ELECTRIC CIRCUITS
ECSE-2010

Lecture 23: Review

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LECTURE 18.1 REVIEW

- Transfer Functions
- Phasors
- Phasor Math

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TRANSFER FUNCTIONS

H(s) = One of the most important things we can find for a circuit

H(s) depends only the circuit

H(s) does NOT depend on the Input

Find H(s) by ASSUMING there is no Initial Stored Energy

and then finding the Ratio of $\frac{\text{Output(s)}}{\text{Input(s)}}$

Will use H(s) to find the Frequency Response of a Circuit

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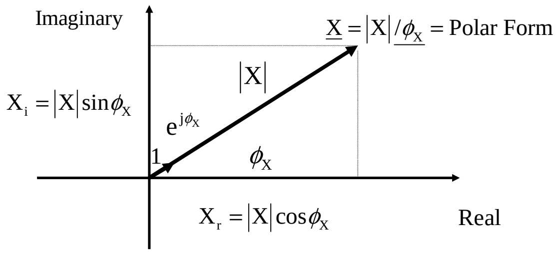
PHASORS



$\underline{X} = X_r + jX_i$ = Rectangular Form

$\underline{X} = |X| e^{j\phi_X}$ = Euler Form

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PHASORS

- **Phasors are Complex Numbers:**
  - **Need to Use Complex Math**
  - **Will Use Complex Math Instead of Solving Differential Equations or Using Laplace Transforms to Find Amplitude and Phase Changes between AC Input and AC Steady State Output**

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PHASORS

- **Phasors are Complex Numbers:**
  - **Will Find that Equations Relating Current Phasors, $\underline{I}$, and Voltage Phasors, $\underline{V}$ for R, L and C will be Linear and Algebraic**
  - **Can Use All Techniques from Unit I to Solve Circuits in the AC Steady State**

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PHASORS

- 3 Ways to Express Phasors
  - Rectangular Form; $\underline{X} = X_r + jX_i$
  - Polar Form; $\underline{X} = |X| \underline{/\phi_X}$
  - Euler Form; $\underline{X} = |X| e^{j\phi_X}$
- Will Need to Be Able to Easily Convert Between the 3 Different Forms

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COMPLEX MATH

- **Addition:**
  - **$\underline{A} + \underline{B} = (a_r + j\,a_i) + (b_r + j\,b_i)$**
    $= (a_r + b_r) + j\,(a_i + b_i)$
- **Subtraction:**
  - **$\underline{A} - \underline{B} = (a_r + j\,a_i) - (b_r + j\,b_i)$**
    $= (a_r - b_r) + j\,(a_i - b_i)$
- **=> Do Addition/Subtraction in Rectangular Form**

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COMPLEX MATH

- **Multiplication:**
  - **Difficult to do in Rectangular Form**
  - **Use Euler or Polar Form**

$$\underline{A} \text{ x } \underline{B} = Ae^{j\phi_1} \text{ x } Be^{j\phi_2} = AB\, e^{j(\phi_1+\phi_2)}$$
$$= A\underline{/\phi_1} \text{ x } B\underline{/\phi_2} = AB\underline{/(\phi_1+\phi_2)}$$

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COMPLEX MATH

- **Division:**

$$\underline{A} \div \underline{B} = Ae^{j\phi_1}/Be^{j\phi_2} = \frac{A}{B}e^{j(\phi_1-\phi_2)}$$
$$= A\underline{/\phi_1} \,/\, B\underline{/\phi_2} = \frac{A}{B}\underline{/(\phi_1-\phi_2)}$$

- **Do Multiplication/Division in Polar Form or Euler Form**

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RATIONALIZATION

- **Division in Rectangular Form:**

$$\underline{C} = \underline{A} \div \underline{B} = \frac{a_r + ja_i}{b_r + jb_i}$$

Want to express as $\underline{C} = C_r + jC_i$

Multiply $\frac{a_r + ja_i}{b_r + jb_i}$ by $\frac{b_r - jb_i}{b_r - jb_i}$

$$=> \underline{C} = \frac{a_r b_r + a_i b_i}{b_r^2 + b_i^2} + j\,\frac{a_i b_r - a_r b_i}{b_r^2 + b_i^2}$$
$$= \quad C_r \quad + j \quad C_i$$

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LECTURE 19.1 AGENDA

- Kirkoff's laws for phasors
- AC steady state impedence

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K'S LAWS FOR PHASORS

- Circuit in AC steady state:

  Input $= x(t) = |X|\cos(\omega t + \phi_X)$

  Express x(t) as a Phasor $\underline{X} = |X| \,/\, \underline{\phi_X}$
- Can express all v's and i's in circuit as phasors:

  $v_1(t) \to \underline{V_1};\ v_2(t) \to \underline{V_2}$

  $i_1(t) \to \underline{I_1};\ i_2(t) \to \underline{I_2}$

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K'S LAWS FOR PHASORS

- KCL:
  - If $i_1 + i_2 = i$; => $\underline{I_1} + \underline{I_2} = \underline{I}$
- KVL:
  - If $v_1 + v_2 = v$; => $\underline{V_1} + \underline{V_2} = \underline{V}$
- K's Laws Work for Phasors!
  - Complex Addition, not Simple Addition

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AC STEADY STATE IMPEDANCE

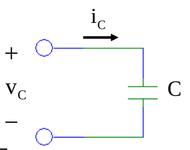
- **Capacitor:**

$$i_C = C\frac{dv_C}{dt}$$

$$v_C(t) = V\cos(\omega t + \phi)$$

$$= \text{Real}\left[V e^{j(\omega t + \phi)}\right]$$

$$= \text{Real}\left[V e^{j\phi}\ e^{j\omega t}\right] = \text{Real}\left[\underline{V_C} e^{j\omega t}\right]$$



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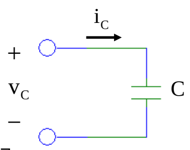

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AC STEADY STATE IMPEDANCE

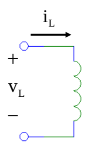
- **Inductor:**

$$v_L = L\frac{di_L}{dt}$$

$$i_L(t) = I\cos(\omega t + \phi)$$

$$= \text{Real}\left[I e^{j(\omega t + \phi)}\right]$$

$$= \text{Real}\left[I e^{j\phi} e^{j\omega t}\right] = \text{Real}\left[\underline{I_L} e^{j\omega t}\right]$$



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AC STEADY STATE IMPEDANCE

$$Z_R = R\,\Omega$$

$$Z_L = j\omega L\,\Omega$$

$$Z_C = -\frac{j}{\omega C} = \frac{1}{j\omega C}\ \Omega$$

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AC STEADY STATE IMPEDANCE

Note:

As $\omega \to 0;\ Z_L = j\omega L \to 0$

Inductor is a Short Circuit for DC

As $\omega \to \infty;\ Z_L = j\omega L \to \infty$

Inductor is an Open Circuit for Very High Frequencies

AC STEADY STATE IMPEDANCE

Note:

As $\omega \to 0;\ Z_C = -\dfrac{j}{\omega C} \to \infty$

Capacitor is an Open Circuit for DC

As $\omega \to \infty;\ Z_C = -\dfrac{j}{\omega C} \to 0$

Capacitor is a Short Circuit for Very High Frequencies

AC STEADY STATE IMPEDANCE

- In General, $\underline{V} = Z\,\underline{I}$ in AC Steady State:
  - Z = AC SS Impedance
  - Units of Ohms
  - Ohm's Law for AC Steady State
- Y = AC Steady State Admittance
  = 1/Z (Units of mhos)

AC STEADY STATE IMPEDANCE

$\underline{V} = Z\underline{I}$; Ohm's Law for AC Steady State

$Z = R(\omega) + jX(\omega) =$ AC Steady State Impedance

$R(\omega) =$ AC Steady State Resistance

$X(\omega) =$ AC Steady State Reactance

$Y = G(\omega) + jB(\omega) =$ AC Steady State Admittance

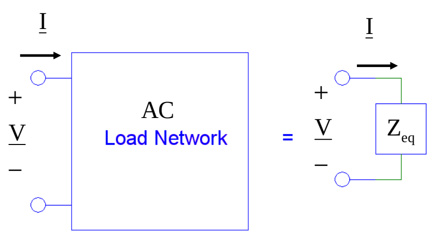
$G(\omega) =$ AC Steady State Conductance

$B(\omega) =$ AC Steady State Susceptance

LECTURE 20.1 AGENDA

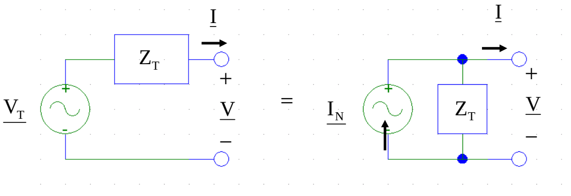
- AC Thevenin/Norton circuits
- AC node equations
- AC mesh equations
- AC bridge circuits

EQUIVALENT IMPEDANCE



$$Z_{eq} = \frac{\underline{V}}{\underline{I}}$$

AC THEVENIN/NORTON



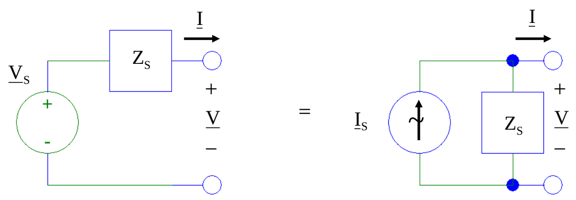
AC Thevenin Circuit

AC Norton Circuit

$\underline{V}_T = \underline{I}_N \, Z_T$

$Z_T = Z_{eq}$ of Dead Source Network

AC SOURCE CONVERSIONS



$$\underline{I}_S = \frac{\underline{V}_S}{Z_S}$$

AC NODE EQUATIONS

Technique to Solve Any AC Steady State Circuit

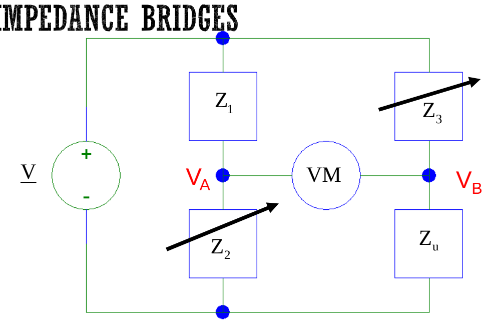
1. Label Unknown Phasor Node Voltages, $\underline{V}_1, \underline{V}_2$, etc.
2. # Unknown Nodes = # Nodes - # Voltage Sources – 1 (Reference)
3. Write a KCL at Each Unknown Node
4. Sum of Phasor Currents OUT of Node = 0
5. Relate Phasor Currents to Phasor Node Voltages using Ohm's Law for AC Steady State
6. Will Always Get the Same Number of Equations as Unknowns
7. Solve Complex Linear Equations for $\underline{V}_1, \underline{V}_2$, etc.

AC MESH EQUATIONS

Technique to Solve Any AC Steady State Circuit

1. Define All Phasor Mesh Currents
   1. Unknown Mesh Currents ($\underline{I}_1, \underline{I}_2, \underline{I}_3$, etc.) and Current Sources (Independent and Controlled)
2. Write KVL around Each Unknown Mesh
3. Sum of Phasor Voltages around Mesh = 0
4. Relate Phasor Voltages to Phasor Mesh Currents using Ohm's Law for AC Steady State
5. Will Always get Same Number of Equations as Unknowns
6. Solve Complex Linear Equations for $\underline{I}_1, \underline{I}_2, \underline{I}_3$, etc.

IMPEDANCE BRIDGES



Balance the Bridge

Accurate Measurement of Z_u

IMPEDANCE BRIDGES

Parallel voltage dividers

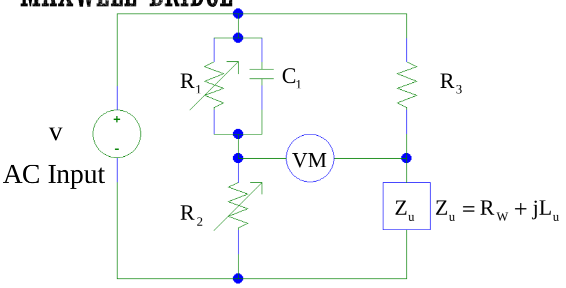
$$V_M = V_A - V_B = \left(\frac{Z_2}{Z_1 + Z_2}\right) \cdot V_s - \left(\frac{Z_u}{Z_3 + Z_u}\right) \cdot V_s$$

$$V_M = \left[\frac{Z_2 \cdot Z_3 - Z_1 \cdot Z_u}{(Z_1 + Z_2) \cdot (Z_3 + Z_u)}\right] \cdot V_s$$

VM is zero when $Z_2 Z_3 = Z_1 Z_u$

$$Z_u = \frac{Z_2 \cdot Z_3}{Z_1} = R_X + jX_X$$

MAXWELL BRIDGE



Measures an Inductive Impedance

IMPEDANCE BRIDGES

- Why 2 Variable Impedances?:
  - Must balance Resistance and Reactance of the Circuit
    - Amplitude and Phase of $\underline{I}_m$, $\underline{V}_m$
    - Real and Imaginary Parts of $\underline{I}_m$, $\underline{V}_m$

LECTURE 21.1

- Review AC Power
  - Complex Power
  - Real Power
  - Reactive Power
  - Apparent Power
- Power Factor

REACTIVE POWER

Define P = "Real Power" $= V_{RMS} I_{RMS} \cos\theta$

P is Measured in Watts

Define Q = "Reactive Power" $= V_{RMS} I_{RMS} \sin\theta$

Q is Measured in VAR's

(Volt-Amperes-Reactive)

REACTIVE POWER

- **Q is a Measure of the Rate of Change of Energy Stored in the Reactive Elements (L, C):**
  - **Power companies must worry about Q since they supplied this energy**
  - **Supplied Q over their Lines => Real Cost**
  - **Power companies want customers to have Low Q**

REACTIVE POWER

$$P = I_{RMS}^2 |Z| \cos\theta = I_{RMS}^2 \; R(\omega) = V_{RMS} I_{RMS} \cos\theta$$

Equivalent ways of expressing Real Power [Watts]

$$Q = I_{RMS}^2 |Z| \sin\theta = I_{RMS}^2 \; X(\omega) = V_{RMS} I_{RMS} \sin\theta$$

Equivalent ways of expressing Reactive Power [VAR's]

REACTIVE POWER

- **Notes on Reactive Power:**
  - **Real Power = P is always $\geq$ 0**
  - **Reactive Power = Q can be $\geq$ 0 or $\leq$ 0**
  - **For Inductive Load, X > 0 => Q > 0**
  - **For Capacitive Load, X < 0 => Q < 0**

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COMPLEX POWER

Define "Complex Power" $= \underline{S} = P + jQ$

$\underline{S}$ is a Complex Number, but not a Phasor

$\underline{S}$ is a Convenient Way to Keep Track of P and Q

$\text{Real}\{\underline{S}\} = P = V_{RMS} I_{RMS} \cos\theta$ => Watts

$\text{Imag}\{\underline{S}\} = Q = V_{RMS} I_{RMS} \sin\theta$ => VAR's

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APPARENT POWER

Magnitude of $\underline{S} = |S| = \sqrt{P^2 + Q^2} = V_{RMS} I_{RMS}$

$|S| =$ "Apparent Power" => [Volt-Amperes]

$|S| =$ Product of V_{RMS} x I_{RMS} at Terminals

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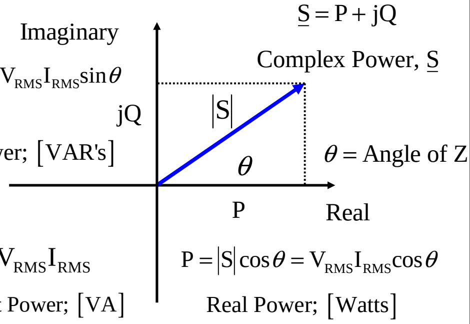
POWER TRIANGLE

$\underline{S} = P + jQ$



$Q = |S| \sin\theta = V_{RMS} I_{RMS} \sin\theta$

Reactive Power; [VAR's]

$|S| = V_{RMS} I_{RMS}$

Apparent Power; [VA]

$P = |S| \cos\theta = V_{RMS} I_{RMS} \cos\theta$

Real Power; [Watts]

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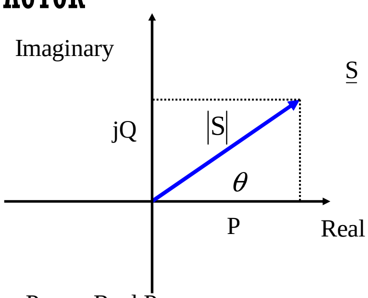
POWER FACTOR



$$\cos\theta = \frac{P}{|S|} = \frac{\text{Real Power}}{\text{Apparent Power}} = \text{Power Factor} = pf$$

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POWER FACTOR

For Inductive Loads, $\theta > 0$; $\cos\theta > 0$

For Capacitive Loads, $\theta < 0$; $\cos\theta > 0$

Need a Way to Distinguish

$$\underline{I} = \frac{\underline{V}}{\underline{Z}} = \frac{|V|\angle\phi}{|Z|\angle\theta} = \frac{|V|}{|Z|}\angle\phi - \theta$$

If $\theta > 0; \Rightarrow$ Lagging Power Factor ($\underline{I}$ lags $\underline{V}$)

If $\theta < 0; \Rightarrow$ Leading Power Factor ($\underline{I}$ leads $\underline{V}$)

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POWER FACTOR

Power Factor:

Define $\text{pf} = \cos\theta; \quad 0 \le \text{pf} \le 1$

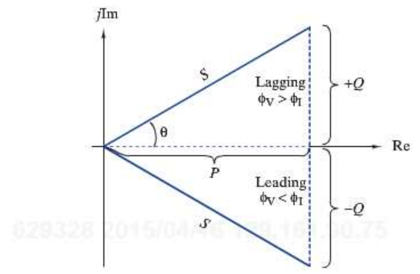
Must distinguish between $\theta \ge 0,\ \theta \le 0$:

$\theta \ge 0;\ X \ge 0;\ Q \ge 0;\ \underline{I}$ lags $\underline{V}$; lagging pf

$\theta \le 0;\ X \le 0;\ Q \le 0;\ \underline{I}$ leads $\underline{V}$; leading pf

e.g: pf = .8 lagging => Inductive Load

pf = .8 leading => Capacitive Load

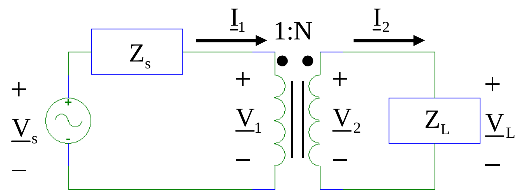


P>0, net energy is transferred from source to load (P<0?)
Lagging (inductive) 1st quadrant
Leading (capacitive) 4th quadrant

LECTURE 22.1

- Coupled Inductors
- Ideal Transformer
- Transformer Circuit
- Power Transfer
- Impedance Matching
- Mutual Inductance (Tee Model)

TRANSFORMER CIRCUIT



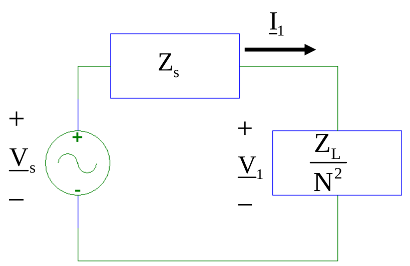
2 Choices for the Equivalent Circuit

Refer Secondary Circuit to the Primary

OR

Refer Primary Circuit to the Secondary

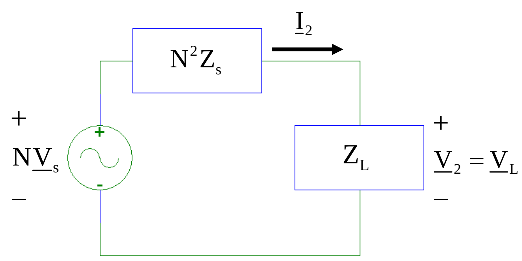
REFERRAL TO PRIMARY



Equivalent to Basic Transformer Circuit

Can Now Do AC Steady State Circuit Analysis

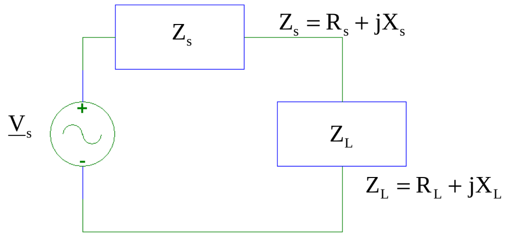
REFERRAL TO SECONDARY



Equivalent to Basic Transformer Circuit

Can Now Do AC Steady State Circuit Analysis

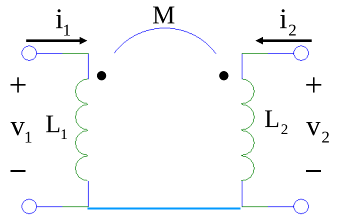
POWER TRANSFER



For Maximum Power to Z_L, Choose $Z_L = Z_s^*$

$\Rightarrow R_L = R_s$ and $X_L = -X_s$

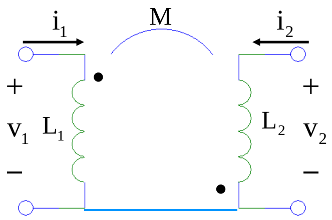
MUTUAL INDUCTANCE



$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

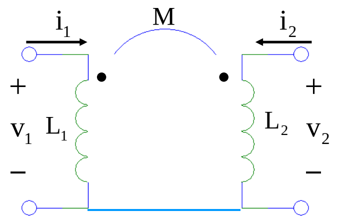
MUTUAL INDUCTANCE



$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

MUTUAL INDUCTANCE

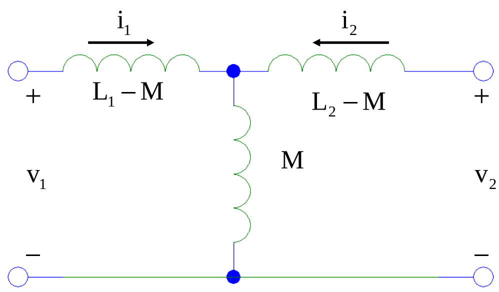


Would Like to Replace with
an Equivalent Circuit that does
NOT have any Mutual Inductance

TEE MODEL

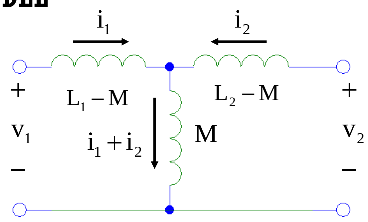
- **Consider a Circuit That Looks Like a "Tee":**
  - **3 Ideal Inductors**
  - **No Mutual Inductance**

TEE MODEL



No Coupling Between Inductors

TEE MODEL



$$v_1 = M\frac{d}{dt}(i_1 + i_2) + (L_1 - M)\frac{di_1}{dt} = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$v_2 = M\frac{d}{dt}(i_1 + i_2) + (L_2 - M)\frac{di_2}{dt} = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

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TEE MODEL

- For a "Tee" Circuit:
  - 3 Ideal Inductors, No Mutual Inductance
  - Same Equations as Before
- => Can Replace Inductors exhibiting Mutual Inductance with "Tee Model" and then do AC Steady State Circuit Analysis :

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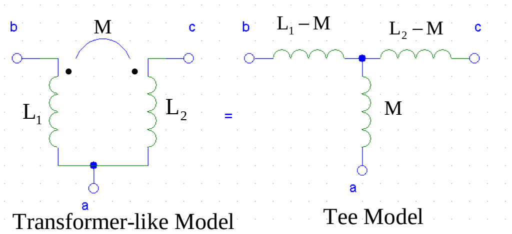
TEE MODEL



Transformer-like Model

Tee Model

If Dots on Opposite Sides => $M \to -M$

Some Inductors in Tee Model May Be Negative!

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