LABORATORY 2: Transient circuits, RC, RL step responses, 2nd Order Circuits, Impedance, s-domain Circuits

Material covered:

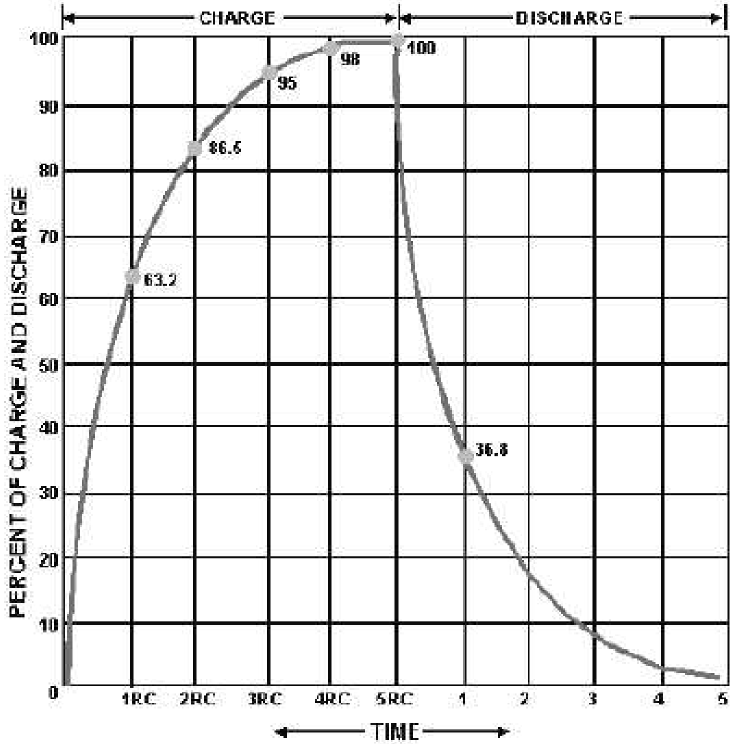
- RC circuits
- 1st order RC, RL Circuits
- 2nd order RLC series circuits
- 2nd order RLC parallel circuits
- Thevenin circuits
- S-domain analysis

Part A: Transient Circuits

RC Time constants: A time constant is the time it takes a circuit characteristic (Voltage for example) to change from one state to another state. In a simple RC circuit where the resistor and capacitor are in series, the RC time constant is defined as the time it takes the voltage across a capacitor to reach 63.2% of its final value when charging (or 36.8% of its initial value when discharging). It is assume a step function (Heavyside function) is applied as the source. The time constant is defined by the equation $\tau = RC$ where

τ is the time constant in seconds
R is the resistance in Ohms
C is the capacitance in Farads

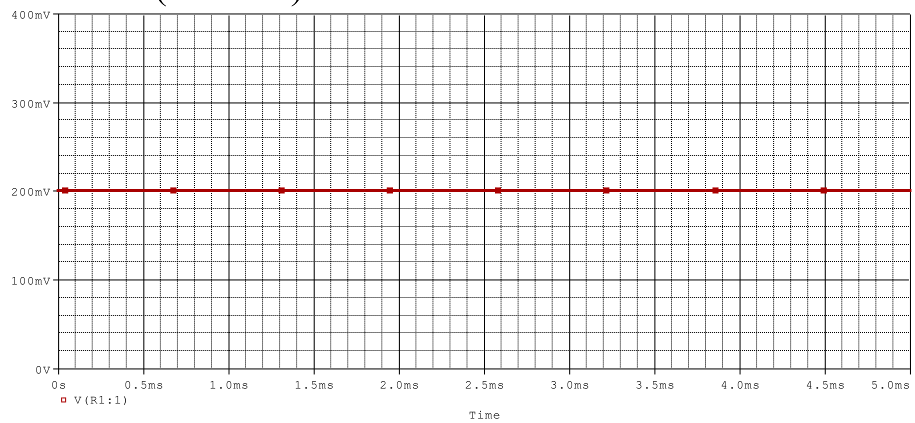
The following figure illustrates the time constant for a square pulse when the capacitor is charging and discharging during the appropriate parts of the input signal. You will see a similar plot in the lab. Note the charge (63.2%) and discharge voltages (36.8%) after one time constant, respectively.


CHARGE
DISCHARGE
PERCENT OF CHARGE AND DISCHARGE
100
90
80
70
60
50
40
30
20
10
0
63.2
86.5
95
98
100
36.8
1RC
2RC
3RC
4RC
5RC
1
2
3
4
5
TIME

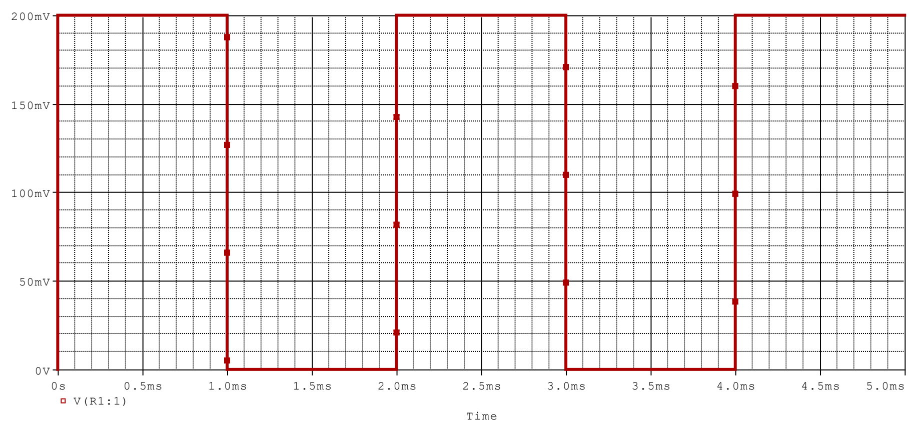
Function Generator: In this lab, we will be looking at DC, sinusoidal, square wave and triangle wave functions. Your Proof of Skills Documentation is a good resource to learn how to make these transient signals for your lab.

Signal types:

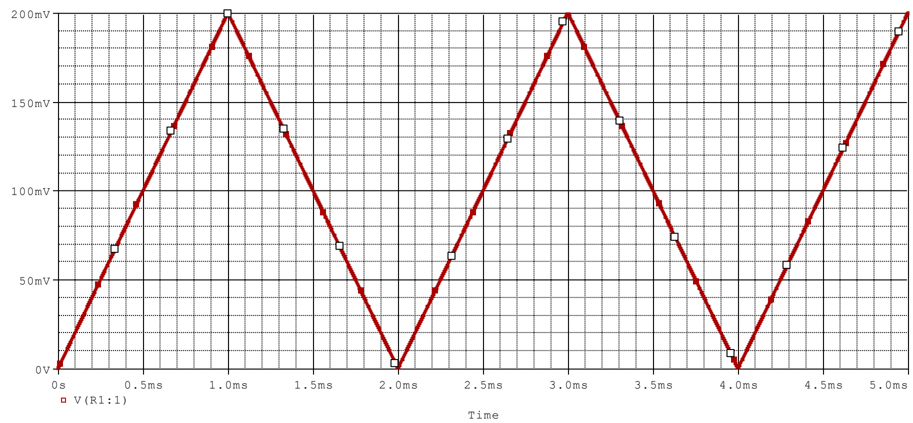
DC source (constant)



Square wave (pulse train)



Triangle wave (try to make this signal…add instructions to Proof of Skills for yoru board if not already there…)



LTSpice:
To generate a square wave, use the voltage component. Use the following settings (do not include the information in parenthesis):

Vinitial[V]: 0 (low voltage in volts)
Von[V]: 2 (high voltage in volts)
Tdelay[s]: 0 (time delay in seconds)
Trise[s]: 0 (rise time in seconds)
Tfall[s]: 0 (fall time in seconds)
Ton[s]: *variable* (pulse width, 50% duty cycle, so should be half the period)
Tperiod[s]: *variable* (as directed in the lab, 1/f, example a 1kHz wave has a 1ms period)
Ncycles: *variable* (can leave blank, number of cycles)

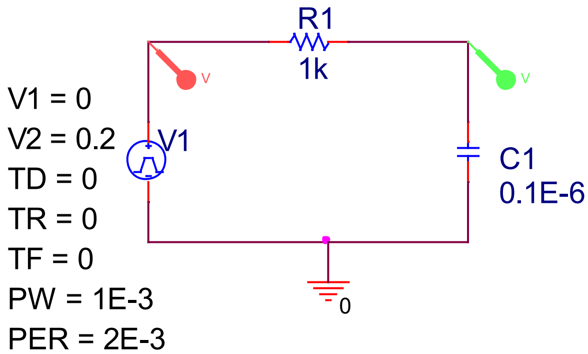
Plot quality/accuracy: When you set your simulation profile the Time Domain (Transient tab) analysis lets you set a few parameters. We will be looking at several periods of the signal. You should be using the following settings:

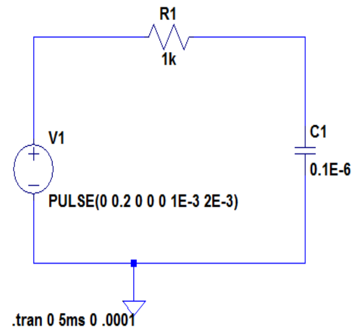
Stop time: (sufficient time for several periods to be displayed) seconds
Time to start saving data: 0 seconds
Maximum timestep: (I usually set this so that there are ~1000 points per period)

Plots: For transient analysis, we can place a Probe on the LTSpice circuit to obtain plots of voltage/current as a function of time. You can place that Probe at a node to obtain plots of voltage as a function of time. Note, this measurement is nodal and is measured relative to the indicated ground. If you are going to measure the voltage drop across a component that is not connected to ground, you can use differential probes, which can be set using the icon with the pair of probes. You place one probe on one side of the component and the other probe on the other side of the component.

Cursors: Cursors are available in LTSpice as well. Right click on the Trace label at the top of the simulation screen. You can choose up to two cursors to use by clicking the pull down menu "Attached Cursor" and selecting 1^{st}, 2^{nd}, or 1^{st} and 2^{nd}. You can drag the cursor along your plot and it will follow the waveform. When you start the cursor, the data window should show up providing voltage levels, time, and frequency if using two cursors as well. More details with pictures are shown here. http://ltwiki.org/index.php?title=Attached_Cursors

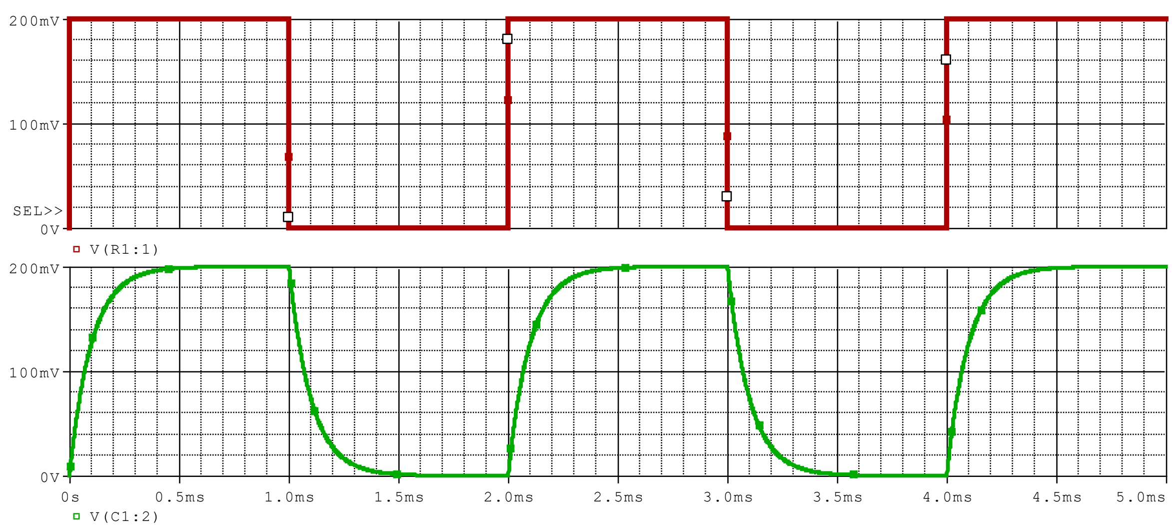
A.1. RC Circuits





Build the RC circuit shown in the top figures (They are the same circuit but the PSpice circuit on the right shows probe position).

1) On your instrumentation board, set your signal to a 1V amplitude (2V peak-to-peak) square wave with a DC offset of 1 Volts. Set the frequency to 1kHz initially. Implement your circuit with R1 = 1kΩ (1E3) and C1 = 0.1µF (0.1E-6). Connect Oscilloscope Channel 1 probes to the source and Oscilloscope Channel 2 probes across the capacitor.
    a. Reduce the frequency such that you can see the voltage across the capacitor reach a constant value (steady state) in each half period of the square wave (approaching the DC condition). You will need to adjust the Base (horizontal axis) of the Oscilloscope to see several periods. (*An example is shown below, where red is the source voltage and green is the voltage across the capacitor.*)



Example of Voltage Output Waveform

b. Estimate the time constant. It will be easier if you use the cursors (described at the start of the lab).
c. Compare your result with the expected time constant. (expected time constant is RC, resistance times capacitance, it will be explained in lecture)
d. Implement your circuit in LTSpice and compare your results.

2) Repeat a-d. for R = 10kΩ and C = 0.01E-6F or any other RC combination.

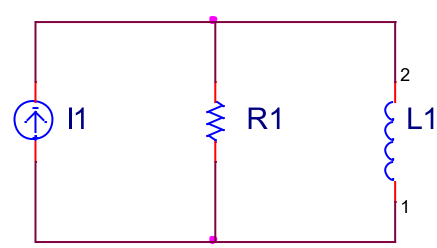
PART A: Proof of Concepts list
A.1: RC Circuit-Prove that the time constant changes with different component values.

Part B: RC, RL Step Responses

Overall notes:

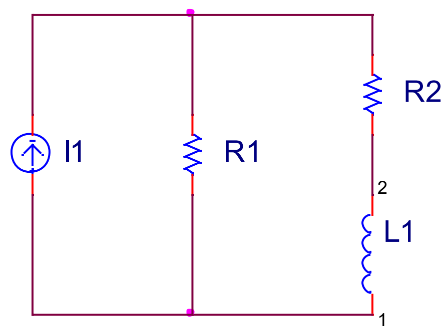
In your plots, you should compare input signals (sources) to outputs signals (component voltage/current).

In Part B, you will investigate an RL parallel circuit.

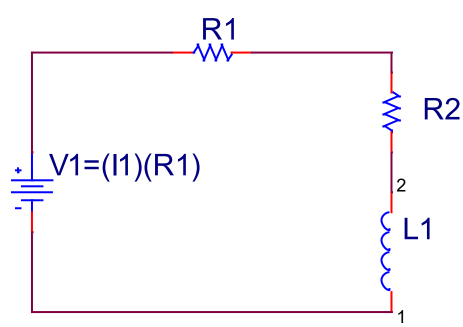


The derivation of the current across the inductor leads to the differential equation, $\frac{L}{R}\frac{di_L}{dt}+i_L=I_s$ For a step function source current, $I_1u(t)$, the solutions to the differential equation take the form $i_L(t)=K_1e^{-t\left(\frac{R}{L}\right)}+K_2$. This form is similar to what we saw for the RC circuit, with a slightly different time constant τ = L/R.

Unfortunately, there are two problems with investigating the above circuit. The first problem is that we don't have access to a current probe. One practical solution is to add a small resistor in series with the inductor. You want the resistor to be small enough such that it does not significantly change the circuit and large enough that you can obtain an accurate voltage reading.



Second, we don't have a reliable current source. However, we can use source transformations to generate an equivalent circuit with a voltage source, as shown in the following circuit.



R2 is the test resistor and should be much smaller than R1, (R2 << R1). In the experiment that follows, you could easily just determine the current by measuring the voltage across R1 without any need for R2. However, we will treat the experiment as representative of a more complex model and use a small R2 resistor.

Note, you may want to prove to yourself that the source transformation circuit results in the same current response in L1.

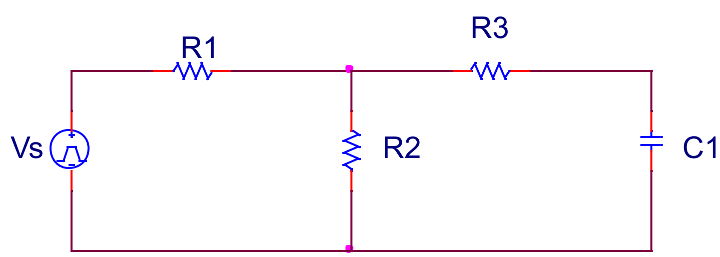
Instrumentation Board

This lab investigates step responses. In Discovery Board, M1K and M2K, we use pulse streams and can't easily make a step function source. To measure an equivalent response using a pulse stream, we make sure that each half cycle is much longer than the time constant for the circuit, $T/2 >> \tau$. By letting the circuit response reach DC steady state in each half cycle, the V=Vo half cycle is equivalent to a source turning on at t = 0 with zero initial conditions and the V = 0 half cycle is equivalent to a source turning off at t = T/2 with initial conditions determined at $t = T/2^-$.

Triggering: Oscilloscopes use a trigger to tell them when to start capturing data. For the Discovery Board, the trigger is available using the Source drop down list located near the top of the display, a bit to the right of center. (Please add to the Proof of Skills Documentation if you find how to do this for the M1K or M2K board!) Often, using the source in the circuit as a trigger is a good choice since that signal is reliable and known. In the experiments, you should connect Oscilloscope Channel 1 to the source to measure input voltages to the circuit and therefore set your Trigger Source to Channel 1 as well. We will want to use rising edge triggering, meaning we start capturing data when the slope of the signal is positive. Just to the right of the Source drop down list is a Cond. drop down list. Set that to Rising. Just below that drop box is the Level setting. This value tells the scope to start capturing data when you cross that voltage. You want to adjust the level somewhere between the maximum and minimum voltages of the channel input, which is again a good reason why you want to use a known signal like the source. With rising edge triggering and pulsed signals, setting the trigger Level slightly higher than low voltage of the pulse is a good choice.

LTSpice
We will use the voltage pulse component introduced above.

B.1. RC Circuits



Build the RC circuit shown in the top figure, with R1 = 6k, R2 = 3k, R3 = 2.7k and C1 = 0.4μF.

1) Using the Discovery Board, set your signal to a 0.5 V Amplitude square wave with a DC offset of 0.5 V (the source should switch from 1 V to 0 V, repeating). Set the frequency such that you see the voltage across the capacitor reach DC steady state response for each half cycle of the square wave. Measure the source signal using Channel 1 of the Oscilloscope and the output signal (capacitor voltage in this case) with Channel 2 of the Oscilloscope.
    a. Analytically, determine the RC time constant for the above circuit. Finding the Thevenin equivalent circuit can help. (A similar circuit was discussed in class.)
    Make sure to include this calculation in the Proof of Concepts…
    b. Determine a differential expression for voltage across the capacitor. Solve the differential equation for a step function voltage source that turns on at t = 0 with a source voltage Vs = 1 V. Plot your result.
    c. In Discovery Board, obtain plots for the voltage (use differential probes) across the capacitor. Compare your results to the expression in part b.
    d. Estimate the RC time constant using the Discovery Board Oscilloscope
    e. Compare your result with the calculated value.
    f. Simulate the circuit in LTSpice using the voltage pulse component as your source and again compare your results.

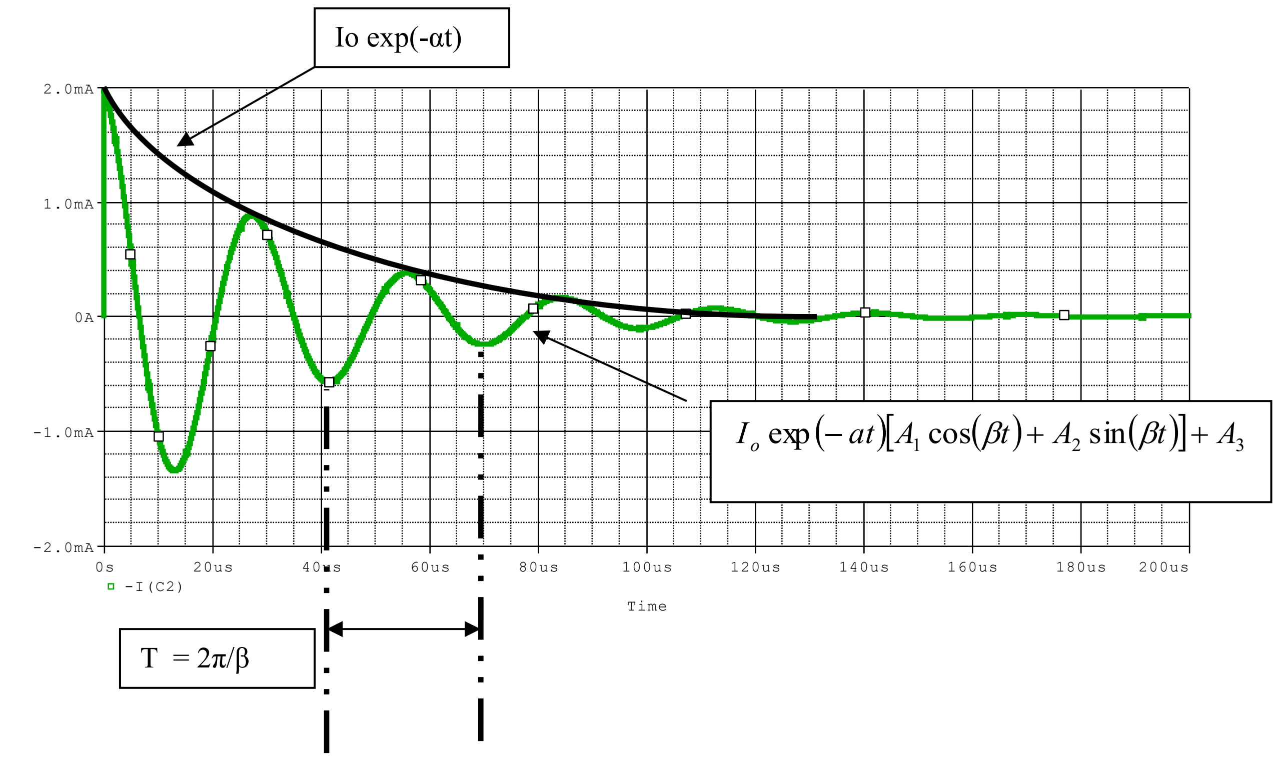
Include screen shots of your results in your Proof of Concept Report.
There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!

PART B: Proof of Concepts list
B.1: RC Circuit: Prove the RC time constant using Thevenin calculations for 1 u(t)

Part C: 2nd Order Circuits

Overall notes:

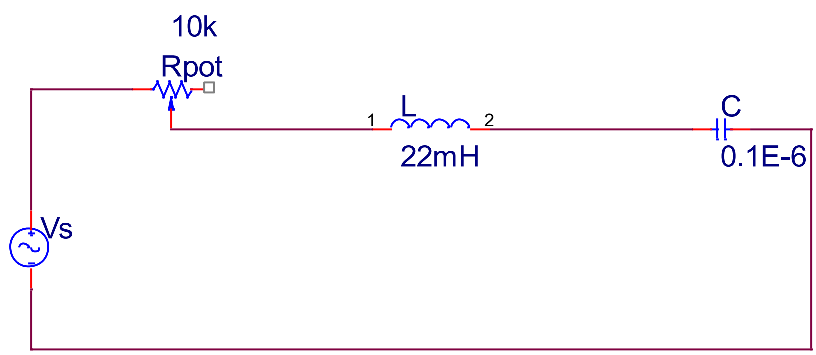
Plots:



In the above figure, an example of an underdamped current response is shown. The curve represents a step response decaying to zero. Using the plot, it is possible to determine the attenuation constant, α, and the oscillation constant, β. The exponential curve shown by the dark black line is defined by an amplitude, Io, and the attenuation constant, α. Both these terms can be obtained by identifying two points on the curve and using two equations/two unknowns to find the terms(for example: one point would be 0.9mA at 35μs). The sinusoidal peaks of the current response (any response) will always lie on the exponential curve. Similarly, the propagation constant, β, can be obtained from the plot by measuring the time between peaks (or minima) and using the relationship β=2π/T. Recall, α and β are determined from the differential equation, where the attenuation constant, α, is the real component of the quadratic solution of the *s* polynomial and the oscillation constant, $j\beta = j\text{sqrt}(\omega_o^2 - \alpha^2)$, is the imaginary component of the quadratic solution of the *s* polynomial.

Overdamped case: A slightly similar discussion may be applied to the overdamped response curve. In this case, typically one of the exponential terms is much larger than the other term and will thus result in a faster decay. With this in mind, it is possible to estimate the smaller exponential term by looking at the region of the curve some time past the initial response. "Some time" depends on how strongly overdamped the circuit is and may be less than a time constant for a strongly overdamped circuit and ~4 time constants for a weakly overdamped circuit.

C.1. RLC Series Circuit



Build the RLC circuit shown in the above figure. The resistor is a 10k potentiometer. Note, one leg of the potentiometer is left floating (not used).

1) On the Discovery Board, set your signal to a 1Vp-p square wave with a DC offset of 0.5Volts. Set the frequency such that you see the voltage across the capacitor reach DC steady state response for each half cycle of the square wave. In the following, consider just one-half cycle of which, as we have seen, is equivalent to the step source circuit.
   a. Adjust the potentiometer so that you can clearly see an overdamped response.
      i. Measure the resistance setting of your potentiometer
      ii. Based on your measured resistance, determine an analytic (calculated) expression for the voltage across the capacitor with a step function source. Remember, for an overdamped circuit
      $$V_c(t) = A_1 \exp(s_1 t) + A_2 \exp(s_2 t) + A_3$$
      iii. Simulate the same circuit in LTSpice. Use the resistance value you measured in place of Rpot (don't use the potentiometer in your LTSpice circuit, floating nodes are bad in LTSpice).

iv. Most likely, your Discovery Board curve is slightly different from the LTSpice and analytic results. In this case, we have ignored some resistances in the circuit. Any ideas on what they were?

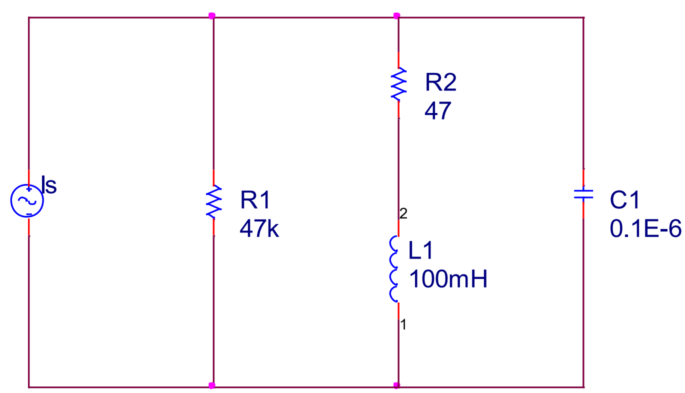
b. Adjust the potentiometer so that you clearly see an underdamped case.

i. Again, compare your result to the analytic and LTSpice results. Remember, for an underdamped circuit

$$V_c(t) = \exp(-\alpha t)[A_1 \cos(\beta t) + A_2 \sin(\beta t)] + A_3$$

Include screen shots of your results in your Proof of Concept Report. ***There are multiple steps here. Arrange in an easy to understand way for grading TAs***!

C.2. RLC Parallel Circuit



Build a circuit equivalent to the RLC circuit shown in the above figure and measure the current through the capacitor. Recall, we don't have a current source but we can build an equivalent circuit using source transformations (see Part B). Also, we use a small probe resistor to obtain a differential voltage measurement which is used to determine the current through the inductor. In your analysis, ignore the probe resistor and treat the circuit as a parallel RLC circuit.

2) On the Discovery Board, set your signal to a 1 V amplitude square wave with a DC offset of 0Volts (no offset). In effect, when considering one-half cycle, you will have nonzero initial conditions for the current through the inductor.

a. Determine a reasonable period for the square wave in your instrumentation board (M1K, M2K, or Analog Discovery board) measurement interface and a reasonable run time for LTSpice. Your square wave should be sufficient that you see the steady state value in one half period, T/2. Measure the voltage across R2 to determine the current through the inductor. Based on the oscilloscope measurements, estimate the attenuation constant and oscillation constant (Refer to the first page of the lab). Ignore the 47Ω resistor in your analysis. You will check if that is reasonable in part d. In the instrumentation board (M1K, M2K, or Analog Discovery board) measurement interface, you will most likely see a DC offset.

b. What type of response do you see (underdamped, overdamped or critically damped)?
c. Using the LTSpice plot, estimate the attenuation constant and oscillation constant. How close is this estimate to your calculated values?
d. In your LTSpice simulation, test the validity of ignoring R2 by running a simulation both with the resistor and without the resistor and comparing the results.

Include screen shots of your results in your Proof of Concept Report. ***There are multiple steps here. Arrange in an easy to understand way for grading TAs***!

C.3. s-domain Analysis with Step Function Input

Apply s-domain analysis to the RLC series in circuit in C.1. Initially, redraw the circuit using impedances (s-domain components). Include the initial value components which act like voltage sources in series with inductors and capacitors. Use symbolic notation in the circuit and the following analysis. Specific component values will be used later.

1. Draw your circuit.

Circuit analysis is applied to obtain to determine voltage across or current through individual components. We will look at the voltage across inductor L1 (You may also choose to look at the current through L1 to prove your concept. Please state which one you've chosen clearly.) In this case, a voltage divider is an appropriate choice. Recall, the voltage across the inductor in the s-domain includes the drop across the inductor and the drop across the initial value voltage source in the your circuit.

2. **Write the transfer function that represents the voltage across inductor L1** when the initial conditions are zero (all the $I_L(0^-)$ terms vanish, though in the Lab itself your circuit will have initial conditions).

3. We will use a source of the form Vs = 2-2u(t). (You may want to draw a sketch of this source.) In order to generate an equivalent source on the Discovery Board, set the function to 2Vpp (1V amplitude) with a 1V DC offset. To obtain a clean signal, you should set one of the oscilloscope channels to the source, set the trigger to that channel and set the Trigger level around 0.15V.
   a. Components: L1 = 100mH, R1 = 1kΩ, R2 = 1kΩ (you may alter values)
   b. Based on the source, determine the initial conditions for the s-domain circuit

   i_{L_1} (0-) =

4. Using the circuit component values and initial conditions, determine the poles and zeroes of the expression on the previous page.

   zeroes:

   poles:
   a. Is the system underdamped, critically damped or overdamped?
   b. Which pole is the most significant in terms of transient behavior?
   c. What is an appropriate time scale when considering plots for the step response? Remember, the attenuation constant(s), $\alpha = 1/\tau$, provides a time scale for the attenuation of the exponential term and the period of oscillation, $T=2\pi/\beta$ provides a time scale for harmonic oscillations if the system is underdamped.
5. Apply partial fraction expansion to the $V_{L1}(s)$ expression.
6. Apply the inverse Laplace transform to the above expression to obtain a time domain expression for your circuit.
7. **Plot your function using any graphing tool** (Excel/**Matlab (for Proof of Skills…)**/etc.)
8. Simulate the circuit in LTSpice and build the circuit. Compare the analytic plot, measured results from the Discovery Board, and simulation results from LTSpice.

PART C: Proof of Concepts list
C.1: RLC Series Circuit: Prove how adjusting the resistance changes the circuit from overdamped to underdamped.
C.2: RLC Parallel Circuit: Prove how the attenuation constant and oscillation constant relates to an underdamped, overdamped, or critically damped circuit. (discuss why/whether you can ignore R2 used to calculate the current through the inductor.
C.3 Using s-domain analysis (Laplace Transforms), prove that you can adjust the circuit from overdamped to underdamped.

Part D: Alpha Laboratories Applications

D.1. Can a Circuit Tell Time, Keep Time, and Create Time?!?

What are timing circuits?

1) Find three different timing circuits and describe the function of each component within it. (You can Google it!)
2) Demonstrate one of those via simulation and DESCRIBE its purpose in a larger circuit. (You may need another component to show it, but you do NOT have to physically build it).

PART D: Proof of Concepts list
D.1: Prove that your chosen timing circuit functions as you expected.

EXTRA CREDIT: Write in your metacognition journal (instructions and template in the link below, feel free to continue to edit a Google doc throughout the course to add entries).

https://ecse.rpi.edu/~ssawyer/CircuitsFall2019_all/Labs/Circuits_OmegaLabDocs/04_Deliverables/05_Circuits_Metacognition%20and%20Reflections.docx

SUMMARY of Concepts

Concept List that must be accounted for in your Proof of Concepts

PART A:

1. **RC Circuit-Prove that the time constant changes with different component values.**

PART B:

1. **RC Circuit: Prove the RC time constant using Thevenin calculations for 1 u(t)**

PART C:

1. **RLC Series Circuit: Prove how adjusting the resistance changes the circuit from overdamped to underdamped.**
2. **RLC Parallel Circuit: Prove how the attenuation constant and oscillation constant relates to an underdamped, overdamped, or critically damped circuit. (discuss why/whether you can ignore R2 used to calculate the current through the inductor.**
3. **Using s-domain analysis (Laplace Transforms), prove that you can adjust the circuit from overdamped to underdamped.**

PART D: Alpha Labs Applications

1. **Prove that your chosen timing circuit functions as you expected.**

Standards Based Assessment:

You will be graded on the following Standards. Please ensure to achieve each standard. If you do not, you can resubmit to the missing standard to the end of the semester. CLEARLY mark the changes you make in you Proof of Concept submission by either Tracking Changes in Word or highlighting changes by writing comments in a different color and/or changing the color of the updated work.

1. **I can change the time constant with different component values in an RC or RL circuit.**
2. **I can use Thevenin to simplify an RC circuit and find its time constant.**
3. **I can adjust the component values of an RLC series circuit to switch between underdamped and overdamped and analyze using both Diff. Eq. and Laplace Transforms.**
4. **I can adjust the component values of an RLC parallel circuit to switch between underdamped and overdamped and analyze it.**
5. **I can explain the use of a resistor in series with an inductor in a parallel circuit for measurement in a built parallel circuit.**
6. **I can design a timing circuit based on what is learned in this lab.**
7. **I can answer for myself "Is this right?" by comparing mathematical calculations to simulation and experimental results.**
8. **I can show plots and diagrams that are easy to read, scaled correctly and clearly labeled.**
9. **I can use consistent variable labels and component values in mathematical calculation, simulation and experimental results for easy comparison.**