

PMU-based Voltage Instability Detection through Linear Regression

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Abstract—This paper proposes a linear regression method using synchrophasor measurement for voltage stability monitoring. The method preprocesses synchrophasor measurements in order to eliminate inconsistencies and errors that embedded in them. This data is then used in the computation of sensitivities suitable for voltage stability monitoring. Some important electrical components such as Over Excitation limiter and On-Load Tap Changer which create discrete changes affecting in voltage instability have been included to assess the method's performance. Moreover, both process and measurement noises, obtained from real-time hardware-in-the-loop simulation and real PMU measurements from the Norwegian network, respectively, have been applied to validate robustness of the proposed methodology.

Index Terms—Voltage Instability Detection, Wide-Area Early Warning Systems, PMU-based applications, Linear Regression

I. INTRODUCTION

Phasor Measurement Units (PMUs) have been adopted to provide a high-sampling rate voltage and current phasors for Wide-Area Monitoring, Control and Protection (WAMPAC) systems [1]. Within WAMPAC systems, are dependent on synchrophasor data-based applications for providing timely information to operators so that preventive and corrective actions can be taken. A number of applications have been implemented to support applications for voltage instability detection. However, the ultimate goals of WAMPAC systems is to go beyond detecting an emergency state and prevent the evolution of a voltage instability that may lead to a system collapse.

The activation of defense plans using the information of PMUs within WAMPAC systems must coordinate with different elements: PMUs, monitoring, control devices and protections. To this aim, once the signals emerging from PMUs are available in a WAMPAC system, appropriate measures indicating the “inception” of instabilities can be determined. In this context, such measures can be considered as “Defense Signals” capable of arming controls or other countermeasures such as load shedding once curative control actions have been exhausted. Nevertheless, the activation of controls and “last resort” countermeasures needs to be coordinated.

In particular, for the case of voltage instability, it has been shown in [2] that appropriate sensitivities can indicate the

inception of long-term voltage instabilities. These sensitivities can be used to arm adaptive load shedding mechanisms [3] by exploiting an important characteristic: once an inception of voltage instability occurs, the sensitivities begin to grow - a change of sign in these sensitivities corresponds to the point of collapse of the network. However, this approach suffers from the assumption of full PMU coverage (all voltage phasors in the HV grid must be monitored). To relax this constraint, a “state reconstruction” approach has been proposed [4], and the use of adaptive load shedding control has been formulated using reconstructed states.

In addition, it is necessary to extract unwanted dynamics and measurement errors from PMU data before performing voltage stability analysis. To this aim, the state reconstruction approach can be used or one can utilize filtering and data processing methods. Some filtering techniques have been proposed, [5], [6]. These techniques were used for removing outliers and oscillations of fast dynamics which are not relevant for long-term voltage instability detection. The above mentioned methodologies serve as a theoretical basis for the generation of defense signals. However, the applicability of these methods is limited due to the need of: *a)* the ability to continuously track the topology at all instances of time, *b)* the availability of state estimator snapshots providing pseudo-measurements for state reconstruction, *c)* the availability of a simplified dynamic model capturing only long-term dynamics (QSS), *d)* the ability of “fitting” the filtered dynamic responses to the designed sensitivities from the QSS model, and *e)* the availability of appropriate computational resources for performing calculations on the designed sensitivities.

The approach proposed in this work therefore seeks to relax some of the requirements from the methodologies above, by improving recent work [7] on sensitivity computation solely relying on synchrophasors. This previous work considered the computation of sensitivities by filtering PMU data, and the determination of “alarms” by comparisons to thresholds computed through off-line analysis. However, filtering methods have significant drawbacks such as specific parameters settings and time delay. To avoid these problems, this paper proposes a method that uses Linear Regression (LR) to compute sensitivities for voltage instability detection. With the advantage of fast computation and a simpler algorithm compared with the filtering approach, the mentioned limitations are resolved.

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The remainder of this paper is structured as follows. Section II describes the proposed LR methodology for pre-processing data for fast and simple sensitivity calculations. Section III presents the test system and simulation results for different scenarios. Section IV shows the applicability of the method on real PMU data from the Norwegian grid. Finally, in Section V, conclusions are drawn and future work is outlined.

II. LINEAR REGRESSION METHOD FOR SENSITIVITY COMPUTATION

A. Linear Regression and Correlation Coefficient

Linear Regression (LR) is an approach to model a linear relationship between a scalar dependent variable y_i and p -vector of regressors x_i , with nonzero slope β_i and an error between the dependent variable and regressors e_i . The generic form of this linear relationship can be written as follows:

$$y_i = \beta_1 x_{i1} + \dots + \beta_p x_{ip} + e_i, \quad i = 1, \dots, l \quad (1)$$

Equation (1) can be written in vector form as $Y = \beta X + e$ where β is also called the *regression coefficients* which is a rate of change of a conditional mean of variables Y with respect to variables X . The linear regression model is the statistical estimation that uses least squares to minimize the *sum of square residuals* in (2) to obtain β .

$$S(\beta) = \sum e^2 = e'e = (Y - \beta x)'(Y - \beta x) \quad (2)$$

The minimum of $S(\beta)$ can be obtained by setting the derivative of $S(\beta)$ equal to zero and β can be defined as:

$$\beta = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^n e_i^2(\beta) \quad (3)$$

However, it is unlikely that the sum of the predicted values, $\sum Y'$ equals to the sum of the observed values, $\sum Y$ (neither the mean of the predicted values, \bar{Y}' equals to the mean of the observed values, \bar{Y}). Therefore, the *good-of-fitness* values, which is derived from the sum of square for linear regression, $\sum(Y - \bar{Y})^2$ and the sum of squares of residuals, $\sum(Y - Y')^2$, is determined to evaluate the relationship between predicted and observed values. This measure is called *correlation coefficient* (see (7)). More details of a general LR approach can also be found in [8].

B. Sensitivity Computation through Linear Regression

The use of sensitivities for voltage instability detection has been proposed previously, see for example [6], [9]. In this study a similar approach is adopted; however, individual components of sensitivities are constructed from the power flow in each transmission line (one direction) instead of injected power flow of the bus (summation of power flow). Moreover, the lines' power flow are calculated solely based on measured voltage and current phasors. This means that the network topology and dynamic models are not required.

The transmitted power on the line can be expressed as:

$$\begin{aligned} \bar{S}_{mn} &= \bar{V}_m \bar{I}_{mn}^* \\ P_{mn} &= \operatorname{Re}(\bar{S}_{mn}), \quad Q_{mn} = \operatorname{Im}(\bar{S}_{mn}) \end{aligned} \quad (4)$$

where

\bar{V}_m = voltage phasor at Bus m .

\bar{I}_{mn} = current phasor from Bus m to Bus n .

\bar{S}_{ik} = complex power transmitted from Bus m to Bus n .

P_{mn} = transmitted real power from Bus m to Bus n .

Q_{mn} = transmitted reactive power from Bus m to Bus n .

To apply the LR method for calculating sensitivities, a rolling window of measured voltage and current phasors (from available PMU buses) will be used to find the *regression line*. Consequently, noise, outliers and oscillations which are produced from fast dynamic components in power system are assumed to have finite variance and are homoscedastic. The LR method to compute sensitivities for voltage instability detection proposed here follows the next steps:

Step 1: Gather voltage & current phasors from all buses with PMUs.

Step 2: Calculate active and reactive powers from the gathered voltage and current phasors using (4).

Step 3: Find the linear correlation (as in (1)) of measured voltage magnitude and calculated powers:

$$V_{m_{t_i}} - V_{m_{t_{i-1}}} = a_i(t_i - t_{i-1}) + b_i \quad (5)$$

$$P_{mn_{t_i}} - P_{mn_{t_{i-1}}} = c_i(t_i - t_{i-1}) + d_i \quad (6)$$

The linear correlation of measured voltage and calculated active power at time instant i (i.e. t_i) are a_i , b_i and c_i , d_i , respectively. The length between t_i and t_{i-1} (so called t_{window}) is selected from the shortest discrete change created by system's components, e.g. smallest time delay between two tap positions of the OLTC.

Step 4: Check the *good-of-fitness* of the measured voltage by calculating the correlation coefficient ($R_{voltage}^2$):

$$R_{voltage}^2 = 1 - SS_{res}/SS_{tot} \quad (7)$$

where

$$\text{Total sum of squares: } SS_{tot} = \sum_{h=t_{i-1}}^{t_i} (V_{m_h} - \bar{V}_m)^2.$$

$$\text{Total sum of square residuals: } SS_{res} = \sum_{h=t_{i-1}}^{t_i} (V_{m_h} - V'_{m_h})^2$$

The value of $R_{voltage}^2$ lies in the [0,1] range. If the value of $R_{voltage}^2$ is small for a time larger than the window size, it means that this window size is too large to capture the linear behaviour of the physical system. Here, if the $R_{voltage}^2 < 0.3$ for $t > t_{window}$, the window size should be reduced to increase the $R_{voltage}^2$ value.

Step 5.1: Replace the oldest measurement by the newest one, maintaining the window size, i.e. t_{window} .

Step 5.2: Check the variation in powers, i.e. $\operatorname{abs}(P_{mn_i} - P_{mn_{i-1}}) < k * \epsilon$, $c_i = c_{i-1}$ where k is a sensitivity factor. The selection of k depends on the *Signal-to-Noise Ratio* (SNR). The lower the SNR (higher noise), the larger k value.

Step 6: Calculate the sensitivity, $\Delta V_i/\Delta P_{mn_i}$:

$$\frac{\Delta V_i}{\Delta t_i} * \frac{\Delta t_i}{\Delta P_{mn_i}} = \frac{a_i}{c_i} \quad (8)$$

Similar expressions can also be derived for $\Delta V_m/\Delta Q_{mn_i}$. Fig. 1 shows the flowchart for sensitivities computation using the proposed LR method.

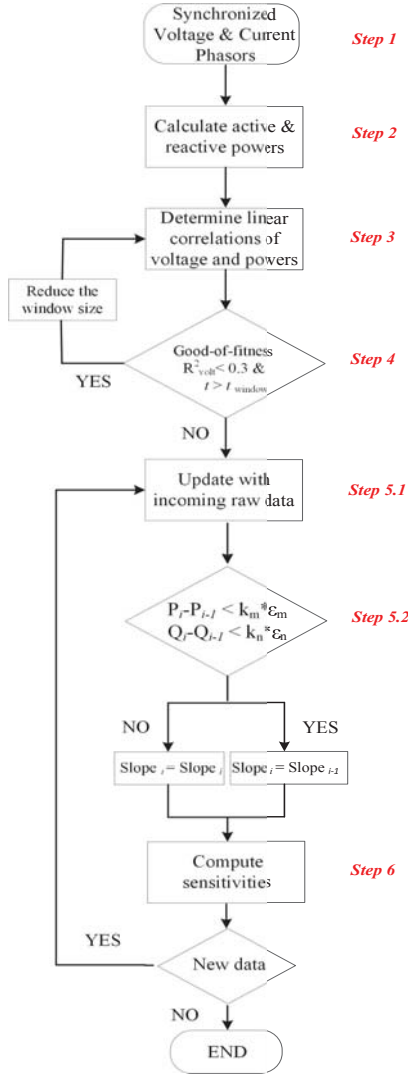


Fig. 1. Flowchart for sensitivities calculation using the LR method

III. DEMONSTRATION

A. Test System and real PMU data

The proposed methodology is applied to simulate phasors from the test system in Fig. 2 (Scenario 1 and 2 in Section III-B) and to PMU data gathered from the Norwegian Grid (in Section IV). The test system includes a salient-pole synchronous generator equipped with a simplified IEEE ST1A excitation system [10], a standard speed governor and a simplified linear model for the hydraulic turbine [11]. The generator is connected at Bus 1 to supply the loads through a 13.8/230 ratio transformer. An Over Excitation limiter (OEL)

is modelled to protect the generator from high excitation levels. An On-Load Tap Changer (OLTC) of 8-tap positions is equipped at Bus 4 to maintain the voltage of the constant power load at Bus 5.

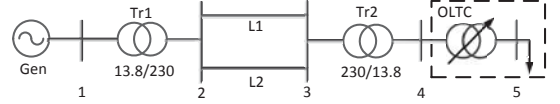


Fig. 2. Test system

B. Study Cases

The voltage instability scenarios used for demonstration are implemented by increasing the load at Bus 5 of the test system shown in Fig. 2. The load is assumed to change as:

$$P_L = P_{Lo}(1 + \lambda) \quad (9)$$

where P_{Lo} is the initial active power and λ is varying parameter representing the active power loading factors. The constant active power load is increased continuously at a rate of **0.1 MW/s**. The impact of system components which lead to voltage instability is studied under two different scenarios for the following cases:

- Case 1: Only OLTC activation
- Case 2: Only OEL activation
- Case 3: OLTC & OEL activation

Scenario 1 – Without random load variations (process noise): In this test scenario, a random load variation (process noise) is **not** applied to the load. This means that SNR equals infinity. The active power load at Bus 5 starts to increase at $t = 50 \text{ sec}$ until the system collapses. Fig. 3 shows the voltage at Bus 5 for three cases when the load is increased. It can be seen that the OLTC (for Case 1 and 3) tries to restore the voltage at the load bus within its deadband [12]. Since the load increases monotonically, the OLTC unsuccessfully attempts to restore the load bus voltage, until it reaches its lower limit. Meanwhile, the OEL (for Case 2 and 3) detects a large field current resulting from the load increase and the generator's voltage is no longer controlled when this current passes beyond the design limit. It can be seen that when OEL reaches its limit, the voltage at Bus 5 in Case 3 declines at a steeper rate (at $t = 130 \text{ sec}$ and $t = 115 \text{ sec}$ for Case 2 and 3, respectively) and the test system collapses sooner (at $t = 145 \text{ sec}$) compared with Case 1 (at $t = 220 \text{ sec}$) where the OEL is not activated.

The activation of OEL and OLTC events can be detected by checking the reduction of R^2 value of measured voltage (a_i). As shown in Fig. 4, it is obvious that these discrete events weaken the linear correlation of measured voltage (a_i), consequently R^2 is drastically reduced. The R^2 value returns to high value (≈ 1 pu, in these test cases) when discrete event is not in the rolling window. In other words, the R^2 value returns to ≈ 1 when the old measurement which contains discrete events are replaced by non-discrete ones (i.e. when OEL and OLTC is not activated).

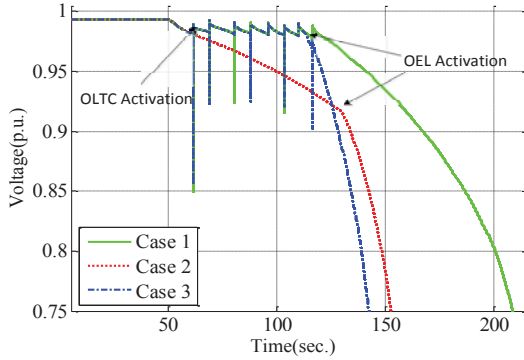


Fig. 3. Voltage at Bus 5 for three cases

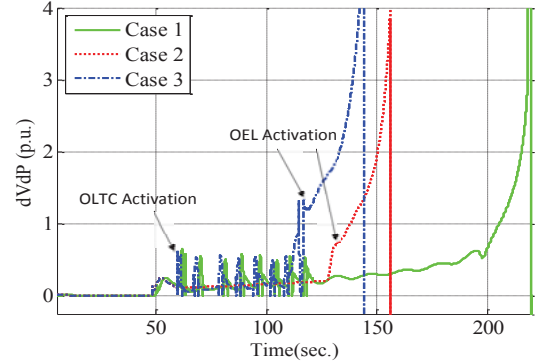
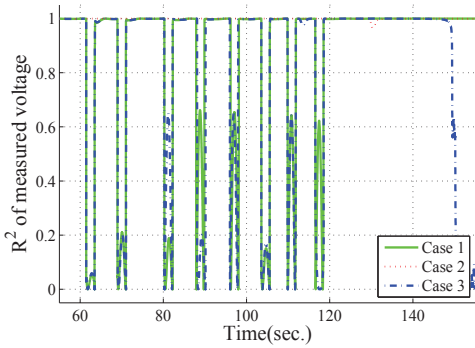
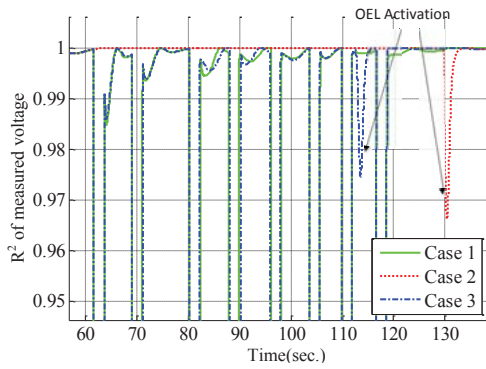


Fig. 5. $\Delta V_5/\Delta P_{45}$ for three cases with non-noised load



(a) R^2 of measured voltage (R^2_{volt})



(b) Zoom of Fig. 4a

Fig. 4. R^2_{volt} for three test cases

Fig. 5 shows the $\Delta V_5/\Delta P_{45}$ for the three cases. The spikes shown in the green and blue lines in Fig. 5 correspond to changes in the OLTC tap position in Case 1 and 3, respectively. There are no visible spikes in the results for Case 2 (the OLTC is not included) but there is an abrupt change in $\Delta V_5/\Delta P_{45}$ caused by OEL activation at $t = 130$ sec (similar to Case 3 at $t = 115$ sec). This means that the proposed method by adopting measurements to calculate sensitivities (in this case, $\Delta V_i/\Delta P_{ik}$) is capable of predicting not only voltage instability but also all discrete events created by system components such as OEL and OLTC.

Scenario 2 – With random load variation (process noise):

Similar experiments as those in Scenario 1 are conducted, however, a random white noise component is added to this load. It is worth noting that there are two types of noise; process noise and measurement noise. Both types of noise can be modeled as Gaussian distribution if they are independent and identically distributed (iid). This means that they are mutually independent and with normal probability distributions. The difference between process noise and measurement noise is that the process noise refers to uncertainties in the system being controlled. Process noise influences directly the system's stability, i.e. driving electromechanical variations and, consequently changing the dynamics of the system. On the other hand, measurement noise relates to the sensitivity of monitoring equipment, such as sensors and instrumentation. In this paper, only process noise is used to model random changes in the load as in (10) [13] where $f(x_{t-1})$ is the load function (represented in (9)) from the previous time step and w_{t-1} denotes process noise with Gaussian random variable.

$$x_t = f(x_{t-1}) + w_{t-1} \quad (10)$$

Measurement noise is not modeled, instead actual PMU measurements are used (in Section IV) to assess the performance of the proposed algorithm under realistic measurement noise. Fig. 6 and 7 show the load increment and voltage profiles (Case 3) of this test scenario, respectively. It is worth noting that since the simulation is performed using simulink/SimPowerSystems EMTP-type models, the obtained results contain outliers (the green dashed lines as seen in Fig. 6) due to the switching of discrete devices (i.e. OLTC).

Fig. 8 shows the plot of R^2_{volt} for load with and without random variations. It can be seen that the R^2_{volt} value of the random load case is lower compared to the non-random's one. However the response of the R^2_{volt} value for both scenarios preserve its important behavior when the OEL and OLTC is activated. Fig. 9 illustrates the $\Delta V_5/\Delta P_{45}$ sensitivity for different SNR values in Case 3. Observe that the sensitivity can detect all activities, load increment at $t = 50$ sec, all 8-tap position steps, and OEL activation at $t = 115$ sec, leading to the collapse ($t = 150$ sec). Fig. 10 depicts the $\Delta V_5/\Delta P_{45}$ for

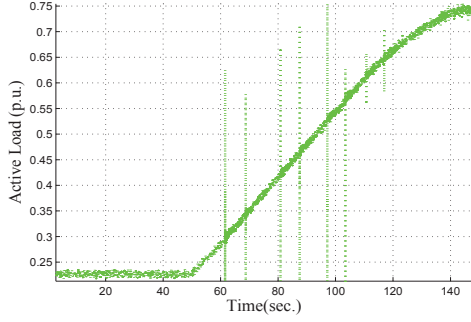


Fig. 6. Load with random variations (process noise)

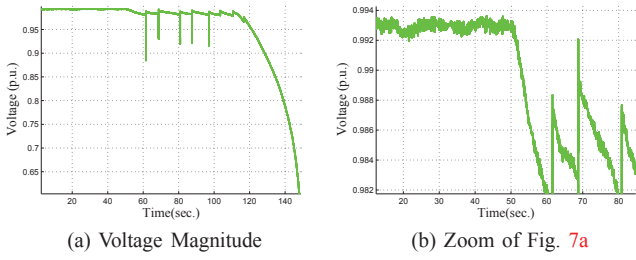


Fig. 7. Voltage at Bus 5 under process noise influence

three cases with the random load variation is increased. As seen in Fig. 8 to 10, it can be concluded that the proposed method for computing sensitivities can be effectively applied when the test system is subjected to process noise.

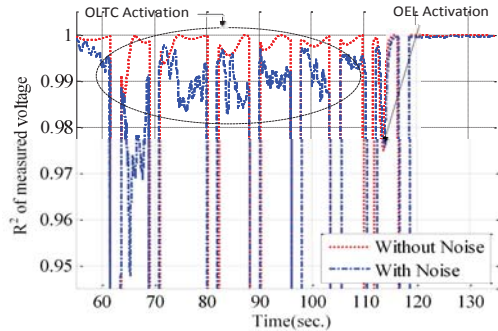


Fig. 8. R^2_{volt} for load with and without process noise (SNR=10 and SNR=inf)

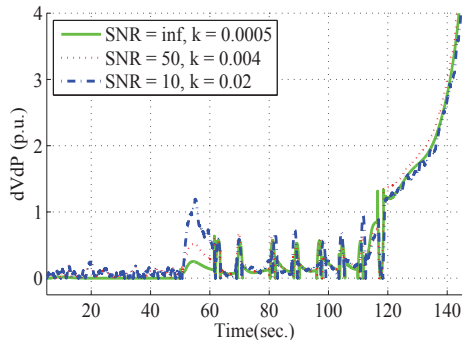
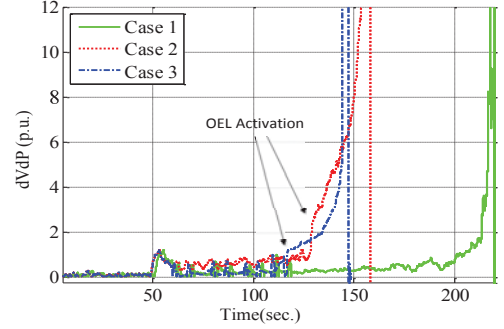
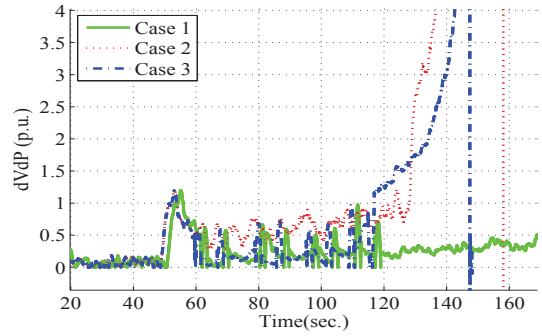


Fig. 9. Case 3: $\Delta V_5/\Delta P_{45}$ for different SNR values



(a) $\Delta V_5/\Delta P_{45}$ for three cases



(b) Zoom of Fig. 10a

Fig. 10. $\Delta V_5/\Delta P_{45}$ for three cases with process noise (SNR = 10)

IV. APPLICATION TO REAL PMU DATA FROM THE NORWEGIAN GRID

In this section, PMU data from January, 2010 was gathered from the Norwegian grid. Fig. 11 shows the approximate location of PMUs installed at high voltage substations.

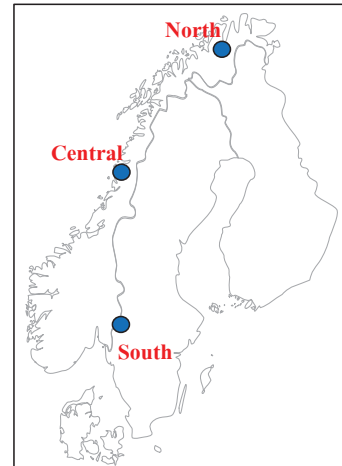


Fig. 11. Approximate PMU locations in the Norwegian Grid

The voltage at the “North” substation started to drop at 16:07 hrs. The OLTC was activated twice to step up the voltage level at 16:12 hrs and 16:15 hrs. However, since the voltage was below allowable limit (≈ 0.93 pu), an industrial load was manually disconnected to prevent a voltage instability. Fig. 12 and 13 depict the voltage profile at the three substations and

the calculated $\Delta V_{m_i}/\Delta Q_{mn_i}$ ¹ sensitivity from the proposed method, respectively.

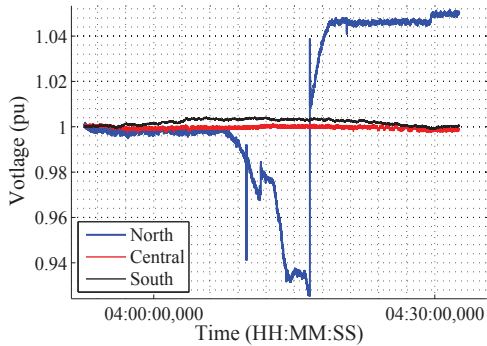


Fig. 12. Measured voltage from three high voltage substations

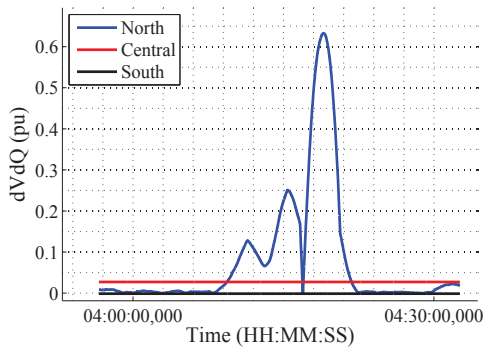


Fig. 13. $\Delta V_{m_i}/\Delta Q_{mn_i}$ for the near-voltage instability in the Norwegian Grid

Observe that the calculated sensitivity, which takes into account both process noise and the measurement noise from actual PMUs, can detect the OLTC's activations and the load disconnection. The authors suspect that the $\Delta V_{m_i}/\Delta Q_{mn_i}$ sensitivity of the "North" substation would have increased steeply if the load was not shedded. In addition, it is worth noting that the sensitivity decreases after the load disconnection and it remains at a positive value which means that the system is stable. A threshold on the sensitivity can be used to activate load shedding mechanisms to prevent voltage instability [14].

V. CONCLUSION

Utilizing the approach proposed in this paper, an impending long-term system collapse can be determined directly from synchrophasor data, in which errors and unnecessary features embedded in measurements can be properly treated. A linear regression method for computing sensitivities was adopted to detect voltage instability, using a rolling window of PMU measurements. It has been shown that this methodology is simple and it can be effectively applied when both process and measurement noise are present. Moreover, activations of system components that capture voltage instability phenomena

¹ ΔQ_{mn} is calculated from the measured voltage and current phasors in the line towards the large industrial load.

[15] (i.e. OEL and OLTC) can be detected efficiently. There are two main advantages offered by this method: network topology and dynamic models are not required. This is beneficial when dealing with large networks and real-time applications where the computational time to decide preventive or corrective actions is critical. Secondly, difficulties related to filtering techniques for preprocessing, such as several cutoff frequencies to be considered in big systems (inter-area and local oscillations) and filter order, are avoided. More importantly, intrinsic time delays from using filtering methods are eliminated.

The ultimate goal of determining sensitivities is not only to use them for wide-area monitoring but also to enable preventive actions and to facilitate cooperation with other controllable devices such as HVDCs [16] and protection systems. Such coordination could use these "defense signals" to activate a number of devices simultaneously and will be the subject of a future publication.

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