# A Framework for Estimation of Power Systems Based on Synchronized Phasor Measurement Data 

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#### Abstract

This paper presents a new approach for synchronized phasor measurement-based state estimation, which starts from the buses with synchronized phasor data and enables highly accurate estimation of portions of the power system with connectivity to these phasor data buses. Observability analysis is used to assemble the buses for this Phasor State Estimator (PSE). The state estimation is formulated as an iterative leastsquares problem, with bus voltage and line current magnitudes and phases as the estimated variables. An important aspect of this formulation is its ability to correct for constant or random phase biases that may exist in some phasor measurement equipment. Beside being a standalone state estimator, the PSE can enhance the reliability of a conventional state estimator. The PSE methodology is demonstrated for a power transfer path in the New York power system.


Index Terms-synchronized phasor measurements, phasor state estimation, observability methodologies, measurement placement.

## I. Introduction

AS conceived during the 1970's [1], [2], [3], [4] State Estimation (SE) provides static estimates of the system states using measurements from units such as RTUs [2]. Currently SEs use the Intercontrol Center Communications Protocol (ICCP) gathering asynchronously fed data with an arrival rate of 1 sample per 4-10 seconds. Even though there have been great efforts to improve the the detection of bad data and topology errors [2], [3], [5], there is still a need to provide better bad data detection and more accurate models for the external systems. With the advancement of synchrophasor technology there has been interests to use Phasor Measurement Unit (PMU) measurements to enhance the estimates provided by SEs used in control centers [6]. In the case when a PMU is applied to every bus on the system, [7] establishes a noniterative least-squares solution to perform state estimation. Assuming far fewer PMUs, [8] suggests to install PMUs at critical buses to enhance the reliability of conventional SEs. ${ }^{1}$
In this paper, we investigate a phasor state estimator (PSE) implementation in a power system where a number of PMUs have been installed on high-voltage (HV) substations, although not necessarily on every HV substation. This new PSE will be

[^0]built on synchronized phasor data only, and its solutions can be used to supplement a conventional SE based on ICCP data. Such a schematic is shown in Fig. 1. This approach is attractive because the PSE is built independent of the conventional SE, allowing the PSE to be implemented without disrupting the role of a conventional SE in control centers.


Figure 1. Hybrid Sate Estimation
The formulation of the proposed PSE allows us to perform a specific type of phasor angle correction that does not exist in conventional SE. We have observed that in some PMUs, the voltage and current phasor angles can exhibit a constant bias due to the signal processing techniques used to determine the phasors. Some other PMUs exhibit random angle jumps, typically multiples of $7.5^{\circ}$, due to unknown reasons [9]. However, the fact that the same angle shift will show up in all measurement channels in a PMU allows the PSE to correct the angle shifts. This is important because such uncorrected phasor data will cause convergence difficulties if used in conventional SEs.

The remainder of this paper is organized as follows. Section II develops the PSE network model and Section III introduces the state estimation solution algorithm. Angle bias correction is discussed in Section IV, which includes an observability analysis. The application of the proposed PSE to actual PMU data recorded in the New York power system is demonstrated in Section V.

## II. PSE Network Model

We first determine the buses and lines in the PSE network model, and the corresponding equations to calculate the voltages and flows in the PSE model. In the sequel, if a PMU is installed on a bus, we will refer to it as a PMU bus. Otherwise, it is a non-PMU bus.

## A. Model Buses and Lines

Consider a power system with $N$ buses. We label these buses as $i=1,2, \ldots, N$, and their corresponding bus voltage phasors as $\tilde{V}_{i}=V_{i} \varepsilon^{j \theta_{i}}, i=1,2, \ldots, N_{1}$. The measurements
for these voltage phasors are denoted as $\tilde{V}_{i m}=V_{i m} \varepsilon^{j \theta_{i m}}$, $i=1,2, \ldots, N_{1}$.

The PMUs also measure a number $L_{1}$ of line current flows between the buses. The current going from a PMU Bus $i$ to another Bus $k$, whether a PMU bus or a non-PMU bus, will be denoted by $\tilde{I}_{i k}=I_{i k} \varepsilon^{j \delta_{i k}}$, whose measurements will be denoted as $\tilde{I}_{i k m}=I_{i k m} \varepsilon^{j \delta_{i k m}}$. Let the total number of nonPMU buses be denoted by $N_{2}$, and without loss of generality, these buses will be labeled as $i=N_{1}+1, N_{1}+2, \ldots, N_{1}+$ $N_{2}$. In addition, let the total number of lines without phasor measurement connecting the PMU and non-PMU buses be $L_{2}$.

Thus the collection of these $N=N_{1}+N_{2}$ buses and $L=L_{1}+L_{2}$ branches connecting these buses comprises the PSE network model. Note that in this PSE model, a branch connecting two PMU buses, whose current flow is measured on both ends, will be counted twice. This would not cause any problem in the subsequent state estimation process.

An example of such a PSE network is shown in Fig. 2. The voltage phasors are measured at the PMU Buses 1, 2, and 3. These PMUs also measure the current phasors on Lines 1-4, 2-5, 3-6, and 3-7. Thus the non-PMU Buses 4, 5, 6, and 7, and Lines 1-7, 2-4, 2-6, 4-5, 5-6, and 5-7 are also included in the PSE network, which has a total of 7 buses and 10 lines. Note that connections from these 7 buses to other buses are not shown in Fig. 2.


Figure 2. PSE Observable Islands (Network 1)

## B. Observability Analysis

From the PSE network model, we can proceed to determine whether there are observable islands within the model. If every bus in the PSE model is connected to all the other buses via the branches available in the model, there the PSE is a single island. Otherwise, one can use an iterative algorithm to isolate the islands, such as those available in [2], [3]. In the sequel, we will assume that the PSE model consists of a single island,
such that all the voltages of the $N$ buses and the currents on the $L$ branches connecting them will be observable.

## C. Network Model

The network model is developed based on circuit equations, using the equivalent circuit in Fig. 3 connecting any two buses in the PSE network. The branch connecting Buses $i$ and $k$ is represented by the transmission line with impedance $\tilde{Z}_{i k}=$ $R_{i k}+j X_{i k}$ and line charging $\tilde{Y}_{i k}=j B_{i k}$. The current phasor leaving Bus i as $\tilde{I}_{i k}$ is related to the bus voltages and line parameters as

$$
\begin{equation*}
\tilde{V}_{k}=\tilde{V}_{i}-Z_{i k}\left(\tilde{I}_{i k}-\frac{1}{2} Y_{i k} \tilde{V}_{i}\right) \tag{1}
\end{equation*}
$$

Note that there will be $L$ such complex equations for the PSE network. Each circuit equation in complex form can be decomposed into 2 equations, one for the real part and the other for the imaginary part, respectively, as

$$
\begin{equation*}
f_{i k \mathrm{re}}=\operatorname{Re}\left(f_{i k}\right)=0, \quad f_{i k \mathrm{im}}=\operatorname{Im}\left(f_{i k}\right)=0 \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
f_{i k}=\left(1+\frac{1}{2} Y_{i k} Z_{i k}\right) \tilde{V}_{i}-Z_{i k} \tilde{I}_{i k}-\tilde{V}_{k} \tag{3}
\end{equation*}
$$

We define the $2 L$ dimensional vector $f$

$$
f=\left[\begin{array}{llll}
\cdots & f_{i k \mathrm{re}} & f_{i k \mathrm{im}} & \cdots \tag{4}
\end{array}\right]^{T}
$$

of all the lines. Furthermore, the branches with current phasor measurements will occupy the first $2 L_{1}$ rows of $f$.


PMU Node $-\Theta \rightarrow$ PMU Current $\triangle$ Estimated Node Network Node
Figure 3. PMU Bus- $i$ Circuit for the PSE Network Model

## D. Measurement Model

At each PMU Bus the available measurements are $V_{i m}$, $\theta_{i m}, I_{i k m}$, and $\delta_{i k m}$. The measurement equations are divided into voltage measurement equations and current measurement equations.

1) Voltage Measurement Equations: are constructed by using the $V_{i m}$ and $\theta_{i m}$ measurements at each PMU-bus and equating them to their corresponding state resulting in

$$
\begin{equation*}
V_{i}=V_{i m}+e_{V_{i}}, \quad \theta_{i}=\theta_{i m}+e_{\theta_{i}} \tag{5}
\end{equation*}
$$

where $e_{V_{i}}$ and $e_{\theta_{i}}$ are measurement errors.
2) Current Measurement Equations: are constructed by using the $I_{i k m}$ and $\delta_{i k m}$ measurements at each PMU bus for Line $i-k$ as

$$
\begin{equation*}
I_{i k}=I_{i_{k} m}+e_{I_{i k}}, \quad \delta_{i k}=\delta_{i k m}+e_{\delta_{i k}} \tag{6}
\end{equation*}
$$

where $e_{I_{i k}}$ and $e_{\delta_{i k}}$ are measurement errors.
We point out that this magnitude and phase measurement error representation is more appropriate than a rectangular coordinate representation because the magnitude and phase are mostly uncorrelated variables as obtained by PMUs.

To summarize, the total number of unknown variables in the PSE model $2 N$ voltage magnitudes and angles, and $2 L$ line current magnitudes and angles. These unknown variables are arranged as a vector

$$
x=\left[\begin{array}{l}
V  \tag{7}\\
I \\
\theta \\
\delta
\end{array}\right]
$$

where the voltage magnitude and angle vectors are respectively defined as

$$
\begin{align*}
V & =\left[\begin{array}{llllll}
V_{1} & \cdots & V_{N_{1}} & V_{N_{1}+1} & \cdots & V_{N}
\end{array}\right]^{T}  \tag{8}\\
\theta & =\left[\begin{array}{llllll}
\theta_{1} & \cdots & \theta_{N_{1}} & \theta_{N_{1}+1} & \cdots & \theta_{N}
\end{array}\right]^{T} \tag{9}
\end{align*}
$$

and the current magnitude and angle vectors are respectively defined as

$$
\begin{align*}
& I=\left[\begin{array}{lll}
\cdots & I_{i k} & \cdots
\end{array}\right]^{T}  \tag{10}\\
& \delta=\left[\begin{array}{lll}
\cdots & \delta_{i k} & \cdots
\end{array}\right]^{T} \tag{11}
\end{align*}
$$

for all the lines where the measured current magnitudes and angles are placed in the first parts of these vectors. These $U_{T}=2(N+L)$ unknown variables are constrained by $E_{T}=$ $2\left(L+N_{1}+L_{1}\right)$ real equations arising from (1), (5), and (6) of the PSE model. Note that from the construction of the PSE network described in Section II, $L_{1} \geq N_{2}$. Strict inequality occurs when, for example, a non-PMU bus is connected to more than one PMU bus and the current phasors on all the connecting branches are measured. Thus

$$
\begin{equation*}
L+N \leq L+N_{1}+L_{1} \tag{12}
\end{equation*}
$$

and the PSE network is observable. As an example, for the PSE network in Fig. 2, $L_{1}=N_{2}=4$. Thus (12) holds as an equality.

## III. Phasor State Estimation

## A. Least-Squares Formulation

The objective of the PSE problem is to find a set of voltage and current phasors satisfying (1) while minimizing the measurement errors in the measurement equations (5) and (6). In the case $L_{1}=N_{2}$, the number of unknown variables is equal to the number of constrained equations. Thus a unique solution for the unknown variables exists, where the measurement errors are taken to be zero. On the other hand, if $L_{1}>N_{2}$, then the PSE can be formulated as a nonlinear
weighted least-squares (WLS) problem [10], [11]. Define the objective function as

$$
\begin{align*}
& q(x)=\frac{1}{2}( \left\|W_{V} e_{V}\right\|^{2}+\left\|W_{\theta} e_{\theta}\right\|^{2}  \tag{13}\\
&\left.+\left\|W_{I} e_{I}\right\|^{2}+\left\|W_{\delta} e_{\delta}\right\|^{2}\right)
\end{align*}
$$

where $\|\cdot\|$ denotes the magnitude of its vector argument, $e_{V}$, $e_{\theta}, e_{I}$, and $e_{\delta}$ are vectors of the measurement errors in (5) and (6). The weighting matrices in (13) are diagonal matrices given by

$$
\begin{align*}
& W_{V}=\operatorname{diag}\left(\cdots, \min \left(1, \frac{1}{V_{i m}}\right), \cdots\right)  \tag{14}\\
& W_{\theta}=\operatorname{diag}\left(\cdots, \min \left(1, \frac{1}{\left|\theta_{i m}\right|}\right), \cdots\right)  \tag{15}\\
& W_{I}=\operatorname{diag}\left(\cdots, \min \left(1, \frac{1}{I_{i k m}}\right), \cdots\right)  \tag{16}\\
& W_{\delta}=\operatorname{diag}\left(\cdots, \min \left(1, \frac{1}{\left|\delta_{i k m}\right|}\right), \cdots\right) \tag{17}
\end{align*}
$$

Most of the weights will be about unity, except for weights on the current magnitude which can be lower for heavily loaded lines.

The WLS problem can be formulated as

$$
\begin{equation*}
\min _{x} q(x), \text { subject to : } f=0 \tag{18}
\end{equation*}
$$

Because $f$ is a nonlinear function of $V, I, \theta$, and $\delta$, we augment the equality constraint $f=0$ to the objective function $q(x)$ to form

$$
\begin{equation*}
q^{\prime}(x)=q(x)+\frac{1}{2}\left\|W_{f} f\right\|^{2}=\frac{1}{2}\|W h(x)\|^{2} \tag{19}
\end{equation*}
$$

where $W_{f}$ is diagonal matrix of large weights,

$$
\begin{align*}
& h=\left[\begin{array}{ll}
f^{T} & e^{T}
\end{array}\right]^{T}, \quad e=\left[\begin{array}{llll}
e_{V}^{T} & e_{\theta}^{T} & e_{I}^{T} & e_{\delta}^{T}
\end{array}\right]^{T}  \tag{20}\\
& W=\operatorname{block}-\operatorname{diag}\left(\begin{array}{ccccc}
W_{f} & W_{V} & W_{\theta} & W_{I} & W_{\delta}
\end{array}\right) \tag{21}
\end{align*}
$$

Thus the constrained WLS problem (18) is transformed into an unconstrained WLS problem of

$$
\begin{equation*}
\min _{x} q^{\prime}(x) \tag{22}
\end{equation*}
$$

## B. Successive Phasor State Estimation Algorithm

There are a number of approaches to solve the nonlinear WLS problem (22). A Newton algorithm would the second derivative of $q^{\prime}$. Here we use the Gauss-Newton Method, which requires only the first derivative of $q^{\prime}$.

In the Gauss-Newton method, starting from the current value $x_{c}$ of the solution vector $x$, the increment in $x$ is computed as

$$
\begin{equation*}
\Delta x=-\left(H\left(x_{c}\right)\right)^{-1}\left(W J\left(x_{c}\right)\right)^{T} h\left(x_{c}\right) \tag{23}
\end{equation*}
$$

where the Jacobian matrix is

$$
\begin{equation*}
J(x)=\partial h(x) / \partial x \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
H=\left(W J\left(x_{c}\right)\right)^{T}\left(W J\left(x_{c}\right)\right) \tag{25}
\end{equation*}
$$

The new solution is updated to $x_{c}+\Delta x$ and the Gauss-Newton iteration (23) is repeated until the solution converges. Note that the convergence of the Gauss-Newton iteration is quadratic when the solution is close to the optimum value of $x$.

## C. Jacobian Matrix Structure

Following the arrangements of the unknown variables in $x$ (7) and the network equations (20), the structure of the Jacobian (24) has the following form

$J=\left[\right.$| $\partial f / \partial x$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $U$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | $U$ | 0 | 0 | 0 |
| 0 | 0 | $U$ | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | $U$ | 0 |$]$

where $U$ is the identity matrix with appropriate dimension, which arises from the partial derivatives of the measurement error vector with respect to its own variables. Note that the Jacobian is very sparse. Algorithms [10] exploiting sparsity can be used solve for the increment $\Delta x$ in (23) for computational efficiency.

## IV. Extension of PSE for Angle Bias Correction

Both persistent and random angle biases in phasor data have been observed in PMUs installed in utilities [9]. A WLS approach is suitable for detecting and correcting such angle biases. The ability for a PSE network to perform angle bias correction requires that there are redundant phasor measurements, that is, $L_{1}>N_{2}$. For example, consider the two PSE networks shown in Figure 4, each with the indicated voltage and current phasor measurements. In Fig. 4a, $\tilde{V}_{3}$ can be computed using the phasor measurements from either Bus 1 or Bus 2. If one of the phasor measurements has an angle bias, the two computed values of $\tilde{V}_{3}$ will be different. If the angle bias is corrected, then all 5 voltages in the network can be accurately computed. A simpler example system in given in Fig. 4b where the voltage phasor measurements at adjacent buses provide a check on the phasor current.

(a) Five-Bus PSE Network (Network 2)

(b) Four-Bus PSE Network (Network 3)

Figure 4. Example networks allowing angle bias correction

## A. Modification of Measurement Equations

There are several potential sources of angle biases. It could be due to a particular signal processing algorithm used by the PMU and the length of the data used for computing the phasor quantities. It could also be attributed to time delays due to long cables or errors in time synchronization. PMU software and firm ware may also contribute to the error.

On the other hand, this kind of angle bias has the characteristic that when large angle bias occurs, the same value appears in all voltage and current angles. Thus in the WLS algorithm formulation, the measurement equations (5) and (6) can be updated to, respectively,

$$
\begin{equation*}
\theta_{i}=\theta_{i m}-\phi_{i}+e_{\theta_{i}} \tag{27}
\end{equation*}
$$

and

$$
\begin{equation*}
\delta_{i k}=\delta_{i k m}-\phi_{i}+e_{\delta_{i k}} \tag{28}
\end{equation*}
$$

Note that we take Bus 1 to be the reference bus, and thus the angle bias will not be applied to its phasor measurements. The angle bias terms form a vector

$$
\phi=\left[\begin{array}{llll}
\phi_{2} & \phi_{3} & \cdots & \phi_{N_{1}} \tag{29}
\end{array}\right]^{T}
$$

Thus the WLS algorithm (22) can be modified to the PSE- $\Phi$ problem of

$$
\begin{equation*}
\min _{x_{\phi}} q^{\prime}\left(x_{\phi}\right) \tag{30}
\end{equation*}
$$

where

$$
x_{\phi}=\left[\begin{array}{ll}
x^{T} & \phi^{T} \tag{31}
\end{array}\right]^{T}
$$

and $e\left(x_{\phi}\right)$ has been modified to incorporate (27) and (28).
Note that the Jacobian matrix is expanded to

$$
J_{\phi}=\left[\begin{array}{l|l}
J & \frac{0}{\partial e(\phi) / \partial \phi} \tag{32}
\end{array}\right]
$$

Note that the Jacobian $\partial e(\phi) / \partial \phi$ is sparse and consists of ones and zeros.

To correct for angle bias, it is necessary to have $L_{1}>N_{2}$. To ensure that all the biases can be corrected, the rank of $J_{\phi}$ must be equal to the number of unknown variables, which when including the angle biases, is $U_{T \phi}=2(N+L)+\left(N_{1}-1\right)$. In the sequel the term PSE- $\Phi$ is used to refer to a network where this conditions hold.

## B. PSE- $\Phi$ Observability Analysis for Test Networks

Next we perform an observability analysis of Network 1 (Fig. 2), Network 2 (Fig. 4a), and Network 3 (Fig. 4b). For all three networks, the rank of the relevant Jacobian matrix $J$ is equal to the unknowns $U_{T}$, as shown in Table I , implying that they are all PSE observable. Now consider correcting potential biases on the PMU buses. For Network 1, $\phi_{2}$ and $\phi_{3}$ are added, resulting in a total of 36 unknown variables. The number of network equations, however, remains at 34, thus implying the rank of $J_{\phi}$ is also 34 . Thus PMU angle biases cannot be corrected for Network 1. For Networks 2 and 3, when $\phi_{2}$ is included in the formulation, increasing the number of unknowns by 1 , the rank of $J_{\phi}$ is also one higher than the rank of $J$, ensuring that these two networks are PSE$\Phi$ observable.

Table I
ObSERVABILITY Analysis Results

| NET. | $E_{T}$ | $U_{T}$ | $\operatorname{rank}(J)$ | Bias Terms | $U_{T \phi}$ | $\operatorname{rank}\left(J_{\phi}\right)$ | Observ. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 34 | 34 | 34 | $\phi_{2}, \phi_{3}$ | 36 | 34 | No |
| 2 | 26 | 24 | 24 | $\phi_{2}$ | 25 | 25 | Yes |
| 3 | 20 | 18 | 18 | $\phi_{2}$ | 19 | 19 | Yes |

## V. Illustration with New York System

Figure 5 shows the one-line diagram of 3 HV buses in the New York power system. This 3-bus network comprises one transmission line and two transformers. PMUs are installed at Buses 1 and 2 providing voltage and current phasors at 6 samples per second. At Bus 1 the current flowing out from Bus 1 to Bus 3 is measured, while the Bus 2 currents flowing through the transformers are measured. With this measurement configuration this network is PSE and PSE- $\Phi$ observable.

Synchronized phasor measurements over a 4-minute period from this network are shown in Fig. 6 along with the solution from the Phasor State Estimation. The voltage and current phasors were corrupted by adding $7.5^{\circ}$ and $15^{\circ}$ to their phase components for short periods of time. The PSE- $\Phi$ algorithm was used, which corrected the biases. Note that in Fig. 6, $\theta_{1}$ is plotted without the angle bias. The result of the bias estimation is shown in Fig. 7, which shows the efficacy of the PSE- $\Phi$ algorithm. The residual error of the least-squares estimation is also shown in Fig. 7. Fig. 8 shows the estimated currents and the measured data, and Fig. 9 shows the absolute error between the measurements and the estimates.


Figure 5. Transmission Path One-Line Diagram

## VI. Conclusions

In this paper, a new approach to state estimation using synchronized voltage and current phasor data is proposed. An algorithm to construct observable phasor state estimator islands is developed. A least-squares approach using synchronized phasor data is formulated, using phasor magnitude and angle variables as the unknowns. The formulation readily extends to the detection and correction of angle biases that may exist in the measured data. Observability analysis using the rank of the Jacobian matrix is proposed and illustrated with several small example networks. The PSE- $\Phi$ method is demonstrated using measured phasor data from the New York power system, and is able to correct the angle biases artificially added to the measured data.


Figure 6. Measured and Estimated Voltages


Figure 7. Angle Bias $(\phi)$ and Residual Error under PMU Data with Bias

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Figure 8. Measured and Estimated Currents


Figure 9. Absolute Error between Measured and Estimated States
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    ${ }^{1}$ Loss of ICCP data at critical buses will result in unobservable islands.

