Effects of Forced Oscillations in Power System Damping Estimation

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Abstract— **This article analyzes the impact of forced power system oscillations on mode damping estimation. Parametric (Yule-Walker) and non-parametric (Welch) methods for mode estimation are tested in the presence of forced power system oscillations. For mode damping estimation based on non-parametric methods, an application of Half Power Point method is proposed. Performances of the mode estimators are evaluated using both simulated and *real* synchrophasor data from the Nordic Grid. The presence of forced oscillations poses difficulties to mode damping estimators, these difficulties are identified, illustrated and explained herein.**

Index Terms—**inter-area oscillations, mode estimation, synchrophasor, mode meter, ambient data, Nordic power system**

I. INTRODUCTION

Real-time monitoring of power system oscillations is of great significance for power system operators; to this aim, software solutions (algorithms) are used for real-time oscillation monitoring, and are commonly referred to as *mode meters* [1, 2]. Mode meters continuously perform digital signal processing on measured synchrophasor signals to provide estimates of mode properties, i.e. mode frequency, damping, and mode shape. Measured synchrophasor signals are composed of slow changing daily patterns[1], superimposed with a system response to ambient noise (ambient response) which is mostly result of random load variations. A frequency spectrum of the system response carries valuable information about system modes (eigenvalues). There are two main groups of algorithms for extraction of mode information from ambient responses:

- *Non-parametric* methods are based on the Discrete Fourier Transform (DFT) computation of the autocorrelation sequence of the random process ("ambient response"). Algorithms from this group are robust and usually straight forward for implementation and tuning [3, 4]. They usually provide only information about the frequency of the modes of oscillation.
- *Parametric* methods rely on a stochastic model of the random process. The usage of this model is advantageous in the case when more information about the process is available, however this requires more effort for proper tuning [4, 5, 6].

Electromechanical oscillations in power systems can be classified into two main groups: 1) "Natural" system mode oscillations and 2) forced oscillations. "Natural" system mode oscillations are excited by random load fluctuations which represent continuous small disturbances to the system. These disturbances excite poorly damped modes causing permanent low amplitude oscillations in the system. Forced oscillations are significantly less investigated in the literature and they are the result of different phenomena in power systems, such as cyclic loads, control loops in power plants, diesel generators etc. [7, 8, 9, 10]. It can be said that this type of oscillations are not result of the general dynamics of the power system, rather they are caused by particular system elements with a distinctive oscillatory behavior.

Mitigating potentially dangerous undamped oscillations requires different control actions in the case of electromechanical modes or forced oscillations. Therefore, mode meter algorithms should be able to clearly distinct these types of oscillations and estimate only "natural" electromechanical modes while forced oscillations, if necessary, should be reported to the operator [11].

The mode meters algorithms reported in literature do not consider existence of the forced oscillations in the system [12, 2]. In this paper we analyze effects of forced oscillations on mode and damping estimation. Two mode meter algorithms, based on the Yule-Walker and Welch-Half Power Point (HPP) spectral estimators, are compared in the presence of forced system oscillations [3, 4, 5]. In the case of *real* synchrophasor data from the Nordic Grid, the Yule-Walker and Welch-HPP methods are compared with the well-known Prony method [13, 14]. The application of the Half-Power Point Method [15] for damping estimation in combination with Welch's method is proposed here, something which was not possible previously using non-parametric estimators. This property can be beneficial in the case when non-parametric estimators provide better results than parametric estimators, usually because of poorly tuned parametric methods or the presence of forced oscillations.

The paper is organized as follows: In Section II, forced oscillations and their spectral characteristics are described. The methodology used for mode estimation is described in Section III, while results of the performed analyzes are given in Section IV. Section V provides conclusions on the effect of forced oscillations on damping estimation.

II. FORCED OSCILLATIONS

Forced oscillation phenomena in power system have sporadically appeared in the literature over the last 40 years [7]. There are different sources of these oscillations. Xuanyin et al. [9] report that the regulation system of steam turbines can cause this kind of oscillatory behavior. Other authors have investigated the impact of cyclic loads in the system [7, 8]. Vournas et al. [10] report diesel generators as one of the possible causes for forced low-frequency oscillations.

Regardless of cause, all types of forced oscillations have some common characteristics which can be used for their identification. Considering that a forced oscillation is a permanent oscillation with a specific frequency, its spectrum is

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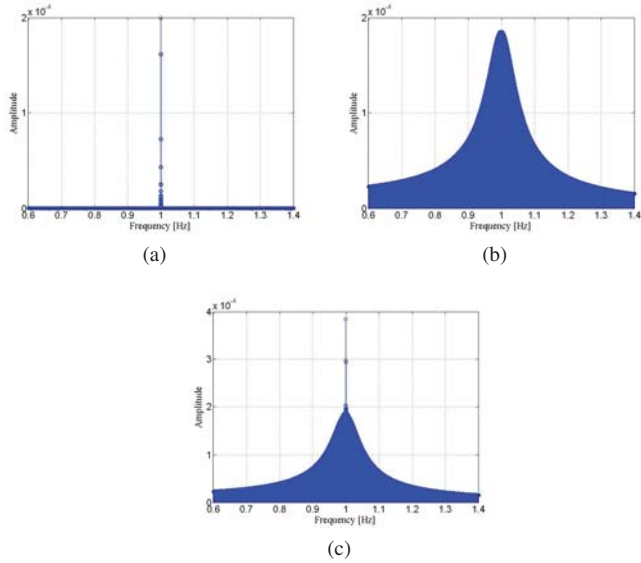[1]Daily load dispatch and load/governor dynamics.

Fig. 1. FFTs computed with simulated data showing (a) forced oscillation (b) damped sinusoid (c) combination of a damped and forced oscillation.

characterized by a very narrow high amplitude peak. This can be concluded from the fact that undamped sine signal is represented by the Dirac delta function in frequency domain. Therefore, a forced oscillation will have most of its spectral content concentrated in one small frequency bin, whereas for a system mode, the spectral content is spread around the main peak (Fig. 1). Observe in Fig. 1a how the spectral response of a forced oscillation at 1 Hz has all of its spectral content in a single frequency bin, while a damped oscillation has a more spread spectral content centered at 1 Hz as shown in Fig. 1b. Finally, it is interesting to observe in Fig. 1c the resulting spectrum from the combination of the two above oscillations. This is similar to the spectrum which one should expect from a forced oscillation overlaying a *true* system mode. It is important to understand this characteristic when estimating power system damping, as explained later. This behavior can be seen in *real* ambient data recorded in the Nordic power system. Fig. 3 shows the computed frequency spectra of four frequency measurements, where narrow peaks in spectra (forced oscillations) can be easily identified. The primary data used for this analysis originates from 24 hours of continuously archived measurements obtained at four different locations in Nordic power system during 2011 (see Fig. 2).

III. MODE DAMPING ESTIMATION

Performing spectral estimation is the most important step in the mode estimation, and it requires special attention from engineers in tuning and data processing.

There are many different methods for mode damping estimation and most of them require a mathematical model of the system. From such a model the complex eigenvalues $\lambda_i = \sigma_i \pm j\omega_i$ can be obtained and the damping ratio of the corresponding i-*th* mode can be calculated as:

$$\zeta_i = \frac{-\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}} \tag{1}$$

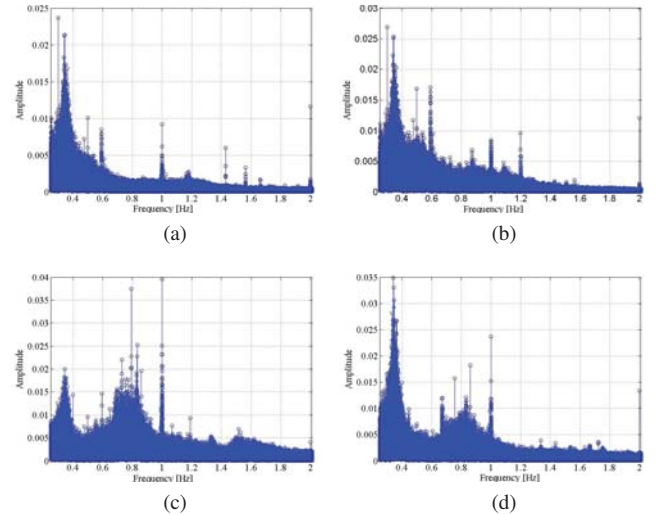Fig. 2. Nordic Power System with locations of PMU measurements

Fig. 3. FFTs containing forced oscillations, computed with 24h of data from (a) Hasle (b) Fardal (c) Alta (d) North1.

However, the process of constructing a highly detailed and accurate model of a complex system, such as a power system, can be very time consuming, and if not done with sufficient precision, may still yield to unsatisfactory results.

Another approach that is frequently used for damping estimation is fitting a transient (or ringdown) response $y(t)$ to a simple function $\hat{y}(t)$ of the following form:

$$\hat{y}(t) = \sum_{i=1}^{P} R_i e^{\lambda_i t} \tag{2}$$

The result of this fitting are eigenvalues, λ_i, and mode amplitudes R_i, whereas damping is calculated using Eq. 1. These methods have limitations, not only because they depend on the existence of transients in the data, but also they depend on
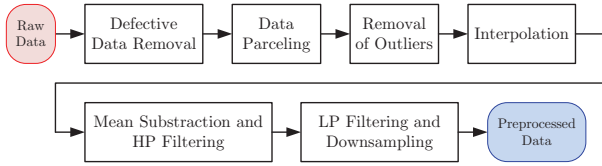
Fig. 4. Preprocessing algorithm

sufficient excitation of the modes of interest.

In the following sections Prony's method is used with the transient data and obtained results (mode frequencies and damping ratios) are used in the comparisons as a reference (accurate) values. On the other hand, Welch's and Yule-Walker spectral estimators use ambient data for continuously tracking of modes properties.

A. Preprocessing

It is not uncommon for synchrophasors to have data quality issues. Prior the spectral estimation, the data should be curated to remove flawed, redundant and irrelevant data. This data preparation is performed by a preprocessing algorithm (Fig. 4) which ensures that consistent data sets are used in mode estimation.

The first step in the preprocessing algorithm is a removal of obviously erroneous samples. This is the case with samples whose time-stamp or measured value is not updated in the new cycle. In the second step, data is split into parcels which are processed separately (Data Parceling in Fig. 4).

In order to obtain a smooth shape of the measured signal, samples whose values deviate more than 3 standard deviations form expected value (mean in the analyzed data block) are removed and missing samples are replaced by interpolated values (Removal of outliers and Interpolation in Fig. 4).

System frequency control causes slow gradual changes in the system's states and consequently in the measured signals (the mean value changes over time). These slow changing components have to be removed to obtain the stochastic signals required for spectral analysis. These components can be efficiently attenuated by a high-pass filter with 20 mHz cut off frequency (Mean Subtraction and HP Filtering in Fig. 4).

The last step in the preprocessing algorithm is low-pass filtering and downsampling (Fig. 4). Sampling rates that are significantly higher than the range of electromechanical dynamics (up to 2 Hz) are filtered out. This results in redundant data and slower computations. In addition to larger computation efforts, the autocovariance matrix which is used in parametric methods, has higher dimensions and is likely to become ill-conditioned. To avoid these problems signals are downsampled to 5 Hz (from original 50 Hz of synchrophasor data).

B. Spectral Estimators

In this paper, two spectral estimators are used:

- *Yule-Walker method (YW)*. This method is the most common spectral estimator from the group of parametric methods. The main assumption used in the Yule-Walker method is that a random process (ambient response) can be accurately described by an Auto-Regressive model. Once the model of the process is determined, computation

of mode frequencies and damping ratios is very straight forward.

- *Welch's method*. This is a non-parametric method i.e. it does not assume any model of the process for which the spectrum should be estimated. These types of methods are advantageous in the case where a model of the random process is not easy to determine, but their disadvantage is that they do not allow straight forward computation of damping ratios. In order to overcome this issue, in this paper, the Half Power Point method for damping ratios is applied.

C. Half-Power Point Method (Welch-HPPM)

By examining the peak width of a mode in the frequency domain the damping can be estimated, a narrow peak means a poorly damped mode while a wide peak means that the mode is well damped. The half-power point method states that the distance between the two half-power points surrounding the peak center is roughly proportional to the mode's damping.

The method assumes that the frequency response of the system can be modeled as:

$$|H(\omega)| = \frac{1}{\sqrt{(1 - (\frac{\omega}{\omega_n})^2)^2 + (2\zeta\frac{\omega}{\omega_n})^2}} \quad (3)$$

where ω_n is the frequency of the analyzed mode (peak in the spectrum) and the damping ζ is calculated using Eq. 4 which is derived from Eq. 3:

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_n} \quad (4)$$

were $|H(\omega_1)| = |H(\omega_2)| = \frac{1}{\sqrt{2}}|H(\omega_n)|$.

In this paper the half-power point method has been applied to spectral estimates computed by Welch's spectral estimator.

IV. EFFECT OF FORCED OSCILLATIONS IN SIMULATED AND REAL SYNCHROPHASOR DATA

A. Analysis using Simulated Data

1) Simulated System and Damping Estimation: The simplified seven bus model of the Southern/Southeastern Brazilian power system shown in Fig. 5 is used for analyzes of forced oscillation effects on mode meter algorithms. The model includes the Itaipu hydro power plant, which is connected by a 765 kV line to the load area. Four 500 kV buses (Buses 1-5), which form a ring with 3 generators, are connected to the Itaipu power station by a 765kV tie-line (line between Bus 5 and Bus 6). All generators are modeled with a fifth order model, and include controls. All model parameter data can be found in [16].

A classical small signal stability study reveals one inter-area mode with frequency of 0.8308 Hz and damping ratio of 6.9316%. These values will be used as references in analyzes of mode meter algorithms below[2].

Ambient data is simulated by adding uniformly distributed pseudo-random values (white noise[3]) in all load buses in the

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[2]While this test system includes other modes, this study is restricted to the 0.8308 Hz inter-area oscillation.

[3]Uniformly distributed pseudo-random valuses are drawn from the standard uniform distribution using Matlab's `rand` command.
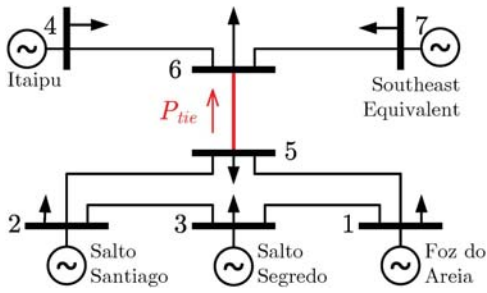
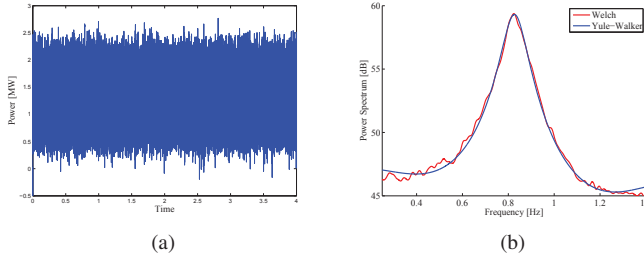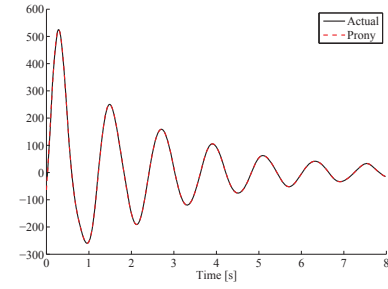Fig. 5.   Single-line diagram of the simulated power system

Fig. 7.   Simulated transient and corresponding Prony estimate

TABLE I
MODE FREQUENCY, DAMPING AND VARIANCE OF DAMPING ESTIMATORS
COMPUTED FROM SIMULATED DATA

Method \ Mode	Frequency[Hz]	Damping[%]	Variance
Expected Value	0.8308	6.9316	-
Yule-Walker	0.8277	6.7001	0.3151
Welch-HPPM	0.8272	6.9364	0.2807
Prony	0.8308	6.9325	-

(a)   (b)

Fig. 6.   (a) 4 h of simulated data, (b) Welch and Yule-Walker spectral estimates for a 10 minutes parcel of pre-processed data.

system. A signal of the tie-line active power, P_{tie} is sampled with frequency of 50 Hz and used as an input to mode meter algorithms (Fig. 6a).

Spectral estimates of the tie-line active power, computed by Welch's and Yule-Walker methods, are shown in Fig. 6b. The spectral estimates are computed with a parcel of 10 minute of simulated data (3000 samples after downsampling). The Fast Fourier Transform length of 512 samples is adopted for Welch's method. In the case of the Yule-Walker method, a random signal is fitted to a low order Auto-Regressive model. A dominant 0.83 Hz pole can be clearly seen in both spectra.

A computation of mode damping ratio is carried out using Prony's method. A transient is generated by applying instant active power load changes in all nodes of the system. The transient response of the system and response of the Prony model are shown in Fig. 7. Note that adequate matching is obtained, leading to a accurate mode and damping estimation using Prony's method.

Results of all three methods used for mode and damping estimation are shown in Table I.   Prony's method shows the best estimate, whereas Welch's method gives accurate estimation. From these results it can be concluded that the non-parametric method (Welch-HPPM) gives more robust results in the case when the random process (ambient response) is not appropriately modeled. In this case this means that the Auto-Regressive model is not fully in accordance with the physical processes. Another important note to make is that parametric spectral estimators will require an adjustable model order in order to be able to handle the changes in system dynamics that occurs in a power system.

2) Real-time mode meter application: For real-time operation, mode and damping ratio estimates must be updated continuously. When updating estimates, data parcels are allowed to overlap in order to perform computations more frequently. In the performed simulations, 10 min. data parcels are used, whereas overlapping ensures that results are updated every

minute. Results of damping estimates through time are shown in Fig. 8.

A variance of estimated damping is slightly lower in the case of Welch's method (0.2807 comparing to 0.3151 for the Yule-Walker method) which shows more reliable estimation in the case of Welch's method.

3) Effect of Forced Oscillations in the Simulated Data: Forced oscillations will, if located nearby a system mode, degrade the accuracy of damping estimators. In this section three characteristic cases are analyzed: 1) Case with no forced oscillation in the system, 2) Case when the frequencies of the forced oscillation and the inter-area mode are the same (forced oscillation is superimposed on the inter-area mode) and 3) Case when the frequency of the forced oscillations is "close" to frequency of the inter-area mode (it is adopted that forced oscillation is "close" to a system mode if frequency difference between them is less than 0.2 Hz).

Spectral estimates in these three cases are shown in Fig. 9., from which following can be concluded[4]:

a. No forced oscillation. In this case, both estimators yield estimates close to the reference value.

b. Superimposed forced oscillation at frequency of 0.83 Hz[5]. In this case, both estimators have decreased damping ratios.

   An increase in peak height results in a decrease of the distance between the half-power points. According to Eq. 4, a small damping ratio is decreased further.

   For the Yule-Walker model there is a decrease in the real part of the eigenvalues, while the imaginary part is relatively unaffected. Consequently, the damping ratio is decreased.

c. Forced oscillation located at 1 Hz[6]. In this case, both estimators have increased damping ratio.

   In general, a forced oscillation located close to the mode peak causes the width of the mode peak to grow. For

[4]For consistency, the model-order of the YW model is fixed in all tests, as well as the block size of the Welch method.

[5]Matching of forced oscillation frequency and electromechanical mode frequency.
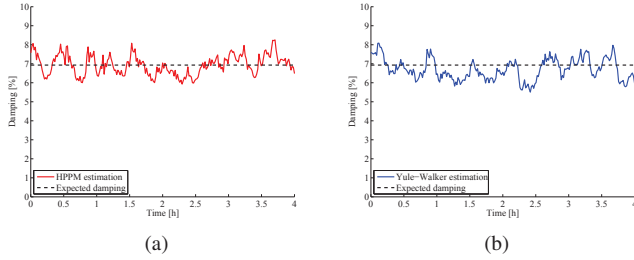
[6]Close to the 0.83 Hz inter-area mode.

Fig. 8. Variation in estimated damping for 4 h of simulated ambient data. (a) Welch-HPPM, and (b) Yule-Walker

TABLE II
EFFECTS OF FORCED OSCILLATIONS (F.O.) ON DAMPING ESTIMATES COMPUTED FROM SIMULATED DATA

	Without F.O.	Superimposed F.O.	F.O. close to the inter-area mode
Yule-Walker	-	Moves real part to the right	Moves real part to the left
Welch-HPPM	-	Increased peak and reduced distance between Half-Power Points	Decreased peak and increase distance between Half-Power Points
Results	Correct	Lower damping	Higher damping

Note: F.O. refers to Forced Oscillation.

Fig. 10. Transient captured from the PMU at Fardal and corresponding Prony Estimate

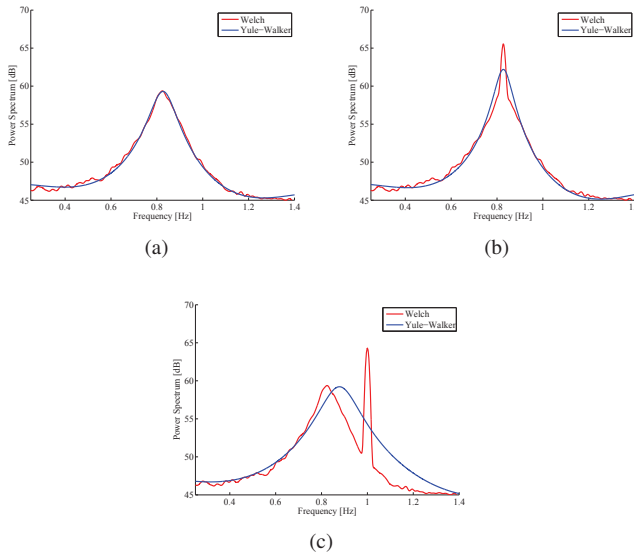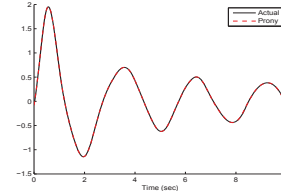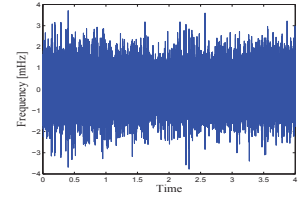Fig. 11. Four hours of pre-processed synchrophasor data from Fardal

Fig. 9. Spectral estimates with (a) no forced oscillation (b) superimposed forced oscillation (c) forced oscillation located close to the inter-area mode

the Welch-HPP estimator, the distance between the half-power points is increased and according to Eq. 4 the damping ratio is increased.

The eigenvalues from the Yule-Walker model have an increase in the real part, while the imaginary part is relatively unaffected, and consequently, it yields too higher damping ratios.

The selection of the model order in parametric methods has a great impact on mode estimation accuracy. Selection of a high order mode estimator leads to over-fitting whereas a low order model leads to the low frequency resolution. The problem of the reduced frequency resolution is especially important in the presence of forced oscillations because they have narrow frequency bands, which requires higher frequency resolution. This analysis yields to the conclusion that usage of a non-parametric methods can be beneficial in the systems with forced oscillations.

The results of performed simulations are summarized in Table II:

B. Analysis using Real Sychrophasor Data

The Nordic Grid has two dominant inter-area electromechanical modes. The following analysis focuses on the inter-area mode located at approximately 0.35 Hz. A reliable estimate of the inter-area mode properties can be obtained

using Prony's method with measured transient data (Fig. 10). Numerical results are shown in Table III.

Forced oscillations occur occasionally in the system, what can be seen from the spectrum time evolution of the selected signal [12]. Consequently, damping estimates change over time. Assuming that the inter-area mode properties remain constant during the period of analysis, it is possible to analyze impact of the forced oscillations on mode damping estimation (it is assumed that only forced oscillations cause changes in damping estimation results).

To demonstrate effects of the forced oscillations four shorter data parcels are selected from the 4 hours long synchrophasor signal (Fig. 11). During these four data parcels, different forced oscillations occurred in the system causing different damping estimations. Computed spectrums of the four analyzed cases are shown in Fig. 12 where following effects can be noticed:

1. No forced oscillations. The data parcel in this case provides a smooth spectrum with a clear peak at the frequency of the dominant inter-area mode. Both methods provide good damping estimates (Half Power Point Method: **6.0860 %**, Yule-Walker: **5.9714 %**).
2. Forced oscillation at 0.35 Hz . The spectrum of the analyzed parcel has a very narrow peak at the frequency of the dominant inter-area mode. This forced oscillation causes a decrease in the estimated damping ratio (Half Power Point Method: **3.8037 %**, Yule-Walker: **3.8117 %**).
3. Forced oscillation at 0.38 Hz The spectrum of the analyzed parcel has a peak at the frequency of the dominant inter-area mode but this peak is wider comparing to case 1. This forced oscillation causes an increase in the estimated damping ratio (Half Power Point Method: **9.6009 %**, Yule-Walker: **9.522 %**).
4. Insufficient spectral content. In this case, the peak in the spectrum is not easily visible for the mode of interest making damping estimation less reliable. Due to reduced

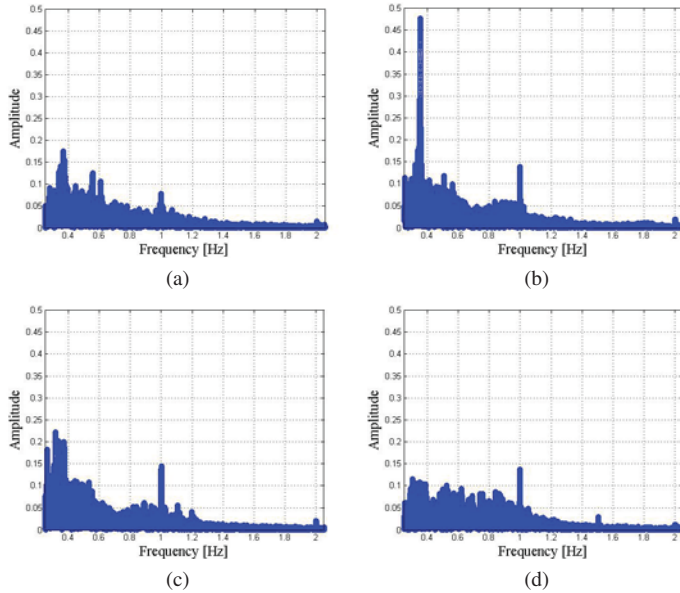| Mode Frequency [Hz]	Damping [%]
0.3561	6.0481

Fig. 12. FFTs computed with 10 min of data from Fardal showing (a) good estimate (b) superimposed forced oscillation (c) forced oscillation located close to mode (d) Insufficient spectral content.

peak height, the estimated damping is significantly higher compared to other cases (Half Power Point Method: **15.7967 %**, Yule-Walker: **15.6316 %**).

V. CONCLUSION

This article analyzes the impact that forced oscillations in PMU data have on damping estimation algorithms, which has been neglected up to now. Several examples are shown where forced oscillations corrupt the accuracy of existing mode meter algorithms. It is shown through simulation studies that the impact of the forced oscillation depends on its location in the frequency spectrum with regards to the inter-area mode whose damping estimate is affecting, and not always decrease the value of the damping estimate (as previously believed [3]).

A forced oscillation superimposed on the inter-area mode will decrease the value of the damping estimate, while a forced oscillation close to the inter-area mode will increase it. This was illustrated through both simulation studies and analysis of *real* PMU data from the Nordic Grid. The most important message to derive from these studies is that prior to the use of mode meters in control centers it is necessary to have a good understanding of the impact of forced oscillations on damping estimates, and that new damping estimation methods that cater to forced oscillations are necessary.

In addition, it is shown that non-parametric spectral estimators are more suitable for spectral analysis in systems with forced oscillations because of their ability to resolve these oscillations as compared to (low order) parametric estimators; another advantage of non-parametric spectral estimators is their comparatively easier tuning. In order to compute mode damping ratios with non-parametric methods, the Half Power Point Method was introduced in this article, being the first method applied for damping estimation using non-parametric estimators.

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