

LABORATORY 3: Transient circuits, RC, RL step responses, 2nd Order Circuits

Note: If your partner is no longer in the class, please talk to the instructor.

Material covered:

- RC circuits
- Integrators
- Differentiators
- 1st order RC, RL Circuits
- 2nd order RLC series, parallel circuits
- Thevenin circuits

Part A: Transient Circuits

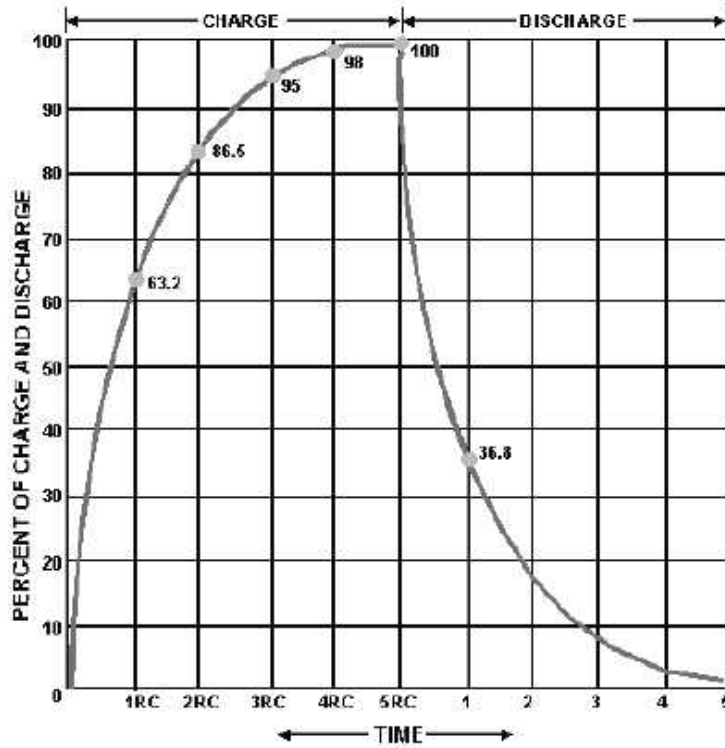
RC Time constants: A time constant is the time it takes a circuit characteristic (Voltage for example) to change from one state to another state. In a simple RC circuit where the resistor and capacitor are in series, the RC time constant is defined as the time it takes the voltage across a capacitor to reach 63.2% of its final value when charging (or 36.8% of its initial value when discharging). It is assume a step function (Heavyside function) is applied as the source. The time constant is defined by the equation $\tau = RC$ where

τ is the time constant in seconds

R is the resistance in Ohms

C is the capacitance in Farads

The following figure illustrates the time constant for a square pulse when the capacitor is charging and discharging during the appropriate parts of the input signal. You will see a similar plot in the lab. Note the charge (63.2%) and discharge voltages (36.8%) after one time constant, respectively.



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Discovery Board:

For most of the remaining class, you will want to compare input and output voltage time varying signals. We will be using the Scope option for measuring voltages now, instead of using the Voltmeter. There are two Scope channels, using the 1+/1- and 2+/2- inputs on the Discovery Board (the same as the Voltmeter probes).

For this lab, you should use Channel 1 for the input signal and Channel 2 for the output signal.

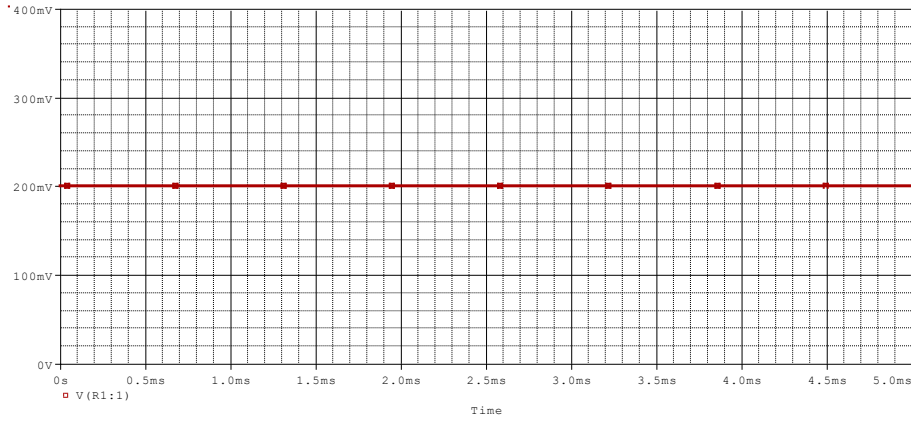
Time: On the upper right of the Scope window, you will see a box labelled Time. This box controls the scale of the horizontal axis. You should make sure that the Pos setting is zero, corresponding to a time of 0 at the far left side of the Oscilloscope screen. The Base setting is the time per division. There are ten horizontal divisions. If you use a Base setting of 5ms/div, the time scale of the whole window is then 50ms. In general, for periodic signals, you will want to display 4-5 periods on the scope. The Base setting needs to be adjusted based on your source frequency (recall $T = 1/f$, where T is the period and f is the frequency).

Channels: The vertical scale is set for each Channel individually. Since both signals are displayed on the same graph, you need to be careful when reading voltage measurements. The C1 and C2 buttons in the upper left can be used to toggle between the different settings. Again, you will need to adjust the Volts/div scale based on your expectations of voltage levels. It is also a good idea to make sure the C1 and C2 offsets are set to zero. The Discovery Board may change the offset settings, which corresponds to where ground is displayed on the scope. I suggest keeping the offset at zero.

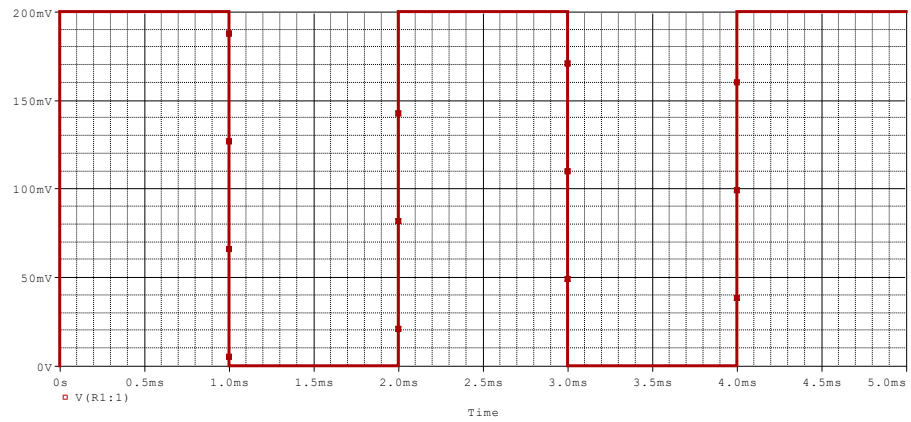
Function Generator: In this lab, we will be looking at DC, sinusoidal, square wave and triangle wave functions. These are all available as icons on the AWG (arbitrary waveform generator) window that we used in the previous lab. Examples of the waveforms are shown on the next page.

Signal types:

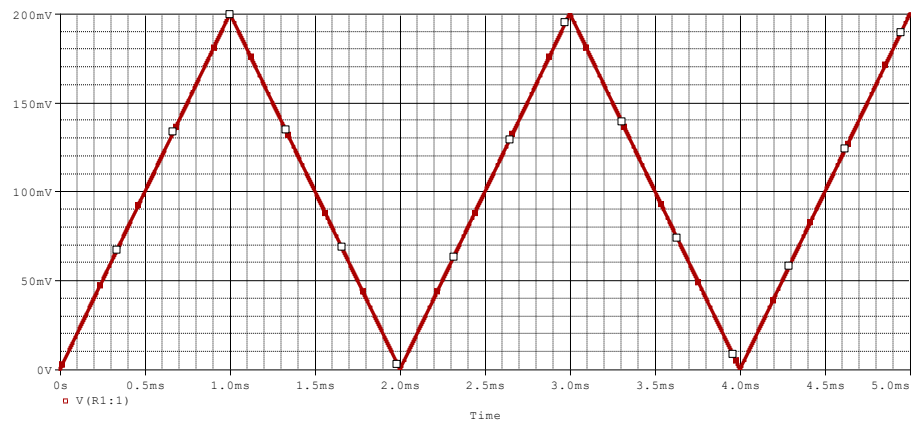
DC source (constant)



Square wave (pulse train)



Triangle wave



LTSpice:

To generate a square wave, use the voltage component. Use the following settings (do not include the information in parenthesis):

Vinitial[V]: 0 (low voltage in volts)

Von[V]: 2 (high voltage in volts)

Tdelay[s]: 0 (time delay in seconds)

Trise[s]: 0 (rise time in seconds)

Tfall[s]: 0 (fall time in seconds)

Ton[s]: *variable* (pulse width, 50% duty cycle, so should be half the period)

Tperiod[s]: *variable* (as directed in the lab, 1/f, example a 1kHz wave has a 1ms period)

Ncycles: *variable* (can leave blank, number of cycles)

Plot quality/accuracy: When you set your simulation profile the Time Domain (Transient tab) analysis lets you set a few parameters. We will be looking at several periods of the signal. You should be using the following settings:

Stop time: (sufficient time for several periods to be displayed) seconds

Time to start saving data: 0 seconds

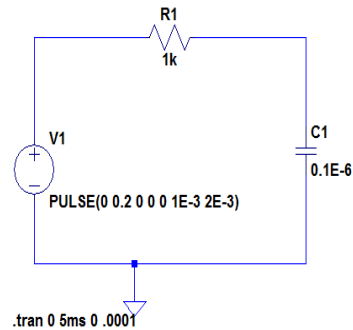
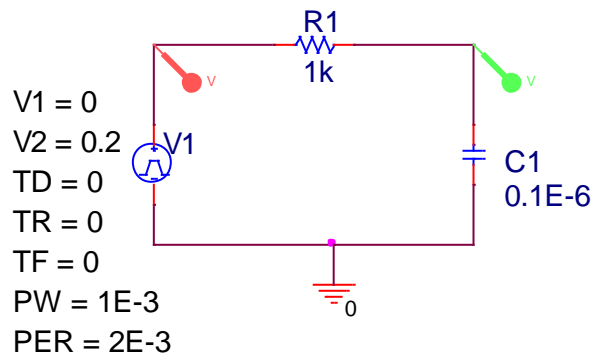
Maximum timestep: (I usually set this so that there are ~1000 points per period)

Plots: For transient analysis, we can place a Probe on the LTSpice circuit to obtain plots of voltage/current as a function of time. You can place that Probe at a node to obtain plots of voltage as a function of time. Note, this measurement is nodal and is measured relative to the indicated ground. If you are going to measure the voltage drop across a component that is not connected to ground, you can use differential probes, which can be set using the icon with the pair of probes. You place one probe on one side of the component and the other probe on the other side of the component.

Cursors: Cursors are available in LTSpice as well. Right click on the Trace label at the top of the simulation screen. You can choose up to two cursors to use by clicking the pull down menu "Attached Cursor" and selecting 1st, 2nd, or 1st and 2nd. You can drag the cursor along your plot and it will follow the waveform. When you start the cursor, the data window should show up providing voltage levels, time, and frequency if using two cursors as well. More details with pictures are shown here.

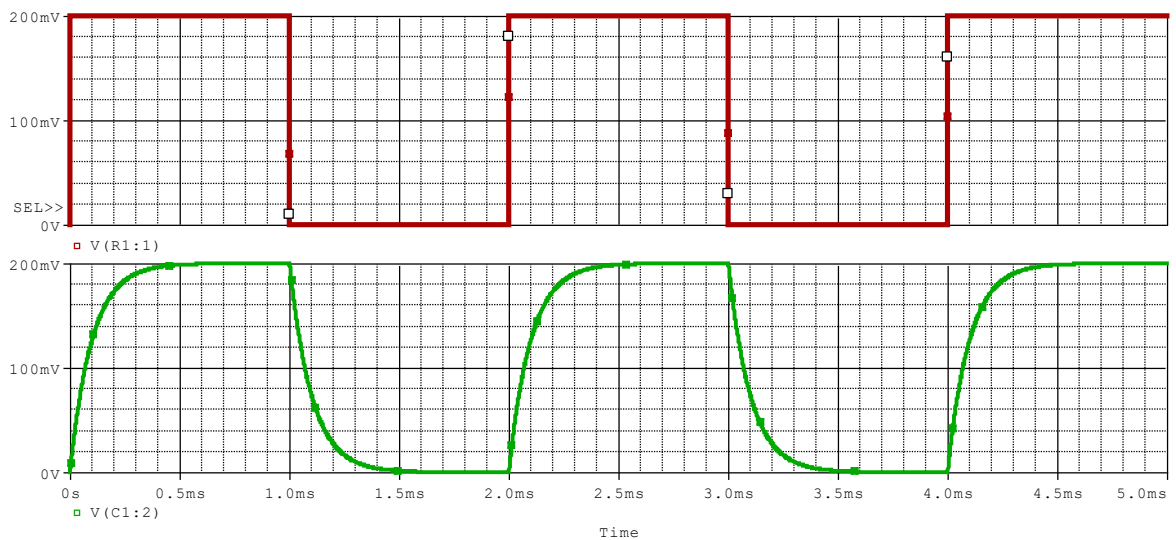
http://ltwiki.org/index.php?title=Attached_Cursors

A.1. RC Circuits



Build the RC circuit shown in the top figures (They are the same circuit but the PSpice circuit on the right shows probe position).

- 1) On the Discovery Board, set your signal to a 1V amplitude (2V peak-to-peak) square wave with a DC offset of 1 Volts. Set the frequency to 1kHz initially. Implement your circuit with $R1 = 1k\Omega$ (1E3) and $C1 = 0.1\mu\text{F}$ (0.1E-6). Connect Oscilloscope Channel 1 probes to the source and Oscilloscope Channel 2 probes across the capacitor.
 - a. Reduce the frequency such that you can see the voltage across the capacitor reach a constant value (steady state) in each half period of the square wave (approaching the DC condition). You will need to adjust the Base (horizontal axis) of the Oscilloscope to see several periods. (An example is shown below, where red is the source voltage and green is the voltage across the capacitor.)



Example of Voltage Output Waveform

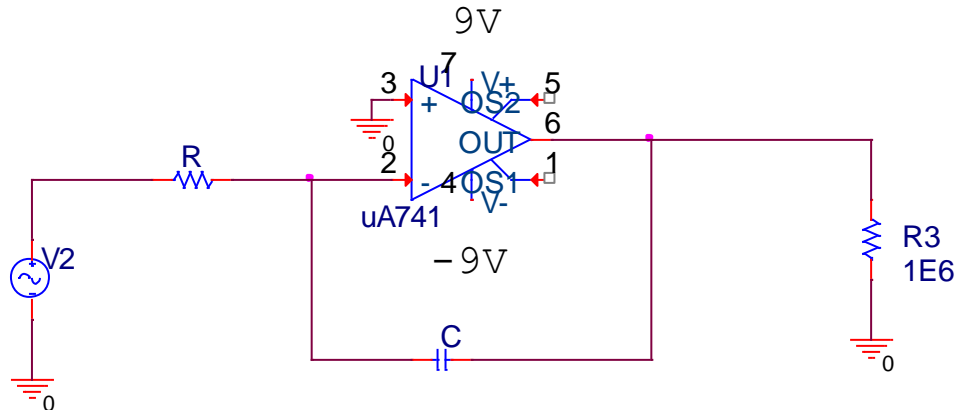
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- b. Estimate the time constant. It will be easier if you use the cursors (described at the start of the lab).
 - c. Compare your result with the expected time constant. (expected time constant is RC , resistance times capacitance, it will be explained in lecture)
 - d. Implement your circuit in LTSpice and compare your results.
- 2) Repeat a-d. for $R = 10\text{k}\Omega$ and $C = 0.01\text{E-}6\text{F}$.

A2: Integrators and Differentiators

The above circuit is called an integrator. The relationship between the input and the output is $V_{out} = -\frac{1}{RC} \int V_{in} dt$.

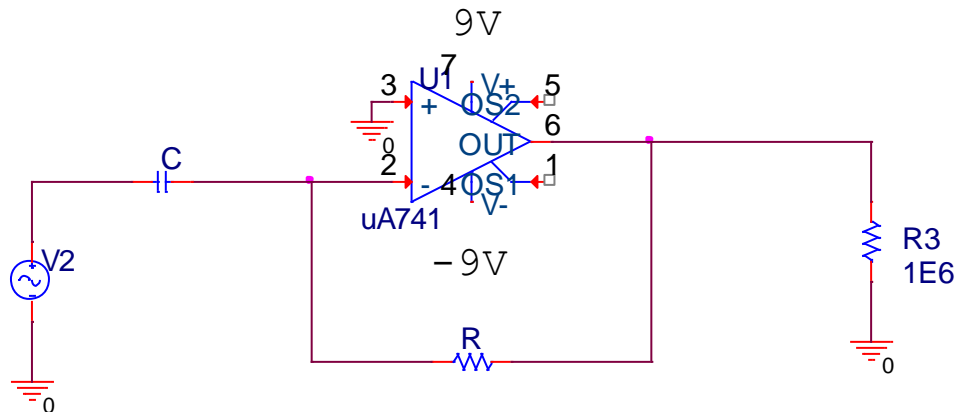
- a) Build the Integrator circuit using $R = 2k\Omega$ and $C = 0.2E-6F$ in LTSpice. You may need to add components together to get the values above.

(Note: The diagram above was made in PSpice but you should be now know how to do it in LTSpice.)

- b) On the Discovery Board, set your signal to a 2.0 V_{p-p} sine wave with a DC offset of 0.0 Volts (no DC component), $V_{in}(t) = V_o \sin(2\pi ft)$. Set the frequency to 1kHz initially. (Discovery Board only)
- For the above settings, mathematically determine $V_{out}(t)$. (yes, you have to do Calculus)
 - Connect Oscilloscope channel 1 input to the source and Oscilloscope channel 2 input to the output of the op-amp. Does the output voltage agree with expectations?
 - Increase and decrease the frequency. Does the waveform change as expected?
 - Change the coupling to DC and set the DC offset to 0.1 V. Explain what you see at the output.
 - Repeat part a-c. with a square wave input. Make sure you set the offset to 0 V. Again, does the output waveform behave as expected?
 - Repeat part a-c with a triangle wave input. Again, does the output waveform behave as expected.

*(LTSpice: The triangle wave simulation is optional and extra credit.)
Clearly label extra credit in the heading of your Concept.*

BE SURE TO DISCUSS all questions especially c. and d. in your proof of concepts...

Differentiators

The above circuit is called an differentiator. The relationship between the input and the output is $V_{out} = -RC \frac{dV_{in}}{dt}$ (we will derive this in class). (*Note: The diagram above was made in PSpice but you should be now know how to do it in LTSpice.*)

Build the Differentiator circuit using $R = 2k\Omega$ and $C = 0.2E-6F$. (Discovery Board only)

- a) On the Discovery Board, set your signal to a 2.0 Vp-p *sine* wave with a DC offset of 0.0 Volts (no DC component), $V_{in}(t) = V_o \sin(2\pi f t)$. Set the frequency to 1kHz initially.
 - a. For the above settings, mathematically determine $V_{out}(t)$.
 - b. Does the output signal agree with expectations?
 - c. Increase and decrease the frequency. Does the waveform change as expected?
 - d. Set the DC offset of your input signal to 0.1 V. Do you notice any change in the output?
 - e. Change the waveform to a triangle wave. Is the output consistent with expectations?
 - f. Change the waveform to a square wave. Is the output consistent with expectations? (*we will discuss what you see in a fair bit of detail when we get to second order transient responses*)

PART A: Proof of Concepts list

A.1: RC Circuit-Prove that the time constant changes with different component values.

A.2: Integrator- Prove that the integrator circuit integrates at least two different signals (i.e. sine wave and square wave) as expected.

A.2: Integrator: Prove/discuss DC offset as it relates to integration of your signals

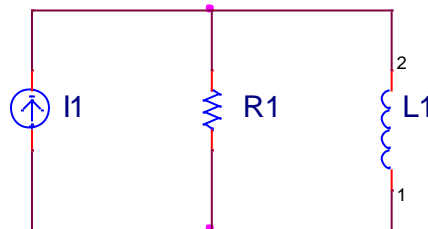
A.2: Differentiator - Prove that the integrator circuit integrates at least two different signals (i.e. sine wave and square wave) as *expected*. Note: Speculate why the square wave and/or triangle output looks the way it does which is likely NOT what you expected!...(mathematical analysis will help but a little physics is needed too...)

Part B: RC, RL Step Responses

Overall notes:

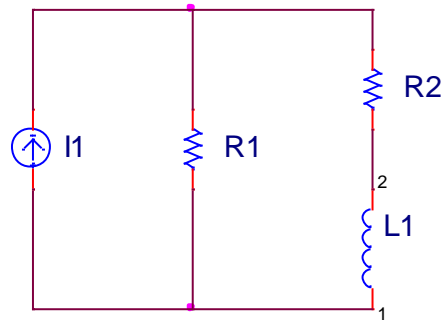
In your plots, you should compare input signals (sources) to outputs signals (component voltage/current).

In Part B, you will investigate an RL parallel circuit.

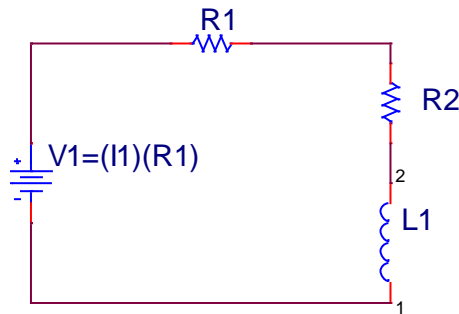


The derivation of the current across the inductor leads to the differential equation, $\frac{L}{R} \frac{di_L}{dt} + i_L = I_s$. For a step function source current, $I_1 u(t)$, the solutions to the differential equation take the form $i_L(t) = K_1 e^{-t\left(\frac{R}{L}\right)} + K_2$. This form is similar to what we saw for the RC circuit, with a slightly different time constant $\tau = L/R$.

Unfortunately, there are two problems with investigating the above circuit. The first problem is that we don't have access to a current probe. One practical solution is to add a small resistor in series with the inductor. You want the resistor to be small enough such that it does not significantly change the circuit and large enough that you can obtain an accurate voltage reading.



Second, we don't have a reliable current source. However, we can use source transformations to generate an equivalent circuit with a voltage source, as shown in the following circuit.



R_2 is the test resistor and should be much smaller than R_1 , ($R_2 \ll R_1$). In the experiment that follows, you could easily just determine the current by measuring the voltage across R_1 without any need for R_2 . However, we will treat the experiment as representative of a more complex model and use a small R_2 resistor.

Note, you may want to prove to yourself that the source transformation circuit results in the same current response in L_1 .

Discovery Board

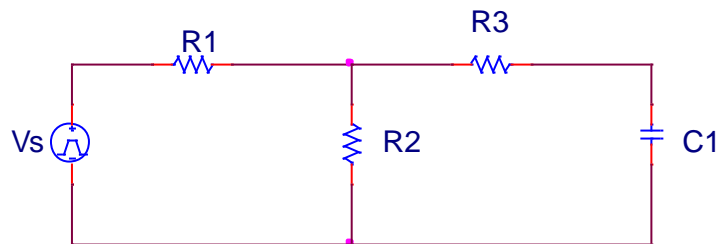
This lab investigates step responses. In Discovery Board, we use pulse streams and can't easily make a step function source. To measure an equivalent response using a pulse stream, we make sure that each half cycle is much longer than the time constant for the circuit, $T/2 \gg \tau$. By letting the circuit response reach DC steady state in each half cycle, the $V = V_0$ half cycle is equivalent to a source turning on at $t = 0$ with zero initial conditions and the $V = 0$ half cycle is equivalent to a source turning off at $t = T/2$ with initial conditions determined at $t = T/2^-$.

Triggering: Oscilloscopes use a trigger to tell them when to start capturing data. For the Discovery Board, the trigger is available using the Source drop down list located near the top of the display, a bit to the right of center. Often, using the source in the circuit as a trigger is a good choice since that signal is reliable and known. In the experiments, you should connect Oscilloscope Channel 1 to the source to measure input voltages to the circuit and therefore set your Trigger Source to Channel 1 as well. We will want to use rising edge triggering, meaning we start capturing data when the slope of the signal is positive. Just to the right of the Source drop down list is a Cond. drop down list. Set that to Rising. Just below that drop box is the Level setting. This value tells the scope to start capturing data when you cross that voltage. You want to adjust the level somewhere between the maximum and minimum voltages of the channel input, which is again a good reason why you want to use a known signal like the source. With rising edge triggering and pulsed signals, setting the trigger Level slightly higher than low voltage of the pulse is a good choice.

LTSpice

We will use the voltage pulse component introduced above.

B.1. RC Circuits



Build the RC circuit shown in the top figure, with $R1 = 6k$, $R2 = 3k$, $R3 = 2.7k$ and $C1 = 0.4\mu F$.

- 1) Using the Discovery Board, set your signal to a 0.5 V Amplitude square wave with a DC offset of 0.5 V (the source should switch from 1 V to 0 V, repeating). Set the frequency such that you see the voltage across the capacitor reach DC steady state response for each half cycle of the square wave. Measure the source signal using Channel 1 of the Oscilloscope and the output signal (capacitor voltage in this case) with Channel 2 of the Oscilloscope.
 - a. Analytically, determine the RC time constant for the above circuit. Finding the Thevenin equivalent circuit can help. (A similar circuit was discussed in class.)

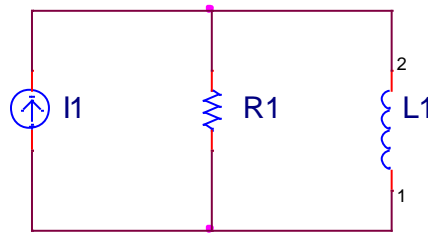
Make sure to include this calculation in the Proof of Concepts...

- b. Determine a differential expression for voltage across the capacitor. Solve the differential equation for a step function voltage source that turns on at $t = 0$ with a source voltage $V_s = 1 \text{ V}$. Plot your result.
- c. In Discovery Board, obtain plots for the voltage (use differential probes) across the capacitor. Compare your results to the expression in part b.
- d. Estimate the RC time constant using the Discovery Board Oscilloscope
- e. Compare your result with the calculated value.
- f. Simulate the circuit in LTSpice using the voltage pulse component as your source and again compare your results.

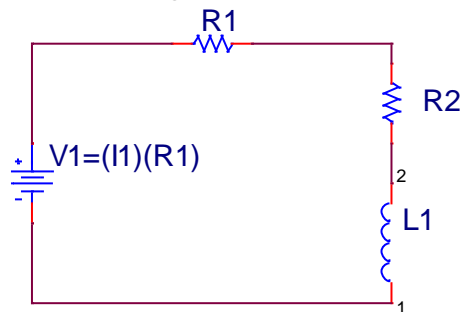
Include screen shots of your results in your Proof of Concept Report.

There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!

B.2. RL Circuits



Original Circuit

Equivalent Circuit with Small Probe Resistance (R_2)

Build the RL circuit shown in the above figure, with $R_1 = 2.2\text{k}\Omega$, $R_2 = 20\Omega$, and $L_1 = 100\text{mH}$.

- 2) In Discovery Board, set your signal to a 2.5 V amplitude square wave with a DC offset of 2.5 Volts (the source pulse switches from 5 V to 0 V, repeating). Set the frequency such that you see the voltage across the inductor reach DC steady state response for each half cycle of the square wave. Your measured voltages may have some spikes when the source square wave has low-high/high-low transitions. Try to ignore the spikes in your discussion of Discovery Board results/calculations. We will talk about why they occur when we discuss second order circuits.
 - a. Analytically, determine the RL time constant for the above circuit.
Make sure to include this calculation in the Proof of Concepts...
 - b. For the original RL parallel circuit, determine a differential expression for current through the inductor. Solve the differential equation for a step function current source (equivalent to a low to high transition).
Plot your result.
 - c. For the original RL parallel circuit, determine a differential expression for voltage across the inductor. Solve the differential equation for a step

function current source (equivalent to a low to high transition). **Plot your result.**

- d. Implement the equivalent circuit and measure the voltage across R2. Scale your result by R2 to estimate the current through L1 as a function of time. Compare your result to part b.
- e. Estimate the RL time constant using the Discovery Board Oscilloscope. Note, as discussed in class, since the solution to the homogeneous differential equation is the same for all components, the RL time constant for this circuit can be found from any voltage or current plot across any component. The experimental result is probably a bit different than the calculated time constant. In the circuit, there is a 'hidden' resistance that is not negligible. Where is that resistance?
- f. Compare your result with the calculated value.
- g. Obtain plots for the voltage across the inductor in Discovery Board. Compare your results to part c.
- h. Simulate the original parallel RL circuit with a current source in LTSpice (include your small probe resistor). Use current pulse component as your source (the component description is similar to voltage pulse component, with currents instead of voltages). Again compare your results.

Include screen shots of your results in your Proof of Concept Report. *There are multiple steps here. Arrange and label with headings in an easy to understand way for grading TAs!*

PART B: Proof of Concepts list

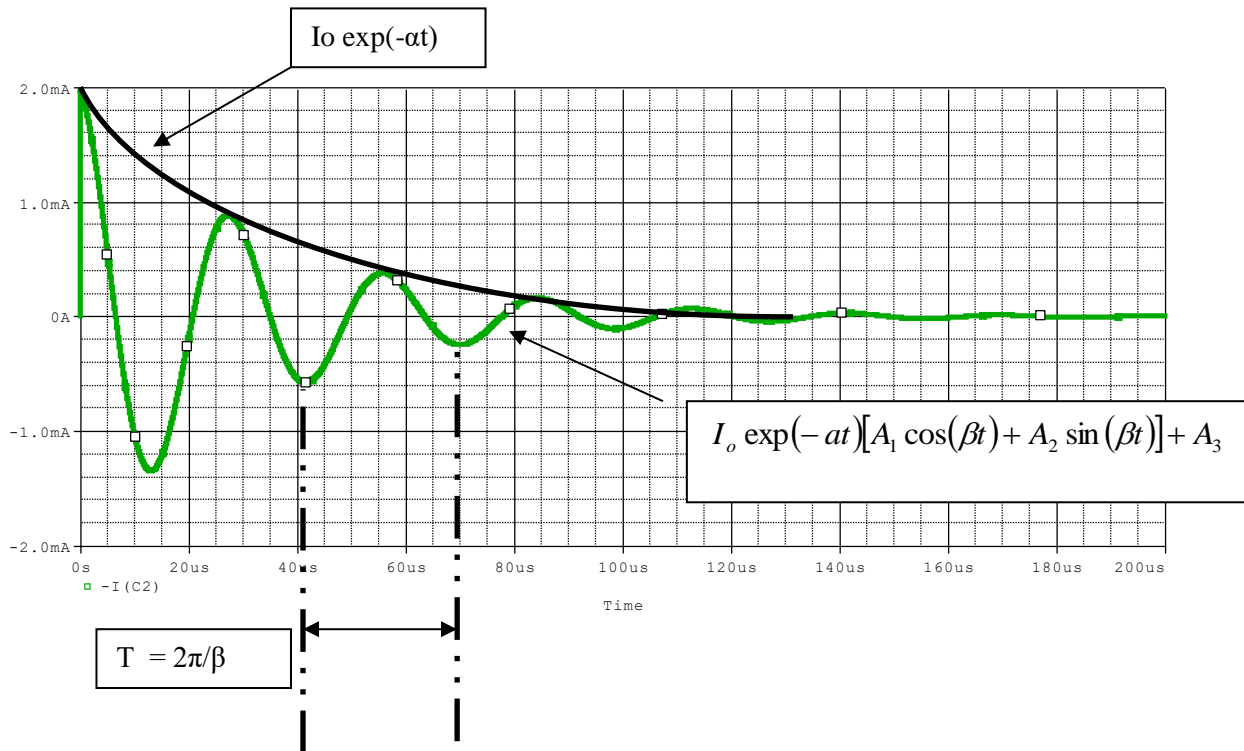
B.1: RC Circuit: Prove the RC time constant using Thevenin calculations for $1 \mu(t)$

B.2: RL Circuit: Prove the RL time constant using an equivalent series circuit and the original parallel circuit. Be sure to also include comparisons to the original parallel circuit in LTSpice.

Part C: 2nd Order Circuits

Overall notes:

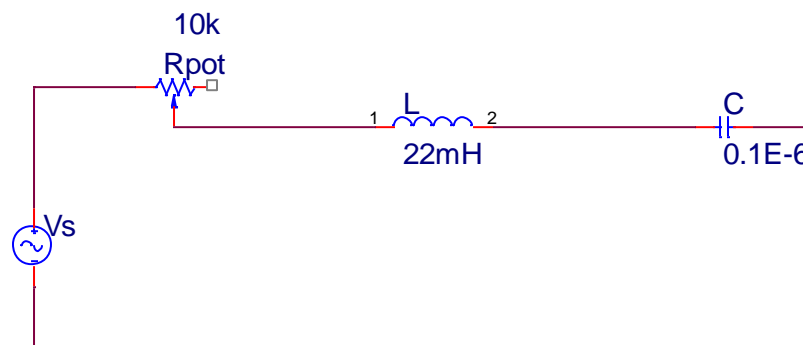
Plots:



In the above figure, an example of an underdamped current response is shown. The curve represents a step response decaying to zero. Using the plot, it is possible to determine the attenuation constant, α , and the oscillation constant, β . The exponential curve shown by the dark black line is defined by an amplitude, I_0 , and the attenuation constant, α . Both these terms can be obtained by identifying two points on the curve and using two equations/two unknowns to find the terms (for example: one point would be 0.9 mA at 35 μs). The sinusoidal peaks of the current response (any response) will always lie on the exponential curve. Similarly, the propagation constant, β , can be obtained from the plot by measuring the time between peaks (or minima) and using the relationship $\beta = 2\pi/T$. Recall, α and β are determined from the differential equation, where the attenuation constant, α , is the real component of the quadratic solution of the s polynomial and the oscillation constant, $j\beta = j\sqrt{\omega_0^2 - \alpha^2}$, is the imaginary component of the quadratic solution of the s polynomial.

Overdamped case: A slightly similar discussion may be applied to the overdamped response curve. In this case, typically one of the exponential terms is much larger than the other term and will thus result in a faster decay. With this in mind, it is possible to estimate the smaller exponential term by looking at the region of the curve some time past the initial response. “Some time” depends on how strongly overdamped the circuit is and may be less than a time constant for a strongly overdamped circuit and ~ 4 time constants for a weakly overdamped circuit.

C.1. RLC Series Circuit



Build the RLC circuit shown in the above figure. The resistor is a 10k potentiometer. Note, one leg of the potentiometer is left floating (not used).

- 1) On the Discovery Board, set your signal to a 1Vp-p square wave with a DC offset of 0.5Volts. Set the frequency such that you see the voltage across the capacitor reach DC steady state response for each half cycle of the square wave. In the following, consider just one-half cycle of which, as we have seen, is equivalent to the step source circuit.
 - a. Adjust the potentiometer so that you can clearly see an overdamped response.
 - i. Measure the resistance setting of your potentiometer
 - ii. Based on your measured resistance, determine an analytic (calculated) expression for the voltage across the capacitor with a step function source. Remember, for an overdamped circuit

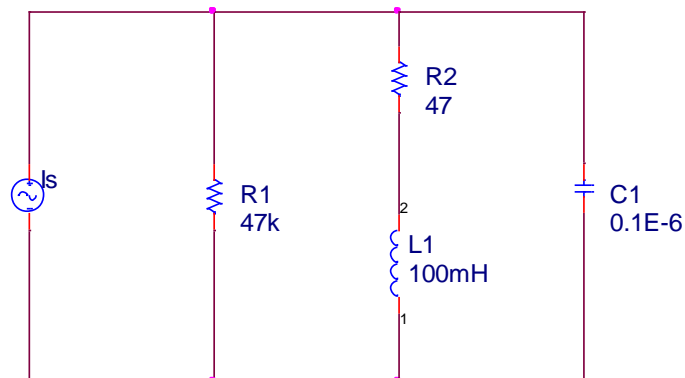
$$V_c(t) = A_1 \exp(s_1 t) + A_2 \exp(s_2 t) + A_3$$
 - iii. Simulate the same circuit in LTSpice. Use the resistance value you measured in place of Rpot (don't use the potentiometer in your LTSpice circuit, floating nodes are bad in LTSpice).

- iv. Most likely, your Discovery Board curve is slightly different from the LTSpice and analytic results. In this case, we have ignored some resistances in the circuit. Any ideas on what they were?
- b. Adjust the potentiometer so that you clearly see an underdamped case.
- i. Again, compare your result to the analytic and LTSpice results. Remember, for an underdamped circuit

$$V_c(t) = \exp(-\alpha t) [A_1 \cos(\beta t) + A_2 \sin(\beta t)] + A_3$$

Include screen shots of your results in your Proof of Concept Report.
There are multiple steps here. Arrange in an easy to understand way for grading TAs!

C.2. RLC Parallel Circuit



Build a circuit equivalent to the RLC circuit shown in the above figure and measure the current through the capacitor. Recall, we don't have a current source but we can build an equivalent circuit using source transformations (see Part B). Also, we use a small probe resistor to obtain a differential voltage measurement which is used to determine the current through the inductor. In your analysis, ignore the probe resistor and treat the circuit as a parallel RLC circuit.

- 3) On the Discovery Board, set your signal to a 1 V amplitude square wave with a DC offset of 0Volts (no offset). In effect, when considering one-half cycle, you will have nonzero initial conditions for the current through the inductor.
- i. Determine a reasonable period for the square wave in the Analog Discovery Board measurement interface and a reasonable run time for LTSpice. Your square wave should be sufficient that you see the steady state value in one half period, $T/2$. Measure the voltage across R2 to determine the current through the inductor. Based on the oscilloscope measurements, estimate the attenuation constant and oscillation

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constant (Refer to the first page of the lab). Ignore the 47Ω resistor in your analysis. You will check if that is reasonable in part d. In the Analog Discovery Board measurement interface, you will most likely see a DC offset.

- j. What type of response do you see (underdamped, overdamped or critically damped)?
- k. Using the LTSpice plot, estimate the attenuation constant and oscillation constant. How close is this estimate to your calculated values?
- l. In your LTSpice simulation, test the validity of ignoring R2 by running a simulation both with the resistor and without the resistor and comparing the results.

Include screen shots of your results in your Proof of Concept Report.
There are multiple steps here. Arrange in an easy to understand way for grading TAs!

PART C: Proof of Concepts list

C.1: RLC Series Circuit: Prove how adjusting the resistance changes the circuit from overdamped to underdamped.

C.2: RLC Parallel Circuit: Prove how the attenuation constant and oscillation constant relates to an underdamped, overdamped, or critically damped circuit.

C2: Prove/discuss why/whether you can ignore R2 used to calculate the current through the inductor.

Part D: Alpha Laboratories Applications

D.1. Can a Circuit Tell Time, Keep Time, and Create Time?!?

What are timing circuits?

- 1) Find three different timing circuits and describe the function of each component within it. (You can Google it!)
- 2) Demonstrate one of those via simulation and DESCRIBE its purpose in a larger circuit. (You may need another component to show it, but you do NOT have to physically build it).
- 3) Have a conversation with a Omega Lab person. Ask them how they plan to use a time circuit. Write their names down and what their application is.

PART D: Proof of Concepts list

D.1: Prove that your chosen timing circuit functions as you expected.

EXTRA CREDIT: Write in your metacognition journal (instructions and template in the link below, feel free to continue to edit a Google doc throughout the course to add entries).

https://ecse.rpi.edu/~ssawyer/CircuitsFall2019_all/Labs/Circuits_OmegaLab Docs/04_Deliverables/05_Circuits_Metacognition%20and%20Reflections.docx

SUMMARY of Concepts

Concept List that must be accounted for in your Proof of Concepts

PART A:

1. **RC Circuit-Prove that the time constant changes with different component values.**
2. **Integrator- Prove that the integrator circuit integrates at least two different signals (i.e. sine wave and square wave) as expected.**
3. **Integrator: Prove/discuss DC offset as it relates to integration of your signals**
4. **Differentiator - Prove that the integrator circuit integrates at least two different signals (i.e. sine wave and square wave) as expected.** Note: Speculate why the square wave and/or triangle output looks the way it does which is likely NOT what you expected!...(mathematical analysis will help but a little physics is needed too....)

PART B:

1. **RC Circuit: Prove the RC time constant using Thevenin calculations for $1 u(t)$**
2. **RL Circuit: Prove the RL time constant using an equivalent series circuit and the original parallel circuit. Be sure to also include comparisons to the original parallel circuit in LTSpice.**

PART C:

1. **RLC Series Circuit: Prove how adjusting the resistance changes the circuit from overdamped to underdamped.**
2. **RLC Parallel Circuit: Prove how the attenuation constant and oscillation constant relates to an underdamped, overdamped, or critically damped circuit.**
3. **Prove/discuss why/whether you can ignore R2 used to calculate the current through the inductor.**

PART D: Alpha Labs Applications

1. **Prove that your chosen timing circuit functions as you expected.**