Designing Model-Free Time Derivatives in the Frequency Domain for Ambient PMU Data Applications

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***Abstract*— Model-free derivatives are essential to several synchrophasor applications. The standard approach to estimate them is to combine a smoothing operation with an ideal derivative computation. However, because the derivative operation increases the signal's content in higher spectral frequencies which can undo the effect of smoothing. Ambient data also brings unique challenges, as mere visual inspection of the estimate in the time domain does not provide insight into the quality of the final estimate. In this regard, the underlying signal's frequency spectrum can provide valuable information for designing a good derivative estimate. This paper introduces a framework for designing a model-free derivative estimate in the frequency domain that accounts for the system's underlying dynamics. The approach is demonstrated on two classic synchrophasor analytics problems on measurements from the Dominion Energy system.**

***Index Terms*— Robust Derivative, Inertial Response, Synchrophasors**

I. Introduction

Reliably estimating derivatives from measurements is a common task in many measurement data-driven applications in power systems. Estimating frequency is fundamentally a problem of estimating the phase angle derivative, and in practice, it involves a smoothing operation to remove fast transients [1]. While Phasor Measurement Units (PMUs) provide a frequency estimate derived from the measured angle, these estimates can vary drastically between manufacturers, even in ideal testing conditions and can be affected by time errors [2]. To this end, alternative means to compute these estimates can be useful for PMU applications that require reliable *n*-th order derivatives. For instance, [3] proposes using the rate of change of frequency (ROCOF) to detect PMU time errors. In addition, data-driven dynamic stability assessment applications involve operating on signal values at successive time steps. With the increasing penetration of renewable generation, coupled with the retirement of conventional generation, there is a need to track inertia, which captures the relationship between power imbalance and ROCOF [4]. Detecting voltage collapse using Lyapunov exponents [5] reduces to finding average voltage magnitude derivative at any given time compared to its value in the past. For detecting generator angular instability, the single and double derivatives of slower electromechanical angular dynamics are often used [6]. Similar ideas apply to frequency stability [7].

Steady-state applications also focus on incremental changes (i.e. derivatives) from an equilibrium. For example, voltage security analysis using Thévenin equivalent [8] involves estimating the derivative of steady-state (slow) voltage-to-current injection changes and therefore involves smoothing/low pass filter to remove dynamics. Along the same lines, there have also been attempts to estimate such derivatives using a polynomial approximation [9] and even power flow Jacobian from ambient data [10], which is a partial derivative of real and reactive power angles and voltages.

Given that the power system operates in ambient conditions [4], extending standard synchrophasor analytics to ambient data for continuous monitoring has sparked substantial interest. For example, there has been a surge in interest in online inertia monitoring [4], which, as previously noted, is fully reliant on a precise estimation of the double derivative of phase angle. Working with ambient data presents a significant challenge because the signal to noise ratio is low. This makes it difficult to visually fine-tune the derivative design, as is commonly done for large signal events.

While multiple applications depend on a signal's derivatives, surprisingly, the problem of computing derivatives hasn't been examined in depth in the power system literature. It has only been investigated on a case-by-case basis in each application scenario and therefore, there is a lack of a generic framework for derivative design that can be applied to any type of synchrophasor signal. Often, not much thought is given to their actual effect on a signal's content. At most, data is smoothed before/after an ideal type of numerical differentiation. Numerical differentiation, if not designed carefully, increases higher frequencies in the estimated derivative's spectrum, thereby undoing the effect of smoothing if applied previously.

The current work proposes a derivative design framework for PMU applications that allows the user to account for the application's validity in relation to the time scale of interest, in order to address the numerous challenges surrounding derivative estimation, as well as a lack of a practical generic framework. Furthermore, because the framework is wholly in the frequency domain, it may be used with spectrum analysis tools [11] to build derivatives for ambient data applications, which has gone entirely unaddressed in the current literature.

This paper is organized as follows. Section II of this paper presents the *n*-th derivative estimation problem, along with a methodology for designing its computation in the frequency domain. The results are obtained for multiple practical use cases on synthetic as well as real PMU data from Dominion System in Section III, while Section IV outlines future work.

II. Derivative Estimate

A. Derivative Estimation Problem in Frequency Domain

The existing practices in power systems often involve using ideal derivatives with or without a smoothing operation (typically, a moving average). The major challenge of reliably computing these derivatives can be understood by analyzing their effect in the frequency domain. An ideal, n^{th} order derivative operation on a signal $y(t)$ results in,

$$\mathcal{F}\left(y^{(n)}(t)\right) = (j\omega)^n \times Y(\omega) \tag{1}$$

where $\mathcal{F}(*)$ denotes the Fourier Transform of any signal. Observe that the gain of an n^{th} order derivative is proportional to ω^n. The higher the order of the derivative estimated, the more distorted the signal's content will be at higher frequencies, resulting in higher frequency noise. Thus, applying numerical differentiation when computing derivatives can often undo the effect of smoothing the signal, as seen in Figure 1.

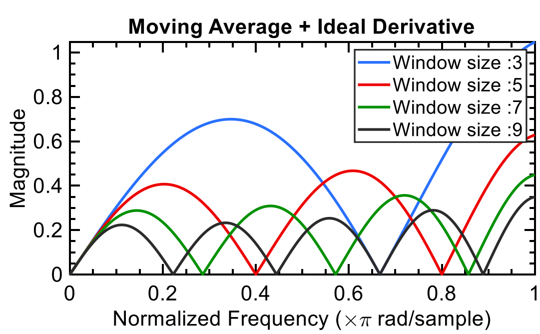

Figure 1. Effect of applying an ideal derivative with moving average

B. Ambient System Characteristics and Derivative Requirements

A derivate estimate procedure should ensure that there is no loss of relevant information as observed in the signal while at the same time suppressing high frequency noise. Now, for ambient conditions, we will show how the information content of the signal can be understood to guide the derivative design. Power system in ambient conditions can be approximately modeled as a linear system driven by random perturbations. The frequency spectrum [11] of any general scalar output measurement y in such systems can be written as,

$$S_{yy}(\omega) = \sum_i \left[\frac{A_i}{j\omega - \lambda_i} - \frac{A_i^*}{j\omega + \lambda_i^*}\right] \tag{2}$$

where $\lambda_i's$ are the system eigen values and $A_i's$ are complex constants. Note that the above can be faithfully estimated from the measurement data and does not require a system model. Now, the eigenvalues $\lambda_i's$ characterize the oscillatory behavior (modes) of the system and therefore, the output signal spectrum can yield insights into the time scales of the underlying system's dynamics as observed in the signal. Thus, it can be used to understand what range of frequencies do not contain any information and therefore can be suppressed in the derivative estimate to reduce the estimate's overall variance. This approach will be later on demonstrated on the real-world measurement data in the results.

Here it is important to mention that while the discussion is focused on linear systems and small signal response of nonlinear systems, the same design approach can be applied for large signal response in nonlinear systems as well. However, in that case, the frequency range to retain depends on the underlying time scales of the system's dynamics that are not straightforward to obtain as in the case of linear systems.

C. Methodology for Designing a Robust Derivative

Once the spectrum is estimated for the measurement signal, the next step is to design how the derivative of the signal is to be estimated. The following two design criteria are desirable:

1. Present a similar behavior to an ideal derive (as shown in (1)) in frequency range where the relevant system dynamics are present. For e.g. < 1 Hz for electromechanical oscillations.
2. Suppressing unwanted high frequency noise/ fast dynamics irrelevant to the application. For e.g. suppress frequencies above 5 Hz in the case of electromechanical oscillations.

To meet these criteria, we expand the derivative design approach in [12], keeping in mind that it will be applied to PMU data. Since the derivative operator is linear in nature, it is realized using a linear, causal FIR filter. The output when a filter with window length N and coefficients $b_0, b_1, \ldots . b_{N-1}$ acts on a signal $y(t)$ is given by,

$$y_{filt}(t) = \sum_{k=0:N-1} b_k y(t - k\Delta t) \tag{3}$$

$$\mathcal{F}\left(y_{filt}(t)\right)|_{\omega} = Y_{filt}(\omega) = \left(\sum_k b_k e^{-jk\omega\Delta t}\right) \times Y(\omega)$$

here, sampling frequency is $f_s = \frac{1}{\Delta t} Hz$. Next, assume we want to realize an n^{th} order derivative by appropriately choosing $b_k's$. We start by obtaining a Taylor series expansion of the exponential terms on the RHS at an arbitrary angular frequency $\omega = \omega^*$ to get a polynomial representation,

$$\sum_k b_k e^{-j\omega k\Delta t} = \sum_i \left(\sum_k \frac{b_k(-k\Delta t)^i e^{-jk\omega^*\Delta t}}{i!}\right)(j\omega - j\omega^*)^i \tag{4}$$

Now, the first design criterion is met by matching the above polynomial to an ideal derivative operation, as given in (1), in the vicinity of $\omega^* = 0$. For this, the first n Taylor coefficients at $\omega^* = 0$ need to be zeroed out and in addition, the coefficient for $i = n$ term need to be equal to 1. This yields $n + 1$ linear equations in filter coefficients b_k's. For a window length of N, the remaining degree of freedom for realizing the second criterion is $m = N - (n + 1)$. The second design criterion is met by zeroing out the first m Taylor coefficients evaluated at the Nyquist angular frequency $\omega^* = \pi f_s$. The classical two-point derivative approach corresponds to $m = 0$, i.e. a minimum window length N and thus, no regards for the second design criterion. Figure 2 shows the transfer function magnitudes for those filters when compared to ideal derivative, which clearly demonstrates the improvement in high frequency suppression properties with increasing window length N. Note that the x axis on the plot is normalized frequency i.e. for higher sampling rates, a longer N will be needed to achieve the same level of suppression in absolute frequency values. For reader's reference, Table I shows the derivative filter coefficients to be used in (3) for first and second order derivatives for $f_s = 1\ Hz$.

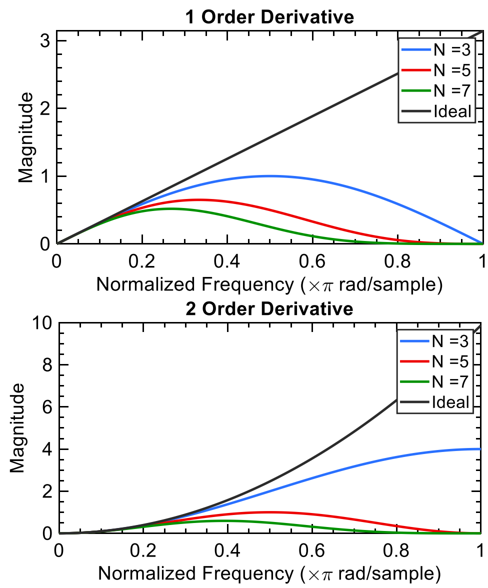

Figure 2. Robust derivatives of 1st and 2nd order for different window sizes

TABLE I DERIVATIVE FILTER COEFFICIENTS b_k

N	First Order	Second Order
2	$[1, -1]$	-
3	$[0.5, 0, -0.5]$	$[1, -2, 1]$
5	$\begin{bmatrix} 0.125, 0.25, 0, \\ -0.25, -0.125 \end{bmatrix}$	$\begin{bmatrix} 0.25, 0, -0.5, \\ 0, 0.25 \end{bmatrix}$
10	$\begin{bmatrix} 0.0039, \ 0.0273, 0.0781, 0.1094, \\ 0.0547, -0.0547, -0.1094, \\ -0.0781, -0.0273, -0.0039 \end{bmatrix}$	$\begin{bmatrix} 0.0078, 0.0391, 0.0625, \\ 0, -0.1094, -0.1094, 0 \\ , 0.0625, 0.0391, 0.0078 \end{bmatrix}$

III. RESULTS

To demonstrate the virtues of the proposed framework, studies are conducted on two popular synchrophasor analytics problems –Thevenin equivalent for steady state voltage security analysis and frequency estimation from phase angle data. However, before showing practical application examples, results are also presented for a synthetic signal to analyze the performance of the methodology under ideal conditions. This is carried out because when using real-world measurements, the ground truth is not known beforehand. When choosing the appropriate window length for ambient data, the frequency spectrum is estimated using Welch's periodogram [11] with Hanning Window and an FFT window length of 2 mins.

A. *Synthetic Ringdown Signal*

For the first test, we create a synthetic ringdown signal with 5 Hz and 2 Hz modes with a high variance additive Gaussian Noise $\epsilon(t) \sim N(0,1)$, sampled at $f_s = 60\ Hz$

$$y(t) = e^{-0.5t}\sin(2\pi 5t) + e^{-0.2t}\cos(2\pi 2t) + \epsilon(t)\,. \qquad (5)$$

This test demonstrates that while the derivative design is focused on suppressing higher frequencies, it provides the additional benefit of suppressing measurement noise due to the low pass filter type effect.

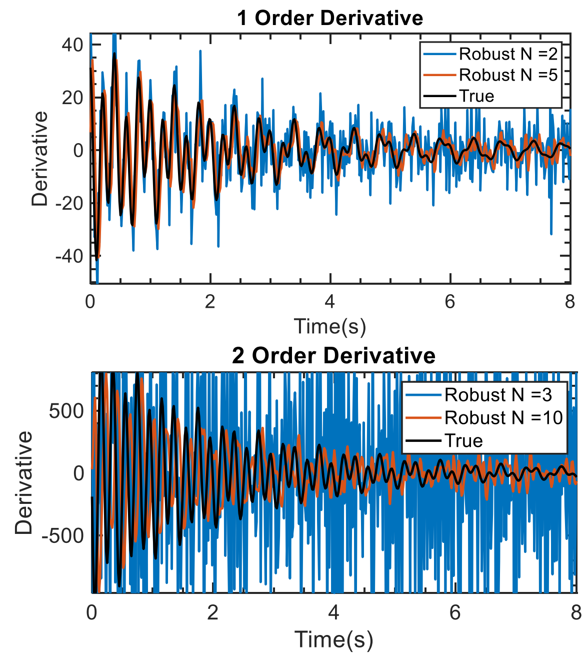

Figure 3. Ringdown Signal Derivative Estimates

The first and second order derivative estimates using the proposed approach for an arbitrary window size are compared against the classical, two-point derivative approach in Figure 3. Also, compared to the quality of the first order derivative estimate, the second order estimates are extremely poor for the 2-point approach. This can be explained using (1), which shows that higher-order derivatives are more prone to enhancing high frequency noise.

B. *Thevenin Equivalent*

This application aims at the online monitoring of steady state voltage stability/security by estimating Thevenin equivalent from synchrophasor measurements. For the stability analysis of a monitored load bus, the idea behind this approach is to represent the remaining power grid using a constant voltage source V_{th} behind a series impedance Z_{th} satisfying,

$$V_{load}(t) = V_{th} - I_{load}(t) Z_{th} \qquad (6)$$

Under normal operation conditions, if no major changes occur in the power grid, (V_{th}, Z_{th}) do not change rapidly and therefore can be treated as a constant in a time window lasting several minutes. Z_{th} can be estimated independent of V_{th} as [8],

$$\frac{\frac{d}{dt}(V_{load}(t))}{-\frac{d}{dt}(I_{load}(t))} = Z_{th} \tag{7}$$

That being said, (V_{th}, Z_{th}) are supposed to represent the response of the grid to slow (quasi-steady state) changes only. However, the power system does not have a flat frequency response. This implies that the estimate for Z_{th} obtained using the above equation is very sensitive to the dynamic nature of the load current $I_{load}(t)$, which poses an issue.

Now, to mimic realistic operating conditions, we create a synthetic data set with the load current $I_{load}(t)$ gradually ramping up along with a small, additive, poorly damped oscillation at 0.8 Hz. The response of the power grid to such a signal is given by a frequency-dependent impedance Z_{th} with a value of 0.2 pu below 0.1 Hz (slow dynamics) and 0.5 p.u. above it with a smooth transition. V_{th} is set to a constant 1 p.u. Gaussian measurement noise $\sim N(0, 10^{-4})$ is added. The results when using a standard two-point derivative $(N = 2)$ are compared against robust derivative with $N = 12$ Firstly, the spectrum for the estimated current derivatives is plotted below.

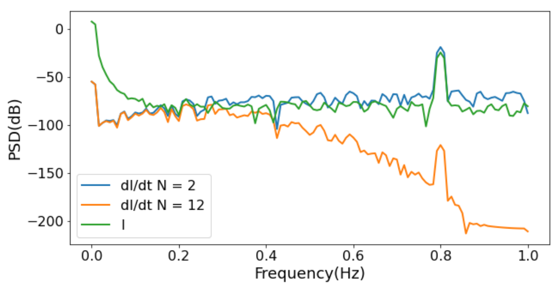

Figure 4. I and d/dt(I) Spectrum

It can be seen from the current magnitude spectrum that the effect of slow ramping is restricted to < 0.2 Hz while the oscillation is around 0.8 Hz. This is typical of practical systems where the underlying dynamics are usually separable in the frequency domain. Also, observe that the derivative estimates are the same up to 0.3 Hz beyond, which the longer window derivative suppresses the faster dynamics.

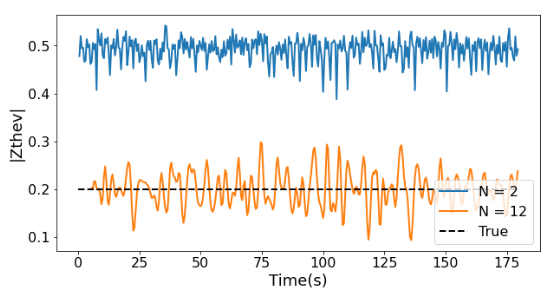

Figure 5 Thevenin Impedance Estimates

Next, we plot the Thevenin Equivalent estimate obtained from (7) using the estimated derivatives. As expected, the two-point derivative gives a wrong estimate for Z_{th} due to it also capturing the interaction of 0.8 Hz components of V and I, while increasing the window size yields a better estimate by filtering out that portion.

C. Frequency Estimation from Phase Angle Data

By definition, frequency represents the first-order derivative of phase angle. While the IEEE C37.118 standard on Synchrophasor Measurements in Power Systems [13] standardizes the phase angle calculation, it does not prescribe how to estimate frequencies. Most PMU algorithms approach frequency estimation by averaging the derivative calculation over a multi-cycle window of data, to remove the effect of measurement noise and transients [1], yielding a response similar to Figure 1. However, there are substantial differences in how each manufacturer implements it resulting in different results even under ideal conditions [2], which become particularly prominent when studying small-signal response in ambient data. Furthermore, frequency estimates can be plagued with significant quantization errors, which makes them even less trustworthy. Therefore, in a practical system with multiple devices from different manufacturers, it is preferable to not directly use frequency estimates from the various PMUs, which makes *a posteriori* frequency estimation from phase angle an important problem.

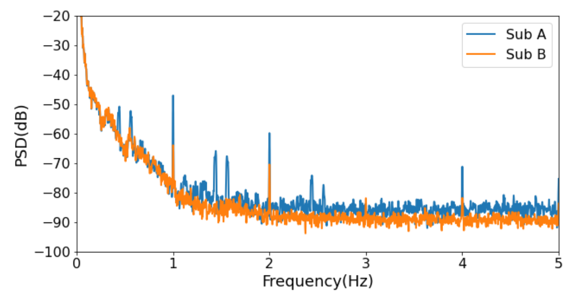

Figure 6. Phase Angle Spectrum

For the present study, a parcel of 20 min phase angle measurements was taken from two synchrophasor device manufacturers at different substations (A and B, respectively). Figure 6 shows the absolute phase angle spectra in the 0-5 Hz range. The initial part, <1 Hz, has a curved spectral baseline due to the absolute phase angle drifting from the 60 Hz power system frequency. There are also sharp peaks at 1, 2 and 4 Hz. Based on *a prior* analysis[14], these are not physical modes but the effect of periodic phase angle corrections made internally by the PMUs to account for clock drift. Note that only frequencies below 5 Hz are of interest.

Substation A's spectrum shows spectral peaks around 0.5 Hz, 1.5 Hz, and 2.5 Hz. These are a result of mechanical rotor oscillations from a combined cycle power generation plant. Below, we compare the frequency estimate encoded inside the device (in blue, DFR) and reported as synchrophasor data with different robust derivative options differing in window lengths.

The estimated gain of the device's internal frequency calculation stays close to an ideal derivative up to 0.4 Hz, increases from 0.4-0.6 Hz, and beyond that, rolls off. The gain continues to bounce beyond 1 Hz, loosely similar to that of a moving average with an ideal derivative (see Figure 1). It neither annihilates the unwanted frequencies nor resembles a derivative in the retained frequency range. A good derivative estimate will closely match an ideal derivative-like operation for the frequency ranges with dynamics of interest, while at the same time annihilating everything at higher frequencies. From the plot in Figure 7, to account for electromechanical oscillations up to 2.5 Hz, $N = 4$ can be a good choice for the robust derivative with better characteristics than the frequency computed from the actual PMU device.

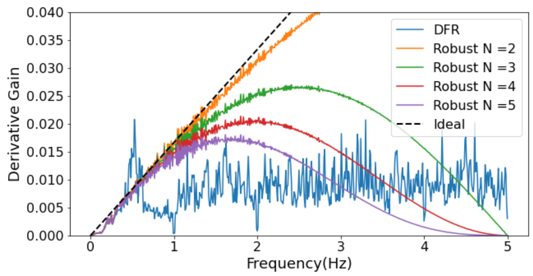

Figure 7 Frequency Estimates Substation A

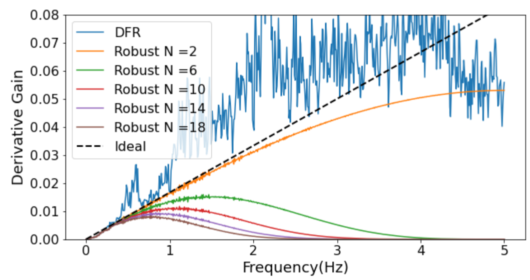

Figure 8 Frequency Estimates Sub B

At Substation B, the device's internal frequency estimate is loosely similar to an ideal derivative up to 0.2 Hz, beyond which it increases until 4 Hz and then rolls off gradually. However, the quality of the derivative is poor. This can be seen in Figure 9 by comparing the estimates in the time domain where even a simple two-point derivative (orange) is less noisy than the device's own estimate.

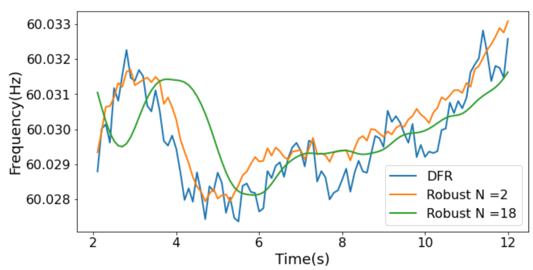

Figure 9 Time derivatives with N=2 & 18 compared to DFR

Finally, based on the phase angle spectrum, since there isn't much information beyond 1 Hz at this location, a more aggressive approach can be taken, using a longer derivative window of $N \geq 18$ for which case, the gain roll-off starts close to 1 Hz, as shown in Figure 8. However, this results in a larger phase shift due to the causal nature of the filter, which is an acceptable tradeoff. That being said it is important to highlight that the requirement for a causal derivative filter is only relevant for online applications. On the other hand, for applications allowing for time delays such as offline analysis or most PMU control center applications, which operate using rolling windows of data, a noncausal derivative can be designed with an additional constraint that minimizes the phase lag.

IV. Discussion and Future Work

In this work, a framework for designing derivatives in the frequency domain is presented. This framework enables designing derivatives guided by the time scale of dynamics of interest in the underlying system while suppressing high-frequency noise. Future work will explore the use of this approach for state estimation for new monitoring applications.

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