

Optimal PMU Placement for Power System Ambient Data-based Mode Estimation Applications

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Abstract—This paper proposes a method for optimal PMU placement for ambient data-based mode estimation. In contrast to traditional methods for PMU placement where the main objective is to maximize observability of the system, the proposed method uses the critical inter-area mode estimation accuracy as the criterion for siting and ranking PMU locations and signals. The criterion is defined as the critical mode’s damping ratio estimation variance. First, this criterion is computed for all signals, and next signal locations are ranked accordingly. Finally, the signal with the lowest criterion value represents the best location for a PMU. Two methods for computing the criterion are presented: 1) Monte Carlo simulation and 2) an approximate expression for the estimation covariance matrix. Furthermore, the paper discusses the possibility of combining two or more synchrophasor signals in order to improve the accuracy of the estimate. The application of the proposed methods is demonstrated using the KTH Nordic 32 test system.

Index Terms—Mode estimation, mode meter, PMU placement, prediction error.

I. INTRODUCTION

Making a decision on where to place Phasor Measurements Units (PMUs) has been of interest in recent years [1]. Generally speaking, the goal is to extract the maximal amount of information about the system’s state with the minimal number of PMUs, which may lead to lower investment cost. This trade-off between observability and investment costs can be handled in different ways depending on application-specific requirements and network characteristics. Some of the criteria considered in this trade-off are numerical and topological observability [2], measurement reliability [3], [4], redundancy [5], bad data detection [6], maintaining observability after islanding and contingencies [4], [5], [7], model order reduction applications [8], data availability from the

perspective of communication networks [9], [10], etc. However, these methods have not provided an answer about the most suitable PMU locations for ambient data-based mode estimation applications. This question is not only important for planning purposes, but also for real-time mode estimation where the use of a reduced number of signals is beneficial from the computational perspective.

This paper proposes a criterion for ranking measured synchrophasor signals (locations) according to their ability to accurately estimate damping ratio of the critical inter-area oscillation. This criterion is defined as a variance of the critical mode’s damping ratio estimate that is obtained by using a signal from a particular location (or combination of locations). Because the criterion is defined for each critical mode, this means that, in general, each mode might require measurements from different locations in order to obtain the most accurate estimate.

This paper presents two methods for calculating the proposed criterion (damping ratio variance) for optimal PMU placement. The first method computes damping ratio variance from a large number of Monte Carlo mode estimation simulations. This method is computationally expensive but provides accurate results. The second method uses an approximate expression of the covariance matrix of the parameters’ estimates (mode frequencies and damping ratios) obtained using the prediction error system identification method [11]. The desired criterion, i.e. critical mode’s damping ratio variance, corresponds to the pertinent diagonal element of the calculated covariance matrix. This makes this approach computationally efficient, but the accuracy of the variance depends on validity of the assumptions used in derivation of the approximation of the covariance matrix.

This paper applies and compares the presented methods using the KTH Nordic 32 test system. The best locations for PMUs are found by evaluating the proposed criterion for signals provided by PMUs (voltage magnitude and phase, current magnitude, active and reactive power).

The remainder of this paper is organized as follows. Section II defines a general algorithm for optimal PMU placement using the proposed criterion. Section III presents algorithms for the criterion's computation. Results are presented in Section IV, whereas conclusions are drawn in Section V.

II. OPTIMAL PMU PLACEMENT ALGORITHM

The general procedure proposed herein consists of ranking possible PMU locations according to the defined ranking criterion. With the obtained ranking, a number of the top-ranked locations (and signals) can be chosen to provide an input for an ambient-based mode estimator or as a location for PMU installation.

A. Global Algorithm

The ranking criterion is defined as a variance of the critical mode's damping ratio estimate that is obtained using a signal from a particular location. The global block diagram of the proposed procedure is shown in Fig. 1.

As shown in the figure, the core of the proposed algorithm is the computation of the criterion defined, i.e. variance of the critical mode's damping ratio estimate. The algorithms for criterion computation are shown in Section III.

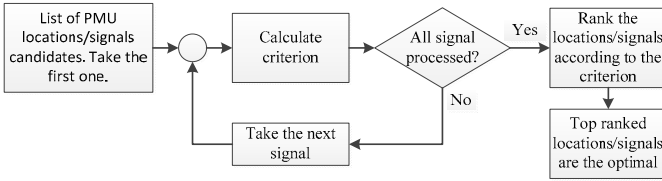


Fig. 1 Block diagram of the proposed algorithm

B. Assumptions and Limitations

- It is assumed that the model of the power system is known, and thus, it is possible to simulate the behavior of the power system which results in ambient PMU data. In the simulations carried out to compute the criterion, it is assumed that the system is excited by random changing loads modeled by white noise.
- The problem of optimal PMU placement with multiple modes is not analyzed in this paper, rather, it is assumed that only one critical mode is of interest. Because the criterion is defined for a single and specific critical mode, when considering multiple modes, a different value of the criterion will be computed for each critical mode in the system. As a result, a final decision on PMU placement has to be made by taking into account all critical modes in the system.
- The proposed algorithm does not provide the number of “required” PMUs because one signal/PMU may be

sufficient for mode estimation (when there is only one critical mode).

- The optimal location(s) is(are) determined for one operating state of the system, which means that the optimal location can be different if a large change in the system's topology occurs. This is not considered in the proposed algorithm.

C. Other Applications

The proposed algorithm can be used to evaluate possible improvements in estimation accuracy obtained by additional PMU installations, as illustrated in Section IV.C.

III. CRITERION COMPUTATION

Two methods for calculating the variance of the critical mode's damping ratio are discussed in the sequel. The first method computes the criterion from a large number of Monte Carlo mode estimation simulations, whereas the second method provides approximate results in a numerically efficient manner.

A. Monte Carlo simulation-based method (MC)

Using the simulated ambient system response, the value of the critical mode's damping ratio can be estimated for each realization of the random excitation using an arbitrary mode estimation algorithm. Here, the prediction error method is used [13].

Let the value of the estimated critical mode's damping ratio in simulation i (i -th mode estimation) be denoted by ζ_i . If N_S simulations are performed with different excitation (load changes) realizations, it is possible to estimate the variance of the critical mode estimate (the defined ranking criterion) as follows:

$$\text{var}(\zeta) = \frac{1}{N_S - 1} \sum_{i=1}^{N_S} (\zeta_i - \bar{\zeta})^2, \quad (0)$$

where $\bar{\zeta}$ represents a mean value of the critical damping ratio over N_S mode estimations. In order to obtain an accurate value of the criterion, it is important to use ζ_i that corresponds to the critical mode of interest¹. To select the appropriate mode the following rules were applied:

- 1) Find the modes with frequencies ω_k that satisfy the following:

$$\omega_{k0} - \varepsilon_\omega < \omega_k < \omega_{k0} + \varepsilon_\omega \quad (0)$$

where ω_k is the frequency of the k -th mode obtained from the mode estimator. ω_{k0} is the true frequency of the k -th critical mode obtained from eigenvalue analysis of the power system model, and ε_ω is the allowed frequency deviation (usually in the range of 5-10 mHz).

¹ A mode estimation procedure provides characteristics of several electromechanical modes (depending on the used model order). Out of these estimated modes, only one mode is selected for analysis and that mode is denoted as the critical mode of interest.

2) Among the modes that satisfy condition (0) find the ones that also satisfy:

$$\zeta_{k_0} - \varepsilon_\zeta < \zeta_k < \zeta_{k_0} + \varepsilon_\zeta \quad (0)$$

where ζ_k is the damping ratio of the k -th mode obtained from the mode estimator. ζ_{k_0} is the k -th true critical mode's damping ratio obtained from the model (eigenvalue analysis) and ε_ζ is the allowed damping ratio deviation (usually in the range of 3-5%).

3) Among the modes that satisfy conditions (0) and (0) find the mode k for which the value defined as $|\omega_k - \omega_{k_0}|$ has a minimal value.

The result of this procedure is that the estimated mode has a high probability of corresponding to the critical system mode. A good property of this approach is that the variance of the critical mode's damping ratio can be estimated using an estimation algorithm that will be used for real time mode estimation, which contributes to better confidence of PMU placement results from the mode estimation algorithm chosen.

B. Approximate method (AP)

Prediction error methods for system identification have a firm mathematical foundation and provide a deep insight into the uncertainty of the estimated model [13]. This means that prediction error methods, in addition to mode estimates, are able to provide estimates of parameter variances. Therefore, it is possible to run mode estimation procedure using the ambient response of a simulation model of the power system, and to obtain the value of the critical damping ratio variance. In this way, the ranking criterion is obtained using only one mode estimation, compared to the Monte Carlo approach, where a large number of mode estimation simulations has to be performed. In the sequel, the method for computing the variance of the critical damping ratio estimate using a prediction error method is described.

Prediction error methods provide a model estimate by minimizing a prediction error criterion over a set of model parameters θ . Formally, this can be written as:

$$\min_{\theta} J = \frac{1}{N} \sum_{t=1}^N \varepsilon^2(t, \theta) = \frac{1}{N} \sum_{t=1}^N (G^{-1}(\theta)y(t))^2, \quad (0)$$

where $y_i(t)$, for $t=1 \dots N$, is the simulated system response, and $G_i(\theta)$ denotes an AutoRegressive Moving Average (ARMA) model structure of the power system with parameters θ .

Under the assumption that the model structure $G(\theta)$ is sufficiently rich to describe the true system (denoted by G_0), the estimate of θ (denoted by $\hat{\theta}$) is a consistent estimate of that true parameter vector θ_0 (note that $G(\theta_0)=G_0$). Moreover, it can be proven that, asymptotically, the estimate $\hat{\theta}$ is an unbiased estimate of θ_0 and that the estimate $\hat{\theta}$ is normally distributed around θ_0 , i.e. $\sqrt{N}(\hat{\theta} - \theta_0) \sim N(0, P_\theta)$ with the covariance matrix P_θ given by [13]:

$$P_\theta = \sigma_e^2 E \left[\psi(t, \hat{\theta}) \psi(t, \hat{\theta})^T \right]^{-1}, \quad (0)$$

where $\psi(t, \hat{\theta}) = - \left. \frac{d\varepsilon(t, \theta)}{d\theta} \right|_{\theta=\hat{\theta}}$, and σ_e^2 is a driving noise (ambient excitation). P_θ can be estimated during the identification process as follows:

$$P_\theta = \frac{1}{N} \sum_{t=1}^N \varepsilon^2(t, \hat{\theta}) \left(\frac{1}{N} \sum_{t=1}^N \psi(t, \hat{\theta}) \psi(t, \hat{\theta})^T \right)^{-1}. \quad (0)$$

It should be noted that (0) is derived using only the first-order derivatives of the function $\varepsilon(t, \theta)$ with respect to the model parameters θ , which means that the expression is valid only for "small" deviations from the true system parameters θ_0 .

Diagonal elements of P_θ represent the estimation variances of the model parameters. However, when using an ARMA model [14], the covariance matrix P_θ does not explicitly contain information about the variance of the critical mode's damping ratio estimate. This variance can be computed by projecting the uncertainty of the ARMA model parameters to the uncertainty of another set of parameters (which in our case includes the critical mode's damping ratio). This is done by mapping the ARMA model parameters θ and the critical mode's damping ratio ζ_i , i.e. $\zeta_i = \zeta(\theta)$:

$$\begin{aligned} \text{var}(\zeta_i) &= E \left[(\zeta_i - \zeta_{i0})(\zeta_i - \zeta_{i0})^T \right] \approx \\ &\approx E \left[\left(\left. \frac{d\zeta(\theta)}{d\theta} \right|_{\theta=\theta_0} (\theta - \theta_0) \right) \left(\left. \frac{d\zeta(\theta)}{d\theta} \right|_{\theta=\theta_0} (\theta - \theta_0) \right)^T \right] = \\ &= \left. \frac{d\zeta(\theta)}{d\theta} \right|_{\theta=\theta_0} E \left[(\theta - \theta_0)(\theta - \theta_0)^T \right] \left. \frac{d\zeta^T(\theta)}{d\theta} \right|_{\theta=\theta_0} = \\ &= \left. \frac{d\zeta(\theta)}{d\theta} \right|_{\theta=\theta_0} P_\theta \left. \frac{d\zeta^T(\theta)}{d\theta} \right|_{\theta=\theta_0} \end{aligned} \quad (0)$$

In (0), the fact that $\zeta_{i0} = f(\theta_0)$ and that the function $\zeta(\theta)$ can be approximated by its first order approximation in proximity of θ_0 are used in the derivation. It should be noted that P_θ is computed implicitly during the prediction error mode estimation process because the derivatives of $\varepsilon(t, \hat{\theta})$ are obtained during the minimization process.

The previous paragraphs show that by using (0) and (0) the ranking criterion can be computed. This method is numerically more efficient because it provides results by running the mode estimator simulation only once.

C. Computational Performance

The main computational complexity of the MC algorithm presented in Section III.A lies in carrying out a total of N_S simulations and computing the mode estimates for each simulation (N_S times). Therefore, the overall computational performance of the MC algorithm corresponds to the time required to obtain a single mode estimate multiplied by the number of mode estimates used in the Monte Carlo

simulations.

In contrast to the MC method, the AP method requires the mode estimation process to be performed only once. In addition to that, the critical damping ratio variance (corresponding to (0)) has to be computed, but the time required for this computation is relatively small in comparison with the time required for mode estimation. Consequently, assuming equal time for all simulations (e.g. $t_S = 1$ sec.), it is obvious that the time for the MC algorithm computation is roughly N_S times larger than the time required for AP computation, where N_S is number of Monte Carlo estimates used in the MC algorithm.

D. Other Methods for Criterion Computation

The value of the defined criterion can also be computed using a bootstrap method [12], however this method is not analyzed here.

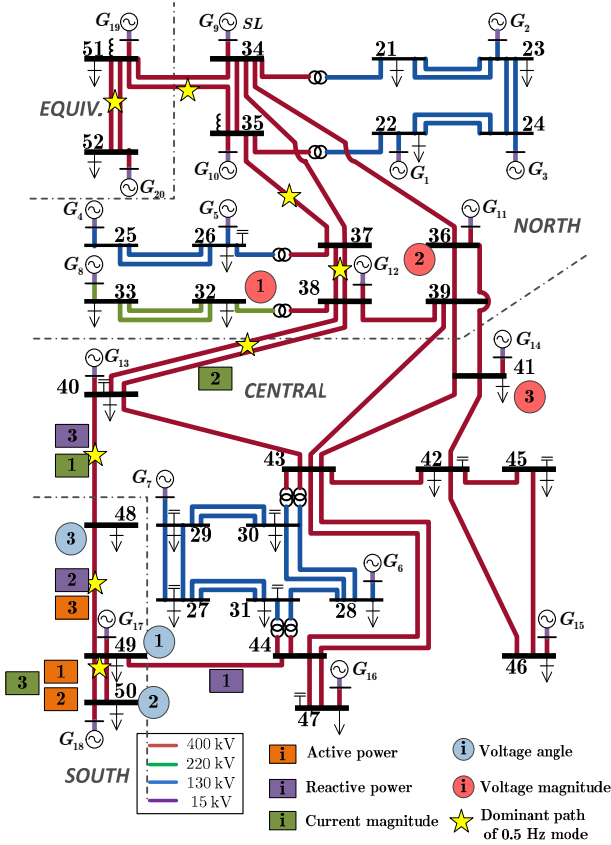


Fig. 2. One-line diagram of the KTH Nordic 32 Test System with locations of the candidate signals for ambient mode estimation².

IV. APPLICATION

In order to illustrate the presented methodology, the KTH Nordic 32 test system is used [15]. This system has one critical inter-area mode at approximately 0.5 Hz (or 3 rad/s), which is closely studied in [16]. The one-line diagram of the system is shown in Fig.2. In the studies below, optimal locations/signals are determined for different signal types

² The numbers in the boxes in the figure are used to designate the ranking order obtained using Monte Carlo simulations

(voltage magnitude and phase, current magnitude, active and reactive power). In addition, a combination of different synchrophasor signals and their contribution to more accurate mode estimation is studied.

A. Optimal PMU locations

The damping ratio variance of the critical mode is estimated using Monte Carlo (MC) method and these variances are used to determine optimal PMU locations for estimating the critical mode in the system. The analyses are performed for different signal types (voltage magnitudes and phases, active and reactive line powers and current magnitudes)³. Monte Carlo simulations are performed through 1000 independent mode estimations. In order to simplify presentation of the results, the obtained values of the criterion are shown only for the three top-ranked signals and these optimal locations are shown in Fig.2. More detailed numerical results are given in the following subsections.

It can be seen that the optimal locations are very different depending on the type of the signals that are considered, which is in accordance to the results presented in [16]. Optimal voltage magnitude signals (buses 32, 36 and 41) are located at the middle of the oscillation path that goes from the north (generators 19 and 20) to the south (generators 17 and 18). This can be explained by the fact that internal generator voltages are kept relatively constant during steady-state periods, which means that voltage magnitude signals at generator buses are not prone to oscillations even in case when the corresponding generators participate in oscillations. However, due to angle differences between the ends of the oscillation path and oscillatory power flow, voltage magnitude signals deeper in the network can be disturbed more severely (buses 32, 36 and 41). This implies that locations deeper in the network might have improved observability of the oscillation mode under analysis.

Generator 20 is an equivalent generator that has large inertia in comparison with other generators, and therefore the rotor's oscillations on this generator are smaller in amplitude. On the other end of the oscillation path, generators 17 and 18 oscillate more strongly, which is the reason why voltage angle signals in the proximity of these generators are the best voltage angles signals to estimate this particular mode. Voltage angle deviations are also reflected in the power flows and currents, and consequently other signal types (active power, current) that are strongly correlated to angle deviations, also show the best results for locations close to generators 17 and 18.

³ The generator buses are excluded from the analysis, as typically PMUs installations and signals from these locations are not accessible to transmission system operators.

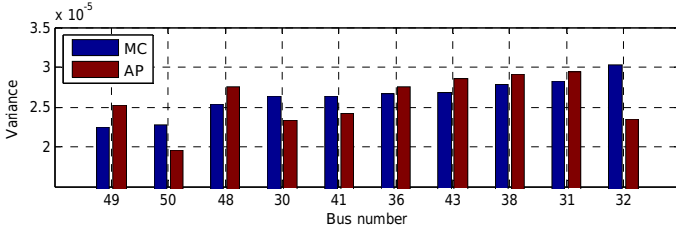


Fig. 3. Estimated variance of the critical mode's damping ratio using voltage phase synchrophasor signals.

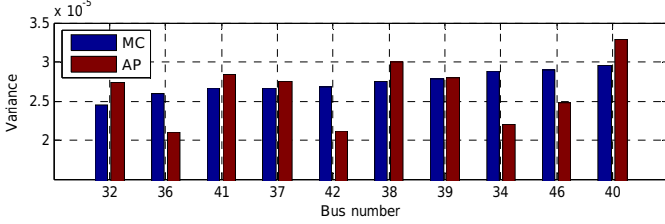


Fig. 4. Estimated variance of the critical mode's damping ratio using voltage magnitude synchrophasor signals.

Generally speaking, voltage magnitude, current and power signals should be chosen somewhere close to the center of the dominant path of the critical mode, whereas voltage phase signals should be chosen at the ends of the dominant path. However, optimal locations also depend on the particular network topology and system parameters. In the analyzed case, most of the oscillatory activity is present around generators 17 and 18, and the optimal signals are located in their proximity (except for voltage magnitude signals).

Further, it can be noticed that voltage phase signals are, in general, able to provide the best mode estimation results (overall lower variance w.r.t. other signal types), whereas voltage magnitude signals are the least sensitive to the signal location (there are several locations that provide similar mode estimation accuracy). In contrast to that, line signals (current magnitudes, active and reactive powers) have to be chosen more carefully because sub-optimal locations can lead to a reduced mode estimation accuracy.

B. Comparison of the Approximate and Monte Carlo Method for PMU placement

The Monte Carlo (MC) method for PMU placement provides good results but it requires large number of mode estimates, which makes it inapplicable if real-life measured data need to be used (due to change in the system's operating conditions during the analysis time-window). In contrast, the Approximate (AP) method requires only one mode estimate to be computed. Hence, the efficiency of the AP method is validated through comparison with the MC method from the previous subsections, and the results are presented in Fig. 3-Fig. 7. It can be seen that the AP method results match relatively well with the results obtained by using MC simulations. The maximal difference between results obtained with these two methods is less than 30%⁴.

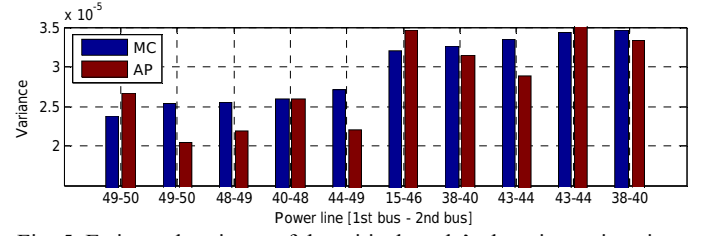


Fig. 5. Estimated variance of the critical mode's damping ratio using active power synchrophasor signals.

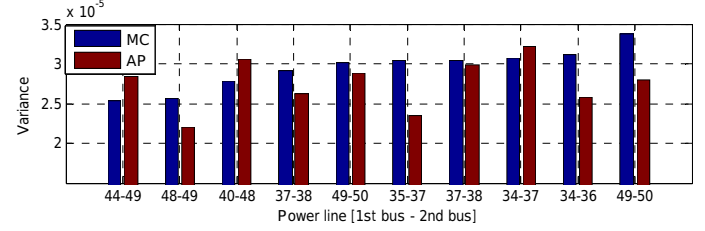


Fig. 6. Estimated variance of the critical mode's damping ratio using reactive power synchrophasor signals.

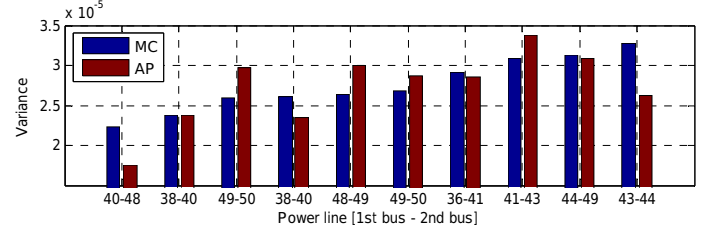


Fig. 7. Estimated variance of the critical mode's damping ratio using current magnitude power synchrophasor signals.

C. Mode estimation using signal combination

It is known that voltage phase signals at the different ends of the dominant path oscillate in opposite phases [14],[16]. This property can be used to increase the accuracy of the mode estimator if the difference of measured voltage phase signals is used as an input to the estimator. In the following studies, 12 additional signals are generated by computing the difference between measured voltage phase signals at buses (34, 35, 36, 37) and buses (48, 49, 50). Using these newly generated signals, the same procedures for the critical damping ratio estimate variance computation are applied. Results are shown in Fig. 8.

It can be noticed that the combination of the voltage phase signals is beneficial in terms of mode estimation accuracy (compare Fig. 4 with Fig. 8). Furthermore, Fig. 8 shows that all signals obtained in this way have a very good accuracy, what makes the selection of optimal PMU locations even easier.

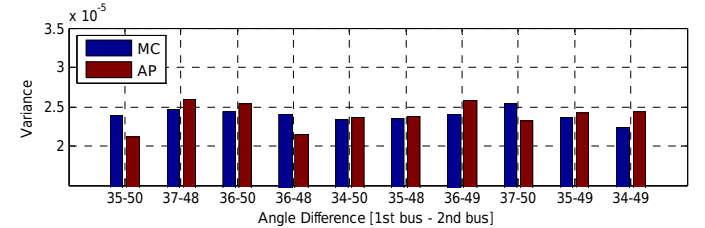


Fig. 8. Estimated variance of the critical mode's damping ratio using voltage phase difference signals obtained.

⁴ Note that, this difference can lead to a different signal ranking, however, the impact on the final estimation accuracy will be insignificant

V. CONCLUSION

This paper proposes an algorithm for optimal PMU placement for ambient data-based mode estimation applications. The optimal locations are determined by ranking the values of the proposed criterion that is associated to each synchrophasor signal. The criterion is defined as the variance of the critical damping ratio estimate. Two methods to compute the defined criterion are presented. The first method uses Monte Carlo type mode estimations, whereas the second method is derived from the theory of prediction error methods for system identification. It is shown that the approximate method requires significantly lower time for computation (complexity is reduced by factor of N_s , where N_s is number of Monte Carlo simulations).

The paper demonstrates application of the proposed PMU placement method using the KTH Nordic 32 test system. The paper discusses suitability of different synchrophasor signals for mode estimation as well as their locations. It is shown that voltage magnitude, active and reactive powers, and current magnitude signals located in the proximity of the dominant path center are able to provide accurate mode estimation results. On the other hand, voltage phase signals, if selected from the end of the dominant mode path provide very accurate mode estimation results. Furthermore, it is shown that combination of different synchrophasor signals can be beneficial with regards to mode estimation accuracy.

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