readily be found after which the model reduction algorithm in Section III can be applied [27].

4) Nonlinear Model: With the system G being reduced to

$$\hat{G}(s) = \left[\begin{array}{c|c} A^{\hat{G}} & B^{\hat{G}} \\ \hline C^{\hat{G}} & D^{\hat{G}} \end{array}\right]$$

it can be reconnected to the nonlinear description of the study area yielding the reduced interconnected system

$$\begin{aligned} \dot{x}^{\hat{G}} &= A^{\hat{G}} x^{\hat{G}} + B^{\hat{G}} u^{\hat{G}} \\ u_2^N &= y^{\hat{G}} = C^{\hat{G}} x^{\hat{G}} + D^{\hat{G}} u^{\hat{G}} \\ \dot{x}^N &= f^N\left(x^N, x_{\text{alg}}^N, u_1^N, u_2^N\right) \\ 0 &= g^N\left(x^N, x_{\text{alg}}^N, u_1^N, u_2^N\right) \\ u^{\hat{G}} &= y_2^N = M^N x_{\text{alg}}^N. \end{aligned}$$

B. Frequency-Weighting

Choosing inputs that excite certain modes of choice and outputs that have high participation in them will result in better model accuracy around the frequencies corresponding to those modes. However, a more general approach for controlling the accuracy in a frequency band is to use frequency filtering of the input and/or output signals [23]. For notational simplicity only filtering of the output will be considered. The Gramians will then be calculated for $\mathcal{F}_l(N, G)$ connected in series with a frequency filter. The Gramians, here the reachability Gramian, would have the structure

$$P = \begin{bmatrix} P_N & P_{NG} & P_{NW} \\ P_{NG}^T & P_G & P_{GW} \\ P_{NW}^T & P_{GW}^T & P_W \end{bmatrix}$$

where W is the frequency-weight, which could for instance be a low-pass filter if good accuracy at lower frequencies has higher priority. The submatrix P_G is then selected for the subsequent model reduction.

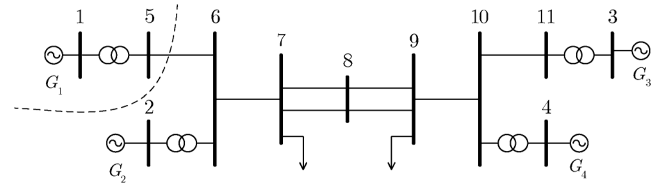

Fig. 3. Klein-Rogers-Kundur 2-area system.

V. Application to Power System Models

A. Klein-Rogers-Kundur 2-Area System

To illustrate how structured model reduction can be applied, we begin by studying a modified version of the Klein-Rogers-Kundur 2-area system which is controlled by one AVR and PSS connected to generator 1. We will define generator 1 and bus 1 and 5 to be the study area (Fig. 3), which is the area from which the system will be controlled. The model is of order 31 out of which 18 states are used for the external area.

1) Eigenvalue Analysis: Similar to the original Klein-Rogers-Kundur 2-area system, this system has one inter-area mode with frequency 0.68 Hz and two local modes, one involving generator 1 and 2 of 1.4 Hz and one with generator 3 and 4 of 0.75 Hz. Since these three modes dominate the electromechanical dynamics, we show how well they are captured for different model orders in terms of their frequency and damping in Table I. We see that the fifth order model of the external area retains the 0.75-Hz local mode well, whereas the model order is insufficient in terms of the 1.4-Hz mode. The inter-area mode is not captured at all, which has to do with the low model order. Adding more states to the model rectifies this and with a model order of 11, the eigenvalues are almost identical to those of the full system.

It is interesting to compare the order in which the modes are captured for the Klein-Rogers-Kundur system in this paper with the order in [36], where some parameter values of the power system were different and no PSS was used. With no PSS the least damped mode was the inter-area mode and being the most controllable and observable mode of the system, it was captured first by the reduced model. This can be contrasted with the outcome of the model reduction presented in Table I, where the inter-area mode is captured last. In this case, due to the presence of the PSS, the inter-area mode is the most damped mode

TABLE I
EIGENVALUES OF A SELECTION OF REDUCED ORDER MODELS COMPARED TO THE FULL-ORDER LINEAR MODEL

Model Order No.	Inter-Area Mode (λ_1)			Local Mode 1 (λ_2)			Local Mode 2 (λ_3)		
	λ_1	f_1 (Hz)	ζ_1 (%)	λ_2	f_2 (Hz)	ζ_2 (%)	λ_3	f_3 (Hz)	ζ_3 (%)
5	—	—	—	-0.445 $\pm$ $j4.90$	0.780	9.04	-1.58 $\pm$ $j7.44$	1.18	20.1
8	-0.691 $\pm$ $j4.26$	0.678	16.0	-0.433 $\pm$ $j4.70$	0.748	9.17	-1.20 $\pm$ $j8.67$	1.38	13.7
11	-0.642 $\pm$ $j4.29$	0.683	14.8	-0.446 $\pm$ $j4.71$	0.750	9.43	-1.10 $\pm$ $j8.72$	1.39	12.5
18 (full system G)	-0.639 $\pm$ $j4.29$	0.683	14.7	-0.447 $\pm$ $j4.71$	0.750	9.45	-1.11$\pm$ $j8.72$	1.39	12.6

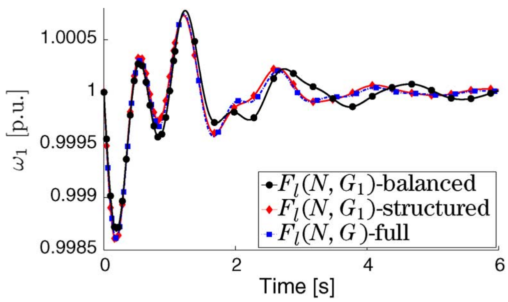

Fig. 4. Transients of the machine speed ω_1 after an initial perturbation to the machine angle δ_1.

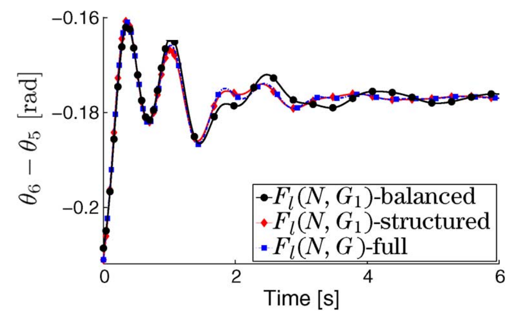

Fig. 5. Transients of the phase difference $\theta_6 - \theta_5$ at the tie-line after an initial perturbation to the machine angle δ_1.

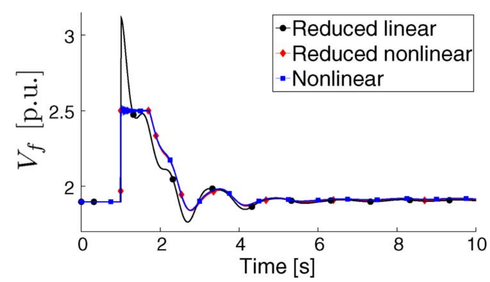

Fig. 6. Transients of V_f after an initial 5% perturbation of V_{ref} of generator 1.

of the three highlighted ones and this will affect the properties of the reduced model.

2) Structured Model Reduction vs. Balanced Truncation: An alternative to using structured model reduction to reduce the external area, would be to apply ordinary balanced truncation without accounting for the effects of the study area. This approach would entail calculating the Gramians for the external system G and base the reduction on those as opposed to calculating the Gramians for the entire system $F_l(N, G)$ and then selecting the sub-Gramians corresponding to the external area G to find the reduced model (Section III).

To compare these two methods we have applied them to the modified Klein-Rogers-Kundur 2-area system. As exogenous input signal to the system the machine angle δ_4 was chosen because of its high participation in the inter-area mode and the 0.75-Hz local mode. No input signal was chosen from the study area, since it cannot be used together with ordinary balanced truncation applied to the external area and we want to have the same input and output signals in both cases to make a fair comparison between the algorithms. The machine speed ω_1 and the voltage V_6 and phase $\theta_6 - \theta_5$ at the tie-line were chosen as output signals.

Under these conditions it was found that the structured model reduction yielded more accurate models. An illustration of this is seen in Figs. 4 and 5, which show the transients of the machine speed ω_1 and the phase difference at the tie-line $\theta_6 - \theta_5$, respectively, after an initial perturbation to δ_1 in the study area. These simulations were done with a 8th order model of the external area. This result is to be expected since balanced truncation ignores the effects of the study area on the external area. Similar results were seen for the WSCC 3-machine 9-bus system [37]. Apart from these examples it can be noted that it is more natural to perform structured model reduction since it permits having input/output signals that are variables of the study area. This is of interest if we want to model a particular variable of the study area with more accuracy.

3) Linear vs. Nonlinear Model Reduction: For small deviations from the steady-state it is known that linearized systems are sufficient to model nonlinearities, but for larger perturbations it is necessary to use a nonlinear description. This motivates the choice of retaining a nonlinear description of the study area while using a reduced linear model of the external area. The idea is that if there is a perturbation in the study area, the effects of it will be greatest in its proximity, thus motivating a nonlinear model of it, while the external area is less affected and consequently it can suffice to have a linear model of it. To demonstrate this idea consider a 5% perturbation of V_{ref} of generator 1 from its steady-state value. We see in Fig. 6 how the saturation in the field voltage is captured by the nonlinear reduced model, whereas it is lost in the linear reduced model. Fig. 7 shows that nonlinearities are also necessary to model the phase angles. It is seen that the trajectories of the full nonlinear model and of the reduced nonlinear model are close to each other.

B. KTH-NORDIC32 System

1) System Description and Simulation Method: We will now consider the KTH-NORDIC32 system, which is a model of the Swedish power system and its neighbors [38]. It is composed of 52 buses, 52 transmission lines, 28 transformers, and 20 generators, of which 12 are hydro generators and 8 are thermal generators. It was noted in [38] that there are three primary modes

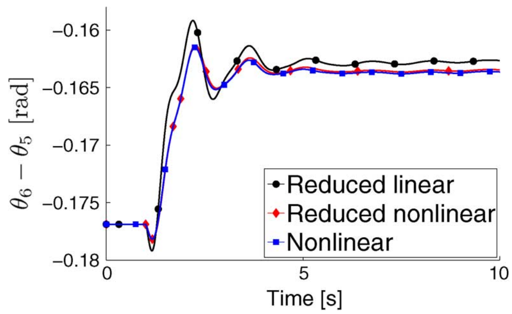

Fig. 7. Transients of $\theta_6 - \theta_5$ after an initial 5% perturbation of V_{ref} of generator 1.

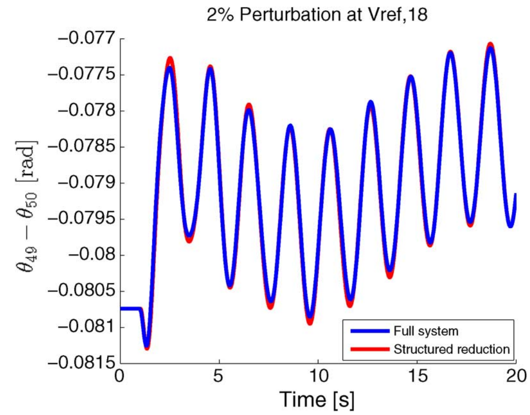

Fig. 8. Responses of $\theta_{49} - \theta_{50}$ after a 2% perturbation to $V_{ref,18}$.

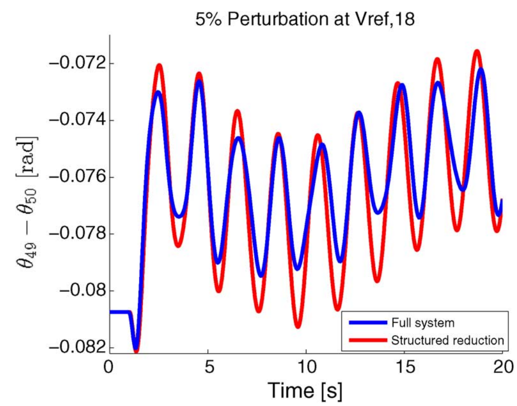

Fig. 9. Responses of $\theta_{49} - \theta_{50}$ after a 5% perturbation to $V_{ref,18}$.

of interest in the KTH-NORDIC32 system: two electromechanical inter-area modes of frequency 0.49 Hz and 0.77 Hz, respectively, and a drift mode of 0.058 Hz stemming from load and turbine/governor dynamics.

To be able to apply the structured model reduction algorithm we define the southern area with generator number 18 and buses 18 and 50 as the study area. The transmission lines between buses 50 and 49 are defined as the tie-lines between the study area, with one generator, and the external area. Originally the external area has 246 states. The reduced external area of model order 17 is chosen here.

To demonstrate the effectiveness of the proposed reduction method, the following studies have been carried out to compare the response of the full nonlinear model of the KTH-NORDIC32 system against a reduced order linear model. The simulations of the full nonlinear model were carried out using PSAT's nonlinear time-domain simulation routine. The linearized matrices of the external area are computed using the proposed model reduction method and while those of the study area are created by using PSAT's small signal stability analysis routine, simulations are performed using MATLAB/Simulink.

2) Sensitivity to the Generator's Control Inputs: To begin with, we will perturb the system by changing the voltage reference value of the AVR of generator 18 at $t = 1$ s and simulate for 20 s. Four case studies have been carried out, they are: 2, 3, 4, and 5% change in the reference value. The monitored variables are voltage angle differences between Bus 49 and Bus 50, $\theta_{49} - \theta_{50}$, which are the interface variables between the study and the external areas.

The results of 2 and 5% step change are presented in Figs. 8 and 9, respectively. Despite some small differences in the amplitude in the latter case, it can be seen that the reduced model matches the full nonlinear model relatively well for both cases in terms of oscillation frequency of the dominant inter-area mode, 0.49 Hz. In addition, the drift mode of 0.058 is also observable in the reduced model in both cases.

The peak-to-peak error is computed and summarized for all 4 cases in Table II. Note that the computation is taken from $t = 5$ s to $t = 15$ s. From the results in the table, the errors are comparatively low except for the 5% step change.

Next, we will apply a step change to the speed reference of the governor of generator 18, $\omega_{ref,18}$. Four cases are considered, they are: 0.2, 0.4, 0.7, and 1% step change in the reference value. The results of 0.2 and 1% step change are presented in Figs. 10 and 11, respectively. It can be seen that the reduced model is able to match the full system relatively well in the first case. On the other hand, in the second case, the reduced model matches the full model in the first 10 s of the responses and starts to deviate afterwards.

TABLE II
ERROR COMPUTATION OF DIFFERENT STEP INPUT TO THE VOLTAGE REFERENCE OF THE AVR

Change in V_{ref}	Abs. Error (%)
2 %	0.0013
3 %	0.0839
4 %	0.0718
5 %	0.39976

The peak-to-peak error is computed and summarized for all 4 cases in Table III. The errors are comparatively low.

3) Sensitivity to Line Removal: In this study, a breaker is placed in one of the parallel lines between Bus 49 and Bus 50. Four case studies are carried out by varying the breaker opening duration; they are 80, 120, 160, and 200 ms. Responses of the voltage angle differences, $\theta_{49} - \theta_{50}$, when the duration are 80 and 200 ms are presented in Figs. 12 and 13, respectively.

According to Fig. 12, it can be seen that, despite some deviations in the peak amplitude, the reduced model preserves the inter-area frequency of oscillation and its responses match those of the full nonlinear model. As the disturbance becomes larger, as in Fig. 13, in addition to peak deviations, a phase shift between the two systems is noticeable. Note that during the first

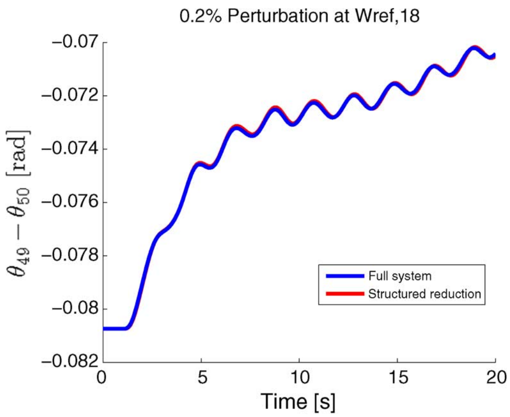

Fig. 10. Responses of $\theta_{49} - \theta_{50}$ after a 0.2% perturbation to $W_{ref,18}$.

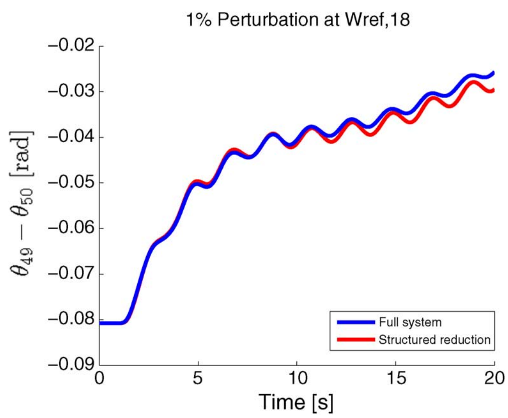

Fig. 11. Responses of $\theta_{49} - \theta_{50}$ after a 1% perturbation to $W_{ref,18}$.

TABLE III
ERROR COMPUTATION OF DIFFERENT STEP INPUT TO THE SPEED REFERENCE OF THE TURBINE GOVERNOR

Change in ω_{ref}	Abs. Error (%)
0.2 %	0.0845
0.4 %	0.4827
0.7 %	0.6601
1 %	1.4389

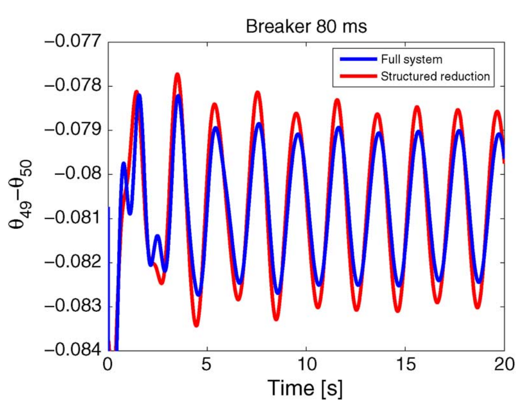

Fig. 12. Responses of $\theta_{49} - \theta_{50}$ after opening a line for 80 ms.

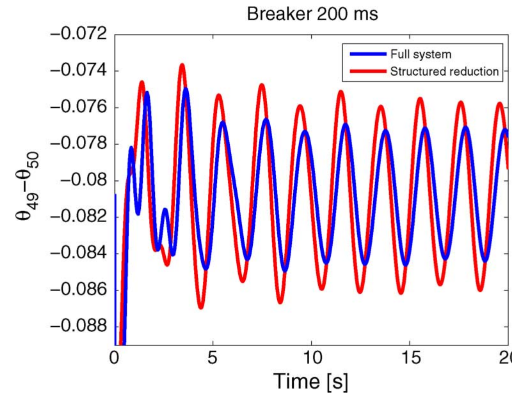

Fig. 13. Responses of $\theta_{49} - \theta_{50}$ after opening a line for 200 ms.

TABLE IV
ERROR COMPUTATION OF DIFFERENT BREAKER DURATION

Breaker Duration (ms)	Abs. Error (%)
80	0.6922
120	1.1088
160	1.6189
200	2.0526

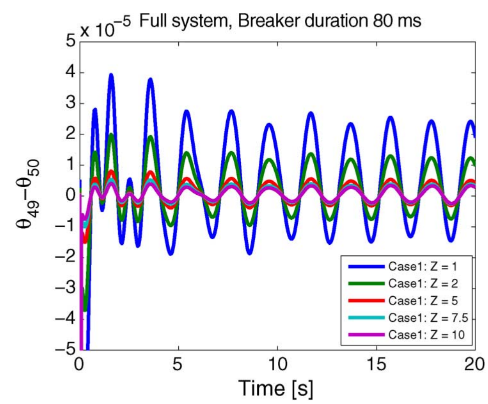

Fig. 14. Sensitivity to line impedance variations: Full system.

cycle after the disturbance, another inter-area mode, 0.77 Hz, is noticeable in the full nonlinear system response and partially captured by the reduced model. Note that inter-area dynamics dominate the response of the system for this type of perturbation, and thus, dominate the response of the reduced linear model also.

The peak-to-peak error is computed and summarized for all 4 cases in Table IV. Although, these values are larger than those of the previous two studies, they are still relatively low.

4) Sensitivity to Line Impedance Variations: A fictitious line with a breaker is added in parallel to the two existing lines between Bus 49 and Bus 50. The impact of line impedance on the full and reduced models is investigated by opening the line for 80 ms and varying the line impedance. Five different line impedance values have been considered here; they are 1, 2, 5, 7.5, and 10 pu. The comparison among 5 cases for the full and reduced models are illustrated in Figs. 14 and 15, respectively.

For both systems, the larger the line impedance, the smaller the amplitude of oscillations becomes (i.e., the network is stiffer). Comparing between the full and the reduced systems, the reduced system manages to preserve the 0.49-Hz inter-area mode but partially captures the 0.77 Hz during the first cycle

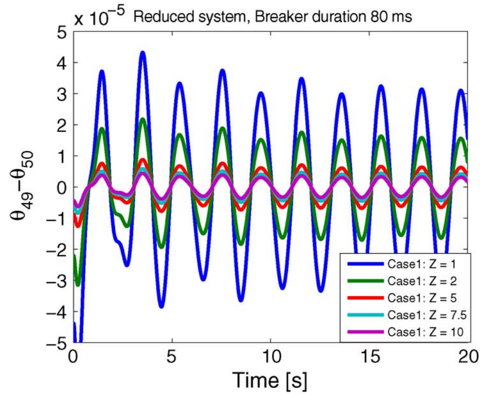

Fig. 15. Sensitivity to line impedance variations: Reduced system.

after disturbance. Overall, the amplitudes of oscillation in the reduced model are larger than those of the full system.

VI. Conclusion

This paper has presented a model reduction algorithm which is applicable to power systems that can be divided into a study area and an external area. The algorithm is based on an extension of balanced truncation which takes the behavior of the full power system into account when reducing the external area. This means that if certain frequency ranges are amplified more by the study area, the reduced model of the external area will, as a result, be more accurate in those frequency ranges.

We have demonstrated the proposed algorithm on the Klein-Rogers-Kundur 2-area system and the KTH-NORDIC32 system. In the latter system, we have demonstrated the validity and sensitivity of the reduced model to different types of disturbances. For small disturbances, the reduced model is capable of matching the responses of the full model nearly well; particularly, the principal modes of oscillations are well-preserved. For larger disturbances, despite some deviations in the peak amplitude of oscillation, the reduced model is sufficient to model the transients following a perturbation. It was shown that the proposed model reduction algorithm is feasible for small and large power systems.

The application of the proposed model reduction method requires to couple the nonlinear model of the study area with a linearized model of the reduced area. The work in this article has been performed using MATLAB/Simulink. Such simulation method is not common in proprietary power system simulation software. To the knowledge of the authors, the only dedicated power system software capable of this type of simulation is DOME [39]. Hence, aspects of software implementation in proprietary and dedicated power system analysis tools remain an open issue.

References

[1] *Power System Coherency and Model Reduction*, J. Chow, Ed.. New York, NY, USA: Springer, 2013.
[2] M. Oatts, S. Erwin, and J. Hart, "Application of the REI equivalent for operations planning analysis of interchange schedules," in *Proc. Power Industry Computer Application Conf., 1989. PICA'89, Conference Papers*, May 1989, pp. 424–430.
[3] F. Milano and S. K. , "Dynamic REI equivalents for short circuit and transient stability analysis," *Elect. Power Syst. Res.*, vol. 79, pp. 878–887, 2009.
[4] F. Ma, X. Luo, and V. Vittal, "Application of dynamic equivalencing in large-scale power systems," in *Proc. IEEE Power and Energy Society General Meeting*, Jul. 2011, pp. 1–10.
[5] J. H. Chow, G. Peponides, P. V. Kokotovic, B. Avramovic, and J. R. Winkelman, *Time-Scale Modeling of Dynamic Networks With Applications to Power System*, J. H. Chow, Ed. New York, NY, USA: Springer-Verlag, 1982.
[6] R. W. de Mello, R. Podmore, and S. K. N. , "Coherency-based dynamic equivalents: Applications in transient stability studies," in *Proc. PICA Conf.*, 1975, pp. 23–31.
[7] A. J. Germond and R. Podmore, "Dynamic aggregation of generating unit models," *IEEE Trans. Power App. Syst.*, vol. PAS-97, no. 4, pp. 1060–1069, Jul. 1978.
[8] J. Chow, R. Galarza, P. Accari, and W. Price, "Inertial and slow coherency aggregation algorithms for power system dynamic model reduction," *IEEE Trans. Power Syst.*, vol. 10, no. 2, pp. 680–685, May 1995.
[9] R. Podmore, Development of Dynamic Equivalents for Transient Stability Studies, EPRI, Report EPRI Project 763, May 1997.
[10] R. A. Date and J. H. Chow, "Aggregation properties of linearized two-time-scale power networks," *IEEE Trans. Circuits Syst.*, vol. 38, no. 7, pp. 720–730, Jul. 1991.
[11] J. H. Chow, "A Nonlinear model reduction formulation for power system slow coherency and aggregation," in *Proc. Workshop Advances in Control and Its Applications*, P. I. H. K. Khalil and J. H. Chow, Eds., 1996, vol. 208, ser. Lecture Notes in Control & Information Sciences, pp. 282–298, New York, NY, USA: Springer-Verlag.
[12] J. H. Chow, "New algorithms for slow coherency aggregation of large power systems," in *Systems and Control Theory for Power Systems*, R. J. T. J. H. Chow and P. V. Kokotovic, Eds., 1995, vol. 64, ser. IMA Volumes in Mathematics and its Applications, pp. 95–115, Institute for Mathematics and its Applications (IMA), New York, NY, USA: Springer-Verlag.
[13] M. Brucoli, M. La Scala, N. Pitrone, and M. Trovato, "Dynamic modelling for retaining selected portions of interconnected power networks," *IEE Proc. Gen., Transm., Distrib.*, vol. 135, no. 2, pp. 118–123, Mar. 1988.
[14] M. Ibrahim, O. Mostafa, and A. El-Abiad, "Dynamic equivalents using operating data and stochastic modeling," *IEEE Trans. Power App. Syst.*, vol. PAS-95, no. 5, pp. 1713–1722, Sep. 1976.
[15] J. Machowski, J. Bialek, and J. Bumby, *Power System Dynamics Stability and Control*. Chichester, U.K.: Wiley, 2012.
[16] J. Machowski, A. Cichy, F. Gubina, and P. Omahen, "External subsystem equivalent model for steady-state and dynamic security assessment," *IEEE Trans. Power Syst.*, vol. 3, no. 4, pp. 1456–1463, Nov. 1988.
[17] F. Ma and V. Vittal, "Right-sized power system dynamics equivalents for power system operation," *IEEE Trans. Power Syst.*, vol. 26, no. 4, pp. 1998–2005, Nov. 2011.
[18] X. Wang, V. Vittal, and G. Heydt, "Tracing generator coherency indices using the continuation method: A novel approach," *IEEE Trans. Power Syst.*, vol. 20, no. 3, pp. 1510–1518, Aug. 2005.
[19] P. Panciatici, Y. Hassaine, S. Fliscounakis, L. Platbrood, M. Ortega-Vazquez, J. Martinez-Ramos, and L. Wehenkel, "Security management under uncertainty: From day-ahead planning to intraday operation," in *Proc. 2010 iREP Symp. Bulk Power System Dynamics and Control (iREP)—VIII (iREP)*, Aug. 2010, pp. 1–8.
[20] W. Sattinger, "Application of PMU measurements in Europe TSO approach and experience," in *Proc. IEEE Trondheim PowerTech 2011*, Jun. 2011, pp. 1–4.
[21] D. Chaniotis and M. Pai, "Model reduction in power systems using Krylov subspace methods," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 888–894, May 2005.
[22] P. Parrilo, S. Lall, F. Paganini, G. Verghese, B. Lesieutre, and J. Marsden, "Model reduction for analysis of cascading failures in power systems," in *Proc. 1999 IEEE American Control Conf.*, 1999, vol. 6, pp. 4208–4212.
[23] D. Enns, "Model reduction with balanced realizations: An error bound and a frequency weighted generalization," in *Proc. IEEE Conf. Decision and Control*, Las Vegas, NV, USA, 1984.
[24] B. Anderson and Y. Liu, "Controller reduction: Concepts and approaches," *IEEE Trans. Autom. Control*, vol. 34, pp. 802–812, 1989.
[25] L. Li and F. Paganini, "Structured coprime factor model reduction based on LMIs," *Automatica*, vol. 41, no. 1, pp. 145–151, Jan. 2005.

[26] A. Vandendorpe and P. Van Dooren, *Model Order Reduction: Theory, Research Aspects and Applications*, ser. Mathematics in Industry Series. New York, NY, USA: Springer Verlag, 2007, ch. Model reduction of interconnected systems.

[27] H. Sandberg and R. M. Murray, "Model reduction of interconnected linear systems," *Optimal Control, Applicat. Meth., Special Issue on Directions, Applications, Methods in Robust Control*, vol. 30, no. 3, pp. 225–245, 2009.

[28] P. Kundur, *Power System Stability and Control*, N. Balu and M. Lauby, Eds. New York, NY, USA: McGraw-Hill, 1994.

[29] T. Glad and L. Ljung, *Control Theory—Multivariable and Nonlinear Methods*. New York, NY, USA: Taylor and Francis, 2000.

[30] J. Sanchez-Gasca, J. Chow, and R. Galarza, "Reduction of linearized power systems for the study of interarea oscillations," in *Proc. 4th IEEE Conf. Control Applications, 1995*, Sep. 1995, pp. 624–630.

[31] K. Glover, "All optimal Hankel-norm approximations of linear multivariable systems and their L_∞-error bounds," *Int. J. Control*, vol. 39, pp. 1115–1193, 1984.

[32] G. Schelfhout and B. De Moor, "A note on closed-loop balanced truncation," *IEEE Trans. Autom. Control*, vol. 41, pp. 1498–1500, 1996.

[33] P. Wortelboer, "Frequency-weighted balanced reduction of closed-loop mechanical servo-systems: Theory and tools," Ph.D. dissertation, Delft University of Technology, TU Delft, Delft, The Netherlands, Jun. 20, 1994.

[34] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*. New York, NY, USA: Wiley, 2007, vol. 2.

[35] G. Rogers, *Power System Oscillations*. Norwell, MA, USA: Kluwer, 1999.

[36] C. Sturk, L. Vanfretti, Y. Chompoobutrgool, and H. Sandberg, "Structured power system model reduction of non-coherent areas," in *Proc. IEEE PES General Meeting, Invited Paper for the Panel Session: "Distributed Control—Bringing Together the Power and Control Communities"*, San Diego, CA, USA, 2012.

[37] C. Sturk, L. Vanfretti, F. Milano, and H. Sandberg, "Structured model reduction of power systems," in *Proc. 2012 American Control Conf. (ACC)*, 2012.

[38] Y. Chompoobutrgool, W. Li, and L. Vanfretti, "Development and implementation of a nordic grid model for power system small-signal and transient stability studies in a free and open source software," in *Proc. IEEE Power & Energy Society General Meeting*, 2012.

[39] F. Milano, "A python-based software tool for power system analysis," in *Proc. 2013 IEEE Power and Energy Society General Meeting*, 2013, pp. 1–5.

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