Power System Frequency Domain Characteristics for Inertia Estimation from Ambient PMU Data

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***Abstract*—Current inertia estimation methods aim at determining a mass or inertia parameter of an underlying power system model by using bus frequency and real power output from PMU time series data. However, this neglects the fact that t inertial response depends of other processes within the grid andhe the power plant control systems. This becomes especially important under the presence of governor dynamics and even more difficult during periods of automatic generation control (AGC) operation. However, due to the turbine governor's time constants being longer, it is hypothesized that the effect of certain low-frequency dynamics can be filtered out of the frequency domain in the ambient data to obtain an inertial response across a specific timescale. An investigation of this hypothesis is presented in this work, aiming to give insight on the frequency range where the filtering can be applied. Furthermore, this work aims to extend the concept of inertia as a constant parameter to a frequency domain inertial response when attempting to estimate the inertia of a power plant. Simulations are performed using a nonlinear model of a single-machine system under ambient load perturbations and comparisons drawn to estimates obtained from an event.**

***Index Terms*—Inertial response, Synchrophasors, Wide area monitoring**

I. INTRODUCTION

With the increasing penetration of renewable generation coupled with conventional generation retirements, power system stability has become a major concern. This is due to an overall reduction in system inertia which plays the role of arresting the frequency changes following major disturbances. Another issue that exacerbates this condition is the tendency of the distributed energy resources to disconnect when under abnormal frequencies [1]. Thus, locational inertia, which defines the relationship between power imbalance and rate of change of frequency (ROCOF), can be used as a metric for system dynamic performance. This motivates the need to track its values as the system changes.

Inertia estimation from disturbance data has been studied in the literature [2] - [4] and is usually carried out by performing inertia estimation using the swing equation. The mechanical input to the generator (P_m) is assumed to be a constant in the first few seconds following the event under the assumption that P_m response is slow. While fairly effective, these approaches are limited to event measurement data (i.e. transient or ringdown responses) and subsequently cannot be used for continuous monitoring/tracking during ambient conditions.

Given the fact that the power system mostly operates in ambient conditions, inertia has traditionally been monitored using generator statuses [5]. The first approach proposed in [6] for estimating inertia from ambient data involved estimating a high order ARMA model between frequency and output power. Using the estimated ARMA model, a step response is applied to it to mimic a transient event. The resulting synthetic data is used to estimate the inertia parameter using conventional event data-based approaches. A similar approach is used in [7]. Meanwhile, in [8], the authors derive a relationship between the inter-area modes and effective inertia utilized for estimation. This derivation is obtained from [9] where it is used on large disturbance data.

Overall, previous work assumes that for periods with negligible automatic generation control (AGC) and prime mover control actions, a linear input-output dynamic model can be estimated between generator power output and frequency, which inherently has the governor dynamics embedded in it. The mechanical input dynamics are a dynamic function of the power set point changes, generator speed, and the remote frequency signal's feedback as part of the area control error in AGC. Because of this it is also doubtful that the mechanical input dynamics can be completely observed and therefore embedded by using only the frequency and real power output measurements. The approach in this paper differs from previous works in that instead of attempting to estimate those dynamics, the goal is to focus on frequency ranges where those dynamics are negligible, i.e. inertia governs the system dynamics. This is carried out by quantifying the mechanical input dynamics and then determining where they can be neglected. Furthermore, this paper explores the idea of defining inertia as a frequency domain response defined by a range of values as opposed to a single, constant value.

The paper is organized as follows. In Section II, the inertia estimation problem is presented. Then, the problem is reformulated in the frequency-domain perspective in Section III. In Section IV results obtained are compared against estimates gathered from a synthetic event. Finally, Section V discusses how the findings of this work affect future direction in this field.

II. Inertia Estimation

Traditionally, the inertial response, or simply the inertia constant, is defined as a scalar metric that links power imbalance in a system to ROCOF. For a single machine system, it can be modeled using the swing equation with damping effects ignored, as follows,

$$M_i \frac{df_i}{dt} = P_{m_i} - P_{g_i} \tag{1}$$

where $M_i =$ the estimated inertia constant metric defined on system base S_{sys}, $P_{m_i} =$ input mechanical power, $P_{g_i} =$ output electrical power in p.u. on system base, $f_i =$ frequency of the i^{th} generator in per unit on a 60 Hz base. Meanwhile, the aggregate inertia constant of an area can then be approximated as,

$$M_{area} \frac{df_{area}}{dt} = \sum_i P_{m_i} - \sum_i P_{g_i} \tag{2}$$

where $f_{area} =$ weighted average of frequencies in the area and $M_{area} =$ area inertial response.

Traditionally, P_{m_i} is assumed constant. This is only valid for very short periods, i.e. during first-swing dynamics. However, under other conditions, when a generator's speed deviation is detected due to power imbalance at the terminals, it drives the error signal of the primary frequency controller (i.e. the governor system) and other control systems in the turbine to adjust P_{m_i} values and restores the power balance.

There is also a provision to change the generator power output setpoint in the turbine governor system. This is done through an outer control loop called the AGC. This is in charge of maintaining the overall system frequency, the correct value of power interchange between multiple areas, as well as each unit's generation. This is an even slower integral type controller and the process is governed by slow timescale dynamics.

III. Frequency Domain Characteristics for Inertial Estimation

A. Data

The first step in inertia estimation is to define areas that can be represented by a lumped equivalent inertia, i.e. buses that can be aggregated. These are mainly comprised of generators that are tightly coupled/coherent. The f_{area} in (2) is taken as,

$$f_{area} = \sum_i w_i f_i \tag{3}$$

where the weights (w_i) are determined according to the contribution of each bus/generator in determining the overall frequency. Traditionally, w_i is set to $\frac{H_i}{\sum H_j}$ for generator buses and 0 for all other buses. Here it is important to note that instead of using f_i measurements directly obtained from phasor measurement units (PMU), bus voltage angles (θ_i) are used through the relation $f_i = \frac{1}{2\pi\times 60}\frac{d\theta_i}{dt}$. This is because the frequency calculations inside a PMU may eliminate faster responses in the phase angles crucial for estimation. From (2), it is also necessary to measure the total generation in the area. Usually, utility companies and transmission systems operators prioritize measuring the generating stations before monitoring the loads. For this reason, such an assumption is valid.

B. Frequency Domain Characteristics for Inertia Estimation

The governing principle behind event-based approaches to inertia estimation is that following a large disturbance the governor takes a longer time to react (due to its longer time constants) than the duration of the event. Therefore, in the first few seconds of the data, P_m can be assumed to be a constant and equal to P_g at time zero and the frequency response to the power imbalance is only impacted by the inertia. This is easily estimated using (1). However, in ambient conditions, the system is continuously being perturbed. Bearing this in mind it is difficult to know when P_m changes and when it does not. In those conditions, the overall generator plus frequency control system can be modeled as a linear system that enables frequency domain analysis.

For a single machine, taking a Fourier transform of (1) results in,

$$M_i(\omega) j\omega f_i(\omega) = P_{m_i}(\omega) - P_{g_i}(\omega) \tag{4}$$

where $f_i(\omega), P_{m_i}(\omega)$, and $P_{g_i}(\omega)$ are Fourier terms corresponding to the frequency value (ω) and not to be confused with the power system frequency of ~60 Hz. Here, it is important to note that in reality, the generator's response to power imbalance from disturbances cannot be a single, constant parameter value M_i as seen in (1) for all types of disturbances. Consequently, M_i in (4) is modeled as a frequency-dependent term, $M_i(\omega)$.

In the existing literature, the single inertia value that is reported is one that captures the relationship between frequency and power imbalance for ideal (i.e. step type) disturbances. These are estimated only leading up to the frequency nadir (i.e. the first few seconds) where the plant's response behaves as a high pass filter to eliminate the slow-moving dynamics in the signals including the effect of P_m. Here it is important to understand that if the disturbance itself is dominantly slow-moving, it is impossible to define such a time window. Under these circumstances such analysis is better handled in the frequency domain. When extending the idea to inertia estimation of a local area inside a larger system, even if the initiating power imbalance is a discrete change, it does not result in a steplike signature because the tie lines respond to such change, and thus are involved in determining that area's frequency. As a result, the value of knowing a single number that captures the relationship between the area's ROCOF and a step disturbance dilutes.

In contrast with existing literature, this work aims to characterize the frequency domain inertial response, i.e. a range of values that can capture the relationship between ROCOF and all possible power imbalances minus the slow frequency control loops. In other words, the values of M_i at different ω help understand the behavior of a machine/area's frequency for any given disturbance signature. This also helps to explain why different events can give different inertia estimates from (1). Moreover, this highlights why it is not sufficient to estimate a single inertia value, but rather we must determine the inertial response or a range of values that gives more insight into the dynamics.

To understand the frequency content of P_m, the typical load frequency control dynamics can be modeled as,

$$P_{m_i}(\omega) = G_{Tgov}(\omega) \times \left(\frac{\Omega_{i_i}(\omega)}{R} + G_{AGC}(\omega) f(\omega) + P_{set_i}(\omega) \right) \tag{5}$$

where P_{set_i} = the generator output setpoint change, R gives the droop rate, Ω_i denotes the generator speed, $G_{Tgov}(\omega)$ is the lumped turbine and governor transfer function, $G_{AGC}(\omega)$ is the AGC transfer function, and f is the vector of frequencies at all buses participating in AGC. Finding upper bound on (5),

$$\begin{aligned} |P_{m_i}(\omega)| &\le |G_{Tgov}(\omega)| \left(\frac{|\Omega_{i_i}(\omega)|}{R} + |G_{AGC}(\omega) f(\omega)| + |P_{set_i}(\omega)| \right) \\ &\le |G_{Tgov}(\omega)| \left(\frac{|\Omega_{i_i}(\omega)|}{R} + \|G_{AGC}(\omega)\| \|f(\omega)\| + |P_{set_i}(\omega)| \right). \end{aligned} \tag{6}$$

It is possible to show how the relatively slow nature of the dynamics at play determine the time scales of P_{m_i}, and can result in tightly bounding the LHS at higher frequencies:

1. It is known that the turbine plus governor dynamics are relatively slow i.e. $\exists \omega_{Tgov}\ s.t. |G_{Tgov}(\omega)| \le \epsilon_{Tgov}\ \forall \omega \ge \omega_{Tgov}$ for a small number ϵ_{Tgov}.
2. The generator speed dynamics $\Omega_i(\omega)$ are also predominantly in the low-frequency range due to the stored kinetic energy in the rotating mass that arrests fast changes. Thus, $\exists \omega_{speed}\ s.t. |\Omega_i(\omega)| \le \epsilon_{speed}\ s.t. \forall \omega > \omega_{speed}$ for a small number ϵ_{speed}.
3. The AGC, an integral type of control, results in an even slower secondary frequency control loop and response i.e. $\exists \omega_{AGC}\ s.t. \|G_{AGC}(\omega)\| \le \epsilon_{AGC}\ \forall \omega \ge \omega_{AGC}$ for a small number ϵ_{AGC}.
4. P_{set_i} is usually changed slowly when ramping the generator output to ensure stability. Thus, $\exists \omega_{set} |P_{set_i}(\omega)| \le \epsilon_{set}\ s.t. \forall \omega > \omega_{set}$ for a small number ϵ_{set}.

Evaluating (6) at $\omega \ge \omega_{min} = \max(\omega_{gov}, \omega_{speed}, \omega_{AGC}, \omega_{set})$,

$$|P_{m_i}(\omega)| \le \epsilon_{Tgov} \left(\frac{\epsilon_{speed}}{R} + \epsilon_{AGC} \|f(\omega)\| + \epsilon_{set} \right). \tag{7}$$

From (4), $M_i(\omega)$ linearly depends on $\frac{P_{m_i}(\omega)}{j\omega f_i(\omega)}$. Hence, dividing (7) by $|\omega f_i(\omega)|$,

$$\frac{|P_{m_i}(\omega)|}{|\omega f_i(\omega)|} \le \frac{\epsilon_{Tgov}}{|\omega|} \left(\frac{\epsilon_{speed}}{R|f_i(\omega)|} + \frac{\epsilon_{AGC} \|f(\omega)\|}{|f_i(\omega)|} + \frac{\epsilon_{set}}{f_i(\omega)} \right). \tag{8}$$

From the above equation, it can be seen that the upper bound given by the RHS is fairly restrictive for higher frequencies. Since $P_{g_i}(\omega)$ is a function of voltage magnitude and angles, it observes both fast and slow dynamics in the system and as such, $\frac{|P_{m_i}(\omega)|}{|\omega f_i(\omega)|}$ becomes almost negligible when compared to $\frac{|P_{g_i}(\omega)|}{|\omega f_i(\omega)|}$ for $\omega \ge \omega_{min}$. This allows us to approximate (4) by the following model to recover a subset of the inertial response values in the frequency domain from data, where both f_i and P_{g_i} are measured,

$$M_i(\omega) = -\frac{P_{g_i}(\omega)}{j\omega f_i(\omega)} \quad \forall \omega \ge \omega_{min}. \tag{9}$$

This approach can easily be extended to (2). However, it is important to note that the governor effect is significant for $\omega < \omega_{min}$ and therefore, without correctly measuring or estimating P_m from data, the power imbalance and consequently the inertial response cannot be estimated. The corresponding frequency value in Hz is denoted by $F_{min} = \frac{\omega_{min}}{2\pi}$.

IV. Results

To illustrate the proposed approach, a one-machine infinite-bus-test system is used, as shown in Fig. 1. The machine can be thought of as an area connected to the rest of the power grid (infinite bus). The line connecting the machine to the infinite bus has $X = 0.1\,pu$. The infinite bus is modeled using a classical machine model GENCLS with a high inertia value of $M = 1000$ pu on a 100 MVA system base in PSS/E.

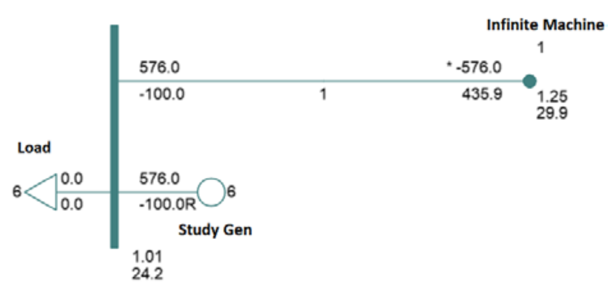

Figure 1. Single machine infinite bus test system

A load is added to the generator bus, which is perturbed in the simulations to emulate events and ambient conditions. This unit represents a 771 MVA combined cycle plant extracted from the Eastern Interconnection model, and modeled in PSS/E using the following models: GENROU— generator, IEEEG1–governor, AC7B—exciter, and PSS2B—power system stabilizer. The parameters are given in the appendix. The value of the generator inertia constant as given in the PSS/E model is $M_{PSSE} = 2 \times 3.17 \times \frac{771}{100} = 48.9$ pu. Note that M_{PSSE} cannot be confused with the inertia constant defined in (1), which will be seen later. M_{PSSE} mainly represents the stored kinetic energy in the spinning mass and satisfies the generator swing equation,

$$M_{PSSE} \frac{d\Omega}{dt} = P_m - P_g - D\Omega \tag{10}$$

where D is a damping coefficient.

To emulate a generator tripping event inside the area, the load at bus 1 P_{load} is stepped up from 0 to 0.5 pu (50 MW). The frequency and P_g plots from the steady-state up to the frequency nadir at ~0.25s are shown in Fig. 2. It can be seen that the generator sees a step increase in loading at its terminal immediately following the event. However, this change does not stay constant because the unit (area) starts to slow down thereby reducing the power flow at the machine's terminal and helps stabilize it. Also, P_m stays fairly constant in that time window and thereby the frequency control dynamics are eliminated from the estimate. For this particular event, using linear regression on (1), M is estimated to be 56.4 pu, which is much more than M_{PSSE}. Because of this, the overall generator inertial response has an effect of generator controls in addition to the mechanical energy. Next, the proposed approach to estimate the inertial response over a range of frequency (ω), illustrates why the estimated value is valid for a range of frequencies.

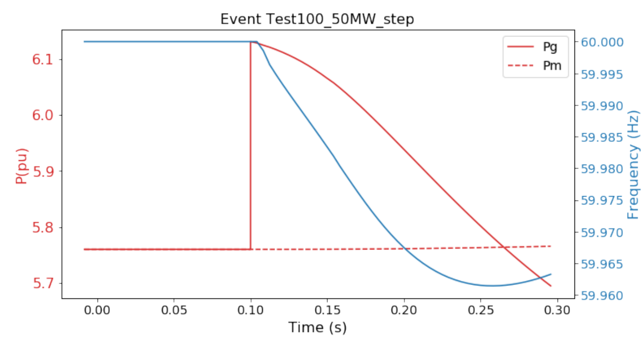

Figure 2. Synthetic event test data

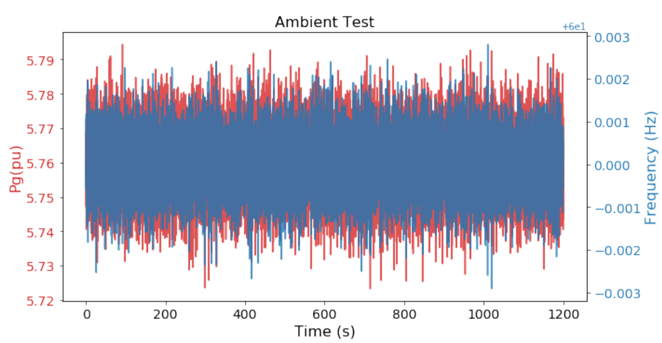

Figure 3. Synthetic ambient data

To emulate ambient conditions, the load is modeled as white noise with $P_{load} \sim N(0, 0.01)$ pu and $Q_{load} \sim N(0, 0.002)$ pu. The load values are sampled at 10 Hz. As a result, the ambient perturbation has a flat spectrum up to 30 Hz. The frequency and the generator output P_g for a 20-minute window used for analysis are plotted in Fig. 3.

To gain better insight into the frequency content that various variables underplay and determine the frequency response of the generator, the power spectral density (PSD) is estimated for P_g, P_m, and $\frac{df}{dt}$ in Fig. 4. These PSDs are calculated using Welch's method [10] with an FFT window size of 2 minutes, a 50% overlap, and a Hanning window. Note that the difference in energy of P_g and P_m signals increases significantly beyond 0.5 Hz owing to the slow nature of the turbine governor dynamics. Moreover, P_g has higher frequency content, this is a result of P_g being influenced by the voltage control loop that is affected by the white-noise-type variation of the load. This energy difference becomes even wider beyond 1.03 Hz. At this point P_m reaches a local maximum and begins to roll off. Accordingly, it is safe to assume that the proposed model in (9) is valid for estimating the inertial response $M(\omega)$ for $\omega \geq 2\pi \times 1.03$ or $F \geq F_{min} = 1.03$ Hz. Another thing to note is that the frequency $f = \frac{1}{2\pi}\frac{d\theta}{dt}$ also has high frequency dynamics and therefore $\frac{df}{dt}$ starts resembling P_g at those frequencies, which we take advantage of in this work.

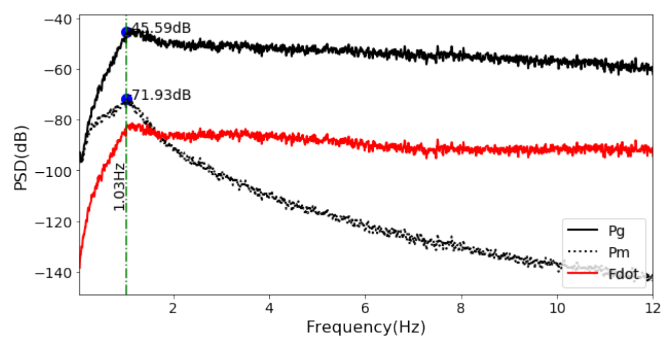

Figure 4. PSDs

Since P_g is also influenced by P_m through generator mechanical dynamics, P_g also observes the same roll-off and allows us to quantify the threshold ω_{min} from measurements to remove the turbine plus governor dynamics from the signals. The value of ω_{min} changes depending on the underlying frequency control dynamics in the turbine governor for the generators under study. The typical ranges for different plant types will be explored in future work.

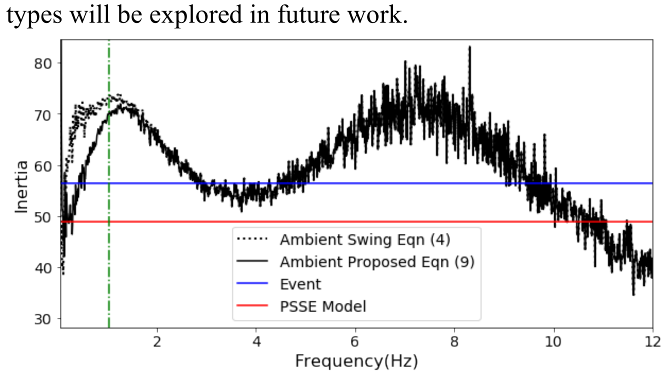

Figure 5. Inertial response estimates

Next, $M(\omega)$ values are estimated from data using the proposed model in (9) and plotted in Fig. 5. For comparison, the estimates obtained using (4) (assuming full knowledge of P_m) are plotted along with the PSS/E model value (red) and the estimate from event data (blue). The first observation is to note that $M(\omega)$ (the actual inertial response values) capture the relationship between power imbalance and frequency and significantly varies in the frequency domain. This implies that depending on the signature of the disturbance seen by the plant, different values are obtained. If a disturbance event results in the excitation of the frequency in the range of 3-5 Hz, the estimated inertia value would be fairly close to that of the step change event studied previously.

It is also evident that M_{PSSE} is lower than the inertial response of the unit for disturbances up to 10 Hz. This is also the range in which the voltage control loop (AVR and PSS) dynamics are active. That is why only taking into account generator statuses can significantly underestimate the inertial response. For faster disturbances of >8 Hz, the inertial response declines steadily, i.e. the generator and its controls stop playing a role in frequency regulation.

Next, it can be seen that beyond $F_{min} = 1.03$ Hz, (when P_m dynamics start becoming negligible in comparison to P_g, as seen previously) the $M(\omega)$ estimates from the proposed model in (9), shown in Fig. 5, closely match the ones from the original model (4). This confirms that the effect of governor dynamics (not accounted for in the inertial response) can be separated.

V. Discussion and Future Work

It is known that the effect of frequency control in generators through the turbine governor imposes a challenge when attempting to estimate the inertial response because of the variations in the unmeasured input mechanical power. In this work, we prove that the effect of these control loops can be filtered out in the frequency domain owing to longer time constants involved. This is especially helpful for continuous monitoring of inertia from ambient data, where unlike event data, there is no clear separation of the different control loop dynamics or between time windows with and without the influence of P_m. Additionally, we take a step back and revisit the concept of inertial response or inertia as a constant value. It is shown that the value estimated depends on the nature of the disturbance and consequently, it is better to treat inertia as a frequency domain response (e.g. as if it is a transfer function) as opposed to a single value.

For future work, the proposed approach will be tested on multi-machine systems with different turbine characteristics, governor models, and AGC. Furthermore, tests will be performed on real-world PMU data collected from Dominion Energy's synchrophasor network.

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Appendix

Model – GENROU, Machine Base – 771 MVA			
Parameter	**Value**	**Parameter**	**Value**
T'do (> 0)	3.9580	Xq	1.6360
T"do (> 0)	0.0290	X'd	0.2760
T'qo (> 0)	0.5400	X'q	0.4610
T"qo (> 0)	0.0530	X"d = X"q	0.2320
H, Inertia	3.1710	Xl	0.1840
D, Speed Damping	0.0000	S(1.0)	0.1060
Xd	1.7080	S(1.2)	0.4530

Model – IEEEG1			
Parameter	**Value**	**Parameter**	**Value**
K	20.0000	K2	0.0000
T1	0.1500	T5	8.2900
T2	0.2000	K3	0.2500
T3 (> 0)	0.2000	K4	0.0000
Uo	0.2700	T6	0.4000
Uc (< 0.)	-0.2500	K5	0.4500
PMAX	0.9000	K6	0.0000
PMIN	0.0000	T7	0.0000
T4	0.2500	K7	0.0000
K1	0.3000	K8	0.0000