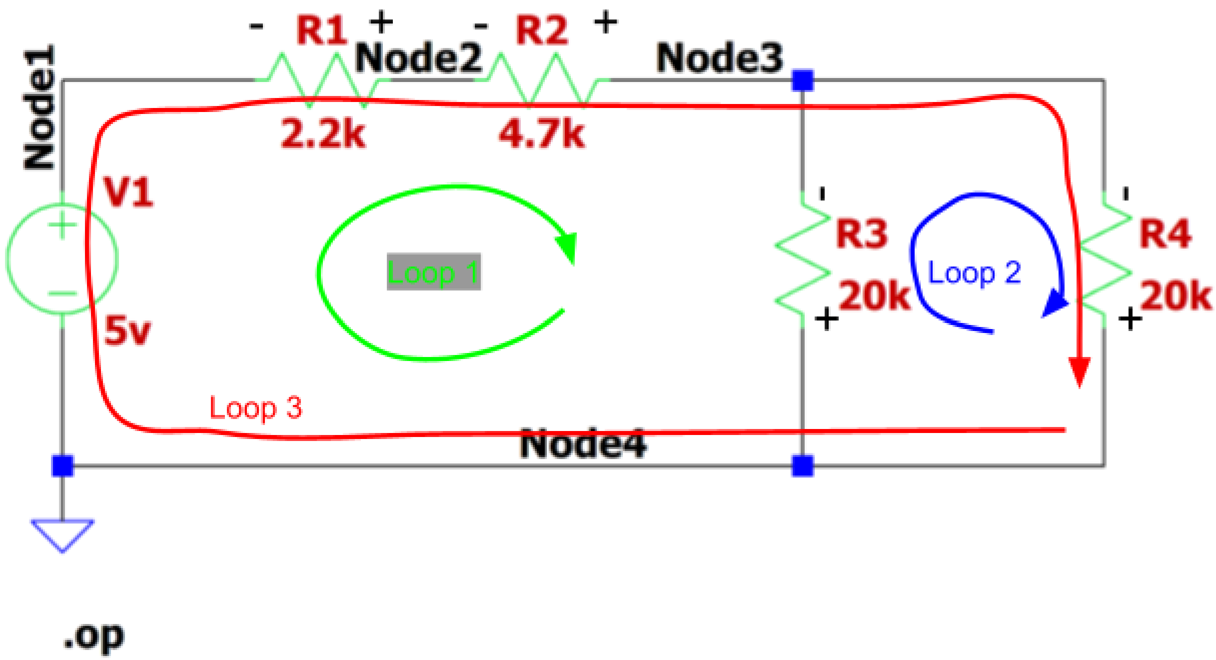
Alpha Proof of Concepts Lab 1

By: Alex Heinle, Frank Martino, Graham Stockard

1. Ohm's Law, Kirchoff's Current Law, and Kirchhoff's Voltage Law

Building Block: Short description and schematic



The above schematic shows a 5V source supplying two resistors (R1 and R2) in series followed by two resistors in parallel (R3 and R4). Nodes 1, 2, and 3 all have the same current since they are in series although the voltage varies. The voltage across R3 and R4 is the same but the current is divided in half.

Analysis:

Equation and short description.

We are able to calculate the voltage across any resistor since we know I and R. Based on this, Kirchoff's Current and voltage laws can be applied by making three loops (shown above by the 3 colored loops). These three loops show how the total voltage adds to 0 volts or the voltage source depending on the loop's location. To show KCL we can create two equations representing the junctions showing how the current in a junction must sum to zero. Since we are creating equations to show KVL which sums up all of the voltages across components we would need to apply Ohm's Law to replace the voltages with current * resistance which we know since V=IR. If the calculations all work, we will have the currents across all of the resistors. After finding these values we can confirm the voltage or current since we know the resistance of each resistor, using Ohm's Law, V=IR which can be manipulated to I=V/R. Once all these equations are found they can be put in a matrix since there are four unknown values and four equations to solve for them.

Total Resistance: R_t = 2.2KΩ + 4.7KΩ + ($\frac{1}{\frac{1}{10K\Omega}+\frac{1}{10K\Omega}}$) = 16.9KΩ

Total Voltage: 5v

Ohm's Law: V=IR

Total Current (manipulation of Ohm's law): V=IR → I=$\frac{V}{R}$ therefore I=$\frac{5v}{16.9K\Omega}$ = 0.295mA

KCL Equations:

Junction 1: I_2-I_3-I_4 = 0

Junction 2: I_3+I_4-I_1 = 0

KVL Equations:

Loop 1: 2.2I_1 + 4.6I_2 + 20I_3= 5

Loop 2: 20I_3 - 20I_4 = 0

Loop 3: = 2.2I_1 + 4.6I_2 + 20I_4 = 5

Matrix:

(Loop 1 and 3 have the same values for resistance; we cannot include loop 3 but it can be replaced with the junction 2 equation. We have 4 unknowns so we need 4 equations)

A:

$$\begin{bmatrix} 2.2 & 4.7 & 20 & 0 \\ 0 & 0 & -20 & 20 \\ -1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

x:

$$\begin{bmatrix} IR1 \\ IR2 \\ IR3 \\ IR4 \end{bmatrix}$$

B:

$$\begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$A(B)^{-1}=x$

IR1: 0.296mA

IR2: 0.296mA

IR3: 0.148mA

IR4: 0.148mA

Using V=IR:

VR1: 0.296mA * 2.2kΩ = 0.651 Volts

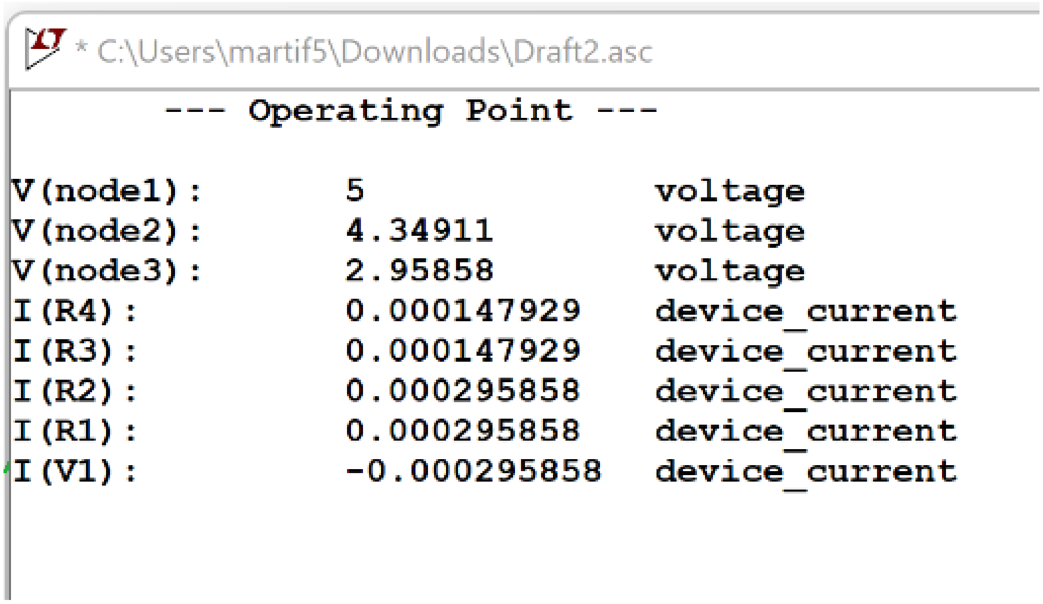
VR2: 0.296mA * 4.7kΩ = 1.391 Volts

VR3: 0.148mA * 20kΩ = 2.96 Volts

VR4: 0.148mA * 20kΩ = 2.96 Volts

Simulation:

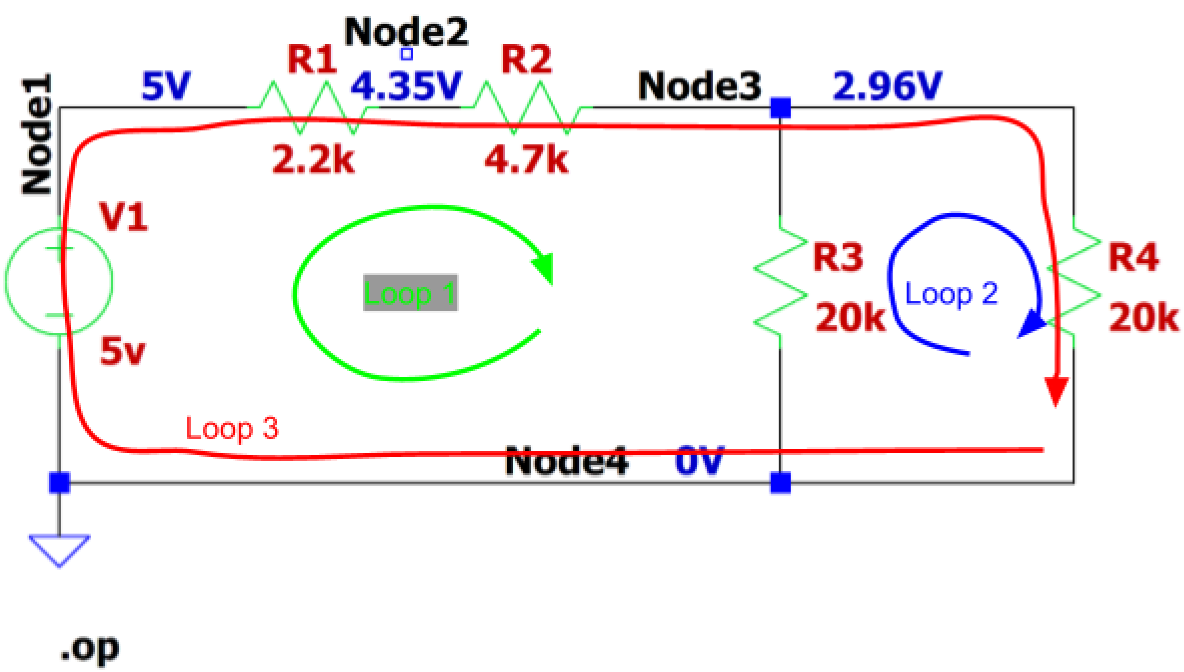
Screenshot of simulation



* C:\Users\martif5\Downloads\Draft2.asc

--- Operating Point ---

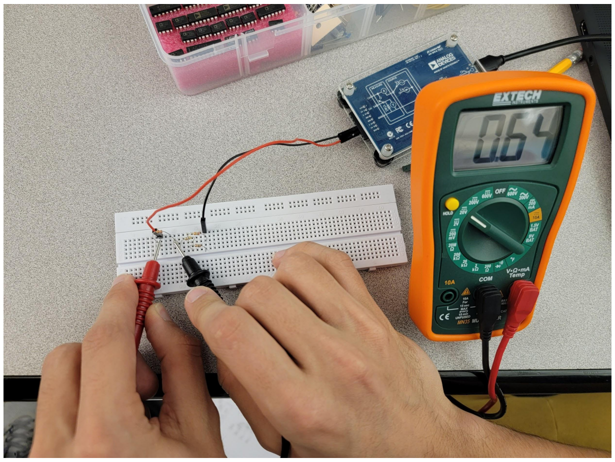
V(node1):	5	voltage
V(node2):	4.34911	voltage
V(node3):	2.95858	voltage
I(R4):	0.000147929	device_current
I(R3):	0.000147929	device_current
I(R2):	0.000295858	device_current
I(R1):	0.000295858	device_current
I(V1):	-0.000295858	device_current

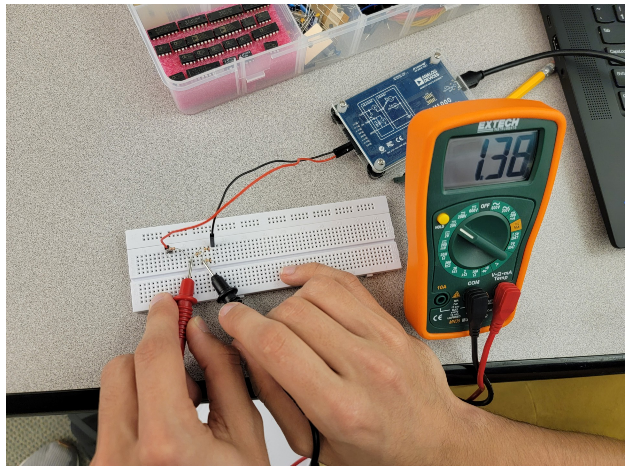


Measurement:

Screenshot of Waveforms output from circuit above.

Remember to clearly show all axes in a measurement plot. Also identify any important portions of the output.





Measured

IR1: 0.3mA

IR2: 0.3mA

IR3: 0.1mA

IR4: 0.1mA

VR1: 0.64 Volts

VR2: 1.38 Volts

VR3: 2.95 Volts

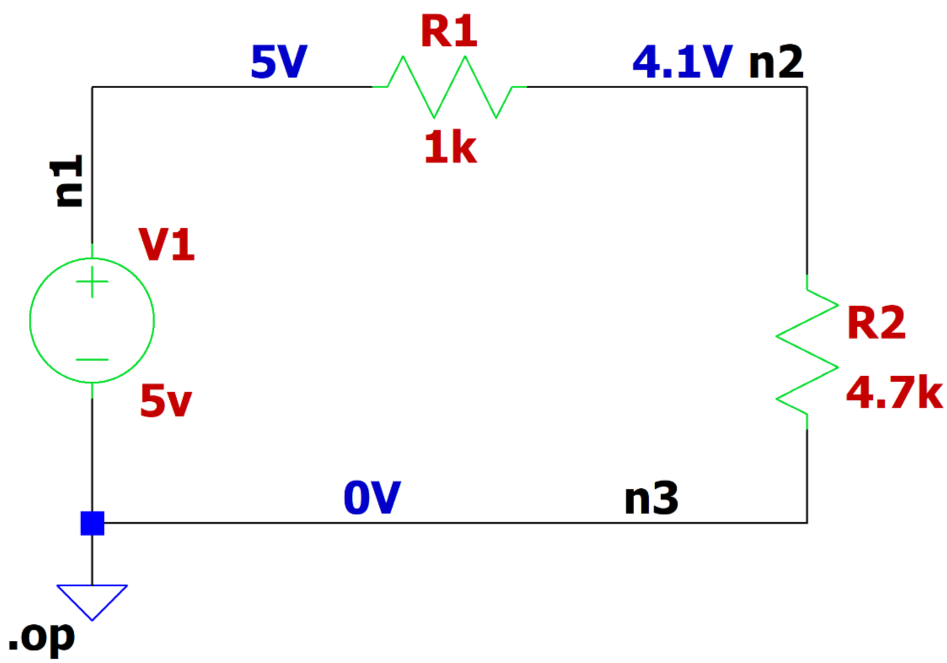
VR4: 2.95 Volts

Discussion (and answer related questions in Alpha Lab):

When looking at our calculated values and the values we measured on the actual breadboard the values were almost all exactly the same and only varied due to the precision of the multimeter. For example, the highest point of precision we could reach when measuring current is the tenths place which is why 0.285mA was rounded up to 0.3mA and 0.148mA was rounded to 0.1mA. When comparing these to the values in LTSpice we had a similar situation where LTSpice could reach very high levels of precision, but our calculators and work were very close.

2. Voltage Divider

Building Block: Short description and schematic



The above diagram shows two resistors in series (R1 and R2), forming a voltage divider.

Analysis:

Equation and short description.

We used the voltage divider equation to measure the voltage drop across the first resistor, which is equal to a voltage drop of 1/5.7 * V1, or 0.877 volts. The node between R1 and R2 should measure 4.123 volts, which leaves a 4.123V drop across R2.

Voltage Divider Equation:

$$Vout \ = \ Vin\frac{R1}{R2+R1}$$

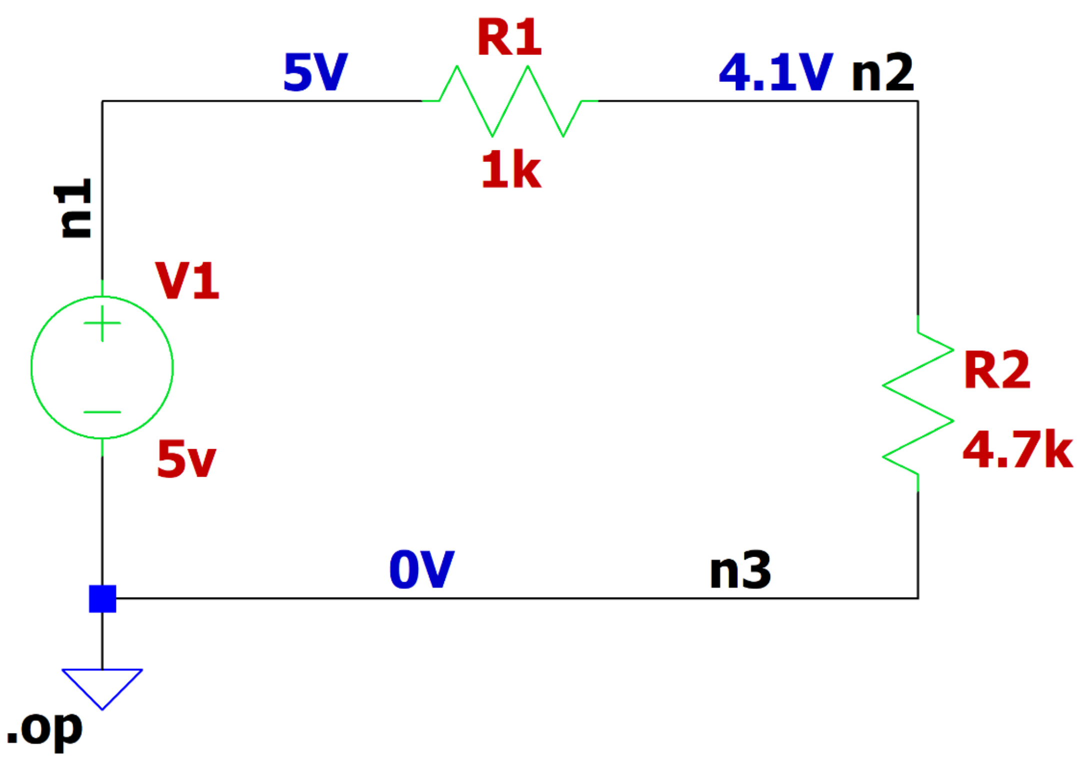
$$VR1 \ = \ 5v\frac{1k}{5.7k} = 0.877v$$

VR2 = 5v - 0.877v = 4.123v (R2 connects to ground, so the voltage drop over R2 must bring the voltage to 0.)

Simulation:

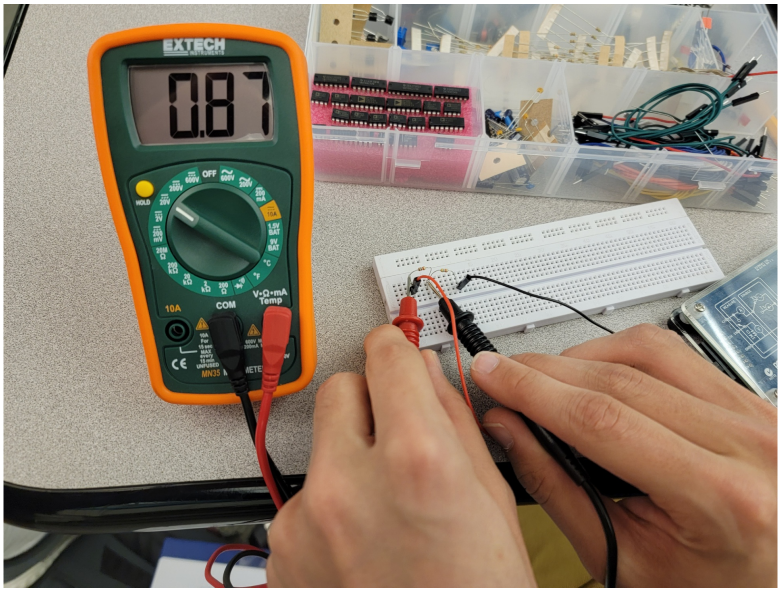
Screenshot of simulation

```
* C:\Users\martif5\Downloads\Draft4.asc

        --- Operating Point ---

V(n1):          5               voltage
V(n2):          4.12281         voltage
I(R2):          0.000877193     device_current
I(R1):          0.000877193     device_current
I(V1):          -0.000877193    device_current
```



Measurement:

Screenshot of Waveforms output from circuit above.



Voltage Measured:

VR1 = 0.87V

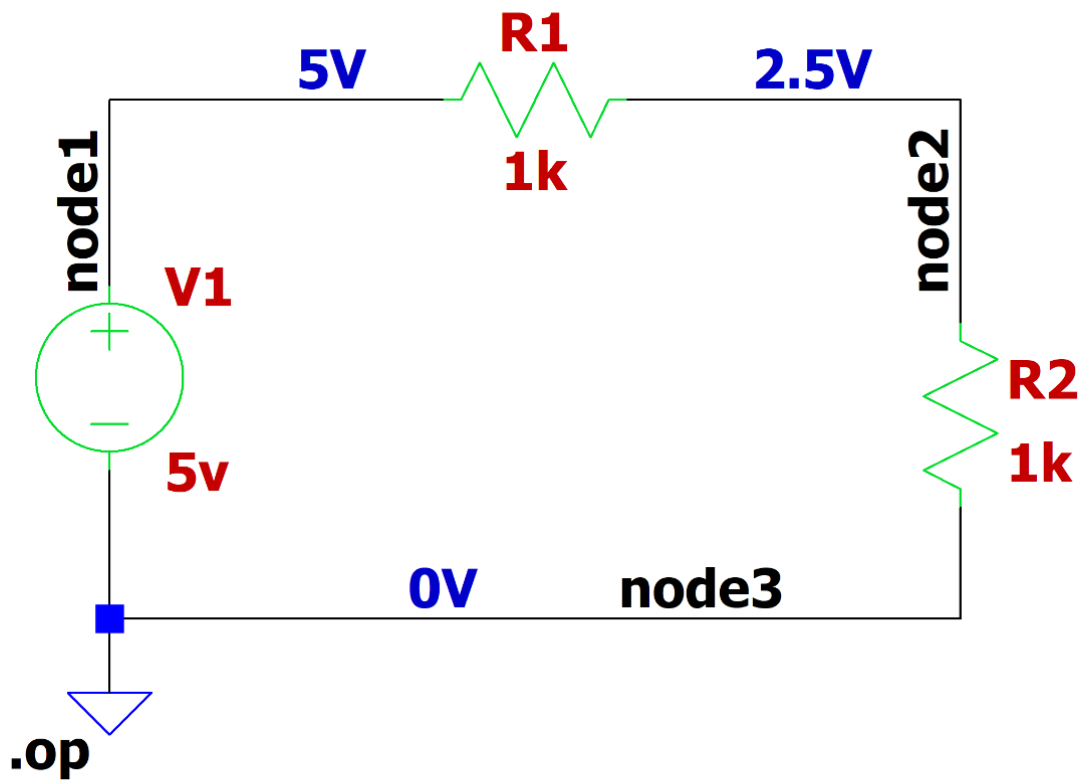
VR2 = 4.11v

Discussion:

With this voltage divider, we predicted the voltage drop across R1 as 0.877V using the voltage divider equation. Our simulation was in accordance with these predictions, and then we created a circuit to determine the actual values. The voltage that we measured across VR1 was 0.87V (dropping one decimal value due to multimeter measurement) which agreed with our predictions. The voltage across VR2 was 4.11, which according to KVL, was roughly what we expected since the sum of all voltage drop within the circuit was 5V and there was a 5V supply. The slight loss (0.02v) was from rounding, an imperfect power supply, and resistor tolerance.

3. Current flow through a series circuit

Building Block: Short description and schematic



The circuit diagram above shows two 1k resistors in series in a 5V circuit.

Analysis:

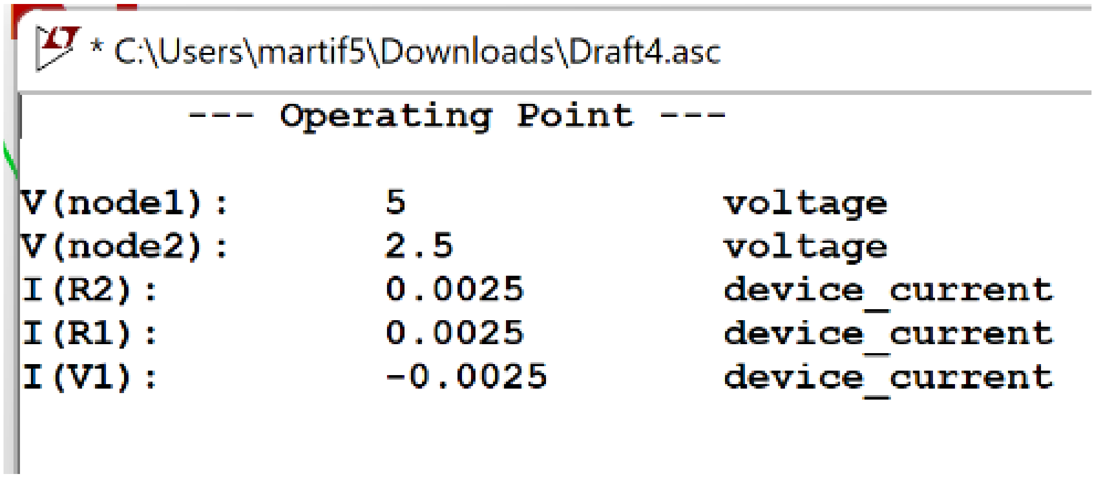
According to Kirchoff's current law, the current entering a node is equal to the current exiting a node. Each of the nodes are labeled in the image above. The current flowing through every resistor will be the same due to the fact that there will always be an equal amount of current exiting and entering, since they are in series.

$I_T = I_{R1} = I_{R2}$

$I_{R1} = 5v / 1k = 5mA = I_{R2}$

Simulation:

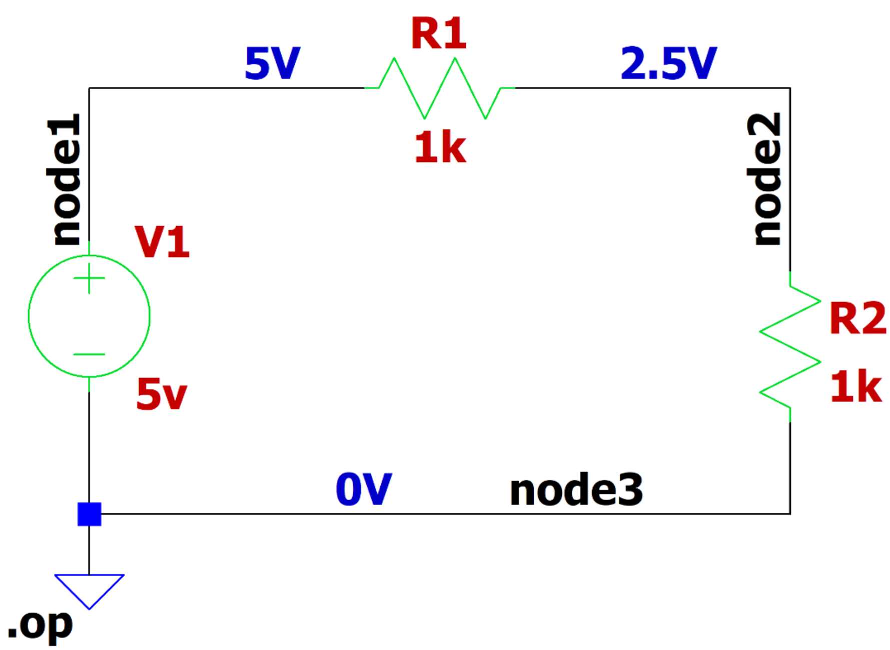
Screenshot of simulation



* C:\Users\martif5\Downloads\Draft4.asc

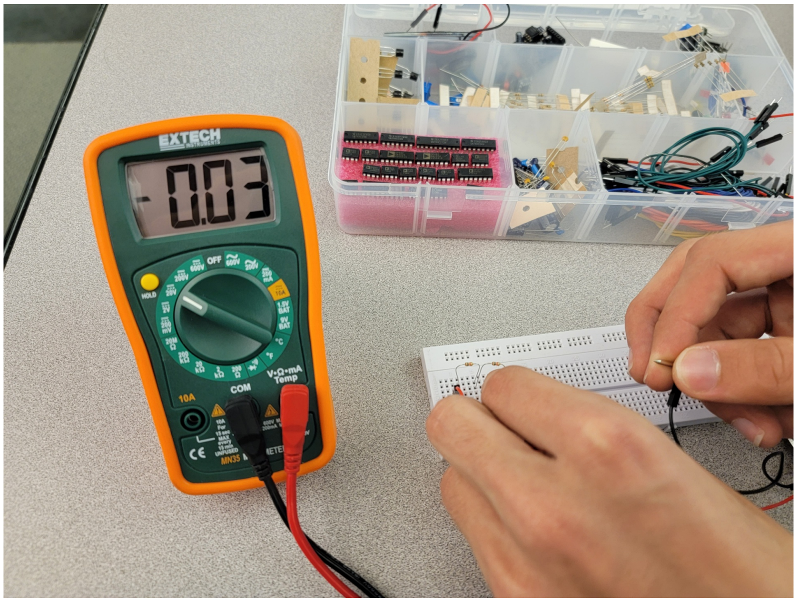
--- Operating Point ---

V(node1):	5	voltage
V(node2):	2.5	voltage
I(R2):	0.0025	device_current
I(R1):	0.0025	device_current
I(V1):	-0.0025	device_current



Measurement:

Screenshot of Waveforms output from circuit above.



Current Measured:

IR1 = 0.03mA

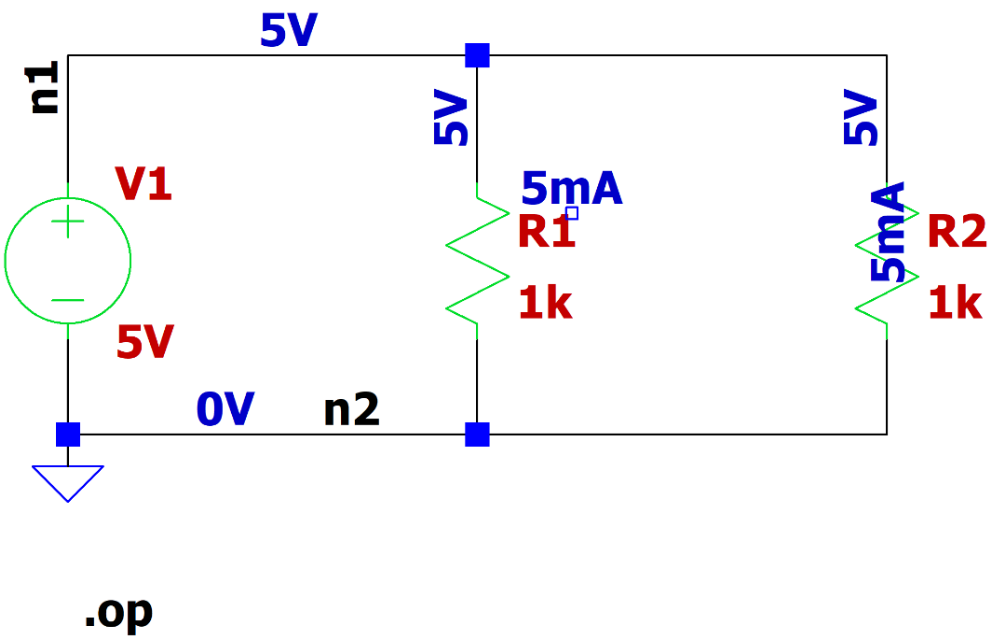
IR2 = 0.03mA

Discussion:

Above shows the previous circuit running and LTSpice simulating the circuit's voltage and current. The solutions the program found are identical to the solutions found using the voltage divider equation. We were able to use the voltage divider equation here since it only contains resistors in series and since they are both of the same value they have the same current and voltage running through them. The current measured through R2, which was 0.03mA, is roughly accurate with our simulation since the multimeter is only accurate to two decimal places.

4. Voltage across a parallel circuit

Building Block: Short description and schematic



The above diagram shows a 5v circuit with two 10k resistors in parallel.

Analysis:

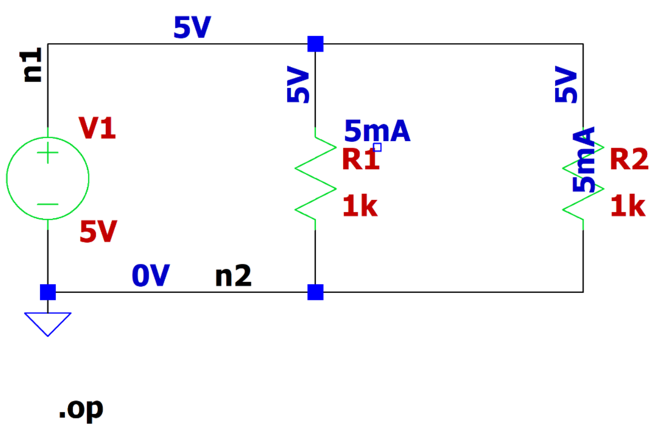
This circuit can be simplified since the two 10k resistors in parallel can be represented by a single 5k resistor. This allows the circuit to be treated as a single loop. Also, due to Kirchoff's voltage law, the voltage measured at the node above the resistors is five on both paths, and the node below the resistors is zero since there must be a 5V drop in order for the sum of voltages in the circuit to be zero. Resistors in parallel share a common voltage.

$$Rtotal = \left(\frac{1}{R1} + \frac{1}{R2} + ... + \frac{1}{Rn}\right)^{-1}$$

Simulation:

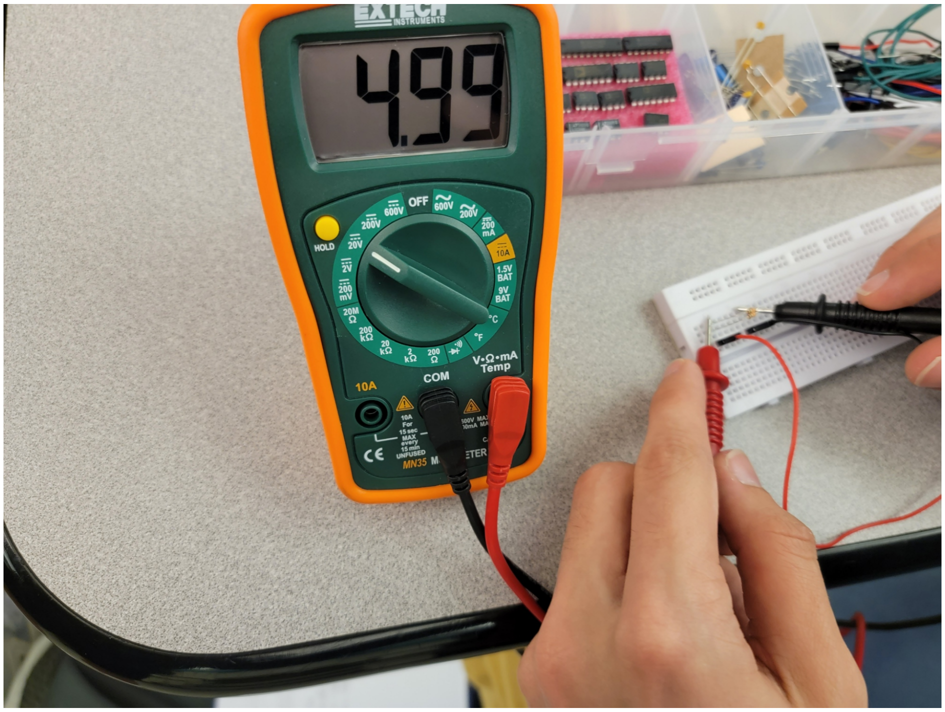
Screenshot of simulation

```
* C:\Users\martif5\Downloads\Draft5.asc
        --- Operating Point ---

V(n1):           5              voltage
I(R2):           0.005          device_current
I(R1):           0.005          device_current
I(V1):           -0.01          device_current
```



Measurement:

Screenshot of Waveforms output from circuit above.



Voltage Measurements:

VR1 = 4.99

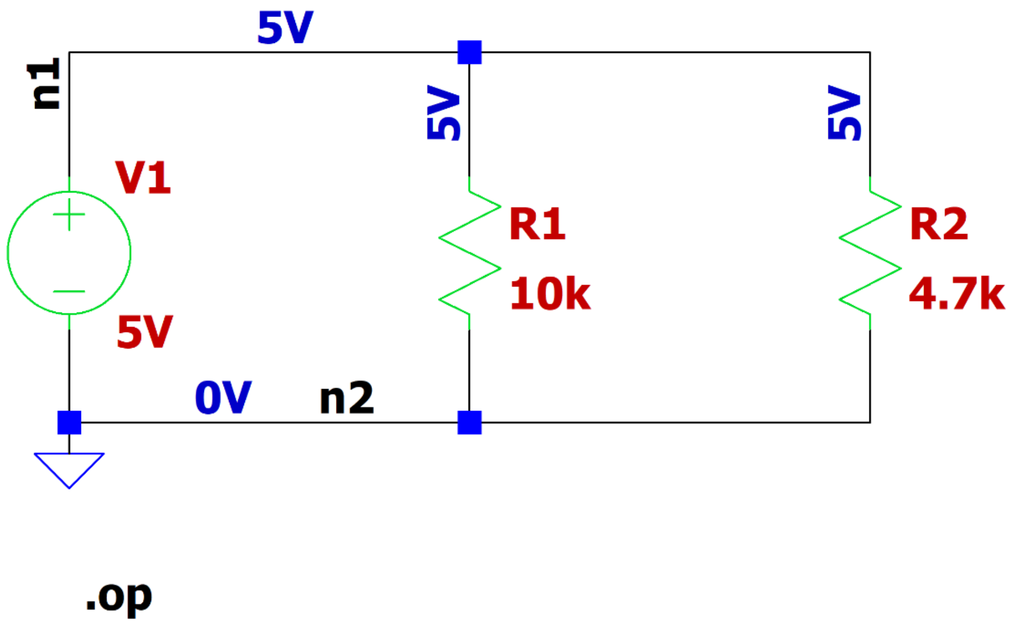
VR2 = 4.99

Discussion:

We created a diagram in LTSpice and calculated that, since the two resistors in parallel are the same resistance, the resistance of the two combined resistors is 5k Ohms. Also, voltage across two parallel resistors is roughly the same as we calculated, since there is a 5V voltage drop from one end (the node shared with the positive lead of the power supply), to the other end (the node shared with the negative lead of the circuit).

5. Current divider in a parallel circuit

Building Block: Short description and schematic



Analysis:

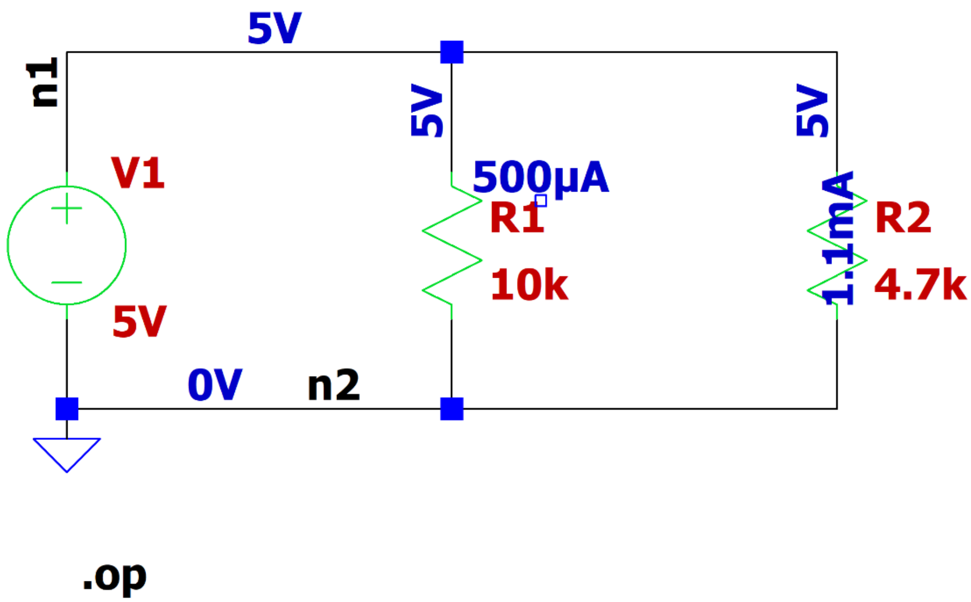
Equation and short description.

This circuit contains two resistors in parallel, R1 and R2. R1 has a resistance of 10k ohms, and R2 has a resistance of 4.7k ohms. The resistors are connected to a 5 volt source, and since they are in parallel, each resistor has an input of 5 volts. The resistance across each resistor can be found simply using ohm's law, as the voltage entering each resistor is already known to be 5 volts. Ohm's law states that I = V / R, so I_{R1} = 5 / 10,000 = .5mA, and I_{R2} = 5 / 4700 = 1.06mA.

Simulation:

Screenshot of simulation

```
       --- Operating Point ---

V(n1):         5              voltage
I(R2):         0.00106383     device_current
I(R1):         0.0005         device_current
I(V1):         -0.00156383    device_current
```



Measurement:

Screenshot of Waveforms output from circuit above.

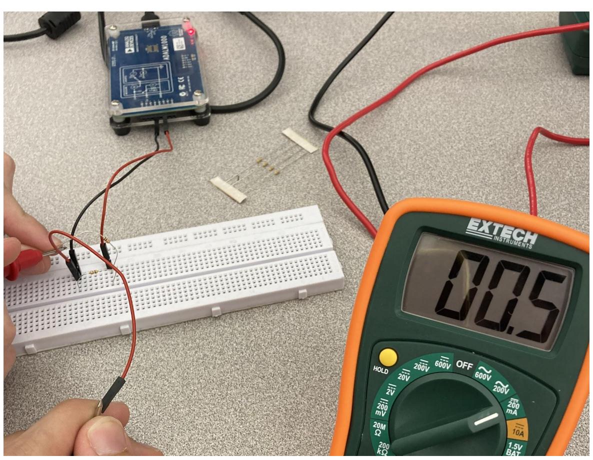
R1 (10k ohms):

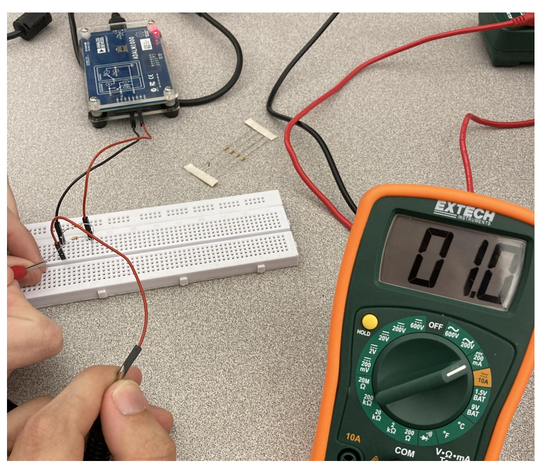
R2 (4.7k ohms):

Currents Measured:

IR1: 0.5

IR2: 1.0





Discussion:

We calculated current across two different value resistors in parallel, and our simulation agreed with our calculations. Then, we created the circuit and measured current through the resistors, which agreed with our findings. The current was greater in the lower value resistor because, since there was less resistance, electricity was able to pass through it more easily. Similar to the other circuits we measured, the current may vary from what is being recorded, but because we can only see up to a certain precision we cannot tell.