Model Order Selection for Probing-based Power System Mode Estimation

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***Abstract*—The paper discusses model order selection for probing mode estimation algorithms. Four methods are analyzed and compared: 1) Residual analysis based model order selection, 2) Model order selection based on singular values, 3) Akaike Information Criterion, and 4) Variance-Accounted-For (VAF) as a measure of optimal fitting between measured data and model response. The methods are assessed using synthetic PMU measurements from the simulation of the KTH Nordic 32 Test system and the IEEE test system with 50 generators and 145 buses.**

***Index Terms*—Model order selection, Akaike Information Criterion (AIC), Variance-Accounted-For, Probing mode estimation.**

I. Introduction

Accurate mode estimation in power systems is one of the most important requirements for a modern wide-area monitoring system (WAMS). Information about critical system modes are extracted from measured synchrophasor data using different control theory and signal processing techniques [1].

Methods for mode estimation can be generally classified into three groups:

1) *Ringdown mode estimation algorithms* [1],[2].
2) *Ambinet-based mode estimation algorithms* [1],[3].
3) *Probing mode estimation* [4].

Ringdown mode estimation algorithms that exploit large system disturbances have been proved to provide accurate mode estimates, however, these methods require the presence of large disturbances in the system. This makes them very useful for postmortem analyses but impractical for continuous system monitoring. Ambient-based mode estimation algorithms make use of the low power ambient excitation which is a result of random load changes in the system. These methods are non-intrusive but are only able to provide modest accuracy due to the low power stochastic excitation.

A compromise between ringdown and ambient-based mode estimation has been found in algorithms that exploit a known low power excitation signal injected at appropriate locations in the system [4]. This type of mode estimation is non-intrusive but is able to ensure better accuracy of the estimates compared to the ambient based methods.

Different mode estimation methods have different set of estimation parameters that have to be chosen carefully in order to obtain accurate estimation results. However, the model order used for fitting the measured system responses is a common parameter for all parametric methods. The problem of optimal model order selection has been investigated extensively in the control and signal processing community [5],[6],[7]. However, the comparison of these methods has not been analyzed explicitly from the perspective of mode estimation in power systems.

This paper gives an overview and a discussion of four methods that can be used as an aid in determining optimal model order. These methods are: 1) Residual analysis for model order selection [7], 2) Model order selection-based on singular values [8], 3) Akaike Information Criterion [9], 4) Variance-Accounted-For (VAF) as a measure of optimal fitting between measured data and model response [8].

The main difference between probing based methods and ambient/ringdown based methods is existence of the known input to the system. This difference affects selection of the model order. The main focus of this paper is model order selection for probing based algorithms; however, similar procedures can be used when either ambient or ringdown-based mode estimation methods are applied.

The model order selection algorithms are assessed using simulation outputs from the KTH Nordic 32 test system and the IEEE test system with 50 generators and 145 buses. The optimal model order is determined for selected input and output signals. The comparisons of the obtained results are used to draw general conclusions about recommended model orders values for mode estimation applications.

The reminder of this paper is organized as follows: Section II provides a description of the different model order selection algorithms. The application of the described

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methods is demonstrated in Section III whereas the results are discussed in Section IV. The paper conclusions are drawn in Section V.

II. Model Order Selection Algorithms

A. Residual analysis based model order selection

As described in [3], an ambient system response measured by a Phasor Measurement Unit (PMU) can be described by a single transfer function $H(z)$ excited by white noise $e(t)$, where the white noise represents random load changes at an aggregated level. This ambient response is also present during probing tests when the system is intentionally excited. Under assumption of the system linearity and using the principle of superposition, the measured synchrophasor signal $y(t)$ can be decomposed into two components: One as a result of ambient excitation ($H(z)e(t)$) and another as a result of probing ($G(z)u(t)$) as shown in Fig. 1. Signal $u(t)$ is the known probing signal and $G(z)$ represents a transfer function between the probing signal $u(t)$ and the measured output signal $y(t)$. It is assumed that $G(z)$ and $H(z)$ have the same denominators which means that an ARMAX model accurately describes the system [10]. This implies that the modes of $G(z)$ or $H(z)$ represent the poles/modes of the system which need to be determined.

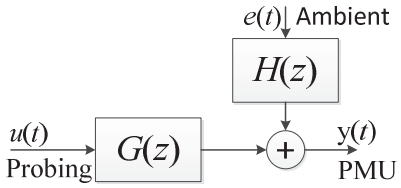

Fig. 1 Power system model during probing tests

By minimizing the prediction error criterion over a set of model parameters, the unknown transfer functions are determined. The prediction error criterion is defined as:

$$I(\theta) = \frac{1}{N}\sum_{t=1}^{N}\varepsilon^2(t,\theta), \quad (1)$$

where $\varepsilon(t) = H^{-1}(z)\left[y(t) - G(z)u(t)\right]$.

In the case of perfect model estimation, it can be shown that $\varepsilon(t)$ is white noise and that any deviation from true model parameters causes increased value of the prediction error criterion. In addition, it is obvious that $\varepsilon(t)$ should not be correlated with the past inputs $u(t)$. These two facts constitute two criteria for model validation and consequently model order selection [7]. These criteria are evaluated by plotting autocorrelation sequence of $\varepsilon(t)$ and cross-correlation between $u(t)$ and $\varepsilon(t)$. If the model order is sufficiently rich to describe the system, the obtained autocorrelation should have the autocorrelation of white noise (peak at 0 and close to zero for other values). In turn, the cross-correlation should be close to zero for all values. Then the optimal model order is the smallest one for which the defined requirements are satisfied. The test on the cross-correlation gives better insight if $G(z)$ is of the right order, while the autocorrelation test gives overall information about $G(z)$ and $H(z)$.

B. Model order selection using singular values

Subspace identification methods are known for being able to provide numerically efficient and robust results even for complex systems [8]. Another benefit of subspace identification methods is that they provide straightforward ways for model order selection. Subspace algorithms aim to extract the column space of the system observability matrix $\mathrm{O}_{obs} = [\mathrm{C}\ \mathrm{CA}\ \mathrm{CA}^2 \ldots \mathrm{CA}^n]^T$ from measured data. Without going into details about different subspace methods, it can be said that the observability matrix column space is determined by performing singular value decomposition (SVD) on an appropriate matrix obtained from measurement data matrices (matrices whose elements are known input data or measured outputs). Determining the dimension of the column spaces is equivalent to selecting the model order. It is done based on the singular values obtained during computation of the observability matrix column space. If the measured signal contains a relatively small amount of process or measurement noise, a number of singular values whose magnitude is relatively large will be exactly equal to the model order. In this paper the MOESP subspace identificaiton method is used to demonstrate applicabilty of singular values for optimal model order selection.

C. Akaike Information Criterion for model order selection

Any information criteria can be viewed as a cost function of two parts: loss function minimization and model complexity penalty. In this context, the Akaike Information Criterion is not an exception. One of its versions can be written in the following way:

$$AIC(\rho) = N\ln(I(\theta)) + \rho n, \quad (2)$$

where: N - number of data samples, n - number of system parameters (characterizing the order of the system) and ρ - regularization coefficient, which is usually equal to 2.

The idea behind such formulation is that AIC puts a penalty on the use of models with orders higher than necessary. The advantage of the AIC is that the minimization can be performed with respect to different model structures. More details about AIC and general identification theory can be found in [9].

D. Variance Accounted For criterion for model order selection

A comprehensive way of evaluating goodness of a fit between measured data and the data obtained by the estimated model is the so-called Variance-Accounted-For (VAF) performance index. This index is defined as:

$$VAF = \max\left\{0, \left[1 - \frac{\frac{1}{N}\sum_{k=1}^{N}(y(k) - \hat{y}(k \mid k-1))^2}{\frac{1}{N}\sum_{k=1}^{N}(y(k))^2}\right]\cdot 100\%\right\}, \quad (3)$$

where $\hat{y}(k \mid k-1)$ is the one step ahead predictor defined as:

$$\hat{y}(k \mid k-1) = H^{-1}(z)G(z)u(t) + \left[1 - H^{-1}(z)\right]y(k)$$

As described in [7], the prediction error method is derived with the assumption that $H^{-1}(z)$ is a proper and monic transfer function with a stable inverse. This means that $\left[1 - H^{-1}(z)\right]y(k)$ depends only on past values of $y(k)$ and not on $y(k)$ itself.

Instead of using one-step ahead predictor $\hat{y}(k \mid k-1)$ in (3), it is possible to use signal $\hat{y}(k)$ simulated with the model (previous measurements are not used to improve the prediction as it was done for $\hat{y}(k \mid k-1)$).

Here, it is also important to make distinction between data used for model identification and data used for validation. The criterion (3) can be obtained in both cases, but if the identification set is used, the criterion can show misleading high values indicating a good fit which are caused by overfitting. Therefore the recommended practice is to use half of data for identification and half for validation. Once the model order is decided, a one more identification can be done using the whole data set.

III. Case Studies

A. *Study using theKTH Nordic 32 Test System*

This section shows results obtained using the KTH Nordic 32 test system. All simulations have been performed using 3000 data samples which correspond to 10 minutes of measurements at 5 Hz sampling frequency (downsampled and pre-processed original PMU stream). In this example, it is assumed that reactive power injection in bus 41 is the input signal, whereas the measured voltage magnitude at bus 48 is considered as the output.

Fig.2-Fig.5 show autocorrelation sequences of the identification residuals (upper part of the figures) and cross-correlations between residuals and input signals (lower parts of the figures).

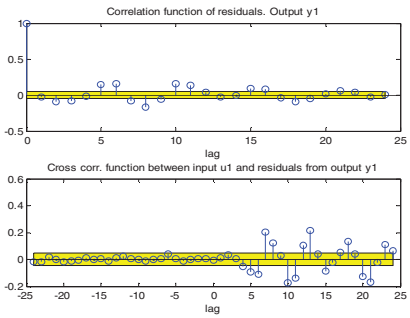

Fig. 2 Autocorrelation of $\varepsilon(t)$ and cross-correlation between $\varepsilon(t)$ and $u(t)$ for model order 6.

A good identification results correspond to very small values in auto and cross-correlation sequences except zero-lag element in the autocorrelation sequence (residual is supposed to be a white noise). An adopted threshold for which the value can be considered small is equal to 0.05. Fig. 3 shows that this criterion is satisfied for model order 12.

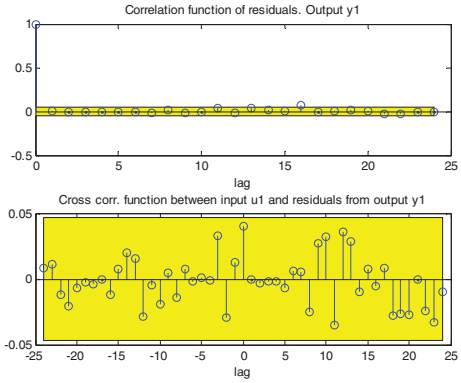

Fig. 3 Autocorrelation of $\varepsilon(t)$ and cross-correlation between $\varepsilon(t)$ and $u(t)$ for model order 12

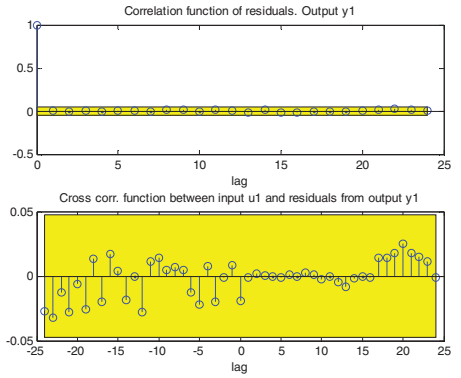

Fig. 4 Autocorrelation of $\varepsilon(t)$ and cross-correlation between $\varepsilon(t)$ and $u(t)$ for model order 18.

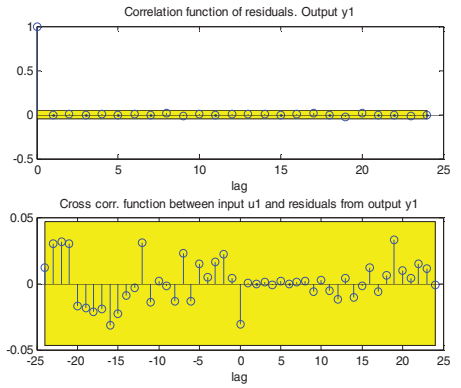

Fig. 5 Autocorrelation of $\varepsilon(t)$ and cross-correlation between $\varepsilon(t)$ and $u(t)$ for model order 22.

Singular values, obtained using the MOESP subspace identification algorithm, are presented in Fig.6. This figure shows how an increase in model order contributes to a better data fit. In other words, smaller singular value (with corresponding model order) implies better fit between the model and underlying process).

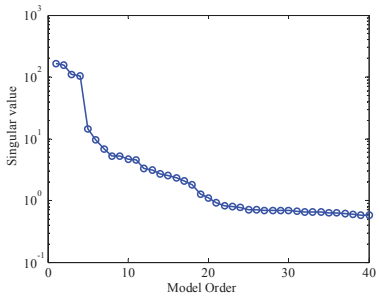

Fig. 6 Singular values obtained from MOESP subspace identification algorithm (KTH Nordic 32 test system).

A similar approach can be used in analyzing the Akaike Information Criterion (AIC).

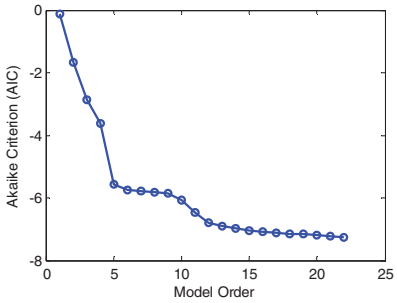

Fig. 7 Akaike Information Criterion (AIC) for different model orders (KTH Nordic 32 test system).

The next model order selection method that is applied is the Variance-Accounted-For (VAF). Four types of this indicator have been calculated and shown in Table I. First, the VAF is computed using the data that are used for the identification process and the one-step ahead predictor (data obtained from the model). This value provides information on how good the fit is between the model and the dataset. Second, instead of using the data used during identification, an independent dataset is used (validation set). This value of VAF gives more information about the fit between the model and the actual underlying process. The third and fourth indicators are computed using fully simulated data in contrast to the one-step ahead predictor. These indicators say more about how good the model fit is over a longer period of time.

TABLE I MODEL FITTING CRITERIA FOR DIFFERENT MODEL ORDERS

	One step ahead VAF		Simulated VAF	
Model order	Identif. data	Validation data	Identif. data	Validation data
6	96.94	96.84	82.07	81.21
12	98.02	97.99	94.97	94.63
18	98.04	97.84	95.44	94.36
22	98.06	98.03	95.01	94.71

The obtained results suggest that model orders higher than 18 do not contribute to better mode estimation. Also, it has to be noted that smaller model orders are more appealing because they will result in faster estimator response, as well as increased computational efficiency. Therefore, it can be concluded that a model order of 12 can be chosen without substantial change in the estimation accuracy.

B. Study using the IEEE Test System with 145 buses and 50 generators

In this subsection a test system with 145 buses and 50 generators is used to evaluate performances of the model order selection methods [11],[12]. A reactive power injection at bus 111 is selected as input and the voltage magnitude of bus 59 is selected as output. Similar studies have been conducted as in the case of KTH Nordic 32 test system (Section III.B), and results are reported in Fig.8-Fig.12. and Table II.

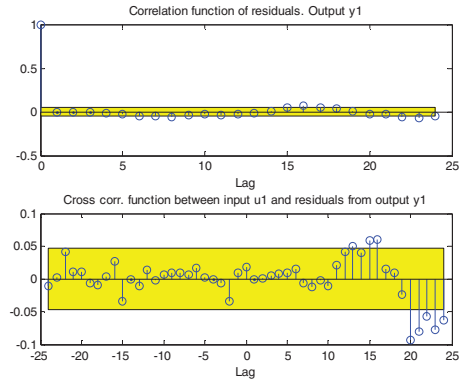

Fig. 8 Autocorrelation of $\varepsilon(t)$ and cross-correlation between $\varepsilon(t)$ and $u(t)$ for model order 18.

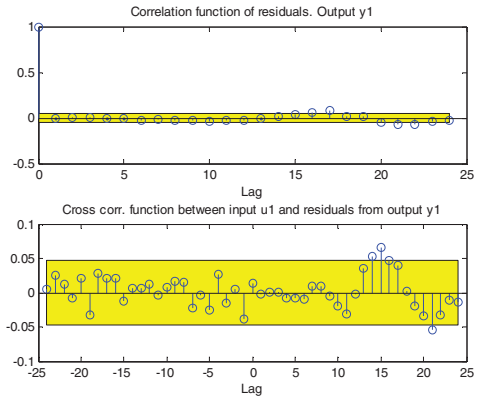

Fig. 9 Autocorrelation of $\varepsilon(t)$ and cross-correlation between $\varepsilon(t)$ and $u(t)$ for model order 22

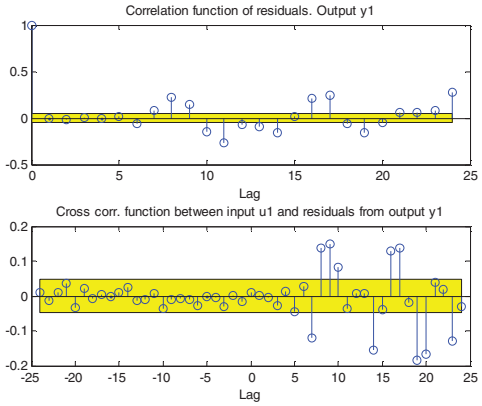

Fig. 10 Autocorrelation of $\varepsilon(t)$ and cross-correlation between $\varepsilon(t)$ and $u(t)$ for model order 6.

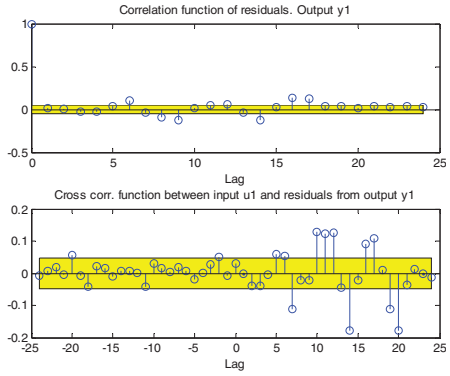

Fig. 11 Autocorrelation of $\varepsilon(t)$ and cross-correlation between $\varepsilon(t)$ and $u(t)$ for model order 12.

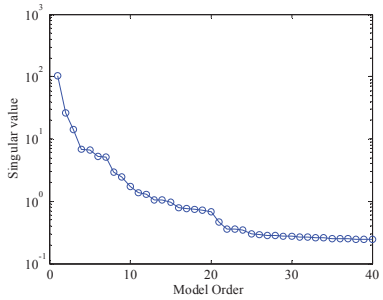

Fig. 12 Singular values obtained from MOESP subspace identification algorithm (IEEE 145 bus test system).

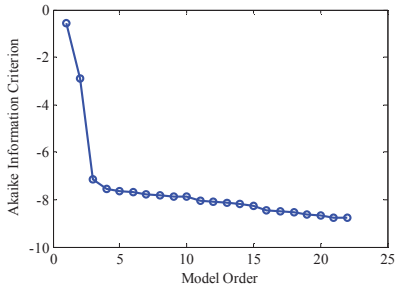

Fig. 13 Akaike Information Criterion (AIC) for different model orders (IEEE 145 bus test system).

TABLE II MODEL FITTING CRITERIA FOR DIFFERENT MODEL ORDERS

Model order	One step ahead VAF		Simulated VAF	
	Identif. data	Validation data	Identif. data	Validation data
6	97.89	97.75	87.23	86.52
12	98.44	98.35	87.91	84.55
18	98.89	98.82	93.92	93.18
22	98.86	98.85	94.41	94.37

The results presented indicate that a model order of 20 describes sufficiently well the system dynamics, whereas lower model orders (around 16) can be selected if faster mode estimation results are required. This is a bit higher model order in comparison to the KTH Nordic 32 test system. This result is understandable taking into account complexity of the system.

IV. CONCLUSION

The paper presents an application of four methods for optimal model order selection for probing based mode estimation. A careful consideration of the results obtained by the described methods gives good insight into the nature of the process that generates low frequency oscillations in the system. However, none of the described methods provides a unique solution to select the best model order, instead, each method contributes to better understanding of the underlying process. Therefore, a detailed analysis of all the obtained results should lead to a set of rules that can be used to derive fully automatized model order selection algorithm, which is a topic of future research.

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