Exam 1 Crib Sheet

Ohm's Law – Linear relationship between voltage and current in a resistor

V = I R

V – Voltage, Volts [V]

I – Current, Amps [A]

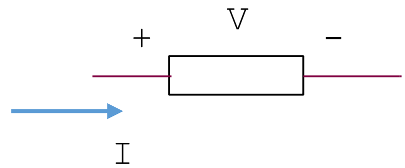
R – Resistance, Ohms [Ω]

Node – a connection between two or more components
Loop – a closed path through which current can flow

Power

P = V I

P – Power, Watts [W]



Using the above polarities (which may ot be correct)

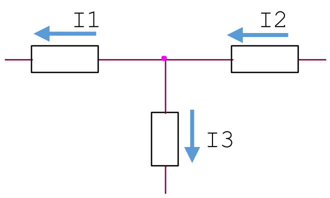
For P > 0, the component consumes power

For P < 0, the component produces power

KCL – Kirchoff's Current Law

$$\sum_{n=1}^{N} I_n = 0$$

The sum of the currents leaving a node is zero (signs determined by polarity).

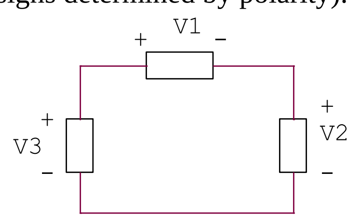


I1 - I2 + I3 = 0

KVL – Kirchoff's Voltage Law

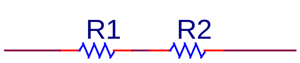
$$\sum_{n=1}^{N} V_n = 0$$

The sum of the voltages around any closed loop is zero (signs determined by polarity).

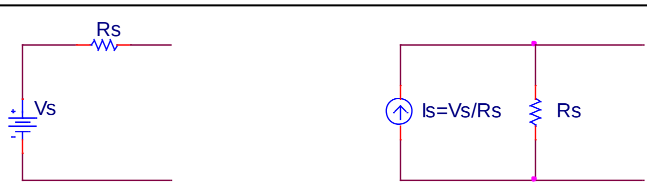


V1 + V2 - V3 = 0

Resistors in series – $R_{EQ} = R1 + R2$



Resistors in parallel - $R_{EQ} = \left(\frac{1}{R1} + \frac{1}{R2}\right)^{-1}$



Source transformation

Voltage divider (two resistors in series)
V_{R1}= Vsource x [R1/(R1+R2)]

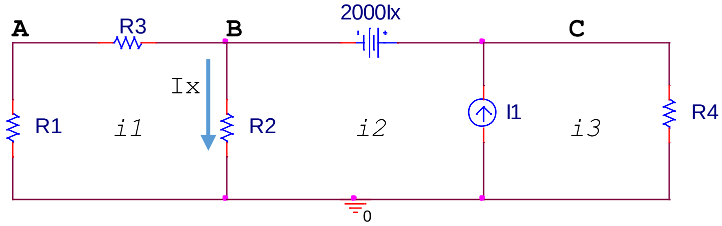
Current divider (two resistors in parallel)
I_{R1} = Isource x [R2/ (R1+R2)]

Superposition – For each **independent** source, turn off all other **independent** sources ***(to turn off: Voltage source becomes a short circuit and Current source becomes an open circuit)*** and find the contribution from that source. Sum the contribution from each source to get the parameter of interest.

Exam 1 Crib Sheet

Node Analysis

# of KCL Equations = Total # of nodes – voltage sources -1



Mesh Analysis

# of KVL Equations = Total # mesh loops – current sources

$$\frac{V_A}{R1}+\frac{V_A-V_B}{R3}=0$$

$$\frac{V_B}{R2}+\frac{V_B-V_A}{R3}-I_1+\frac{V_C}{R4}=0$$

$$V_C-V_B=2000I_x$$

$$\frac{V_B}{R2}=I_x$$

$$(i_1)R1+(i_1)R3+(i_1-i_2)R2=0$$

$$-2000I_x+(i_3)R4+(i_2-i_1)R2=0$$

$$i_3-i_2=I_1$$

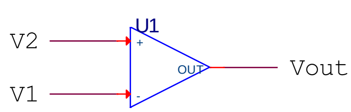
$$i_1-i_2=I_x$$

Example includes a Current Controlled Voltage Source (CCVS) as a dependent source and I1 as an independent source.

Thevenin voltage (**V_{TH}**) – **Open** circuit the load, find the voltage across the load nodes
Norton current (**I_N**)– **Short** circuit the load, find the current through that short circuit
Thevenin resistance (**R_{TH}**) – Turn off all **independent** sources, replace the load with a test voltage (Vtest), find the current (Itest) through the test voltage, R_{TH} = Vtest/Itest.

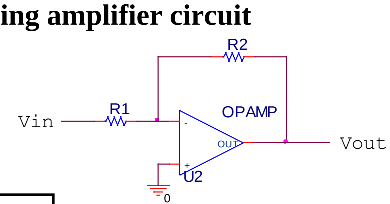
$V_{TH} = I_N R_{TH}$ (Ohm's Law relationship)

Comparator



If V1 < V2 , Vout = $V^{+}_{saturation}$
If V1 > V2 , Vout = $V^{-}_{saturation}$

Inverting amplifier circuit



$$Vout=-\frac{R2}{R1}Vinn$$

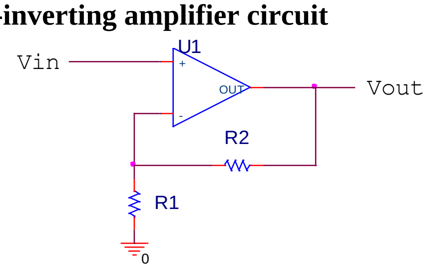
<u>Ideal op amp equations</u>

$I_N = I_P = 0$ no current draw Rin = ∞

$V_P = V_N$ (A -> ∞)

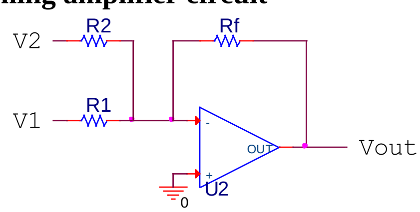
Rout = 0

Non-inverting amplifier circuit



$$Vout=\left(1+\frac{R2}{R1}\right)Vinn$$

Summing amplifier circuit



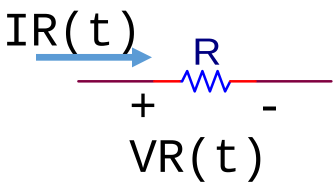
$$Vout=-\frac{Rf}{R1}V1-\frac{Rf}{R2}V2$$

Exam 2 Crib Sheet

IV Characteristics – Time domain

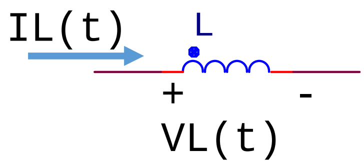
Resistors –

$$V(t) = I(t)R$$



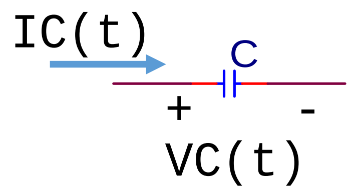
Inductors –

$$V_L(t) = L\frac{dI_L}{dt}$$



Capacitors –

$$I_C(t) = C\frac{dV_C}{dt}$$



Continuity conditions

$$I_L(t_o^-) = I_L(t_o^+)$$

$$V_C(t_o^-) = V_C(t_o^+)$$

IV Characteristics – Laplace domain

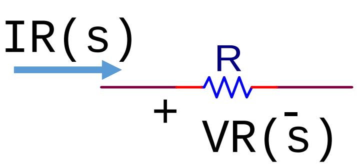
$$Z_R = R$$

$$Z_L = sL$$

$$Z_C = \frac{1}{sC}$$

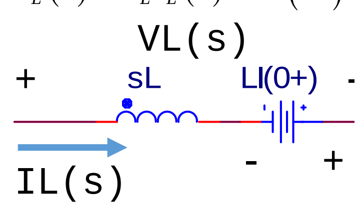
Resistors –

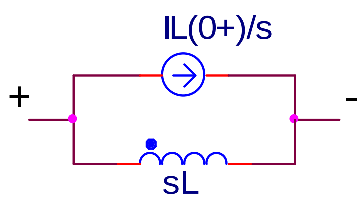
$$V(s) = Z_R I(s)$$



Inductors –

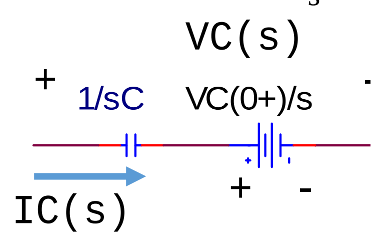
$$V_L(s) = Z_L I_L(s) - LI(0^+)$$

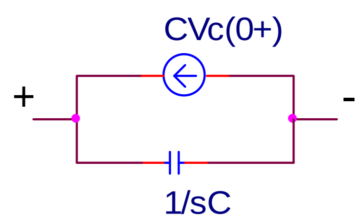




Capacitors –

$$V_C(s) = Z_C I_C(s) + \frac{V_C(0^+)}{s}$$





Impedance, Z [Ω], properties have the same characteristics as resistance

Impedances in series add, $Z_{EQ} = Z_1 + Z_2$

Impedances in parallel have an inverse relationship, $Z_{EQ} = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} = \frac{Z_1 Z_2}{Z_1 + Z_2}$

Initial Value Theorem $\lim_{s\to\infty}\{sF(s)\} = f(t=0^+)$	Final Value Theorem $\lim_{s\to 0}\{sF(s)\} = f(t\to\infty)$

Exam 2 Crib Sheet

First order circuits

Differential equation: $\tau \frac{dy}{dt} + y = f(t)$, with solution $y(t) = y_h(t) + y_p(t)$

$f(t)$ represents a source function or n[th] derivative of the source function, with appropriate coefficients

$y_h(t)$ represents the homogeneous/transient part of the solution

For first order circuits, the homogeneous solution always takes the form $y_h(t) = Ae^{\frac{-t}{\tau}}$

$y_p(t)$ represents the particular/forced part of the solution.

The particular solution is always the same type of function as the source.

τ is the time constant

For RC circuits, $\tau = RC$

For RL circuits, $\tau = L/R$

Second order circuits

Differential equation: $\frac{d^2y}{dt^2} + 2\alpha \frac{dy}{dt} + \omega_o^2 y = f(t)$, with solution $y(t) = y_h(t) + y_p(t)$

s-domain $s^2Y(s) + 2\alpha sY(s) + \omega_o^2 Y(s) = F(s)$

$y_h(t)$ represents the homogeneous/transient part of the solution

The form of the homogeneous solution depends on the damping

$y_p(t)$ represents the particular/forced part of the solution.

The particular solution is always the same type of function as the source.

$f(t)$ represents a source function or n[th] derivative of the source function

$F(s)$ represents the Laplace transform of the function f(t)

Overdamped ($\alpha > \omega_o$)	$y_h(t) = A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}$	$-\alpha_1, -\alpha_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$
	$y(0^+) = A_1 + A_2 + y_p(0^+)$	$\frac{dy(0^+)}{dt} = -\alpha_1 A_1 - \alpha_2 A_2 + \frac{dy_p(0^+)}{dt}$
Critically Damped ($\alpha = \omega_o$)	$y_h(t) = A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$	α from the differential equation
	$y(0^+) = A_1 + y_p(0^+)$	$\frac{dy(0^+)}{dt} = -\alpha A_1 + A_2 + \frac{dy_p(0^+)}{dt}$
Underdamped ($\alpha < \omega_o$)	$y_h(t) = e^{-\alpha t}\left[A_1 \cos(\beta t) + A_2 \sin(\beta t)\right]$	α from the differential equation $\beta = \sqrt{\omega_o^2 - \alpha^2}$
	$y(0^+) = A_1 + y_p(0^+)$	$\frac{dy(0^+)}{dt} = -\alpha A_1 + \beta A_2 + \frac{dy_p(0^+)}{dt}$

RLC series circuit $\alpha = \frac{1}{2}\frac{R}{L}$ $\omega_o = \frac{1}{\sqrt{LC}}$	RLC parallel circuit $\alpha = \frac{1}{2}\frac{1}{RC}$ $\omega_o = \frac{1}{\sqrt{LC}}$

Exam 2 Crib Sheet

Partial Fraction Expansion

Simple Real Poles:

In General:

$$\text{Expand: } F(s) = \frac{A_1}{s-p_1} + \frac{A_2}{s-p_2} + \frac{A_3}{s-p_3} + \ .....$$

$$A_n = [(s-p_n)F(s)]\big|_{s=p_n} \ ; \quad \text{Cover-Up Rule}$$

$$=> \ f(t) =)A_1 e^{p_1 t} + A_2 e^{p_2 t} + A_3 e^{p_3 t} + \ .....) \quad t \geq 0$$

Real, Equal Poles – Double Pole:

- Real, Equal Poles – Double Pole:

$$\text{Expand } F(s) = \frac{A_1}{s-p_1} + \ ..+ [\frac{A_{n1}}{s-p_n} + \frac{A_{n2}}{(s-p_n)^2}]$$

$$A_{n2} = \left[(s-p_n)^2 F(s)\right]\Big|_{s=p_n} \ ; \ \text{Cover-Up Rule}$$

Usually Find A_{n1} from evaluating F(0) or F(1)

$$=> f(t) = (A_1 e^{p_1 t} + \ .... + A_{n1} e^{p_n t} + A_{n2} t e^{p_n t}) \quad t \geq 0$$

Simple Poles   Repeated Poles

Complex Conjugate Poles

In General:

$$\text{Expand } F(s) = \frac{A_1}{s-p_1} + \ .... + \frac{A}{s+\alpha-j\beta} + \frac{A^*}{s+\alpha+j\beta}$$

Find A_1 and $A = |A|\underline{/\phi}$ from Cover-Up Rule

$$=> f(t) = A_1 e^{p_1 t} + \ .... + 2|A|e^{-\alpha t}\cos(\beta t + \phi) \quad t \geq 0$$

Simple Poles   Complex Poles

LAPLACE TRANSFORMS

Signal	f(t)	F(s)
Impulse	$\delta(t)$	1
Step	u(t)	$\frac{1}{s}$
Constant	Au(t)	$\frac{A}{s}$
Ramp	tu(t)	$\frac{1}{s^2}$

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LAPLACE TRANSFORMS

Signal	f(t)	F(s)
Exponential	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
Damped Ramp	$[te^{-\alpha t}]u(t)$	$\frac{1}{(s+\alpha)^2}$
Cosine Wave	$[\cos\beta t]u(t)$	$\frac{s}{s^2+\beta^2}$
Damped Cosine	$[e^{-\alpha t}\cos\beta t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$

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LAPLACE TRANSFORMS

Time Domain	s-Domain
$Af_1(t)+Bf_2(t)$	$AF_1(s)+BF_2(s)$
$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$\frac{df(t)}{dt}$	$sF(s)-f(0^-)$
$e^{-\alpha t}f(t)$	$F(s+\alpha)$
$t\,f(t)$	$-dF(s)/ds$
$f(t-a)u(t-a)$	$e^{-as}F(s)$

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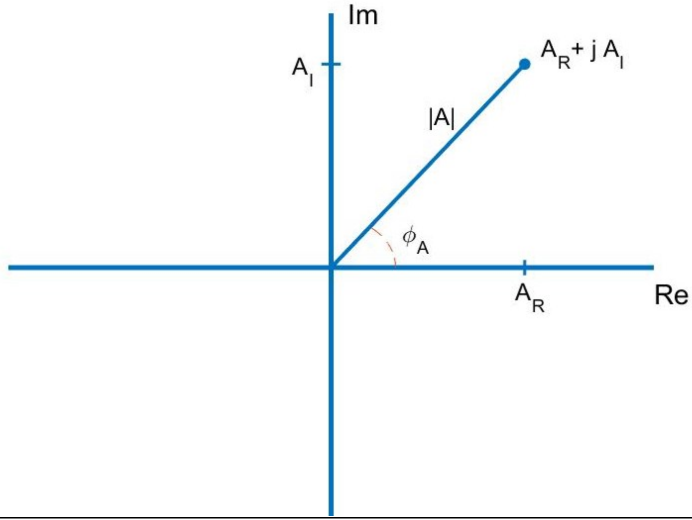
LAPLACE TRANSFORMS

Signal	Time Domain	S Domain
Impulse	$\delta(t)$	1
Step	$u(t)$	s^{-1}
Constant	$Au(t)$	As^{-1}
Ramp	$tu(t)$	s^{-2}
Exponential	$e^{-\alpha t}u(t)$	$(s+\alpha)^{-1}$
Damped ramp	$te^{-\alpha t}u(t)$	$(s+\alpha)^{-2}$
Cosine	$\cos(\beta t)u(t)$	$\frac{s}{s^2+\beta^2}$
Damped cosine	$e^{-\alpha t}\cos(\beta t)u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\beta^2}$
Sum	$Af_1(t)+Bf_2(t)$	$Af_1(s)+Bf_2(s)$
Integral	$\int_0^t f(\tau)d\tau$	$s^{-1}f(s)$
Derivative	$\frac{df(t)}{dt}$	$sf(s)-f(0^-)$
Exponential × function	$e^{-\alpha t}f(t)$	$f(s+\alpha)$
t × function	$tf(t)$	$-\frac{df(s)}{ds}$
Shifted function	$f(t-a)u(t-a)$	$e^{-as}f(s)$

NOTATION: $\mathcal{L}\{f(t)\}(s) = f(s)$ and $\mathcal{L}^{-1}\{f(s)\}(t) = f(t)$

Exam 3 Crib Sheet

Complex Numbers



Rectangular form**:**

$A = A_R + jA_I$

Polar form:

$|A|\angle\varphi_A$

Rectangular to polar

$|A| = \sqrt{(A_R)^2 + (A_I)^2}$

$\varphi_A = tan^{-1}\left(\frac{A_I}{A_R}\right)$

Polar to rectangular

$A_R = |A|\,cos(\varphi_A)$

$A_I = |A|\,sin(\varphi_A)$

Euler's Law: $e^{j\theta} = \boldsymbol{cos}(\theta) + j\,\boldsymbol{sin}(\theta)$

Mathematics with complex number

Addition/Subtraction – Rectangular form

$A + B = (A_R + B_R) + j(A_I + B_I)$

$A - B = (A_R - B_R) + j(A_I - B_I)$

Complex conjugate

$A = A_R + jA_I$ $\qquad A^* = A_R - jA_I$

Multiplication/Dvision – Rectangular form

$AB = |A||B|\angle(\varphi_A + \varphi_B)$

$\frac{A}{B} = \frac{|A|}{|B|}\angle(\varphi_A - \varphi_B)$

Complex conjugate

$A = |A|\angle\varphi_A$ $\qquad A^* = |A|\angle - \varphi_A$

AC Steady State signals

Time domain signals

$\mathrm{F(t)} = \mathrm{A}_{\boldsymbol{o}}\mathbf{cos}\,(\boldsymbol{\omega}\mathrm{t} + \boldsymbol{\theta})$

A_o – amplitude

ω – radial frequency, 2πf

Θ – phase

Phasor signals

$\tilde{F} = A_o\angle\theta$

A_o – amplitude

Θ – phase

(Rectangular form) $F(t) = A_o\,\mathbf{cos}(\omega t + \theta) \leftrightarrow A_o\mathbb{R}\{e^{j(\omega t+\theta)}\} \leftrightarrow A_o e^{j\theta} \leftrightarrow A_o\angle\theta$ **(Phasor form)**

Impedances – Laplace domain (zero initial conditions)

$Z_R = R$	$Z_L = sL$	$Z_C = \frac{1}{sC}$

Impedances – AC steady state

$Z_R = R$ $Z_R = R\angle 0°$	$Z_L = j\omega L$ $Z_L = \omega L\angle 90°$	$Z_C = \frac{1}{j\omega C}$ $Z_C = \frac{1}{\omega C}\angle - 90°$

Impedance, Z [Ω], properties have the same characteristics as resistance

In <u>series</u> add, $Z_{EQ} = Z_1 + Z_2$ $\qquad$ ***In <u>parallel,</u> inverse relationship,*** $Z_{EQ} = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1} = \frac{Z_1 Z_2}{Z_1 + Z_2}$

Admittance, Y [mho], properties have characteristics that are the 'inverse' of impedance

In <u>parallel,</u> add, $Y_{EQ} = Y_1 + Y_2$ $\qquad$ ***In <u>series,</u> inverse relationship,*** $Y_{EQ} = \left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)^{-1} = \frac{Y_1 Y_2}{Y_1 + Y_2}$

AC Steady State Power

$S = P + jQ$

S – Complex power

Using Ohm's Law relationships for impedances (Z)

Exam 3 Crib Sheet

P – Real power, [W]
Q – Reactive power, [VAR]

|S| –Apparent Power, [VA]

$$P = I_{RMS}^2 |Z| \cos\theta = I_{RMS}^2 R(\omega) = V_{RMS} I_{RMS} \cos\theta$$

Equivalent ways of expressing Real Power [Watts]

$$Q = I_{RMS}^2 |Z| \sin\theta = I_{RMS}^2 X(\omega) = V_{RMS} I_{RMS} \sin\theta$$

Equivalent ways of expressing Reactive Power [VAR's]

If using V_{RMS}^2 version of equations also divide by |Z| (phasor form) *cos or sin θ OR must use complex conjugate of Z (rectangular form)

Capacitive reactance is negative (Q < 0)

Inductive reactance is positive (Q > 0)

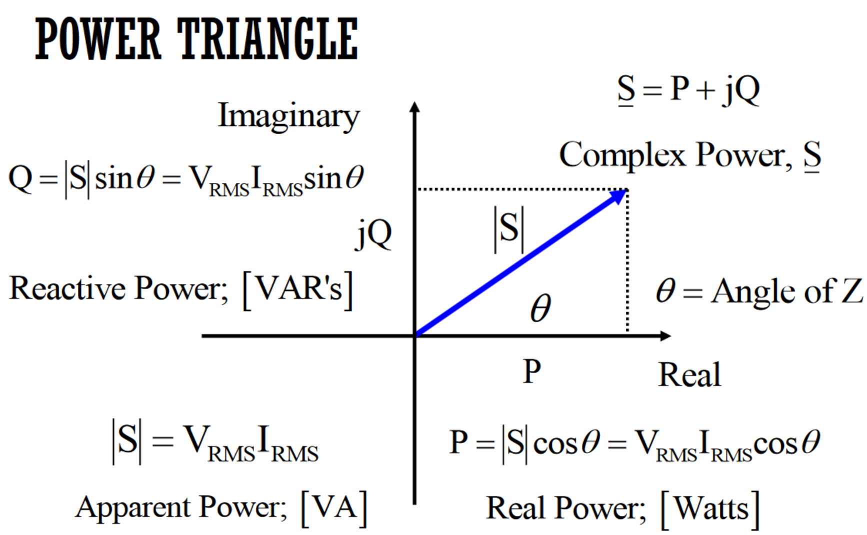
Power produced by the source(s) is equal to the sum of the power produced/stored for each impedance in the circuit

Power factor – a metric over how efficient power consumption/production appears to be

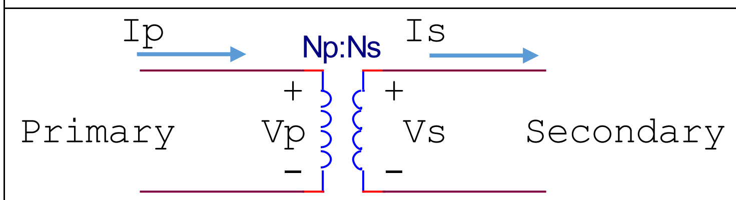
$$0 < \text{power factor} < 1$$

$$\text{Power factor} = \frac{P}{|S|} = cos(\varphi_S)$$

Power Triangle



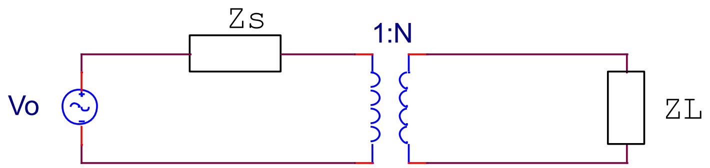
Ideal Transformers



Np : number of windings on the primary
Ns : number of windings on the secondary

Primary: source side of the transformer
Secondary: load side of the transformer
The winding ratio, $N = \frac{Ns}{Np}$
Voltage relationship, $Vs = NVp$
Current relationship, $Is = \frac{Ip}{N}$

Exam 3 Crib Sheet



Refer to secondary (voltage source):

$$Vo_{eq} = NVo \qquad Zs_{eq} = N^2 Zs$$

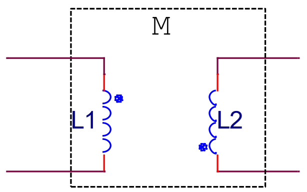
Refer to secondary (current source):

$$Io_{eq} = \frac{Io}{N} \qquad Zs_{eq} = N^2 Zs_{eq}$$

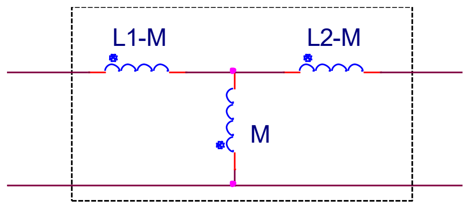
Refer to primary:

$$ZL_{eq} = \frac{ZL}{N^2}$$

Mutual Inductance



Coupled Inductors



Tee Model

The Tee model for coupled inductors represents an equivalent circuit.

M is the mutual inductance, the coupling between the two inductors.

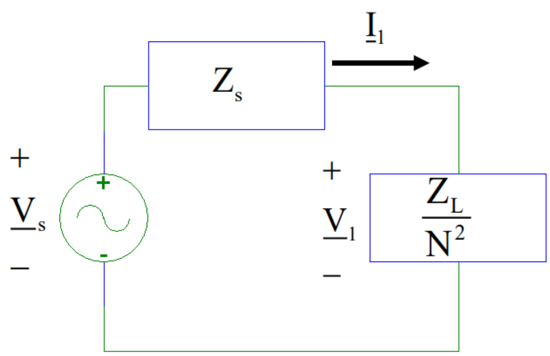
$$M = k\sqrt{L_1 L_2}$$

where k is the coupling coefficient

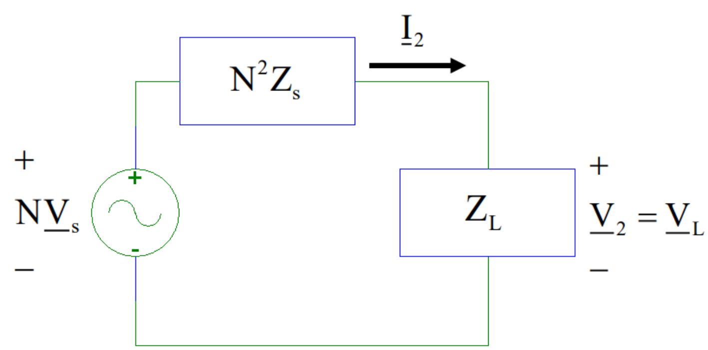
$0 < k < 1$

Student Requested Add-ons

REFERRAL TO PRIMARY



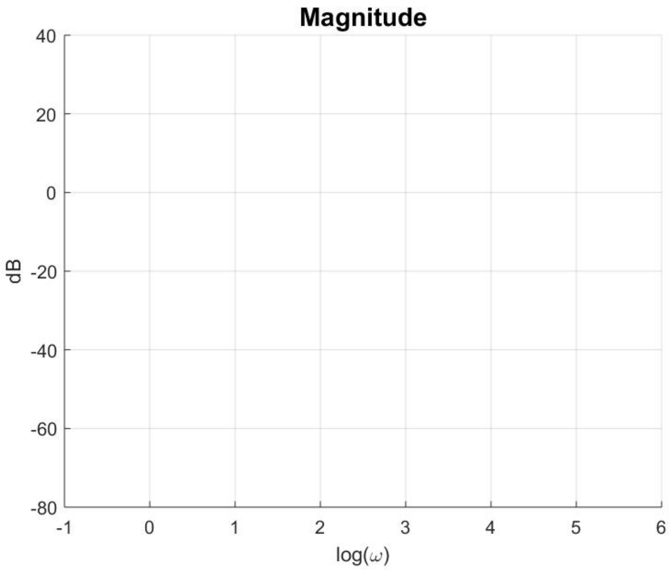
REFERRAL TO SECONDARY



Complex Power	
$P = I_{RMS}^2 \lvert Z \rvert cos\theta$	$Q = I_{RMS}^2 \lvert Z \rvert sin\theta$
$P = I_{RMS}^2 R(\omega)$	$Q = I_{RMS}^2 X(\omega)$
$P = V_{RMS} I_{RMS} cos\theta$	$Q = V_{RMS} I_{RMS} sin\theta$
Notes	
$R(\omega) = Z_{REAL}$	$X(\omega) = Z_{IMG}$
$\theta = Angle\ of\ Impedance$	$\theta = tan^{-1}(\frac{Z_{IMG}}{Z_{REAL}})$
$\theta > 0$ => $I\ lags\ V\ (Ind.)$	$\theta < 0$ => $I\ leads\ V\ (Cap.)$

Bode Plots Crib Sheet

Bode Plots



Decade – a change in frequency by one order of magnitude, for example

100 rad/s → 1000 rad/s

10^4 Hz → 10^5 Hz

dB – decibel

dB = 20 log |F(jω)|

Note the argument of the logarithm is a magnitude expression

A change of 20dB corresponds to a of |F(jω)| by one order of magnitude

Bode plot magnitude approximations

$H(s) \propto s^n$	Slope +20dB/decade
$H(s) \propto \frac{1}{s^n}$	Slope -20dB/decade
$H(s) \propto K$	'Flat', dB = 20log|K|

Sketching Bode plot magnitudes (real poles and zeros)

Crossing an n-pole: Slope **changes** by -20*n dB/decade Crossing an n-zero: Slope **changes** by +20*n dB/decade	**'n'** indicates the number of poles or zeros 'Crossing' rules apply when going from a lower frequency to a higher frequency

Sketching Bode plot phases (real poles and zeros)

Crossing an n-pole: Phase **changes** by $-n*\frac{\pi}{2}$ Crossing an n-zero: Phase **changes** by $+n*\frac{\pi}{2}$	Phase changes are 'spread out' over two decades, one decade on either side of the pole or zero

Corrections for Bode plot magnitudes (real poles and zeros)

At an n-pole: The 'real' dB valule is -3n dB 'below' the asymtote At an n-zero: The 'real' dB valule is +3n dB 'above' the asymtote	The asymptote is the straight line approximation of the Bode plots 'Far away' from poles and zeros, the asymptotes are an accurate representation of the Bode plot

Bode Plots Crib Sheet

Second Order Circuits	
Damping ratio, $\delta = \frac{\alpha}{\omega_o}$, a metric of the damping α is the attenuation constant ω_o is the resonant frequency	$\delta > 1$, overdamped $\delta = 1$, critically damped $\delta < 1$, underdamped

Lowpass/Highpass filters

Overdamped and critically damped cases, the Bode plots follow the procedure on the previous page

Underdamped cases, use the critically damped approximation, add a 'correction' of $20\log\left|\frac{1}{2\delta}\right|$ at the resonant frequency, ω_o

Bandpass filters

Overdamped, the Bode plots follow the procedure on the previous page

Critically damped and underdamped cases

At the resonant frequency, the magnitude Bode plot is 0dB

The vertex where the stopbands meet is $20\log|2\delta|$

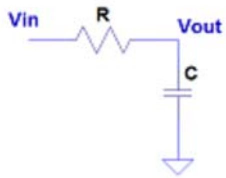
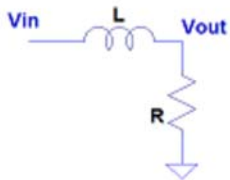
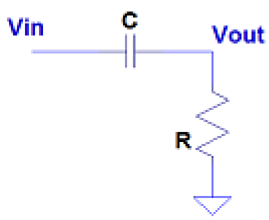
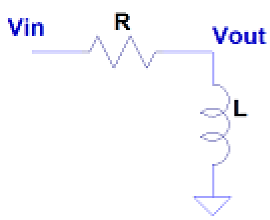
Note: The above discussion is for second order circuits only. If there is a gain stage, the Bode plot moves 'up' or 'down' and the dB value of the gain determines the reference for adding corrections/stopband vertices

Cascaded Filters – Magnitude Bode Plots
$H(s) = H_1(s)H_2(s)H_3(s)$ (three stages) → dB = $20\log|H_1(j\omega)H_2(j\omega)H_3(j\omega)| = 20\log|H_1(j\omega)| + 20\log|H_2(j\omega)| + 20\log|H_3(j\omega)|$ angle = $\angle\left[H_1(j\omega)H_2(j\omega)H_3(j\omega)\right] = \angle H_1(j\omega) + \angle H_2(j\omega) + \angle H_3(j\omega)$

Bode Plots Crib Sheet

First order filters

$\omega_c = \frac{1}{RC}$ $\omega_C = \frac{R}{L}$

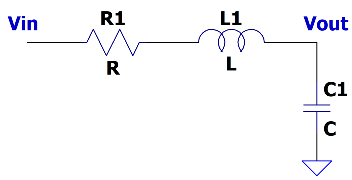
Filter name	Schematic(s)	H(s)	pole/zero ID
Low pass filter	 	$\frac{\omega_c}{s + \omega_c}$	1 pole
High pass filter	$\omega_c = \frac{1}{RC}$  $\omega_C = \frac{R}{L}$ 	$\frac{s}{s + \omega_c}$	1 zero at zero 1 pole

Bode Plots Crib Sheet

Second order filters

Filter name | Schematic(s) | H(s) | pole/zero ID

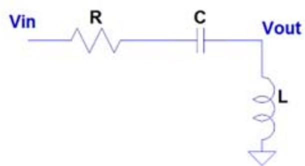
Low pass filter



$$H(s) = \frac{\omega_o^2}{s^2 + 2\alpha s + \omega_o^2}$$

2 poles

High pass filter

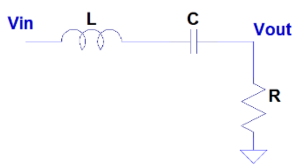


$$H(s) = \frac{s^2}{s^2 + 2\alpha s + \omega_o^2}$$

2 zeros at zero

2 poles

Bandpass filter



$$H(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_o^2}$$

1 zero at zero

2 poles

Bandstop filter

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + 2\alpha s + \omega_o^2}$$