Equation-Based Modeling of Three-Winding and Regulating Transformers using Modelica

Mohammed Ahsan Adib Murad, Francisco José Gómez
KTH, Royal Institute of Technology,
Stockholm, Sweden
maamurad@kth.se , fragom@kth.se , luigiv@kth.se

Luigi Vanfretti
KTH Royal Institute of Technology, Stockholm, Sweden
Statnett SF, Oslo, Norway
luigi.vanfretti@kth.se, luigi.vanfretti@statnett.no

***Abstract*—The simulation of power transformer models is important when analyzing the dynamic behavior of power systems, in particular, when considering voltage magnitude or phase regulation controls. This paper reports results of extending the library of transformers in the iTesla Modelica Power Systems Library. Three transformer models have been implemented: a three-winding transformer, an under-load tap changing transformer (ULTC) and a phase shifting transformer (PST). An IEEE 14-Bus, power system test model was also implemented, both in Modelica and PSAT, to assess the performance of the models. Software-to-software validation is carried out against PSAT, a quantitative and qualitative assessment of the validation results between PSAT and Modelica is given.**

***Index terms* – Modelica, PSAT, Power Transformer, Simulation Software, Power System Simulation.**

I. INTRODUCTION

Adequate modeling of conventional and controllable power transformers allows studying the dynamic behavior of power network under different operating conditions. In the literature, classical transformer models have been studied [1]. Different transformer models have been developed, each focusing on a particular application or to represent specific physical phenomena. Generally, transformer models are classified according to their application: lightning overvoltage studies or the purpose of elements of the model, e.g., models based on leakage inductance, transmission line modeling, etc.

From the models above, those used in phasor time-domain simulations can be easily implemented using equation-based modeling languages. These kinds of languages allow engineers to implement models directly using mathematical equations. The Modelica equation-based modeling language is object-oriented and standardized, which allows model implementation directly from mathematical equations. This is an important characteristic, which implicitly decouples the model from the mathematical solver, thus providing unambiguous simulation results among different tools [2]. The attractive features of this language have been successfully exploited in different areas such as the automotive and aerospace industry [3].

European transmission system security handling is becoming a challenge due to the growing complexities of the pan-European power network. To overcome these complexities, the FP7 iTesla (Innovative Tools for Electrical System Security within Large Areas) project was initiated to develop a toolbox that will support the operation of the European transmission network [4]. The iTesla project has adopted the Modelica language for modeling of power system dynamic components and a Modelica power system library [5] compatible with Modelica tools has been developed.

The purpose of this work is to improve this power system library with the implementation of new Modelica models of conventional power system components (transformers) for phasor time-domain simulation. To implement these models the PSAT implementation is taken as reference [6]. PSAT is a Matlab-based power system analysis tool, its performance depends on Matlab, but however its validity for power system analysis has already been proven [7]. To prove that Modelica models of transformers have the expected behavior, software-to-software validation was performed by implementing all the models in the IEEE 14-Bus test system, taken PSAT as a software reference for this validation. Finally, a quantitative assessment between the simulation results of Modelica and PSAT is given.

II. DETAILS OF TRANSFORMER MODELS

This work reports the implementation and validation of Modelica models for two regulating transformers and a three winding transformer. The regulating transformers considered herein are: Under Load Tap Changing (ULTC) and Phase Shifting Transformer (PST) transformer models. A two winding transformer was already implemented in the iTesla power system library [5]. Load Tap Changing and Phase Shifting transformers are widely used for voltage regulation without interrupting the load. Three Winding Transformers are used for cost savings. As the power system library will be used to model complete power system networks, there is a

---

This work was supported in part by the EU-funded FP7 iTESLA project and by Statnett SF, the Norwegian Transmission System Operator, under grant agreement n°283012. iTESLA (Innovative Tools for Electrical System Security within Large Areas), online: www.itesla-project.eu

need to add these transformers models to enrich the power system library so it can be used to represent different networks.

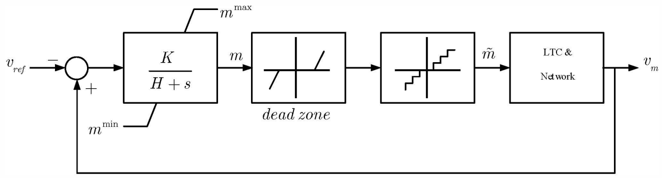

Figure 1: Secondary voltage control scheme of ULTC [6].

A. ULTC (Under Load Tap Changer)

ULTC is a regulating transformer that controls the voltage or reactive power at the secondary side of the transformer by varying the tap ratio. The regulator used to control the secondary voltage is shown in Fig.1. The ULTC transformer is modeled as an equivalent PI-circuit as depicted in Fig. 2.

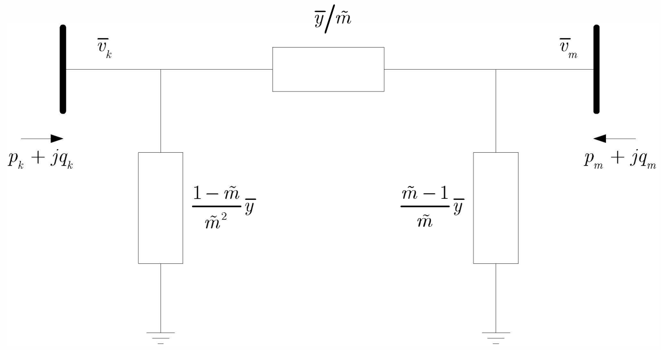

Figure 2: Equivalent pi circuit of ULTC [6].

The current injection at Bus k (i_k) and Bus m (i_m) are calculated from:

$$\begin{bmatrix} \bar{\imath}_k \\ \bar{\imath}_m \end{bmatrix} = \bar{y} \begin{bmatrix} \frac{1}{m^2} & -\frac{1}{m} \\ -\frac{1}{m} & 1 \end{bmatrix} \begin{bmatrix} \bar{v}_k \\ \bar{v}_m \end{bmatrix} \tag{1}$$

where, $\bar{y} = (r_T + jx_T)^{-1}$ is the series admittance of the transformer, m is the off nominal tap ratio, r_T and x_T are transformer resistance and reactance. The tap ratio m is the output of the regulator shown in Fig. 1. The tap ratio step Δm is taken as zero, then

$$\tilde{m} = m \tag{2}$$

To model the secondary voltage control the differential equation used (calculated from the controller shown in Fig. 1) is:

$$\dot{m} = -Hm + K(v_m - v_{ref}) \tag{3}$$

where, H is the integral deviation, K is the inverse time constant, v_m is secondary bus voltage and v_{ref} is the reference voltage.

B. PST (Phase Shifting Transformer)

Phase Shifting Transformer is used to control the active power flow by varying the phase angle. It can reduce the congestion on some transmission lines and, in addition, it can redistribute the active power flow through transmission lines. The regulator used to control the active power is shown Fig. 3.

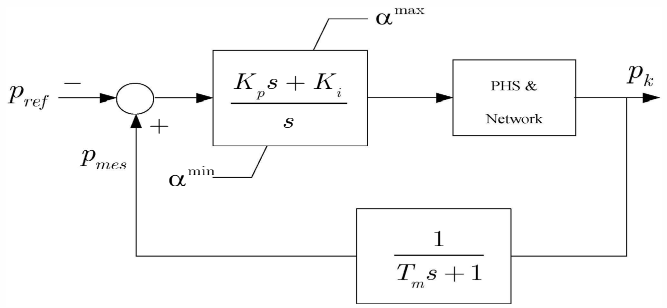

Figure 3: Control scheme of phase shifting transformer [6].

The differential equations that describe the PST are given by

$$\dot{\alpha} = \frac{K_p(p_k - p_{mes})}{T_m} + K_i(p_{mes} - p_{ref}) \tag{4}$$
$$\dot{p}_{mes} = (p_k - p_{mes})/T_m$$

where, α is the phase angle, p_{mes} is the measured power flow, p_k is the real power flow. K_i, K_p, T_m are integral gain, proportional gain and measurement time constant, respectively. The equivalent PI-circuit of PST is shown in Fig. 4.

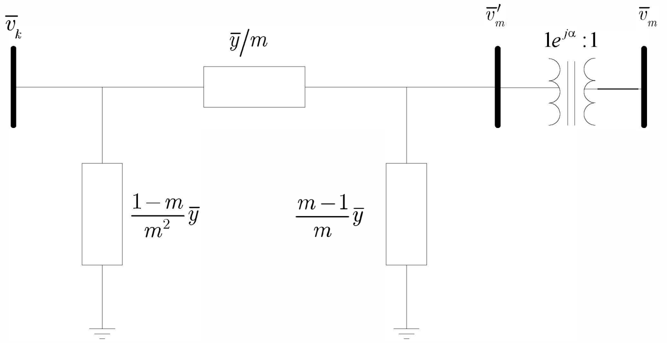

Figure 4: Equivalent pi circuit of a PST [6].

C. Three Winding Transformer (TWT)

The three winding transformer model is described as three two-winding transformers in a star connection, shown in Fig. 5.

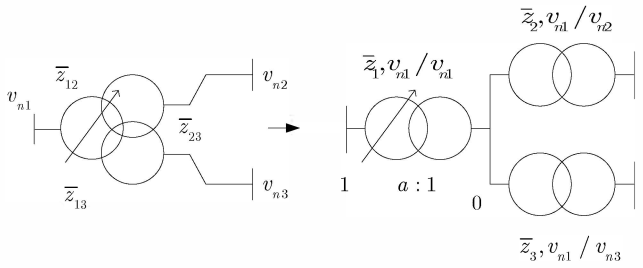

Figure 5: Three Winding Transformer equivalent circuit [6].

The branch impedances with the resulting star impedances are given by

$$\begin{aligned} \bar{z}_{12} &= \bar{z}_1 + \bar{z}_2 \\ \bar{z}_{13} &= \bar{z}_1 + \bar{z}_3 \\ \bar{z}_{23} &= \bar{z}_2 + \bar{z}_3 \end{aligned} \tag{5}$$

Hence,

$$\begin{aligned} \bar{z}_1 &= (\bar{z}_{12} + \bar{z}_{13} - \bar{z}_{23})/2 \\ \bar{z}_2 &= (\bar{z}_{12} + \bar{z}_{23} - \bar{z}_{13})/2 \\ \bar{z}_3 &= (\bar{z}_{13} + \bar{z}_{23} - \bar{z}_{12})/2 \end{aligned} \tag{6}$$

these impedances are to be computed internally by the Modelica model.

III. IMPLEMENTATION IN MODELICA

Modelica language allows implementing Modelica models using different class stereotypes. One of these stereotypes defines a Connector class, which is used to connect components. The iTesla power system library uses the connector class known as PwPin [5]. This class has four variables, real voltage and current (*vr* and *ir*), imaginary voltage and current (*vi* and *ii*). To implement these three transformers this connector class is used.

A. ULTC & PST in Modelica

The continuous model of the ULTC has been implemented in Modelica, taking the tap ratio step as $\Delta m = 0$. To calculate the current variables of the connector, equation (1) is used and Modelica implementation is given below.

```
R*p.ir-X*p.ii= (1/m^2)*p.vr- (1/m)*n.vr;
R*p.ii+X*p.ir= (1/m^2)*p.vi- (1/m)*n.vi;
R*n.ir-X*n.ii= n.vr- (1/m)*p.vr;
X*n.ir+R*n.ii= n.vi- (1/m)*p.vi;
der(m)= -(H*m)+K*(vm-vref);
```

The PST is implemented using two sub-models. The fixed tap ratio of the PST is modeled in the same way as an ULTC, in one sub-model. In another sub-model, the angle alpha of the PST is modeled using the relation $\bar{v}_m^{'}: v_m = 1e^{j\alpha}:1$ (see Figure 4). The implementation of the angle relationship in Modelica is given below.

```
der(alpha)= (Kp*(pk-pmes)/Tm)+Ki*(pmes-pref);
der(pmes)=(pk-pmes)/Tm;
p.vr=n.vr*cos(alpha)-n.vi*sin(alpha);
p.vi=n.vr*sin(alpha)+n.vi*cos(alpha);
p.ir+n.ir=0;
p.ii+n.ii=0;
```

Then these two sub models are added together to implement the complete model. All the limiters of the controllers of both transformers are included using `if..else` statements.

B. TWT in Modelica

The two winding transformer in Modelica is modeled as a transmission line with only series impedance without iron losses. Three Winding Transformer is implemented by using the method of equivalent three two-winding transformers (see the three branches of transformer in Fig. 5), but in the case of Three Winding Transformer the impedances are taken as a resulting star impedance (equation 6); in the first branch a fixed tap ratio is taken into account. Finally these three branches are joined together to complete the whole transformer model and shown in Fig. 6.

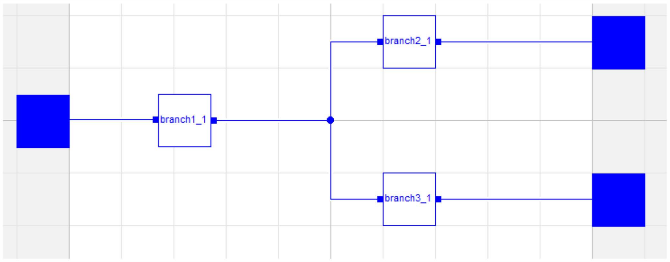

Figure 6: Three winding transformer in Modelica.

IV. VALIDATION OF TRANSFORMER MODELS

A. Test System

These transformers models here tested in the IEEE 14-Bus test system. The single line diagram with the data of the IEEE 14-Bus test system is taken from [6] and [8].

To simulate the test system networks one of the available Modelica simulation environment, Dymola by Dassault Systemes, is used.

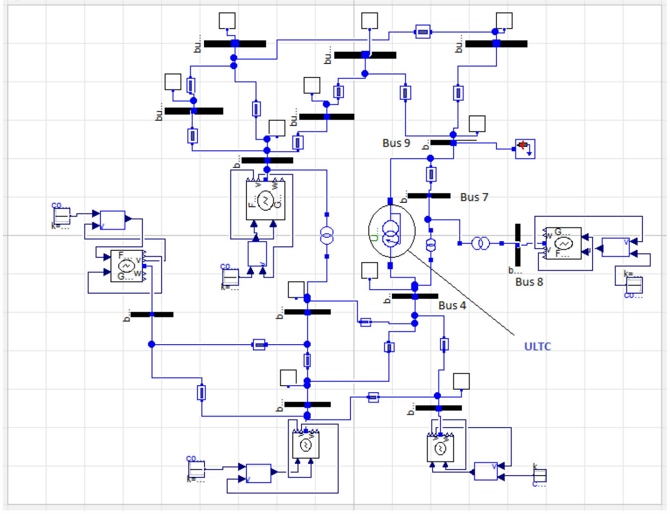

Figure 7: IEEE 14-Bus test system in Modelica with ULTC.

The ULTC and PST were placed in between Bus 4 and Bus 9 (see Fig. 7) in two different IEEE 14-Bus test systems. In the case of the TWT, the TWT is placed between Bus 4, Bus 8 and Bus 9 in another IEEE 14-Bus system (shown in Fig. 8). In this case Bus 7 is not used as it becomes a fictitious bus inside the TWT. All these test systems were also implemented in PSAT.

To test the dynamic behavior of the ULTC, PST and static behavior of TWT three kinds of tests were carried out. The perturbations applied in these test systems are given in Table I.

Table I: Test cases for the validation.

Test #	Perturbations applied
1	Three phase fault applied at Bus 9, at 10s with clearing time 100ms.
2	Active and Reactive load increased by 10% in bus 9, starting from 5s.
3	Active and Reactive load decreased by 10% in bus 9, starting from 5s.

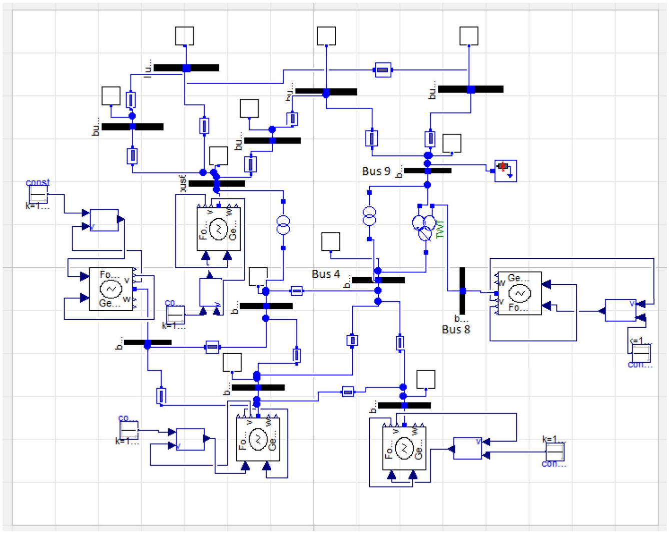

Figure 8: IEEE 14-Bus test system in Modelica with Three Winding Transformer.

B. *Quantitative comparison*

The qualitative observation only provides an insight of the validity of a model. In contrast, a quantitative assessment allows to "measure" the validity of a model response against its reference in numerical metrics. To validate the implementation of the Modelica models in section III, results of two different software packages are analyzed both graphically and numerically. The quantitative assessment is carried out using the Root Mean Square Error (RMSE) [9]. The RMS value of the error is calculated using the equation.

$$Z_{RMSE} = \sqrt{\frac{1}{n}[(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_n - y_n)^2]} \qquad (7)$$

Here, $x_1, x_{2,} \dots, x_n$ are the discrete measurement points at time $t_1, t_{2,} \dots, t_n$ for software package (a) and $y_1, y_{2,} \dots, y_n$ are the discrete measurement points at time $t_1, t_{2,} \dots, t_n$ for software package (b). Z_{RMSE} is the RMS value of the error of Z variable.

C. *Simulation and Results*

Time domain simulations were performed in both software packages with the same initialization and simulation configuration. Power flow computations were performed in PSAT and the same power flow solution is used in Dymola to initialize the Modelica test system. The simulation set up is given in the Table II.

Table II: Simulation set up

Set Up	PSAT	Modelica
Simulation Environment	Matlab	Dymola
Integration Algorithm	Trapezoidal Rule	Rkfix2[a]
Time step	0.001	0.001
Tolerance	1×10^{-5}	1×10^{-5}
Simulation Time	25s	25s

a. Rkfix2 is Runge-Kutta, second order, fixed time step method.

Figure 9 illustrates the comparison of the tap ratio and voltage at Bus 9, where the ULTC is connected. Figure 10 and 11 show the comparison for test case 2 and 3 of ULTC. Figure 12 illustrates the two state variables of PST for the test case 1. Figure 12 shows the internal bus and bus 4 voltages of the three-winding transformer when the TWT is connected for the test case 1. The RMSE error between the simulation results for all the cases is given in the Table III.

The simulations executed for 25 s, with a time step of 0.001s. The RMS value of the error is calculated using 25000 points from both simulation results. The RMSE calculated for the ULTC measures the error from dynamic tap ratio (M_{RMSE}), for the PST it measures the error from alpha (α_{RMSE}) and from active power ($Pmes_{RMSE}$) and for the TWT it measures the internal bus voltage (V_{RMSE}). The RMS error calculations are given in Table III.

Table III: RMSE calculations using Equation (7)

Test scenario	1	2	3
M_{RMSE}	3.5439e-06	3.0717e-06	3.2029e-06
α_{RMSE}	6.7955e-04	7.4991e-04	5.9284e-04
$Pmes_{RMSE}$	6.7955e-04	4.8702e-04	4.2581e-04
V_{RMSE}	7.5164e-04	6.2314e-05	6.0375e-05

From all the graphical comparison it is evident that the simulations have a satisfactory match. In the case of the PST (Fig. 12), the difference is noticeable, but from Table III the RMS error indicates that the errors are within tolerance range. The visible difference can be further improved by more efficient initialization of the PST model.

V. CONCLUSIONS AND FUTURE WORK

The three transformers were successfully implemented using Modelica, which proofs the simulation capabilities of Modelica as a modeling language for power systems, using equation based modeling, for time-domain simulation.

The simulation time has been measured for both Dymola and PSAT tools. The simulation time measured in the first tool lasts an average time of 144 s for the simulation of IEEE 14-Bus system during 25 s. Whereas the same simulation performed in PSAT is completed with an average time of 312 s. In case of the ULTC, the model implemented in this work is a continuous model. The ULTC model will be improved by modeling its discrete step operation using discrete elements available in Modelica standard library in future work.

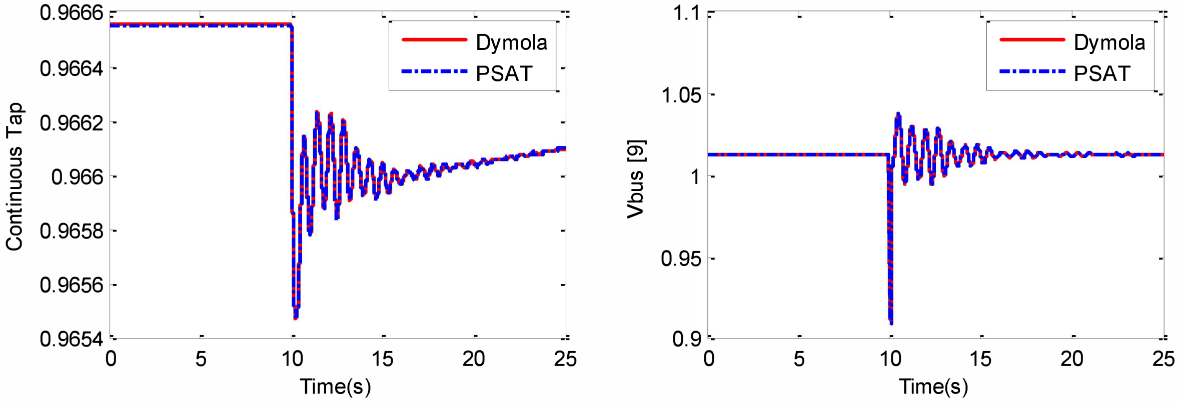

Figure 9: Illustration of the continuous tap ratio of ULTC and bus voltage (9) with three phase fault at 10s.

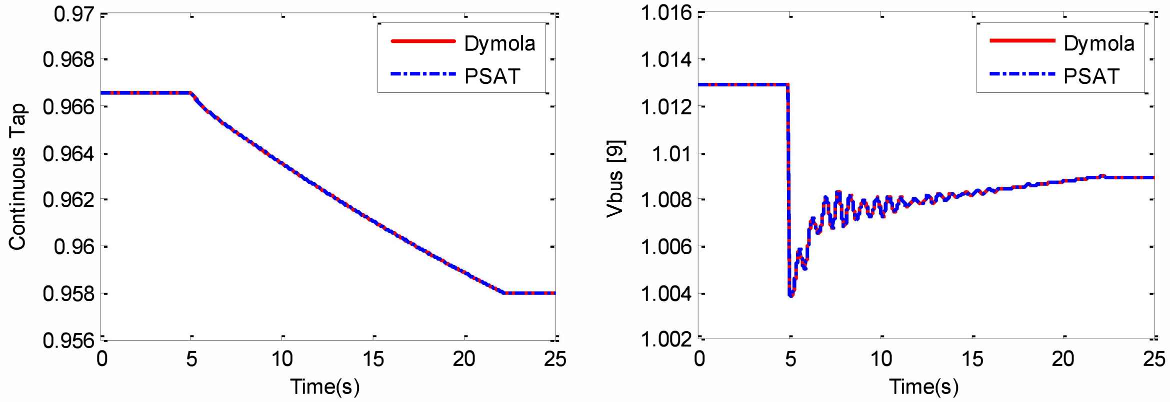

Figure 10: Illustration of the continuous tap ratio of ULTC and bus voltage (9) with 10% active and reactive load increased at 5s.

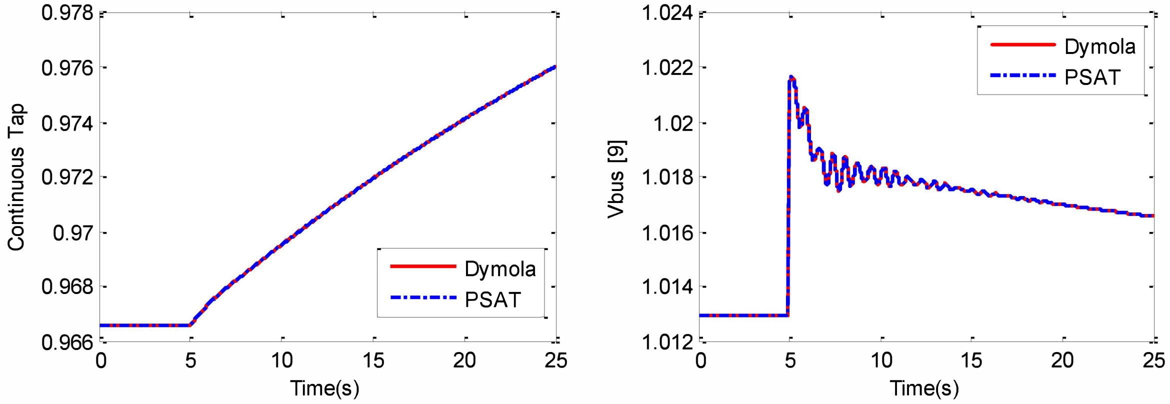

Figure 11: Illustration of the continuous tap ratio of ULTC and bus voltage (9) with 10% active and reactive load decreased at 5s.

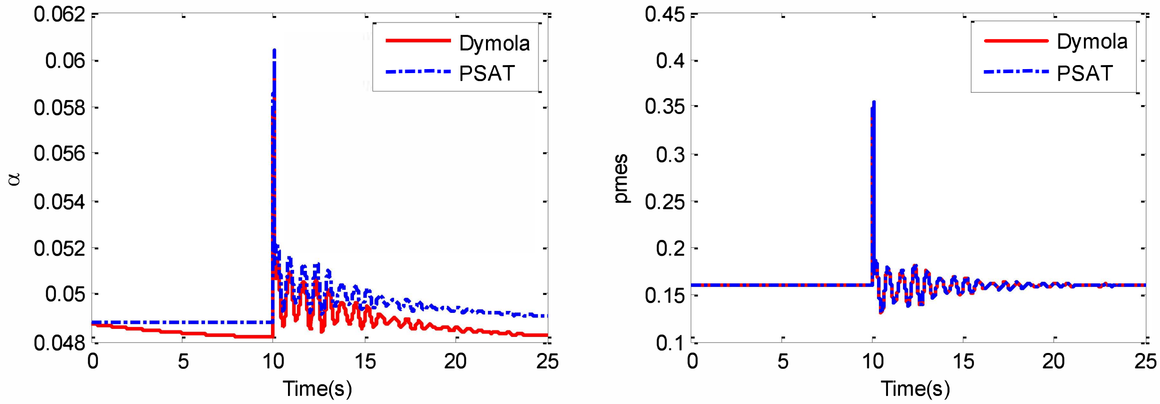

Figure 12: Illustration of the α and p_{mes} of PST with three phase fault at 10s.

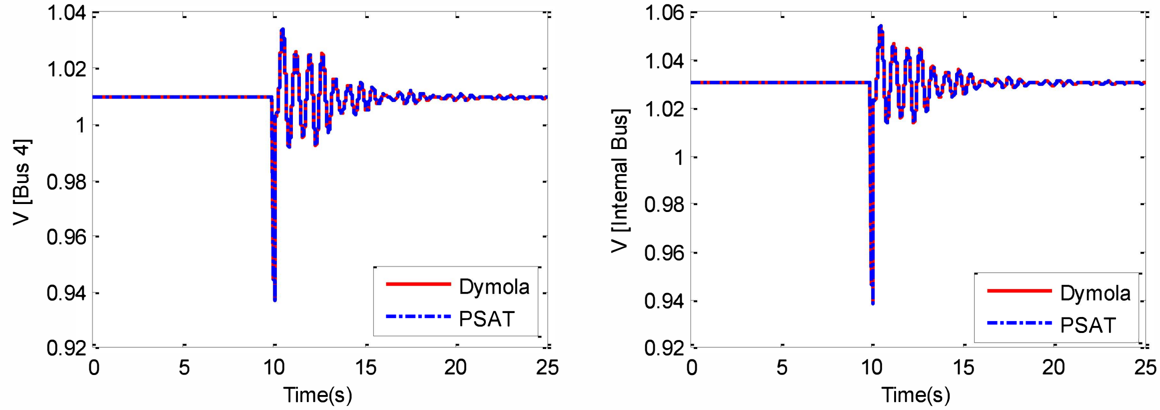

Figure 13: Illustration of the Bus voltage (4) and internal Bus of TWT with three phase fault at 10s.

VI. REFERENCES

[1]. C. González. "Power transformer modeling analysis and survey by means of the frequency response". Master Thesis Dissertation. Carlos III de Madrid University, September, 2009.

[2]. Vanfretti, L.; Li, W.; Bogodorova, T.; Panciatici, P., "Unambiguous power system dynamic modeling and simulation using modelica tools," *Power and Energy Society General Meeting (PES), 2013 IEEE* , vol., no., pp.1,5, 21-25 July 2013.

[3]. P. Fritzson, *Introduction to Modeling and Simulation of Technical and Physical Systems with Modelica*. Wiley-IEEE Press, 2011. ISBN: 978-1-118-01068-6.

[4]. iTesla: Innovative Tools for Electrical System Security within Large Areas. [Online]. Available: http://www.itesla-project.eu/

[5]. Bogodorova, T.; Sabate, M.; Leon, G.; Vanfretti, L.; Halat, M.; Heyberger, J.B.; Panciatici, P., "A modelica power system library for phasor time-domain simulation," *Innovative Smart Grid Technologies Europe (ISGT EUROPE), 2013 4th IEEE/PES* , vol., no., pp.1,5, 6-9 Oct. 2013

[6]. F. Milano, Power System Analysis Toolbox Documentation for PSAT. version 2.1.8, 2013.

[7]. Milano, F., "An Open Source Power System Analysis Toolbox," *Power Systems, IEEE Transactions on* , vol.20, no.3, pp.1199,1206, Aug. 2005

[8]. Kodsi, S. K. M., Caizares, C. A. "Modelling and Simulation of IEEE 14 bus System with FACTS Controllers". Technical report, 2003. University of Waterloo.

[9]. Rogersten, R.; Vanfretti, L.; Wei Li; Lidong Zhang; Mitra, P., "A quantitative method for the assessment of VSC-HVdc controller simulations in EMT tools," *Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), 2014 IEEE PES* , vol., no., pp.1,5, 12-15 Oct. 2014