Static Stability Indexes for Classification of Power System Time-domain Simulations

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***Abstract*—Two new static indexes that use time series from power system dynamic simulations are presented on this paper. The proposed indexes are used to check whether the time series from dynamic simulations will converge to a safe and stable equilibrium point and to measure the quality of the stability. They can also be used to classify simulations according to the severity of the contingency applied. The Integral Square Generator Angle (ISGA) index is used as a dynamic index to assess the validity of the proposed indexes and their use for classification of time-domain simulations. An illustrative example using the Klein-Rogers-Kundur's two-area system is presented and then the static indexes are validated trough several nonlinear simulations using the KTH-Nordic32 system.**

***Index Terms*—overload, under/over voltage, transient stability, severity index and impact assessment.**

I. Introduction

TODAY, methods to determine the likelihood of catastrophic system failures are necessary, as indicated by the negative impact of large-scale power outages in recent years. The FP7 iTesla project aims to build a software toolbox to cope with evolving power system operation challenges from 2-days ahead up to near real-time. Dynamic impact assessment of detailed time-domain simulations is part of the off-line analysis workflow within the iTesla toolbox[1]. The aim is to develop offline criteria to support the online analysis functions.

After performing a dynamic simulation for a specific contingency, an appropriate post-contingency severity index is determined in order to classify the impact of the contingency. To do so, a set of scalars (namely severity indexes), provide a measure of how severe the contingency is. The proposed indexes have been developed to satisfy requirements such as fast computation because several contingencies have to be evaluated for each operating condition, and at the same time they must provide a good measure of how severe the contingency is. These indexes can help to classify different time-domain simulations, this classification determines if a contingency will result in a *safe* operating condition or if there are *mild* or *severe* violations to specified operation criteria. The requirements mentioned before and the need for simple methodology, differentiates the indexes proposed here to those described in [1], [2] and [3]. To support and validate the results of the proposed indexes, the Integral Square Generator Angle (ISGA) index is also calculated to monitor transient stability problems, more details about this index can be found in [4].

II. Static Stability Indexes

An index is a scalar, vector, a matrix of numbers or ratio indicating specific characteristics or properties, in this case, of the steady state stability of a power system. To assess a given operating condition under different contingencies, simulation outputs can be analyzed to determine if a particular contingency will result in an acceptable operating condition. The indexes are used to classify the condition of the system as safe or unsafe through specific operational criteria. In this section, two static indexes are proposed, namely the overload and the under/over voltage index.

A. Overload Index

The time series of active and reactive power in the transmission network just after an outage has occurred can be calculated from simulation outputs. This calculations can be used to compare against the capacities of different devices in order to observe if the calculated post-fault time series of active and reactive power through the lines exceeds the capacity of any component in the network. If one or more components of the network are overloaded, the overload index can be used to measure the associated severity of the overload. The equation describing this index is

$$f_x = \sum_{i=1}^{N_l} w_{f_i} \left(\frac{S_{mean,i}}{S_{max,i}} \right)^p \tag{1}$$

where f_x is the overload performance index for the operating point x, N_l is the number of transmission lines, $S_{mean,i}$ and $S_{max,i}$ are the average and maximum apparent power flows of the ith line, respectively, w_{f_i} is a weighting factor of the ith transmission line, which can be defined by the best judgment of the system operator, for instance $w_f = [1, 1, ..., 1]$ for unitary weight in all the lines. Finally p is an exponent to reduce masking effects, which means that a high value of the exponent will scale the effects of an overload resulting in a higher index value. Table I shows how each parameter is defined.

The final value of the overload index f_x is a scalar, and its interpretation is as follows:

$$\begin{array}{lcl} f_x = 1 & \rightarrow & \text{All lines are within the limits} \\ f_x > 1 & \rightarrow & \text{At least one line has violated limits} \\ f_x \gg 1 & \rightarrow & \text{A severe violation has ocurred} \end{array} \tag{2}$$

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[1] iTESLA(Innovative Tools for Electrical System Security within Large Areas), online: www.itesla-project.eu

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TABLE I
OVERLOAD INDEX PARAMETERS

Variable	Description	Dimension	Units
f_x	Actual overload index	$scalar$	-
N_l	Number of transmission lines	$scalar$	-
S_{mean}	Mean apparent power flow	$\Re^{1\times N_l}$	MVA, p.u.
S_{max}	Max apparent power flow	$\Re^{1\times N_l}$	MVA, p.u.
w_f	Weighting factor of lines	$\Re^{1\times N_l}$	-
p	Exponent	$scalar$	-

B. Under/Over Voltage Index

Following a disturbance in the power network, e.g. a line outage, the power flow trough the transmission lines is affected causing changes in other variables of the system. For instance, voltages across the system can be depressed or increased. Data from a simulation will contain information about faults, this be used to determine if any device has violated the acceptable operational limits. For the case of bus voltages, it is possible to measure the severity ratio of violations (under and over operational limits) as follows:

$$v_x=\sum_{i=1}^{N_b} w_{v_i}\left(\frac{v_{init,i}-v_{mean,i}}{\Delta v_i}\right)^q,\ \Delta v_i=\frac{v_{max,i}-v_{min,i}}{2} \quad (3)$$

where v_x is the performance index for the operating point x. It indicates if any bus in the system has surpassed the operational limits. N_b is the number of buses to be analyzed, $v_{init,i}$ is the initial voltage at the ith bus before any disturbance has occurred (pre-fault value), $v_{mean,i}$ is the average voltage of the post-fault data at the ith bus. w_{v_i} is a weighting factor of the ith bus, which can be defined by the best judgment of the system operator, for instance $w_v = [1, 1, ..., 1]$ for unitary weight in all buses. $v_{max,i}$ and $v_{min,i}$ are the upper and lower voltage limits for the ith bus, respectively and q is an exponent to reduce masking effects, which means that a high value of the exponent will scale the effects of violations in the voltage limits resulting in a large index value.

TABLE II
UNDER/OVER VOLTAGE INDEX PARAMETERS

Variable	Description	Dimension	Units
v_x	Under/over Voltage index	$scalar$	-
N_b	Number of buses	$scalar$	-
v_{init}	Nominal voltage (pre-fault)	$\Re^{1\times N_b}$	V, p.u.
$v_{mean}*$	Mean voltage (post-fault)	$\Re^{1\times N_l}$	V, p.u.
v_{max}	Maximum voltage allowed in bus	$\Re^{1\times N_l}$	V, p.u
v_{min}	Minimum voltage allowed in bus	$\Re^{1\times N_l}$	V, p.u
w_v	Weighting factor of buses	$\Re^{1\times N_l}$	-
q	Exponent	$scalar$	-

The final value v_x is a scalar, and its interpretation is as follows:

$$\begin{aligned} v_x &= 1 &\rightarrow&\quad \text{All buses are within the limits} \\ v_x &> 1 &\rightarrow&\quad \text{At least one bus has violated limits} \\ v_x &\gg 1 &\rightarrow&\quad \text{A severe violation has ocurred} \end{aligned} \quad (4)$$

III. CLASSIFICATION OF TIME-DOMAIN SIMULATIONS

We define the three classes in which time-domain simulations can be classified:

$$1 = \psi_x < \kappa_0 \quad \textit{safe} \quad (5a)$$

$$1 < \psi_x < \kappa_0 \quad \textit{mild} \quad (5b)$$

$$1 < \psi_x > \kappa_0 \quad \textit{severe} \quad (5c)$$

where ψ_x is a real integer representing one of the static indexes and κ_0 is a real integer that represents the boundary which define the class of time domain simulation as *safe*, *mild* and *severe* limit violation. Defining the precise value of κ_0 is not simple, it is subject to the system under analysis and the settings used in the static index such as limits and the masking exponent.

If for a given contingency the bounds of a static index are within the *safe* (5a) or *mild* (5b) classification, further assessment of the simulation results are not required. On the contrary, if the contingency is classified as *severe* (5c), the simulation results require a deeper inspection, e.g application of a dynamic index. In this form, static indexes serve to classify time domain simulations under different contingencies.

IV. SUPPORTING DYNAMIC INDEX

In this section, the transient stability index ISGA is presented to assess the validity of the indexes described on section II. From the system theory point of view, transient stability is a strongly nonlinear, high-dimensional problem [5]. To assess this problem, numerical integration methods or ”time domain” (TD) simulation methods are used. To analyze time domain simulations the ISGA index [4] can be used as follows:

$$J = \frac{1}{M_{tot}T}\int_0^T \sum_{i=1}^{N_m} M_i\left(\delta_i(t)-\delta_{coa}(t)\right)^2\ dt, \quad (6)$$

$$\delta_{coa}(t) = \sum_{i=1}^{N_m} M_i\delta_i(t) \Big/ \sum_{i=1}^{N_m} M_i, \qquad M_{tot} = \sum_{i=1}^{N_m} M_i$$

The index can be used to judge the severity of stable and unstable transient events in simulations. The term $\delta_i(t)$ represent the generator angle of the ith machine as a function of time. The constant M_i is the inertia of the ith machine and $\delta_{coa}(t)$ is the center of angle or inertia of all the machines. Table III describes the parameters used by the ISGA index.

TABLE III
TRANSIENT STABILITY INDEX PARAMETERES

Variable	Description	Dimension	Units
J	ISGA index	$scalar$	-
N_m	Number of machines	$scalar$	-
T	Number of simulation seconds	$scalar$	-
M	Inertia of the machines (2*H)	$\Re^{1\times N_m}$	-
M_{tot}	Total Inertia of all machines	$scalar$	-
δ	Generator angles	$\Re^{nt\times N_m}$	p.u.
δ_{coa}	Center of angle or inertia	$\Re^{nt\times 1}$	p.u.

$nt =$ Length of analyzed window of time

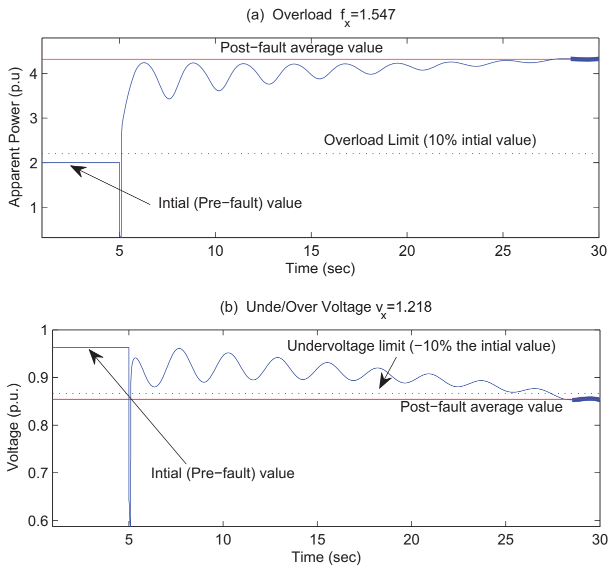

Fig. 1. (a) The apparent power flow through line 3 increases after the trip of line 8. (b) Voltage at bus 7 drops more than 10% of its initial value.

The final ISGA index value J is a scalar, and its interpretation is as follows:

$$\begin{array}{lll} J < 1 & \rightarrow & \text{All machines are transient stable} \\ J > 1 & \rightarrow & \text{Transient instability} \\ J \gg 1 & \rightarrow & \text{Sever transient instability} \end{array} \tag{7}$$

V. ILLUSTRATIVE EXAMPLE

In this section, two case studies using the Klein-Rogers-Kundur test system [6] are shown to illustrate the interpretation of the proposed indexes.

A. Case study 1: Illustration of static indexes

In this case study, a 3-phase fault was applied at 5 sec in bus 8, then line 8 was tripped at 5.09 sec by opening the breaker of bus 8 and finally the fault at the bus was cleared at 5.1 sec. The simulation length was set to 30 sec and the result of applying the static indexes described in Section II are shown in Figures 1 (a) and (b), respectively. The outage of line 8 in the circuit produced an overload in the power flow through line 3 as shown in Figure 1 (a). The overload index ($f_x = 1.54$) indicates that the power flow through line 3 surpassed more than 10% of the initial value. Similarly, the voltage at bus 7 dropped more than 10% of its initial value ($v_x = 1.21$), as seen in Figure 1 (b). Both, the overload and under/over voltage indexes captured the problem by showing values grater than zero.

Remark. Note that the limits used here for both indexes were set to 10% of the initial value. These limits can be defined by the best judgement of the system operator.

B. Case study 2: Illustration of the transient stability Index

In order illustrate the ISGA index described in Section IV, the following simulation was performed using the system described in [6]: line 1 was tripped at 5 sec by opening the breaker of bus 6. As result the system was divided in two subnetworks; one with machine 3 and other with all other machines. The machine angles are displayed in Figure 2. From this figure it can be observed that machine 3 deviates from the rest of the machines. Table IV summarizes the results of applying the ISGA index J at different time instants of the simulation. From the table can be observed how J grows slowly during the first 7 seconds in a rate of 0.01, indicating that the machines are close to their center of inertia. However, at 8 seconds, the index changes drastically from $J = 0.54$ to $J = 2.87$ in one second and continues to grow exponentially in the subsequent seconds (highlighted in bold) indicating a severe transient problem.

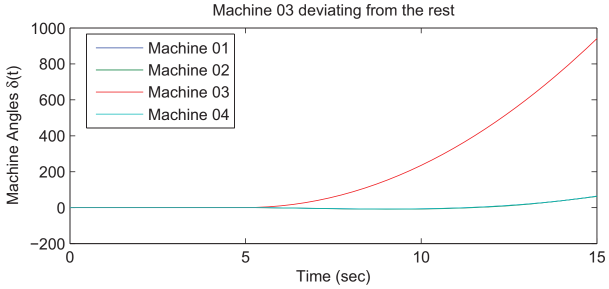

Fig. 2. From $t = 6$ sec approx, machine 3 deviates from the rest of the machines in the network.

Remark The ISGA index J is not calculated only for the time instants shown in Table IV, it is calculated from a window of data starting at time $t = 1$ sec, to the time specified in Table IV.

TABLE IV
TRANSIENT STABILITY INDEX (J) AT DIFFERENT TIMES

Time (sec)	4	5	6	7	8	9	10	11	12
Index J	0.01	0.02	0.03	0.54	**2.87**	**9.29**	**23.12**	**48.19**	**89.30**

VI. APPLICATION TO THE KTH-NORDIC32 SYSTEM

In this section the effectiveness of the proposed indexes is presented. Different case studies on the KTH-Nordic32 power system were considered and the results are summarized in Table V.

A. System Description

The KTH-Nordic32 system was constructed from the data proposed in [7], further details are available in [8]. The one-line diagram is shown in Figure 3 and it is comprised of 52 buses, 80 transmission lines and 20 generators. There are 12 hydro generators located in the North and equivalent areas, the rest are thermal generators located in the Central and South areas. This weakly coupled system exhibits lightly damped low frequency inter-area oscillations. The two lowest damping modes have a frequency of 0.46 Hz and 0.78 Hz, respectively and a damping ratio of 4.08% and 4.98%, respectively.

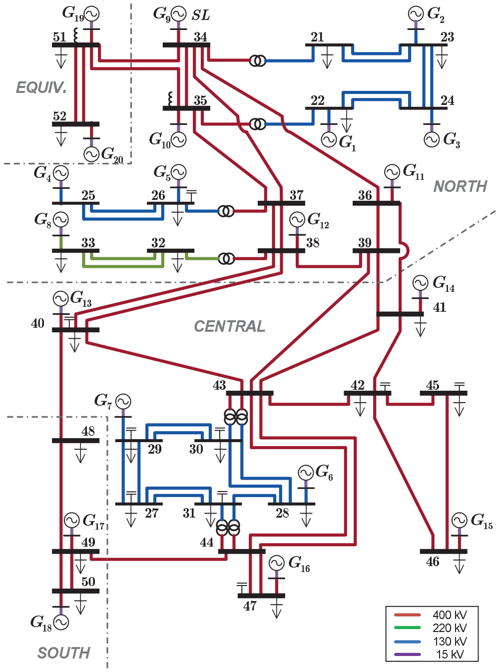

Fig. 3. The KTH-Nordic32 power system.

B. Considerations for the analysis

As mentioned before, the system has 80 transmission lines from which 20 correspond to low voltage corridors of 15, 130 and 220 kV, respectively and 20 that correspond to lines connecting each of the generators to the main grid. The system has 40 major 400 kV tranmission lines, which transport most of the power flows from the north to the south (red lines shown in Figure 3). For the analysis of the overload index, only the most significant corridors were analyzed to investigate if the power flow surpassed a pre-defined limit following the loss of any major transmission line. Note that although only the most relevant lines were monitored for limit violations, all lines were considered when faults were applied.

Similarly, from the 52 buses that the system has, 20 connect the generators, 13 link the low voltage lines and 19 connect the high voltage corridors (400 kV lines). To investigate the under/over voltage problem in the network, only the 19 buses where the 400 kV lines are connected were considered in the analysis. All generators were evaluated to investigate the transient stability phenomena.

C. Simulation Results

18 case studies were analyzed and the results are presented in Table V. On each experiment, a 3-phase fault at a different bus was applied at 5 sec and then cleared at 5.1 sec. Column 2 on Table V, indicates the bus number where the 3-phase fault was applied for each case study. To clear the 3-phase fault, different lines were tripped by opening a circuit breaker at 5.05 sec in the corresponding faulty bus. The third column on Table V, indicates the line tripped on each case. In all the experiments, the simulation time was set to 40 sec to give enough time to rich steady state. For each case study, the overload (f_x), under/over voltage (v_x) and ISGA (J) indexes were calculated following the methodology described in sections II and IV. The results of applying the indexes are presented in columns 5, 6 and 7 of Table V. For the f_x and v_x indexes, the initial (pre-fault) values correspond to the value of the signal at 2 sec and the limits were set to $+20\%$ and $\pm 5\%$ of the initial value for the overload and the under/over voltage, respectively. Unitary weighting factors were considered and the exponents to reduce masking effects were set to $p = 3$ and $q = 3$, respectively. Based on simulation experience of the KTH-Nordic32 system and the limits defined before, the f_x and v_x boundaries were set to $\kappa_1 = 3$ and $\kappa_2 = 5$, respectively. Finally the post-fault data used to calculate the indexes was set to 80 samples prior to the end of the simulation time to capture a significant section of the signal.

C.1 Safe Cases (no limits violation)

From the results of case studies 1 to 6 on Table V, we can observe that both the overload and under/over voltage indexes are equal to one and that the ISGA index is practically zero. A recall of inequalities (2), (4) and (7) shows that none of the monitored 400 kV lines surpassed 20% of the pre-fault value, any bus surpassed or dropped $\pm 5\%$ of its initial voltage and all the machines were transiently stable. These set of contingencies are classified as *safe* according to inequality (5a), on these cases the contingencies applied did not affect the operation of the system and calculation of dynamic indexes like ISGA are not relevant, as indicated by the values on column 7 on Table V.

On these simulations, only 130 and 220 kV lines were tripped and because they are not significant corridors on the system, the effects of the faults are not reflected in any of the indexes. Although, there exist minimal changes in some parameters of the system caused by the simulated fault (i.e increase of load in neighbor lines and fluctuations of voltage in neighbor buses), the proposed indexes cannot capture the small variations because of the parameters/limits used in the index calculation. Relaxing the overload and the under/over voltage limits can reflect the changes in the signals. However, regarding the limits in the proposed static indexes, the system will always be transient stable and the ISGA index will be always close to zero for these case studies.

C.2 Mild Violations

From the results of case studies 7 to 13 on Table V, we can observe that the overload and under/over voltage indexes are grater than one but less than the bounds ($\kappa_1 = 3$ and $\kappa_2 = 5$) and that the ISGA index is approximately zero. Again, recalling inequalities (2), (4) and (7) indicates that at least one line and one bus have violated the pre-defined limits but that the system continues to be transiently stable. In these case studies, faults were applied at important lines of the system. For instance, is possible to observe from case study 7 that tripping one of the parallel lines connecting Bus 40 with Bus 38 in central west, increases uniformly both static indexes ($v_x = 1.23$ and $f_x = 1.26$). The fault causes an overload of more than 20% on neighbor lines (Bus 38-Bus 40 and Bus 39-Bus 43) and a drop of more than 5% of voltage on surrounding buses. Based on inequality (5b), these case

TABLE V
Application of the static and transient stability indexes to the Nordic power system in different case studies

Case Number	**3-phase fault at bus**	**Line Tripped from-to**	**Simulation Time (sec)**	**Overload** f_x	**Under/over voltage** v_x	**ISGA** J
1	24	22-24	40	1.00	1.00	0.3×10^{-3}
2	21	21-23	40	1.00	1.00	0.4×10^{-3}
3	27	27-31	40	1.00	1.00	0.3×10^{-3}
4	24	23-24	40	1.00	1.00	0.3×10^{-3}
5	29	27-29	40	1.00	1.00	0.3×10^{-3}
6	28	28-30	40	1.00	1.00	0.3×10^{-3}
7	40	38-40	40	1.23	1.26	0.4×10^{-3}
8	48	38-39	40	1.49	2.60	0.5×10^{-3}
9	43	43-47	40	1.05	2.73	0.2×10^{-3}
10	34	34-51	40	1.31	1.00	0.5×10^{-3}
11	42	42-43	40	1.01	1.04	0.3×10^{-3}
12	41	36-41	40	1.51	2.46	0.5×10^{-3}
13	47	44-47	40	1.02	1.38	0.2×10^{-3}
14	49	48-49	11.39	3.56	29.51	5.23
15	34	34-37	23.53	5.90	167.97	1.22
16	44	44-49	23.71	4.48	183.75	1.40
17	32	32-33	30.61	17.70	754.13	10.97×10^{3}
18	25	25-26	40	13.89	580.2	21.57×10^{3}

Overload boundary $\kappa_1 = 3$, Under/Over voltage boundary $\kappa_2 = 5$

studies illustrate the effectiveness of the proposed indexes to classify the dynamic simulations as *mild* violations. In this classification, the contingencies perturb the system in such a way that some limits are violated but the system continues operating on a critical state. On this case, dynamic indexes still not required as indicated by the values of column 7 on Table V.

C.3 Severe Violations

In the last case studies 14 to 18, severe violations were simulated. The tripped lines were selected in such a way that the system was transiently unstable to demonstrate the effectiveness of the proposed indexes in relation with the ISGA index. In all the cases the fault caused isolation of one machine or a group of machines and as a consequence, first the loss of synchronism and then the system collapse causing large index violations. In most of the cases, the simulation did not reached the pre-defined simulation ending time (40 sec). Case study 14 represents the tripping of the important line from Bus 48 to Bus 49 in the south west. The fault isolated generators 17 and 18 causing its deviation from the center of inertia, in this case the ISGA index captured the transient stability problem with a number greater than one $J = 5.23$ and the static indexes also reflects the severity of the instability with a large value ($f_x = 3.56$ and $v_x = 29.51$). Static indexes are out of bonds, which correspond to case (5c), the contingencies are classified as *severe* and values of column 7 on Table V confirms the results with large index numbers.

VII. Conclusions

In this paper, two novel static indexes for the classification of power system time domain simulations have been presented. The proposed indexes rely on time-domain simulations and were developed to accomplish fast computation. To support the results, the Integral Square Generator Angle (ISGA) index was applied in combination with the proposed indexes to demonstrate that the former can be used to filter or classify dynamic simulations according to the quality of the stability or severity of instability. Three classes to classify contingencies were defined based on boundaries that depend on the system itself and static index specifications such as limits and masking exponents. The simulation results indicate that limits of the proposed indexes can be adjusted to capture violations and discard minor problems. The input parameters to the indexes play a key roll on the calculation. In this paper only the most significant lines and buses of the Nordic system were monitored. The severity of the contingency is directly related to the magnitude of the index.

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