

# Identifying Low-Order Frequency-Dependent Transmission Line Model Parameters

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**Abstract**—This paper describes the modeling and parameter identification of a frequency dependent transmission line model from time-domain data. To achieve this, a single-phase transmission line model was implemented in OpenModelica where the frequency dependent behavior of the line was realized by a series of rational functions using the Modelica language. Next, the developed line model was exported as a Functional Mock-up Unit (FMU). The RaPID toolbox was then used for automated parameter optimization of the model within the FMU that was interfaced to RaPID via the FMI Toolbox for MATLAB. Given a reasonable starting guess of the set of parameters, the toolbox improved the model’s response significantly, resulting in a good approximation even though low-order representations were used for the identification process. It was found that even though the process was straightforward, it can be enhanced by exploiting the physical/numerical properties of this specific problem.

**Index Terms**—EMTP, Parameter estimation, Power system modeling, System identification, Transmission lines.

## I. INTRODUCTION

The application of simulation and modeling has consistently increased over the last couple of decades and is nowadays an indispensable tool in power system engineering. For simulation results to be accurate, the models and their parameters must reflect the behavior of the actual components within the power grid. One way of achieving this is by model calibration using real-life measurements, thus adapting model parameters to best represent field measurements.

Parameter estimation of transmission line models has recently received more attention with the growing availability of data from Phasor Measurement Units (PMU). Previous authors have presented a variety of techniques to estimate the electrical parameters, both for real-time estimation [1], [2] and off-line processing [3]. These are typically meant to estimate the impedance and admittance of the line, thus being valid at frequencies close to the fundamental.

For detailed studies of complex devices that rely on short time-scale control and protection actions, time-domain simulations are generally required. One such application being dynamic simulations involving HVDC converters whose modeling requires detailed representation of control and protection functions activated in very short time-scales [4]. For accurate representation of transmission lines within these time-scales, frequency dependent traveling wave models must be used. To

the knowledge of the authors, previous research has been limited to comparing frequency dependent models with measured data in power cables [5], [6], their main focus being on the modeling itself and with relatively less attention to the process of fitting the frequency dependent parameters to best represent the measured data.

An issue with the increased use of modeling comes with the development of new software platforms. This often requires that models are re-implemented for different software targets where each implementation complies to the requirements of each specific platform. Thus, models are generally not exchangeable. In an effort to overcome this limitation, the Modelica language has been developed [7]. Modelica is an object-oriented language based on the idea that models should consist of their mathematical equations, thus enabling consistent representation among different platforms.

In previous work [8], [9], a system identification tool named *Rapid Parameter Identification toolbox* (RAPID) was presented. The toolbox is publicly available for download at [10]. The software is developed in MATLAB, which allows for an open and modular design such that the user can make modifications when required. Examples of such modifications could be the definition of custom objective functions or development of additional interfaces that allow the use of external optimization solvers. Out of the box, RAPID interfaces with the FMI Toolbox [11] in SIMULINK, thus allowing any model that complies with the FMI standard [12] to be used in the parameter identification workflow.

In this paper, the characterization of a single-phase transmission line model by its frequency dependent parameters is performed through system identification using time-domain reference data generated using EMTP-RV. The paper is arranged as follows. In Sections II and III, the equivalent circuit model is derived and implemented. Then, the problem-specific modifications and assumptions are discussed in Section IV, followed by the results and discussions in Section V. Finally, the conclusions are given in Section VI.

## II. TRANSMISSION LINE MODELING

### A. Multi-conductor transmission lines

Multi-conductor transmission lines consisting of  $n$  conductors are characterized by their per-unit length shunt admittance

and series impedance matrices,  $\mathbf{Y}$  and  $\mathbf{Z}$  respectively. Both matrices are of dimensions  $n \times n$  where the off-diagonal elements represent the coupling between conductors.

The phase domain representation can be greatly simplified by applying modal decomposition, such that the resulting  $n$  modes can be treated as independent single-phase transmission lines. This is achieved by calculating

$$\lambda = \mathbf{T}_i^{-1} \mathbf{Y} \mathbf{Z} \mathbf{T}_i = \mathbf{T}_v^{-1} \mathbf{Z} \mathbf{Y} \mathbf{T}_v \quad (1)$$

where  $\lambda$  is a diagonal matrix in which each element  $\lambda_i$  corresponds to the eigenvalue of mode  $i$ . The square matrices  $\mathbf{T}_v$  and  $\mathbf{T}_i$  are the transformation matrices which relate voltages and currents between the two domains, respectively.

Considering the distributed nature of parameters in transmission lines, the voltage and current at any point  $x$  along the line are related by a set of second order differential equations, known as the Telegrapher's equations. For each mode  $i$ , the single-phase solution can be written as

$$I_i^m(x) = e^{-\sqrt{\lambda_i}x} I_i^{m+} + e^{+\sqrt{\lambda_i}x} I_i^{m-} \quad (2)$$

$$V_i^m(x) = Z_{c,i}^m \left( e^{-\sqrt{\lambda_i}x} I_i^{m+} - e^{+\sqrt{\lambda_i}x} I_i^{m-} \right) \quad (3)$$

where  $I_i^{m+}$ ,  $I_i^{m-}$  are the forward and backwards traveling waves, respectively. The characteristic impedance for each mode  $i$  is the reciprocal of the characteristic admittance and is calculated as

$$Z_{c,i}^m = (Y_{c,i}^m)^{-1} = \sqrt{\frac{Z_i^m}{Y_i^m}} \quad (4)$$

where  $Z_i^m$  and  $Y_i^m$  are the  $i$ -th element of the diagonalized matrices  $\mathbf{Z}$  and  $\mathbf{Y}$ , respectively [13].

### B. Circuit equivalent

For the modeling of transmission lines, the voltages and currents at an arbitrary position  $x$  is of little significance. The relevant behavior is at the two terminals of the line, i.e. at  $x = 0$  and  $x = \ell$  where  $\ell$  is the length of the transmission line. The terminal voltages and currents for both ends of the can be obtained by (2)-(3) and results in

$$\underline{x=0}: \quad I_i^m(0) = I_i^{m+} + I_i^{m-} \quad (5)$$

$$V_i^m(0) = Z_{c,i}^m (I_i^{m+} - I_i^{m-}) \quad (6)$$

$$\underline{x=\ell}: \quad I_i^m(\ell) = H_i^m I_i^{m+} + (H_i^m)^{-1} I_i^{m-} \quad (7)$$

$$V_i^m(\ell) = Z_{c,i}^m \left( H_i^m I_i^{m+} - (H_i^m)^{-1} I_i^{m-} \right) \quad (8)$$

where the propagation function of mode  $i$  for the entire line length is denoted  $H_i^m$  and is calculated as

$$H_i^m = e^{-\sqrt{\lambda_i}\ell}. \quad (9)$$

Multiplying both sides of voltage in (6) by the characteristic admittance  $Y_{c,i}^m$  and subtracting it to the currents in (5) yields

$$I_i^m(0) - \underbrace{Y_{c,i}^m V_i^m(0)}_{\hat{I}_i^m(0)} = \underbrace{2I_i^{m-}}_{\tilde{I}_i^m(0)} \quad (10)$$

which relates three currents and can be interpreted as Kirchoff's current law. For the other end, the characteristic admittance is applied to both sides of (8) and added to the current in (7) which yields

$$I_i^m(\ell) + \underbrace{Y_{c,i}^m V_i^m(\ell)}_{\hat{I}_i^m(\ell)} = \underbrace{2H_i^m I_i^{m+}}_{\tilde{I}_i^m(\ell)}. \quad (11)$$

The results in (10) and (11) allows for the transmission to be modeled as controllable current sources in parallel at each side, shown in Fig. 1.

The controllable current sources at both sides,  $\tilde{I}_i^m(0)$  and  $\tilde{I}_i^m(\ell)$  represent the arriving wave that originates at the other side. At the sending side,  $x = 0$ , this can be calculated by multiplying the voltage in (8) with the characteristic admittance and subtracting it from the current in (7),

$$I_i^m(\ell) - \underbrace{Y_{c,i}^m V_i^m(\ell)}_{\hat{I}_i^m(\ell)} = 2(H_i^m)^{-1} I_i^{m-} \quad (12)$$

which by multiplication with  $H_i^m$  results in

$$\tilde{I}_i^m(0) = 2I_i^{m-} = H_i^m \left( I_i^m(\ell) - \hat{I}_i^m(\ell) \right). \quad (13)$$

Similarly at the receiving side,  $x = \ell$ , the positive direction traveling current wave can be determined by multiplying the characteristic admittance to (6) and adding it to the current in (5) according to

$$I_i^m(0) + \underbrace{Y_{c,i}^m V_i^m(0)}_{\hat{I}_i^m(0)} = 2I_i^{m+} \quad (14)$$

which by substitution into the right side of (11) results in

$$\tilde{I}_i^m(\ell) = 2H_i^m I_i^{m+} = H_i^m \left( I_i^m(0) + \hat{I}_i^m(0) \right). \quad (15)$$

The single-phase transmission line can therefore be modeled by connecting the characteristic admittance in parallel to a controllable current source, which represents the wave that enters the other line end and is delayed and attenuated/distorted by the propagation function.

### C. Frequency dependent parameters

In practice transmission lines are lossy (i.e. not lossless). Thus, the characteristic admittance  $Y_{c,i}^m$  and propagation function  $H_i^m$  do contain both real and imaginary parts that vary

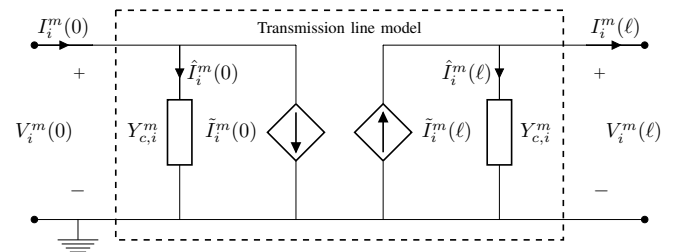


Fig. 1. Interfacing circuit at the terminals of the transmission line according to Kirchoff's current law.

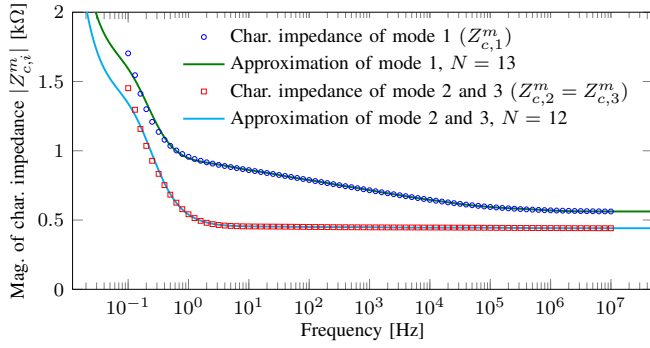


Fig. 2. Example of the approximated characteristic impedance in the modal domain where data points are approximated by continuous function. Note that not all frequency points are shown in the figure.

with frequency. Because time-domain simulations require accuracy over a wide range of frequencies, transient simulation programs often utilize frequency domain curve fitting. First, the frequency dependent parameters are calculated for a finite set of frequencies. These points are then fitted by an approximation in a continuous form that allows for a convenient translation into the time-domain.

Various forms of approximations exist, but in this paper a series of first-order rational functions will be used. This can be written as

$$f(s) = \sum_{k=1}^N \frac{c_k}{s + a_k} + d \quad (16)$$

where  $N$  is the order of the approximation,  $d$  is a real constant and the pair  $(c_k, a_k)$  are the  $k$ -th residue and pole, respectively.

An example of the fitted representation is shown in Fig. 2 where the characteristic impedance is represented by a continuous function, fitted to a set of data points.

#### D. Propagation delay

In addition to attenuation and distortion, that reshape the wave as it is traveling down the line, the propagation function also represents the delay, i.e. the time it takes for a wave to reach from one side to the other.

For accurate representation in the time-domain, the propagation function is therefore treated as two separate parts connected in series. One being the rational approximation in the form given in (16) and the other as a pure delay. Because the frequency domain representation of a time delay is a phase-shift  $e^{-s\tau}$ , the representation can be written as

$$H_i^m(s) \approx e^{-s\tau_i} \sum_{k=1}^N \frac{c_k}{s + a_k} \quad (17)$$

where  $\tau_i$  is the propagation delay of the  $i$ -th mode.

### III. IMPLEMENTATION USING MODELICA

In this paper OpenModelica [14], a software tool compliant with the Modelica language, has been chosen for the transmission line model implementation. In addition to being *free/libre* and open source, it also provides a graphical interface and comes with an integrated compiler, thus allowing for quick and

TABLE I  
FREQUENCY DEPENDENT TRANSMISSION LINE PARAMETERS

	Parameter	Description
$Y_c(s)$	$N$	Order of the characteristic admittance approx.
	$(a_k, c_k)$ $k = 1 \dots N$	Pole-residue pair for each $k$ of the rational functions used in the $N$ -th order approximation
	$d$	Constant in approx. of characteristic admittance
$H(s)$	$\tau$	Propagation delay of transmission line
	$N$	Order of propagation function approx.
	$(a_k, c_k)$ $k = 1 \dots N$	Pole-residue pair for each $k$ of the rational functions used in the $N$ -th approximation

effort-less simulation during model development. Furthermore, OpenModelica also supports direct export conforming to the Functional Mock-up Interface (FMI) [12] that allows for models to be exchanged for co-simulation with an embedded solver, or as in this paper, to be explicitly simulated in a different environment using its own solvers (i.e. *FMI for Model Exchange*).

#### A. Transmission line parameters

Because the modal domain approach is used in this paper, a single-phase model is sufficient. In this model the transmission line's behavior is defined by the propagation function ( $H_i^m$ ) and characteristic admittance ( $Y_{c,i}^m$ ). The input parameters to the model are thus the parameters used in the frequency dependent approximation of  $Y_{c,i}^m$  and  $H_i^m$ . The denotation of the parameters is given in Table I and the model is implemented according to the circuit and reference directions shown in Fig. 1.

#### B. Characteristic admittance implementation

The frequency dependent behavior of the characteristic admittance is modeled by the rational approximation according to (16). Because the approximation is a series of admittances, it can be interpreted as a series of parallel connected shunts, as illustrated in Fig. 3. Thus, the characteristic admittance is implemented as a component that internally connects  $N + 1$  branches in parallel.

For the real constant in the approximation ( $d$ ), the implementation was carried out using a resistor where the time-domain current flowing through it is given by

$$i_0(t) = d \cdot v_i^m(t). \quad (18)$$

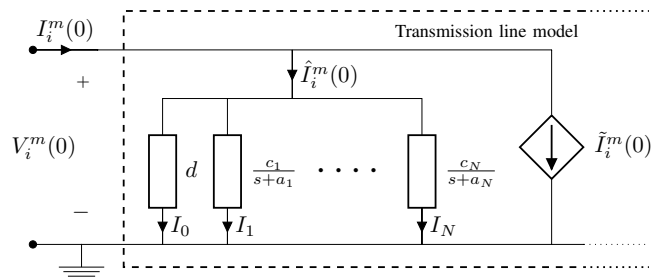


Fig. 3. Approximation of the characteristic admittance realized as a series of parallel connected branches.

For the  $k$ -th parallel branch, the frequency-domain current according to Fig. 3 is given by

$$I_k(s) = \frac{c_k}{s + a_k} V_i^m(s). \quad (19)$$

Solving (19) for the voltage and translating it into the time-domain, the resulting differential equation suitable for input into Modelica becomes

$$v_i^m(t) = \frac{1}{c_k} i_k'(t) + \frac{a_k}{c_k} i_k(t). \quad (20)$$

### C. Propagation function implementation

The propagation function relates the wave that enters at one end to the wave that arrives at the other end. In the model, this is implemented by calculating the wave by combining the current that enters at the terminal of the model ( $I_i^m$ ) and the current flowing through the shunt admittance ( $\hat{I}_i^m$ ) according to (13) and (15) for the sending and receiving end, respectively. Note that the propagation is only a control function that provides input to the current source at the opposite end and does not interface with the electrical circuit, differentiating it from the shunt admittance as explained previously.

The representation given by (17) corresponds to a series of parallel first-order low-pass filters where the results are added and finally delayed before the output is provided as a control signal for the current source at the other end. This is illustrated in Fig. 4.

For the  $k$ -th first order low-pass filter, the input and outputs are related by the differential equations

$$y_k'(t) + a_k y_k(t) = c_k u(t). \quad (21)$$

The output of all blocks are then added and the result is delayed by  $\tau$  seconds using the Modelica function `delay(y, tau)`. The output is connected to the current source corresponding to the other side of the transmission line.

### D. FMU compilation and packaging

The objective of this work is to perform time-domain parameter estimation given reference input/output signals. Therefore, the implemented transmission line model needs to be formulated in terms of input/output signals that can be used in the parameter identification process.

To achieve this, the terminal voltages at both ends were chosen as input signals and the current at both sides as the output. Therefore, ideal controllable voltage sources were connected at both ends of the transmission line where each of the external control signals were connected to instances of `RealInput`

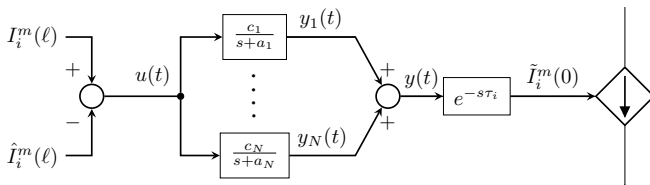


Fig. 4. Implementation of the propagation function in the single-phase transmission line model.

interfaces from the Modelica Standard Library (MSL). To provide output signals, current sensors were connected to two instances of `RealOutput` interfaces.

To export the model in a FMU, the circuit model was compiled directly from the OpenModelica Connection Editor (OMEdit) by right-clicking the model and choosing the option to export the model as an FMU.

## IV. PARAMETER IDENTIFICATION USING RAPID

### A. Reference Data Generation from EMTP-RV

To obtain the reference input/output data, against which the parameter estimation is performed, EMTP-RV is used. A small circuit was created consisting of the transmission line, a single-phase ideal voltage source was connected to the first phase together with passive components at the other terminals. The reason being to allow for non-ideal reflections at the terminals (i.e. neither open-circuited nor short-circuited).

The transmission line was simulated as a 300 km long 3-phase frequency dependent (FD) model where the conductors are placed in a "flat" geometry above ground. The conductors are assumed to be perfectly transposed. The magnitude of the step voltage, applied to phase A at  $t = 20$  ms, is 400 kV. The simulated phase voltages and currents were recorded and saved for input into MATLAB.

Once the voltages and currents had been loaded into MATLAB, the phase domain quantities required transformation into their respective modal components. This was achieved by applying the phase domain quantities to the inverse of the transformation matrices of voltage and current,  $\mathbf{T}_v^{-1}$  and  $\mathbf{T}_i^{-1}$  respectively. Because the transmission line is assumed to be perfectly transposed, the transformation matrices (eigenvectors) are real and constant with regard to frequency.

Finally, data for each mode were packaged and saved in separate files because their parameter identification are handled separately. The two row-vectors (one signal for each end of the line) of modal voltages at both ends are combined into a matrix designated *input* and the two row-vectors of modal currents are combined into a matrix named *output*. Also a row vector for the *time* is included in each of the modal output files.

### B. Propagation delay

Even though the time delay  $\tau_i$ , is a parameter of the model and therefore should be included in the identification process, it was challenging to obtain a good fit because small variations in the estimated delay had a very large impact on the model's response. Therefore, the time-delays from the original EMTP model were used directly in the FMU model, i.e.  $\tau_1 = 1.105284$  ms and  $\tau_2 = \tau_3 = 1.005756$  ms.

### C. Constraining the propagation function

The propagation function is a dimensionless quantity and represents the delay, attenuation and distortion of the wave as it travels down the line. Because it is a passive and physical quantity, the losses in the line (resistive) result in a magnitude less than unity; otherwise it would imply that the wave gains

energy during its path to the opposite side. The RAPID toolbox is obviously unaware of this application-specific problem and might therefore produce parameters which result in magnitudes larger than one. This cannot be prevented by setting the default min/max constraints for each of the parameters because the magnitude at each frequency is a combination of all poles and residues. When attempting to perform simulations, it was noticed that this problem sometimes cause the simulation to crash. Therefore, code was added directly into RAPID's source code to handle this. At each iteration, the propagation function magnitude at 0 Hz ( $h_0$ ) was calculated according to

$$h_0 = \lim_{s \rightarrow 0} |H_i^m(s)| = \sum_{k=1}^N \frac{c_k}{a_k}. \quad (22)$$

If  $h_0 > 1$ , all the residues are scaled by the calculated magnitude  $c_k^{\text{new}} = c_k^{\text{old}}/h_0$  which result in the propagation function having unity magnitude at 0 Hz.

#### D. Objective function

Even though the available pre-written objective functions in RAPID resulted in sufficiently accurate results, a new more easily interpretable objective function was implemented. Because the model contained two outputs, one for each end, the error rms current for each of the currents were calculated as

$$\epsilon_i = \sqrt{\frac{1}{N} \sum_{n=1}^N (y_i[n] - \tilde{y}_i[n])^2} \quad (23)$$

where  $N$  is total number of samples,  $y[n]$  and  $\tilde{y}[n]$  are the reference current from EMTP-RV and the result from the FMU's simulation in SIMULINK, respectively. The total objective function output was simply implemented as the sum of both rms currents,

$$\epsilon = \epsilon_1 + \epsilon_2 \quad (24)$$

where  $\epsilon_1$  and  $\epsilon_2$  corresponds to the sending end ( $x = 0$ ) and receiving end ( $x = \ell$ ) error rms current.

## V. RESULTS

It is commonly recognized that non-linear optimization problems in most cases require a good initial guess of the parameters in order to converge. Identification of the frequency dependent transmission line parameters were no exception. The RAPID toolbox is equipped with different algorithms, both algorithms that cover a large search-space and more localized search-methods (gradient based) that rely on finding the local minimum. The local minimum algorithms were found to provide good results, and thus, only their results are presented herein. Furthermore, only the result of the first mode ( $i = 1$ , the ground mode) is presented due to space limitations.

The results of the iterative parameter identification of mode 1 are summarized in Table II where each attempt implies a restart using the outputs from the previous optimization as initial parameters. In Fig. 5 the reference and current generated

by the model at each iteration is shown. Only the current at one of the sides (the sending end,  $x = 0$ ) is shown.

As summarized in the table, the initial guess of the model parameters ( $p_0$ ) were used as input to the optimization. Applying these parameters in the model yielded the current as shown in green in Fig. 5. The magnified plot shows that the initial parameters resulted in a current that was in quite close agreement with the reference data during the first couple of milliseconds. After that, the current from the initial parameters deviates significantly. This implies that the initial guess was in quite good agreement with the reference model for higher frequencies but not at lower frequencies.

For the first execution of the identification process (iteration), the algorithm used was the conjugate gradient method (CG). As shown by the parameters in Table II, the first iteration only resulted in adjustments of the parameters used in the characteristic admittance. Both residues  $c_1$ ,  $c_2$  and the constant  $d$  were adjusted. This adjustment provided a significant improvement in the fitness ( $\epsilon$ ), which was reduced from 1695 to 152. The improvement can be seen by the red curve in Fig. 5 that shows a better agreement with the reference over a longer period of time, where the initial guess diverged significantly. However, there is still some mismatch as the estimated current (red trace) is initially below the reference current in the lower figure but when the system is close to reaching its steady-state at approximately  $t = 0.1$  s, the current is instead higher than the reference current (blue trace). This is even more evident after the negative edge in voltage after  $t = 0.2$  s.

The second iteration used the Nelder-Mead (NM) algorithm,

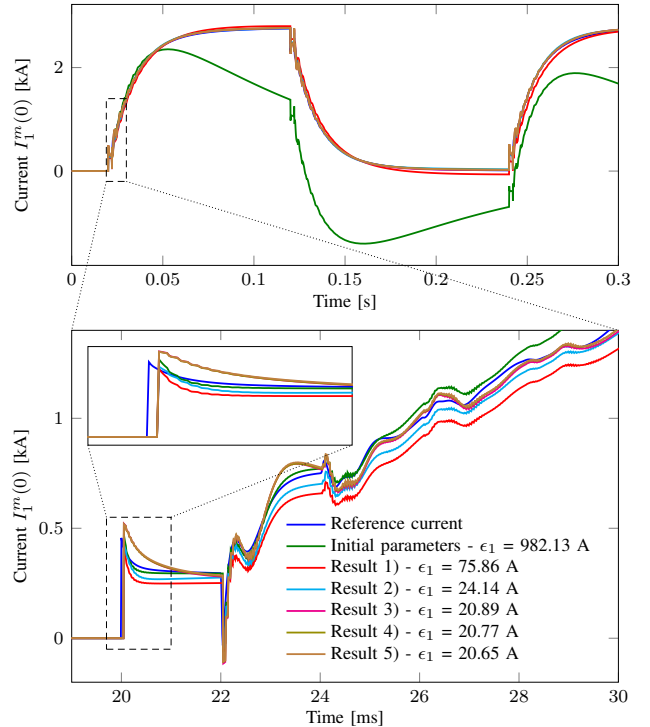


Fig. 5. Results of the parameter identification in terms of the mode 1 current at the sending end ( $x = 0$ ) of the transmission line.

TABLE II  
RESULTS FOR THE PARAMETER IDENTIFICATION OF MODE 1.

Parameters	$Y_{c,1}^m(s), N = 2$					$H_1^m(s), N = 2$				Fitness		
	$a_1$	$a_2$	$c_1$	$c_2$	$d$	$a_1$	$a_2$	$c_1$	$c_2$	$\epsilon_1$	$\epsilon_2$	$\epsilon$
Min	1.0	1000	-100	-100	0.001	1000	1000	-10000	-10000	-	-	-
Max	100	100000	1000	1000	0.003	100000	100000	100000	100000	-	-	-
$p_0$	2.3	14000	-0.002	-6.7	0.0017	4500	38000	5200	-6600	982.13	983.15	1695.27
1) CG	2.30000	14000.0	0.00960	-6.70000	0.001497	4500.03	38000.0	5200.00	-6600.00	75.86	75.95	151.81
2) NM	2.07279	7627.82	0.02688	-3.26625	0.00151	4553.79	36650.6	5233.13	-7083.49	24.14	25.55	49.69
3) NM	1.97822	1182.26	0.01715	-0.90443	0.00182	4480.87	32524.1	5244.19	-6511.54	20.89	18.45	39.34
4) NM	2.05775	1201.14	0.02148	-0.88970	0.00181	4493.65	32606.8	5241.17	-6598.36	20.77	16.62	37.39
5) NM	2.04272	1120.77	0.02050	-0.83378	0.00181	4486.23	32089.3	5244.28	-6532.42	20.65	16.65	37.30

which further improved the fitness to below 50. The same method was then repeated, each time with a small improvement until no further progress could be made.

It should be noted that even though the initial parameters had a closer agreement with the reference data during the very first couple of milliseconds (as seen in the bottom figure), the parameter identification process makes a trade-off to achieve an overall better estimation. This comes naturally from the use of a limited order approximation, the parameters are optimized such that the overall agreement is as good as possible.

It should be mentioned that the first mode transmission line model in EMTP-RV used to generate the reference data were of the 13th and 21st order for the characteristic impedance and propagation function, respectively. These are of significantly higher orders than those used in the identification process (second order). It is therefore unreasonable to assume that a perfect agreement can be achieved. However, as seen in Fig. 5, the identified parameters improve the behavior of the simulated transmission line quite well as compared to the set of initial parameters.

## VI. CONCLUSION

This paper has applied the RAPID Toolbox for parameter identification to the problem of obtaining low-order frequency dependent transmission line parameters using time-domain reference data. Even though alternative methods exist, the toolbox offers a very general approach to system identification and model validation because the toolbox requires no information about the internal implementation of the model. The user simply provides a model together with the reference data and once the parameters have been setup, the toolbox allows for identification using a variety of algorithms with little intervention from the user. Furthermore, because the toolbox is written in MATLAB, it is very easy to modify/add functionalities which might be required for specific problems. In the work presented in this paper, a constraint had to be implemented such that the wave did not gain energy when traveling down the line.

Not unexpectedly, it was found that the initial guess was very important for a solution to converge. However, when such a initial guess was used, local (gradient) search methods managed to optimize the agreement between the model and reference data. Even though a low-order approximation were used in the identification process, a set of parameters which showed a good fit with the overall response were found.

Requiring a reasonable initial set of parameters is not a large drawback because in most cases, the transmission line structure (e.g. spacing between conductors) is known. This and other available information could be combined with a set of default values for the unknowns, thus allowing for generation of model parameters to be used as a starting point for the parameter estimation process. The typical case might very well be that some parameters are known whereas others can only be roughly estimated, e.g. ground resistivity.

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