ECSE 6961
The Wireless Channel

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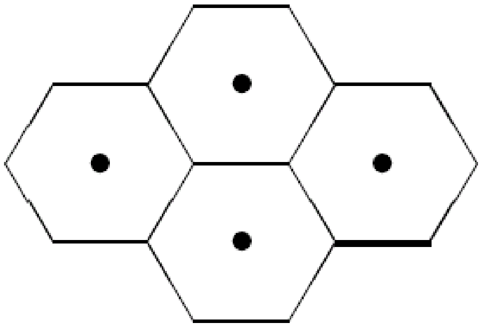
Slides based upon books by Tse/Viswanath, Goldsmith, Rappaport, J.Andrews etal

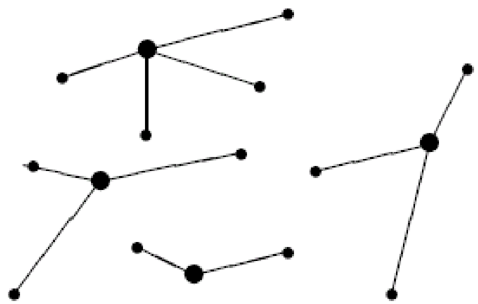
Wireless Channel is Very Different!

- Wireless channel "feels" very different from a wired channel.
  - Not a point-to-point link
  - Variable capacity, errors, delays
  - Capacity is shared with interferers
- Characteristics of the channel appear to change randomly with time, which makes it difficult to design reliable systems with guaranteed performance.
- Cellular model vs reality:



(a)

Part (a): an oversimplified view in which each cell is hexagonal.



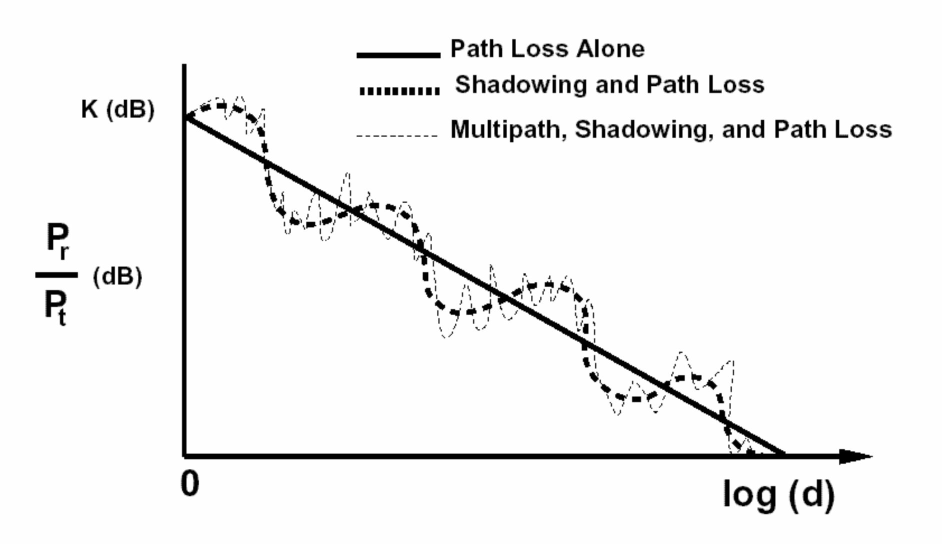
(b)

Part (b): a more realistic case where base stations are irregularly placed and cell phones choose the best base station

Cellular system designs are **interference-limited**, i.e. the interference dominates the noise floor

Basic Ideas: Path Loss, Shadowing, Fading

- Variable decay of signal due to environment, multipaths, mobility



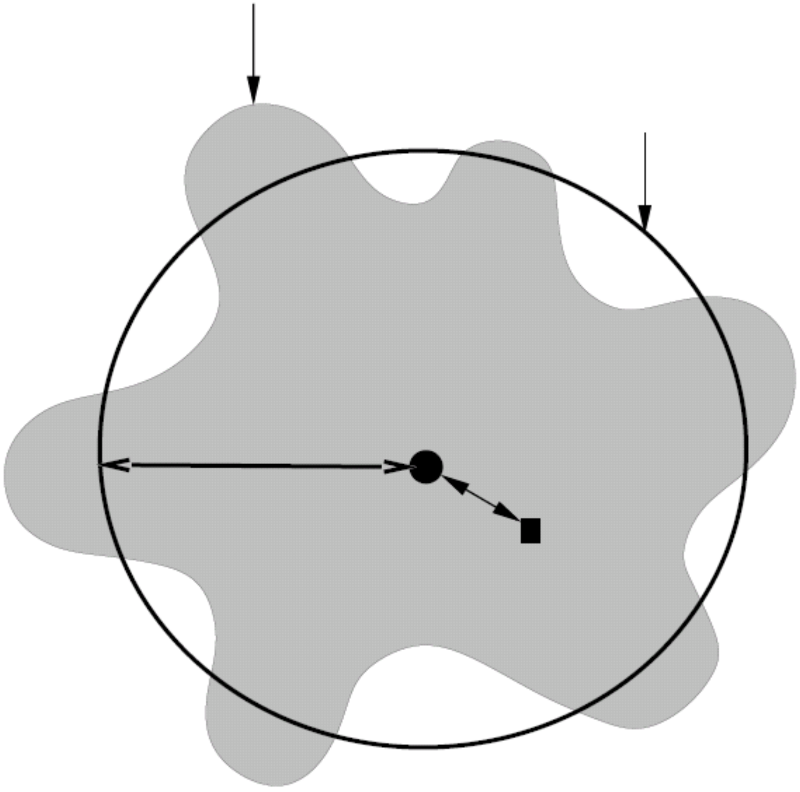
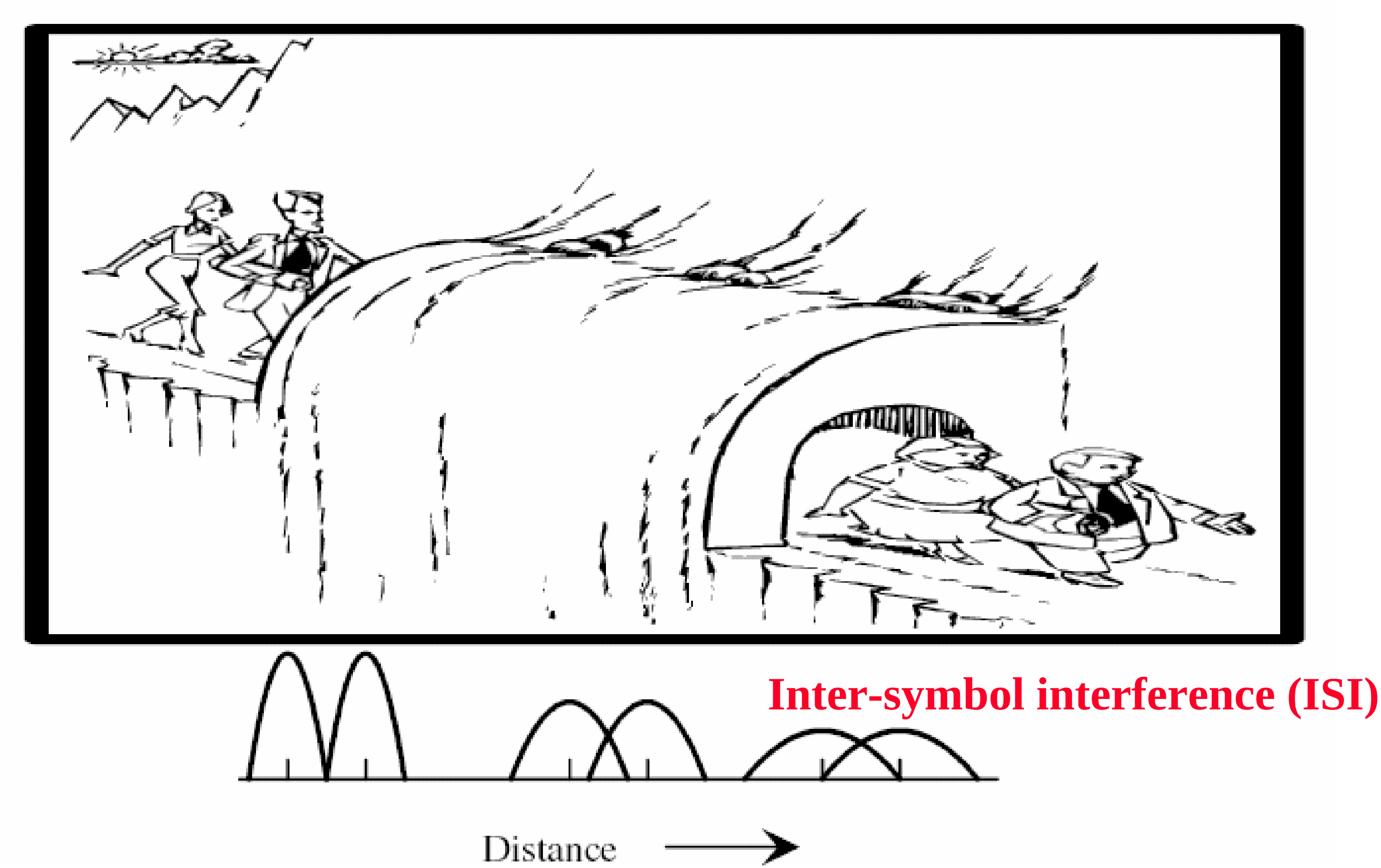
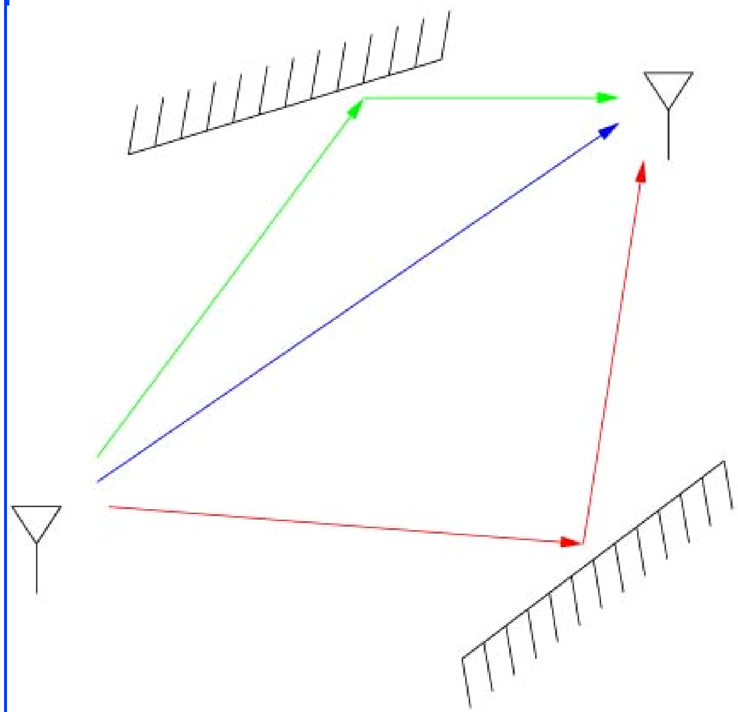


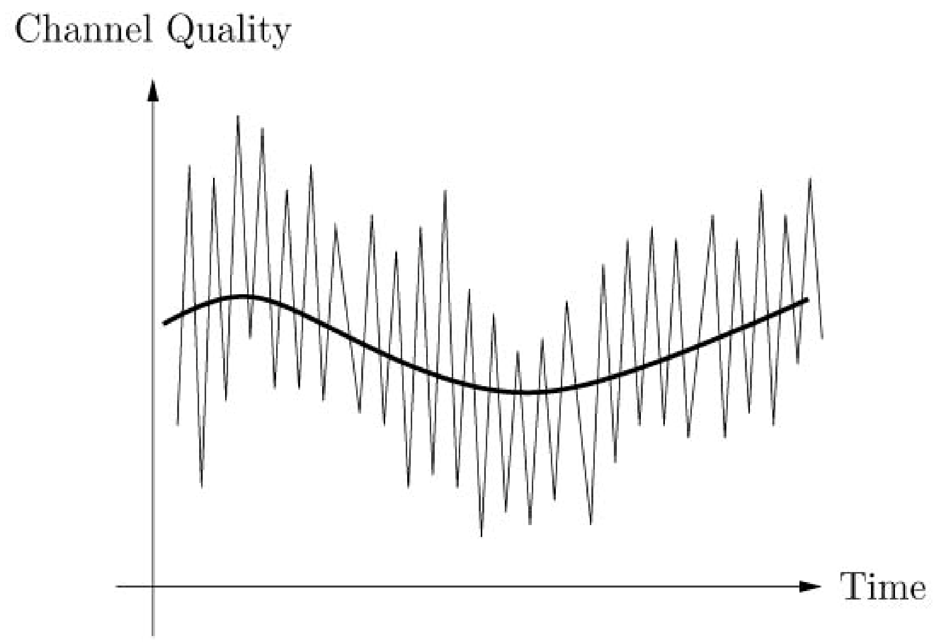
Figure 2.10: Contours of Constant Received Power.

Attenuation, Dispersion Effects: ISI!



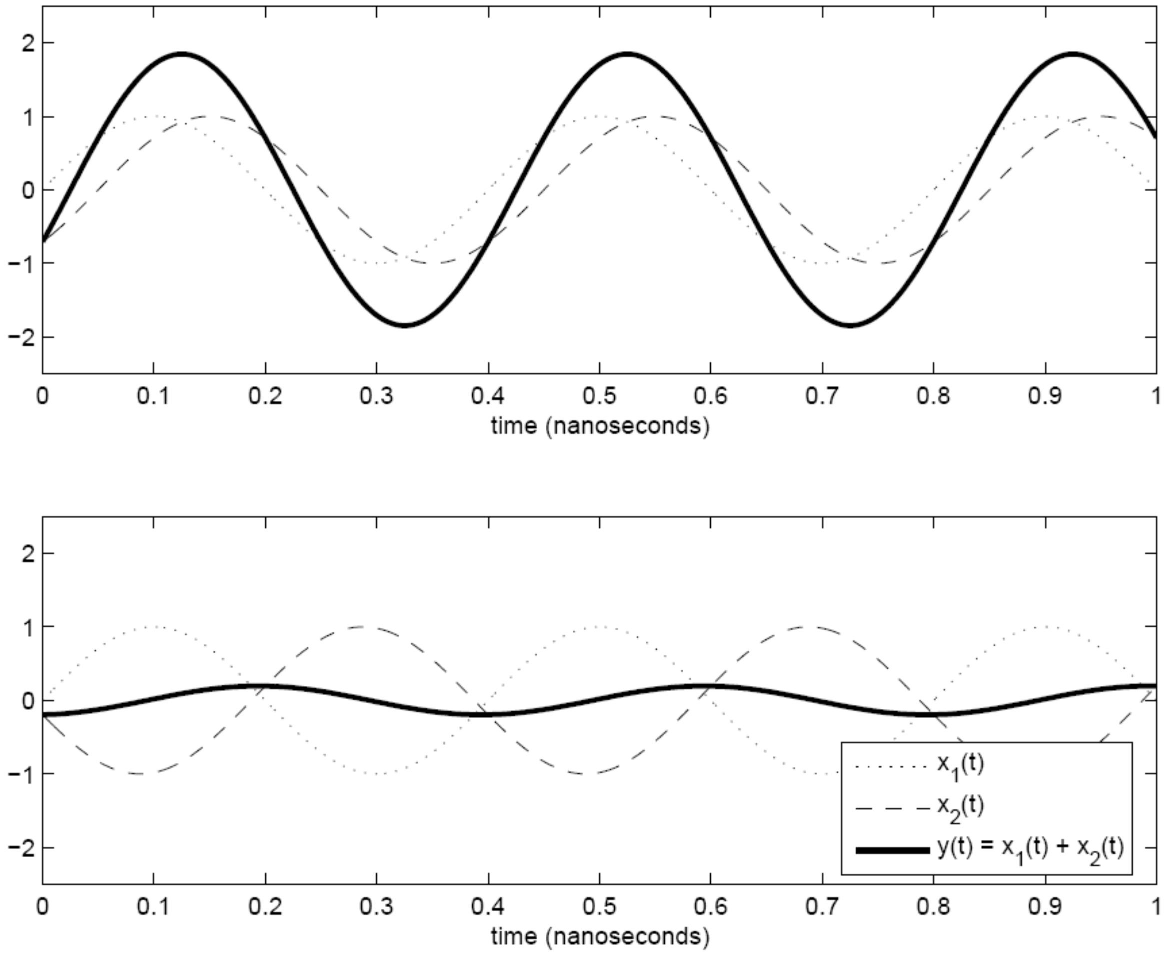
Wireless Multipath Channel





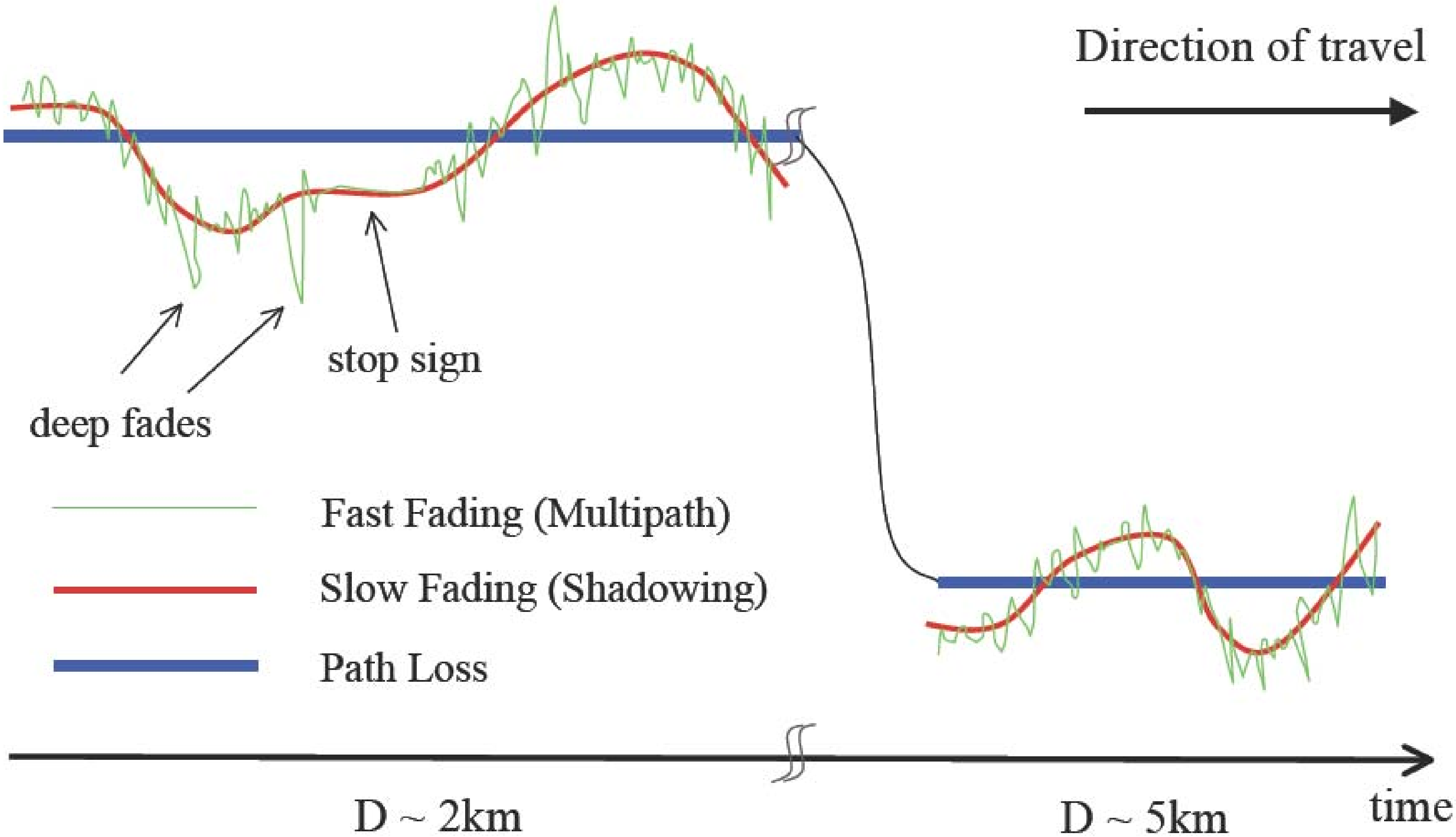
Channel varies at two spatial scales:
 * Large scale fading: path loss, shadowing
 * Small scale fading:
   Multi-path fading (frequency selectivity, coherence b/w, ~500kHz),
   Doppler (time-selectivity, coherence time, ~2.5ms)

MultiPath Interference: Constructive & Destructive



The difference between constructive interference (top) and destructive interference (bottom) at $f_c = 2.5$ GHz is less than 0.1 nanoseconds in phase, which corresponds to about 3 cm.

Mobile Wireless Channel w/ Multipath



Game plan

- We wish to understand how physical parameters such as
  - carrier frequency
  - mobile speed
  - bandwidth
  - delay spread
  - angular spread

  impact how a wireless channel behaves from the cell planning and communication system point of view.
- We start with deterministic physical model and progress towards statistical models, which are more useful for design and performance evaluation.

Large-scale Fading: Path Loss, Shadowing

Large-scale fading: Cell-Site Planning

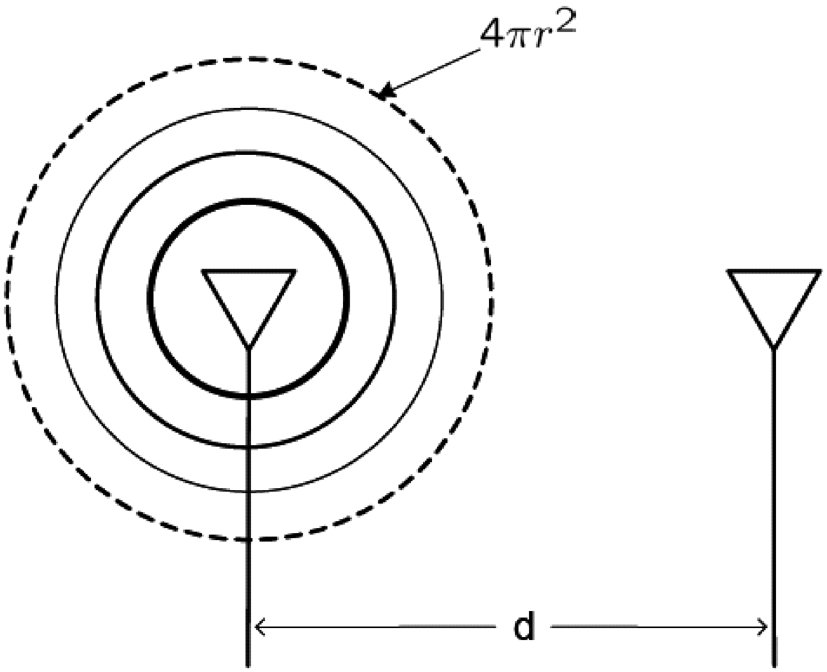
- In free space, received power attenuates like $1/r^2$.
- With reflections and obstructions, can attenuate even more rapidly with distance. Detailed modelling complicated.
- Time constants associated with variations are very long as the mobile moves, many seconds or minutes.
- More important for cell site planning, less for communication system design.

Path Loss Modeling

- Maxwell's equations
  - Complex and impractical
- Free space path loss model
  - Too simple
- Ray tracing models
  - Requires site-specific information
- Empirical Models
  - Don't always generalize to other environments
- Simplified power falloff models
  - Main characteristics: good for high-level analysis

Free-Space-Propagation

- If oscillating field at transmitter, it produces three components:
  - The electrostatic and inductive fields that decay as $1/d^2$ or $1/d^3$
  - The EM radiation field that decays as $1/d$ (power decays as $1/d^2$)
-



Electric (Far) Field & Transfer Function

- Tx: a sinusoid: cos 2πft
- Electric Field: source antenna gain (α_s)

$$E(f,t,(r,\theta,\psi)) = \frac{\alpha_s(\theta,\psi,f)\cos 2\pi f(t - \frac{r}{c})}{r}.$$

- Product of antenna gains (α)

$$E_r(f,t,u) = \frac{\alpha(\theta,\psi,f)\cos 2\pi f(t - \frac{r}{c})}{r}$$

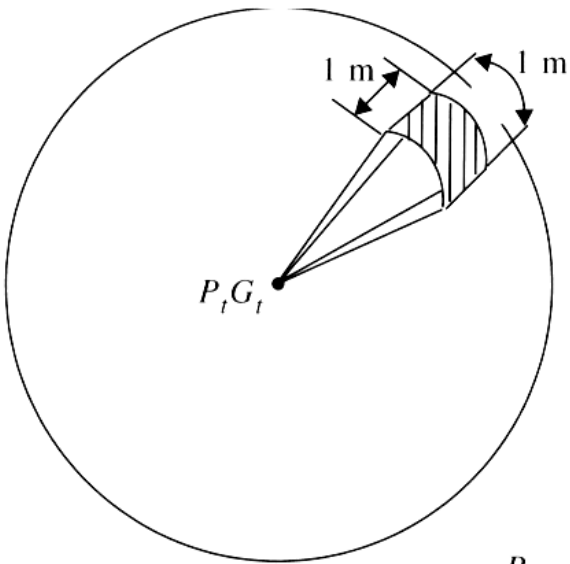
- Consider the function (transfer function)

$$H(f) := \frac{\alpha(\theta,\psi,f)e^{-j2\pi fr/c}}{r}.$$

- The electric field is now: $\boxed{E_r(f,t,u) = \Re\left[H(f)e^{j2\pi ft}\right].}$

Linearity is a good assumption, but time-invariance lost when Tx, Rx or environment in motion

Free-space and received fields: Path Loss



(a)

$$P_d = \frac{P_t G_t}{4\pi d^2} = \frac{EIRP}{4\pi d^2} = \frac{|E|^2}{120\pi} \quad \text{W/m}^2$$

(power flux density P_d)

<u>Note</u>: Electric Field (E) decays as *1/r*, but
Power (P_d) decays as $1/r^2$

Path Loss in dB: $P_L \text{ dB} = 10 \log_{10} \frac{P_t}{P_r} \text{ dB}.$ $\quad \frac{P_r}{P_t} = \left[\frac{\sqrt{G_l}\lambda}{4\pi d} \right]^2.$

$\sqrt{G_l}$ is the product of the transmit and receive antenna field radiation patterns in the LOS direction.

Decibels: dB, dBm, dBi

- **dB** (Decibel) = 10 $\log_{10}$ (Pr/Pt)
  Log-ratio of two signal levels. Named after Alexander Graham Bell. For example, a cable has 6 dB loss or an amplifier has 15 dB of gain. System gains and losses can be added/subtracted, especially when changes are in several orders of magnitude.

- **dBm (dB milliWatt)**
  Relative to 1mW, i.e. 0 dBm is 1 mW (milliWatt). Small signals are -ve (e.g. -83dBm).

  Typical 802.11b WLAN cards have +15 dBm (32mW) of output power. They also spec a -83 dBm RX sensitivity (minimum RX signal level required for 11Mbps reception).

  For example, 125 mW is 21 dBm and 250 mW is 24 dBm. (commonly used numbers)

- **dBi (dB isotropic) for EIRP (Effective Isotropic Radiated Power)**

  The gain a given antenna has over a theoretical isotropic (point source) antenna. The gain of microwave antennas (above 1 GHz) is generally given in dBi.

- **dBd (dB dipole)**
  The gain an antenna has over a dipole antenna at the same frequency. A dipole antenna is the smallest, least gain practical antenna that can be made. A dipole antenna has 2.14 dB gain over a 0 dBi isotropic antenna. Thus, a simple dipole antenna has a gain of 2.14 dBi or 0 dBd and is used as a standard for calibration.

  The term dBd (or sometimes just called dB) generally is used to describe antenna gain for antennas that operate under 1GHz (1000Mhz).

dB calculations: Effective Isotropic Radiated Power (EIRP)

- EIRP (Effect Isotropic Radiated Power): effective power found in the main lobe of transmitter antenna.
  - EIRP = $P_t G_t$
  - In dB, EIRP is equal to sum of the antenna gain, Gt (in dBi) plus the power, Pt (in dBm) into that antenna.

- For example, a 12 dBi gain antenna fed directly with 15 dBm of power has an Effective Isotropic Radiated Power (EIRP) of:

  12 dBi + 15dBm = 27 dBm (500 mW).

Path Loss (Example 1): Carrier Frequency

Example 2.1: Consider an indoor wireless LAN with $f_c = 900$ MHz, cells of radius ~~100~~ 10m m, and nondirectional antennas. Under the free-space path loss model, what transmit power is required at the access point such that all terminals within the cell receive a minimum power of 10 μW. How does this change if the system frequency is 5 GHz?

Solution: We must find the transmit power such that the terminals at the cell boundary receive the minimum required power. We obtain a formula for the required transmit power by inverting (2.7) to obtain:

$$P_t = P_r \left[\frac{4\pi d}{\sqrt{G_l}\lambda} \right]^2 .$$

Substituting in $G_l = 1$ (nondirectional antennas), $\lambda = c/f_c = .33$ m, $d = 10$ m, and $P_r = 10\mu$W yields $P_t = 1.45$W = 1.61 dBW (Recall that P Watts equals $10\log_{10}[P]$ dbW, dB relative to one Watt, and $10\log_{10}[P/.001]$ dBm, dB relative to one milliwatt). At 5 GHz only $\lambda = .06$ changes, so $P_t = 43.9$ ~~KW~~ W = 16.42 dBW.

- Note: effect of frequency f: 900 Mhz vs 5 Ghz.
  - Either the receiver must have greater sensitivity or the sender must pour 44W of power, even for 10m cell radius!

Path Loss (Example 2), Interference & Cell Sizing

Example 3.1 *Consider a user in the downlink of a cellular system, where the desired base station is at a distance of 500 meters, and there are numerous nearby interfering base stations transmitting at the same power level. If there are 3 interfering base stations at a distance of 1 km, 3 at a distance of 2 km, and 10 at a distance of 4 km, use the empirical path loss formula to find the signal-to-interference ratio (SIR, i.e. the noise is neglected) when* $\alpha = 3$*, and then when* $\alpha = 5$.

- Desired signal power: $$P_{r,d} = P_t P_o d_o^3 (0.5)^{-3},$$
- Interference power: $$P_{r,I} = P_t P_o d_o^3 \left[3(1)^{-3} + 3(2)^{-3} + 10(4)^{-3}\right].$$
- SIR: $$SIR(\alpha = 3) = \frac{P_{r,d}}{P_{r,I}} = 28.25 = 14.5dB,$$ $$SIR(\alpha = 5) = 99.3 = 20dB,$$
- SIR is much better with higher path loss exponent ($\alpha = 5$)!
- Higher path loss, smaller cells => lower interference, higher SIR

Path Loss: ***Range*** vs ***Bandwidth*** Tradeoff

- Frequencies < 1 GHz are often referred to as "beachfront" spectrum. Why?

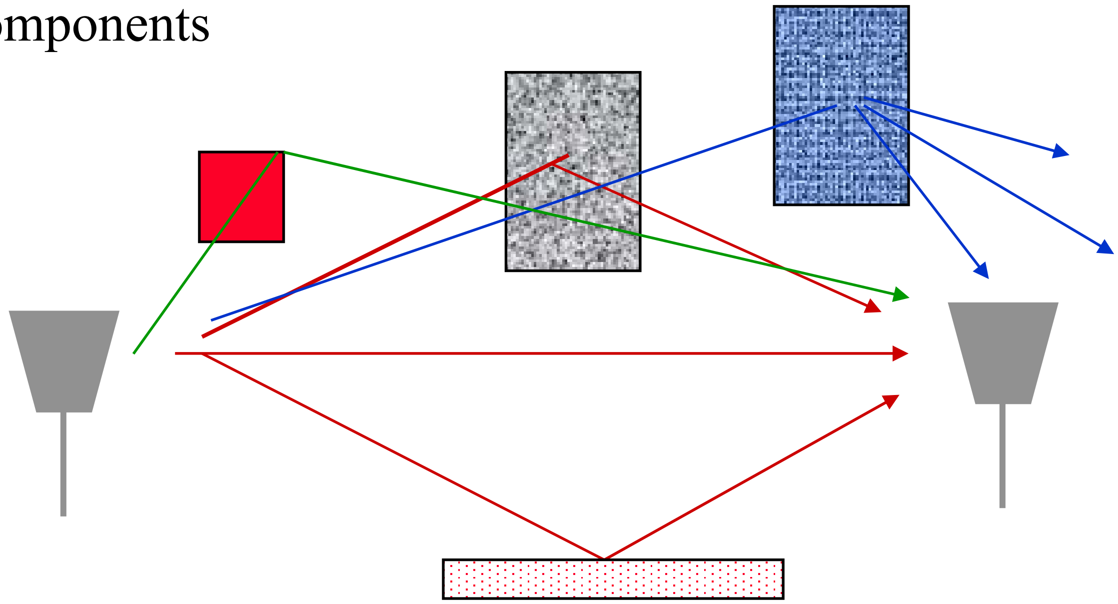
- **1.** High frequency RF electronics have traditionally been harder to design and manufacture, and hence more expensive. [less so nowadays]
- **2.** Pathloss increases ~ $O(f_c^2)$
  - A signal at 3.5 GHz (one of WiMAX's candidate frequencies) will be received with about 20 times less power than at 800 MHz (a popular cellular frequency).
  - Effective path loss exponent also increases at higher frequencies, due to increased absorption and attenuation of high frequency signals

- Tradeoff:
  - Bandwidth at higher carrier frequencies is more plentiful and less expensive.
  - Does *not* support large transmission ranges.
  - (also increases problems for mobility/Doppler effects etc)

- WIMAX Choice:
  - Pick any two out of three: *high data rate, high range, low cost.*

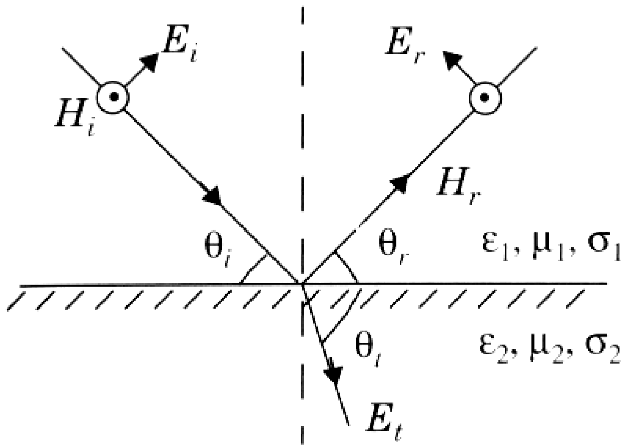
Ray Tracing

- Models all signal components
  - Reflections
  - Scattering
  - Diffraction

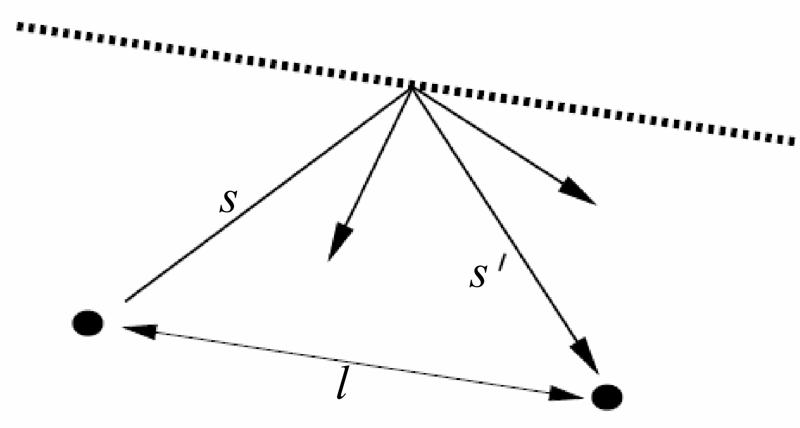


- Diffraction: signal "bends around" an object in its path to the receiver:
  - Diffraction Path loss exceeding 100 dB
- Error of the ray tracing approximation is smallest when the receiver is many wavelengths from the nearest scatterer, and all the scatterers are large relative to a wavelength and fairly smooth.
  - Good match w/ empirical data in rural areas, along city streets (Tx/Rx close to ground), LAN with adjusted diffraction coefficients

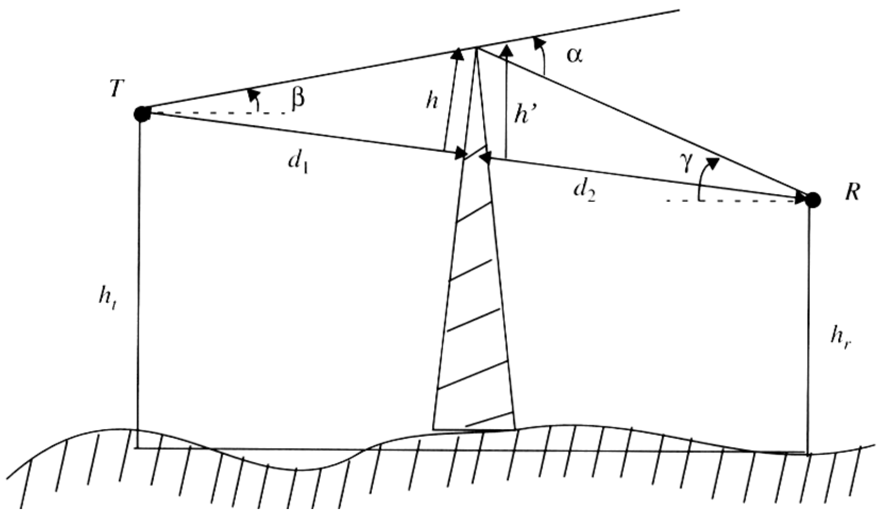
Reflection, Diffraction, Scattering



<u>Reflection/Refraction</u>: large objects (>>λ)



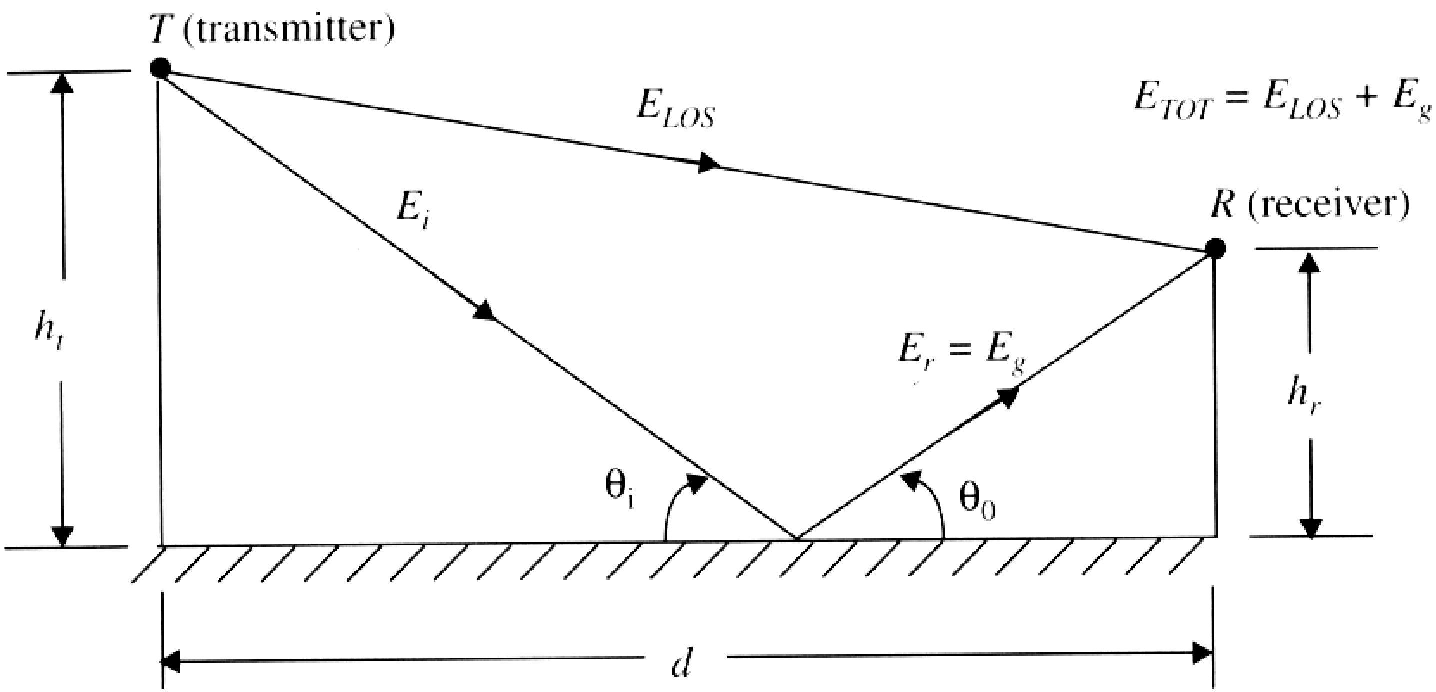
<u>Scattering</u>: small objects, rough surfaces (<λ): foilage, lamposts, street signs



<u>Diffraction/Shadowing:</u> "bending" around sharp edges,

- 900Mhz: λ ~ 30 cm
- 2.4Ghz: λ ~ 13.9 cm
- 5.8Ghz: λ ~ 5.75 cm

Classical 2-ray Ground Bounce model



$$P_r \approx \left[\frac{\lambda\sqrt{G_l}}{4\pi d}\right]^2 \left[\frac{4\pi h_t h_r}{\lambda d}\right]^2 P_t = \left[\frac{\sqrt{G_l} h_t h_r}{d^2}\right]^2 P_t,$$

$$P_r \text{ dBm} = P_t \text{ dBm} + 10\log_{10}(G_l) + 20\log_{10}(h_t h_r) - 40\log_{10}(d).$$

2-ray model observations

- The electric field flips in sign canceling the LOS field, and hence the path loss is *$O(d^{-4})$ rather than $O(d^{-2})$.*
- The *frequency effect disappears*!
  - Similar phenomenon with antenna arrays.
- Near-field, far-field detail explored in next slide:
  - Used for cell-design

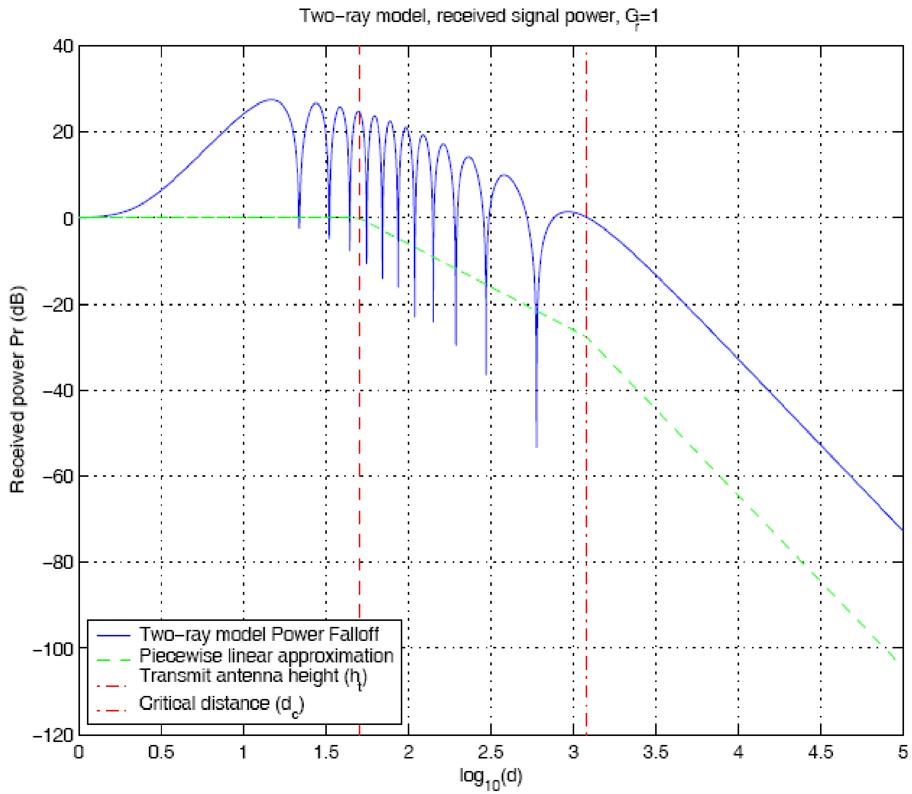
2-ray model: distance effect, critical distance

- $d < h_t$: constructive i/f
- $h_t < d < d_c$: constructive and destructive i/f (multipath fading upto critical distance)
- $d_c < d$: only destructive interference

$$d_c = 4h_t h_r / \lambda,$$

- Piecewise linear approximation w/ slopes 0, -20 dB/decade, -40 dB/decade

$f = 900\text{MHz}$, R=-1, $h_t = 50\text{m}$, $h_r = 2\text{m}$, $G_l = 1$, $G_r = 1$



Received Power versus Distance for Two-Ray Model.

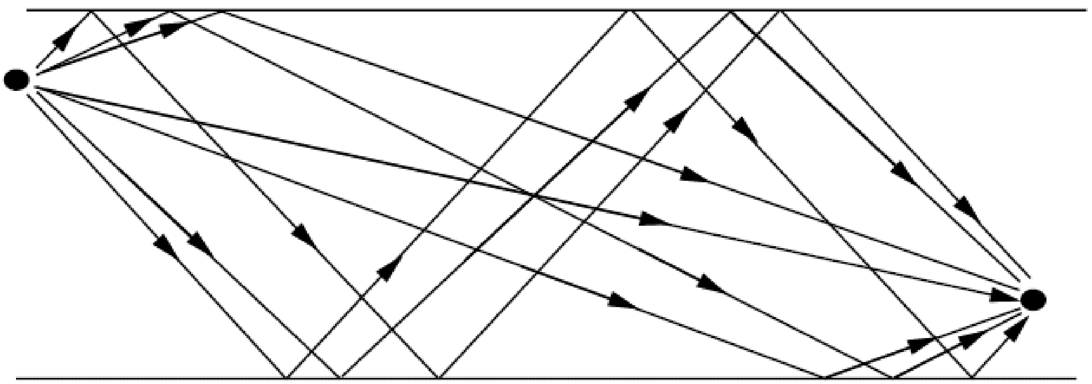
2-ray model example, cell design

Example 2.2: Determine the critical distance for the two-ray model in an urban microcell ($h_t = 10$m, $h_r = 3$ m) and an indoor microcell ($h_t = 3$ m, $h_r = 2$ m) for $f_c = 2$ GHz.

Solution: $d_c = 4h_t h_r/\lambda = 800$ meters for the urban microcell and 160 meters for the indoor system. A cell radius of 800 m in an urban microcell system is a bit large: urban microcells today are on the order of 100 m to maintain large capacity. However, if we used a cell size of 800 m under these system parameters, signal power would fall off as d^2 inside the cell, and interference from neighboring cells would fall off as d^4, and thus would be greatly reduced. Similarly, 160 m is quite large for the cell radius of an indoor system, as there would typically be many walls the signal would have to go through for an indoor cell radius of that size. So an indoor system would typically have a smaller cell radius, on the order of 10-20 m.

- Design the cell size to be < critical distance to get ***O(d^{-2}) power decay in cell and O(d^{-4}) outside!***
- Cell radii are typically much smaller than critical distance

10-Ray Model: Urban Microcells



- Ground and 1-3 wall reflections
- Falloff with distance squared (d^{-2})!
  - Dominance of the multipath rays which decay as d^{-2}, …
  - … over the combination of the LOS and ground-reflected rays (the two-ray model), which decays as d^{-4}.
- *Empirical studies:* $d^{-\gamma}$, where γ lies anywhere between two and six

Simplified Path Loss Model

- Used when path loss dominated by reflections.
- Most important parameter is the path loss exponent γ, determined empirically.

$$P_r = P_t K \left[\frac{d_0}{d}\right]^{\gamma}, \qquad 2 \le \gamma \le 8$$

- <u>Cell design impact:</u> If the radius of a cell is reduced by half when the propagation path loss exponent is 4, the transmit power level of a base station is reduced by 12dB (=10 log 16 dB).
  - Costs: More base stations, frequent handoffs

Typical large-scale path loss

Path Loss Exponents for Different Environments

Environment	Path Loss Exponent, *n*
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

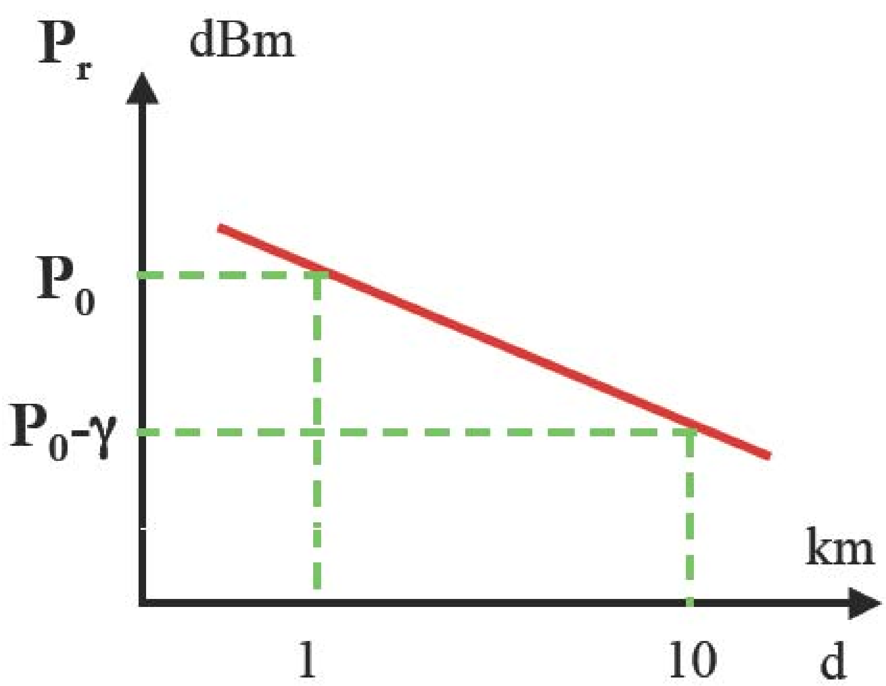
Empirical Models

- Okumura model
  - Empirically based (site/freq specific)
  - Awkward (uses graphs)
- Hata model
  - Analytical approximation to Okumura model
- Cost 136 Model:
  - Extends Hata model to higher frequency (2 GHz)
- Walfish/Bertoni:
  - Cost 136 extension to include diffraction from rooftops

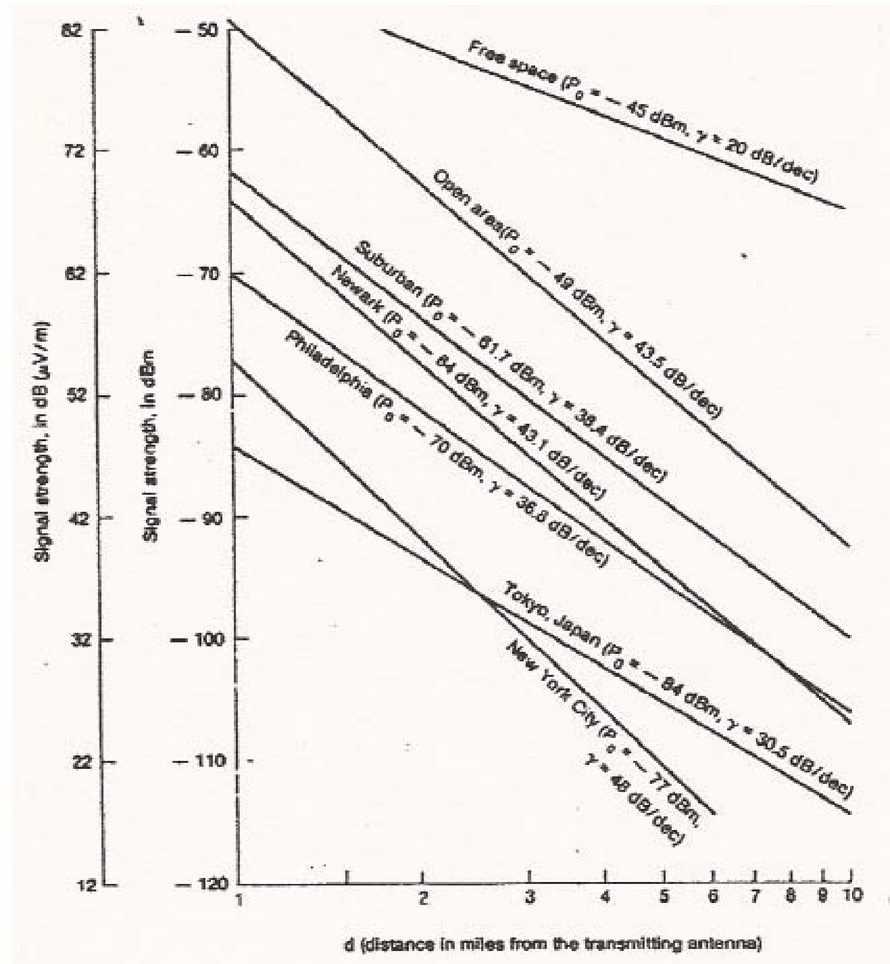
Commonly used in cellular system simulations

Empirical Model: Eg: Lee Model

Lee's Model on a Log-Log scale



Standard Deviation : 7 to 10 dB



Empirical Path Loss: Okamura, Hata, COST231

- Empirical models include effects of path loss, shadowing and multipath.
  - Multipath effects are averaged over several wavelengths: local mean attenuation (LMA)
  - Empirical path loss for a given environment is the average of LMA at a distance d over all measurements
- **Okamura**: based upon Tokyo measurements. 1-100 lm, 150-1500MHz, base station heights (30-100m), median attenuation over free-space-loss, 10-14dB standard deviation.

$$P_L(d)\ \text{dB} = L(f_c,d) + A_{mu}(f_c,d) - G(h_t) - G(h_r) - G_{AREA}$$

- **Hata**: closed form version of Okamura

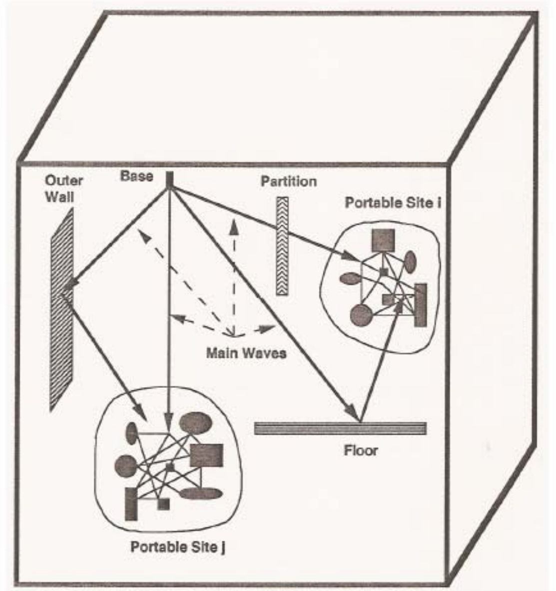
$$P_{L,urban}(d)\ \text{dB} = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d). \quad (2.31)$$

- **COST 231:** Extensions to 2 GHz

$$P_{L,urban}(d)\text{dB} = 46.3 + 33.9\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d) + C_M, \quad (2.34)$$

Indoor Models

- 900 MHz: 10-20dB attenuation for 1-floor, 6-10dB/floor for next few floors (and frequency dependent)
- Partition loss each time depending upton material (see table)
- Outdoor-to-indoor: building penetration loss (8-20 dB), decreases by 1.4dB/floor for higher floors. (reduced clutter)
- Windows: 6dB less loss than walls (if not lead lined)



Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

Path Loss Models: Summary

- Path loss models simplify Maxwell's equations
- Models vary in complexity and accuracy
- Power falloff with distance is proportional to d^2 in free space, d^4 in two path model
- General ray tracing computationally complex
- Empirical models used in 2G/3G/Wimax simulations
- Main characteristics of path loss captured in simple model $P_r = P_t K [d_0/d]^\gamma$

Shadowing

$$P_r = P_t P_o \chi \left(\frac{d_o}{d} \right)^{\alpha},$$

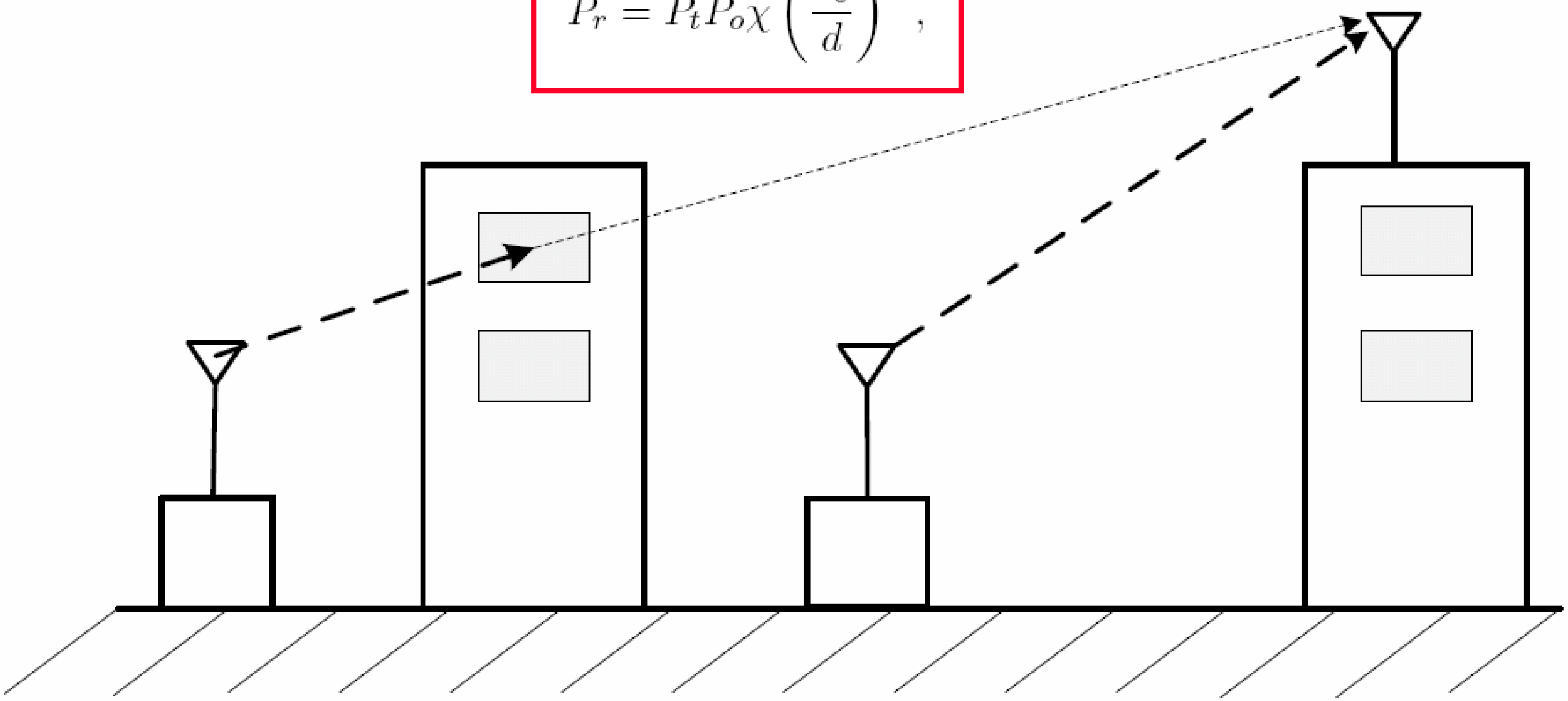


Figure 3.3: Shadowing can cause large deviations from path loss predictions.

- Log-normal model for shadowing r.v. (χ)

Shadowing: Measured large-scale path loss

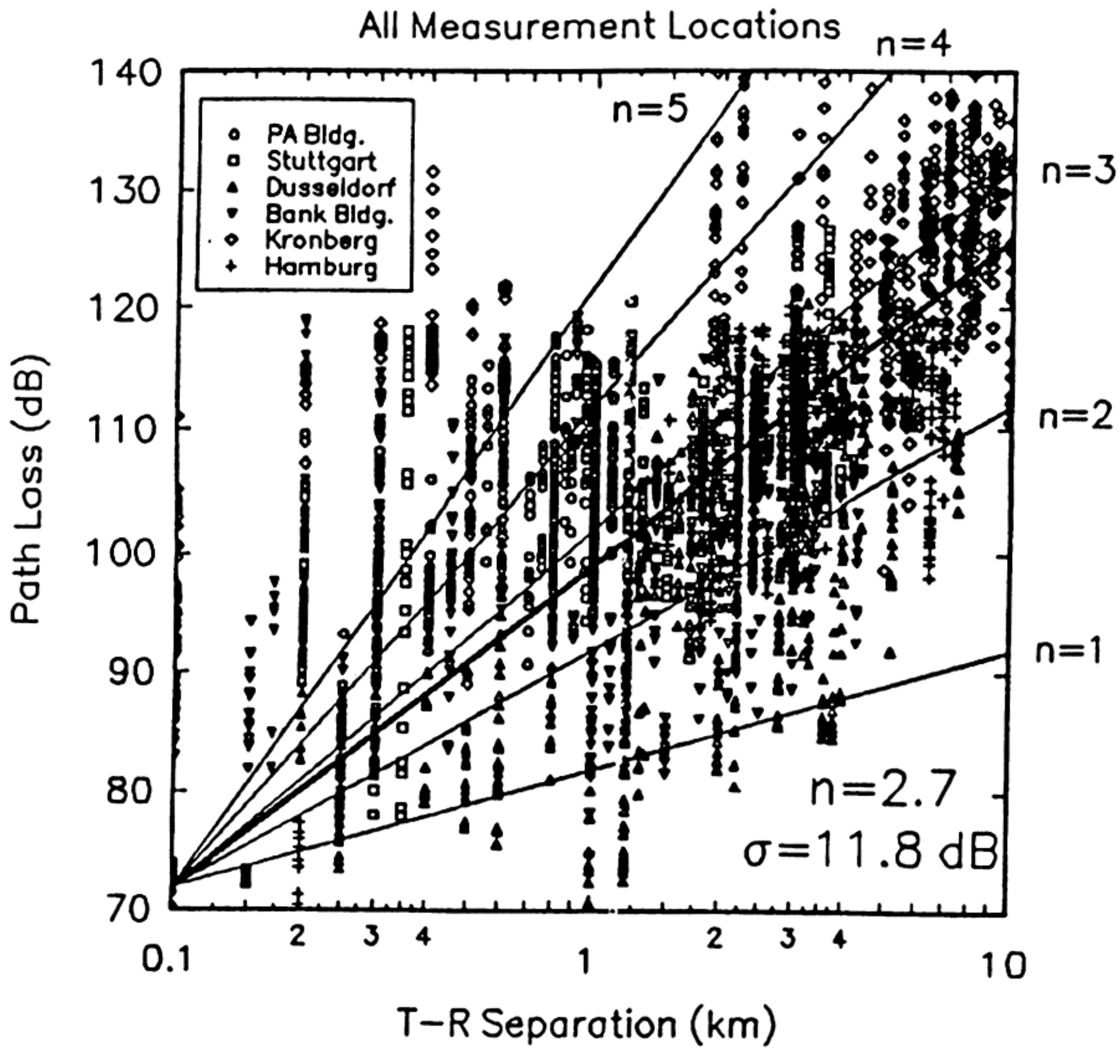


Figure 4.17 Scatter plot of measured data and corresponding MMSE path loss model for many cities in Germany. For this data, $n = 2.7$ and $\sigma = 11.8$ dB [from [Sei91] © IEEE].

Log-Normal Shadowing

- Assumption: shadowing is dominated by the attenuation from blocking objects.
- Attenuation of for depth *d:*

  $$s(d) = e^{-\alpha d},$$

  (α: attenuation constant).
- Many objects:

  $$s(d_t) = e^{-\alpha \sum di} = e^{-\alpha dt},$$

  $d_t = \sum d_i$ is the sum of the random object depths
- Cental Limit Theorem (CLT): $\alpha d_t = \log s(d_t) \sim \mathrm{N}(\mu, \sigma)$.
  - $\log s(d_t)$ is therefore log-normal

Area versus Distance Coverage model with Shadowing model

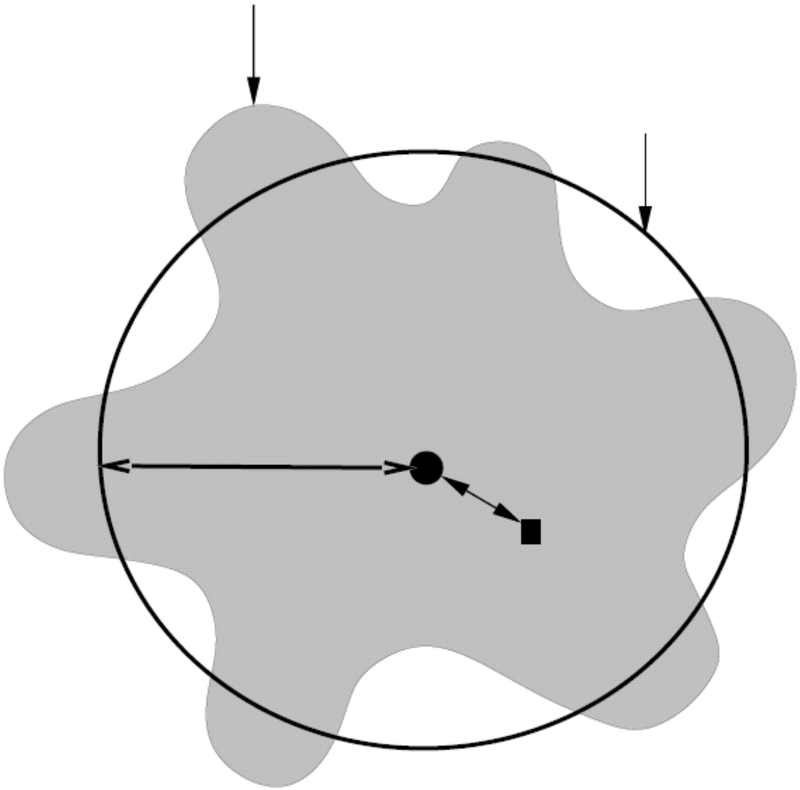
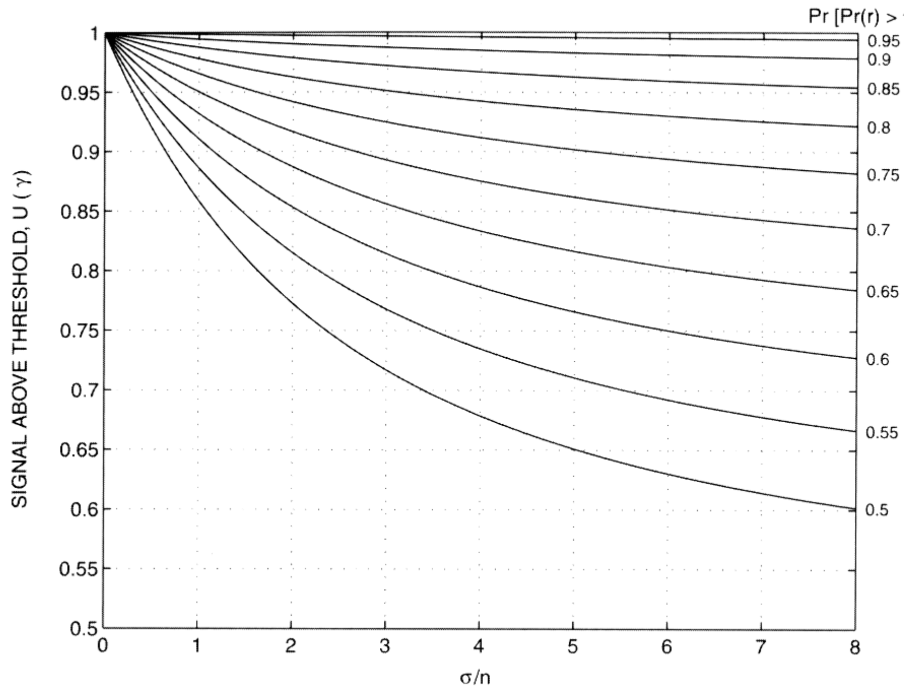


Figure 2.10: Contours of Constant Received Power.



4.18 Family of curves relating fraction of total area with signal above threshold, $U(\gamma)$ as on of probability of signal above threshold on the cell boundary.

Outage Probability w/ Shadowing

$$p(P_r(d) \leq P_{min}) = 1 - Q\left(\frac{P_{min} - (P_t + 10\log_{10} K - 10\gamma\log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right),$$

Example 2.5:

Find the outage probability at 150 m for a channel based on the combined path loss and shadowing models of Examples 2.3 and 2.4, assuming a transmit power of $P_t = 10$ dBm and minimum power requirement $P_{min} = -110.5$ dBm.

Solution We have $P_t = 10$ mW = 10 dBm.

$$\begin{aligned} P_{out}(-110.5\text{dBm}, 150m) &= p(P_r(150m) < -110.5\text{dBm}) \\ &= 1 - Q\left(\frac{P_{min} - (P_t + 10\log_{10} K - 10\gamma\log_{10}(d/d_0))}{\sigma_{\psi_{dB}}}\right). \\ &= 1 - Q\left(\frac{-110.5 - (10 - 31.54 - 37.1\log_{10}[150])}{3.65}\right) \\ &= .0121. \end{aligned}$$

An outage probabilities of 1% is a typical target in wireless system designs.

- Need to improve receiver sensitivity (i.e. reduce Pmin) for better coverage.

Shadowing: Modulation Design

Consider a WiMAX base station (BS) communicating to a subscriber, with the channel parameters $\alpha = 3$, $P_o = -40dB$, $d_0 = 1m$, $\sigma_s = 6dB$. We assume a transmit power of $P_t = 1$ Watt (30 dBm), a bandwidth of $B = 10$ MHz and due to rate $1/2$ convolutional codes, a received SNR of 14.7 dB is required for 16QAM, while just 3 dB is required for BPSK[4]. Finally, we consider only ambient noise with a typical power spectral density of $N_o = -173dBm/Hz$, with an additional receiver noise figure of $N_f = 5dB$[5].

The question is this: At a distance of 500 meters from the base station, what is the likelihood that the BS can reliably send BPSK or 16 QAM?

- Simple path loss/shadowing model: $\boxed{P_r = P_t P_o \chi \left(\frac{d_o}{d}\right)^\alpha}$
- Find Pr:

$$\begin{aligned} P_r(dB) &= 10\log_{10} P_t + 10\log_{10} P_o - 10\log_{10} d^\alpha + 10\log_{10}\chi \\ &= 30dBm - 40dB - 81dB + \chi(dB) = -91dBm + \chi(dB) \end{aligned}$$

- Find Noise power:

$$\begin{aligned} I_{tot}(dB) &= N_o + N_f + 10\log_{10} B \\ &= -173 + 5dB + 70 = -98dBm \end{aligned}$$

Shadowing: Modulation Design (Contd)

- SINR: $$\gamma = -91dBm + \chi(dB) + 98dBm = 7dB + \chi(dB).$$
- *Without* shadowing ($\chi = 0$), BPSK works 100%, 16QAM fails all the time.
- *With* shadowing ($\sigma_s = 6$dB):

BPSK:

$$\begin{aligned} P[\gamma \geq 3dB] &= P[\frac{\chi+7}{\sigma} \geq \frac{3}{\sigma}] \\ &= P[\frac{\chi}{6} \geq -\frac{4}{6}] \\ &= Q(-\frac{4}{6}) = 0.75 \end{aligned}$$

16 QAM

$$\begin{aligned} P[\gamma \geq 14.7dB] &= P[\frac{\chi+7}{\sigma} \geq \frac{14.7}{\sigma}] \\ &= Q(\frac{7.7}{6}) = .007 \end{aligned}$$

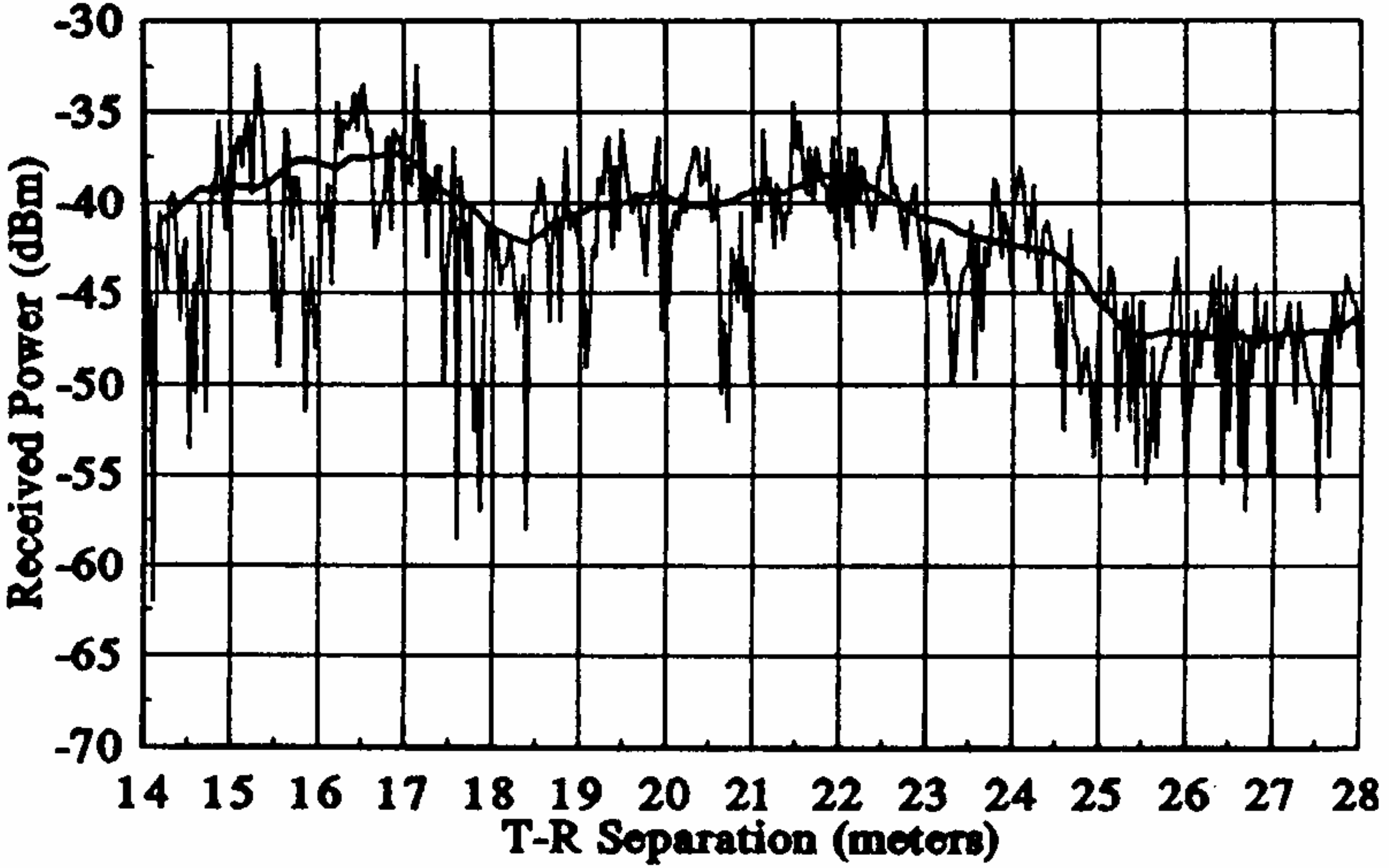
- 75% of users can use BPSK modulation and hence get a PHY data rate of 10 MHz · 1 bit/symbol ·1/2 = 5 Mbps
- Less than 1% of users can reliably use 16QAM (4 bits/symbol) for a more desirable data rate of 20 Mbps.
- Interestingly for BPSK, w/o shadowing, we had 100%; and 16QAM: 0%!

Small-Scale Fading: Rayleigh/Ricean Models, Multipath & Doppler

Small-scale Multipath fading: System Design

- Wireless communication typically happens at very high carrier frequency. (eg. f_c = 900 MHz or 1.9 GHz for cellular)
- Multipath fading due to constructive and destructive interference of the transmitted waves.
- Channel varies when mobile moves a distance of the order of the carrier wavelength. This is about 0.3 m for 900 Mhz cellular.
- For vehicular speeds, this translates to channel variation of the order of 100 Hz.
- *Primary driver* behind wireless communication system design.

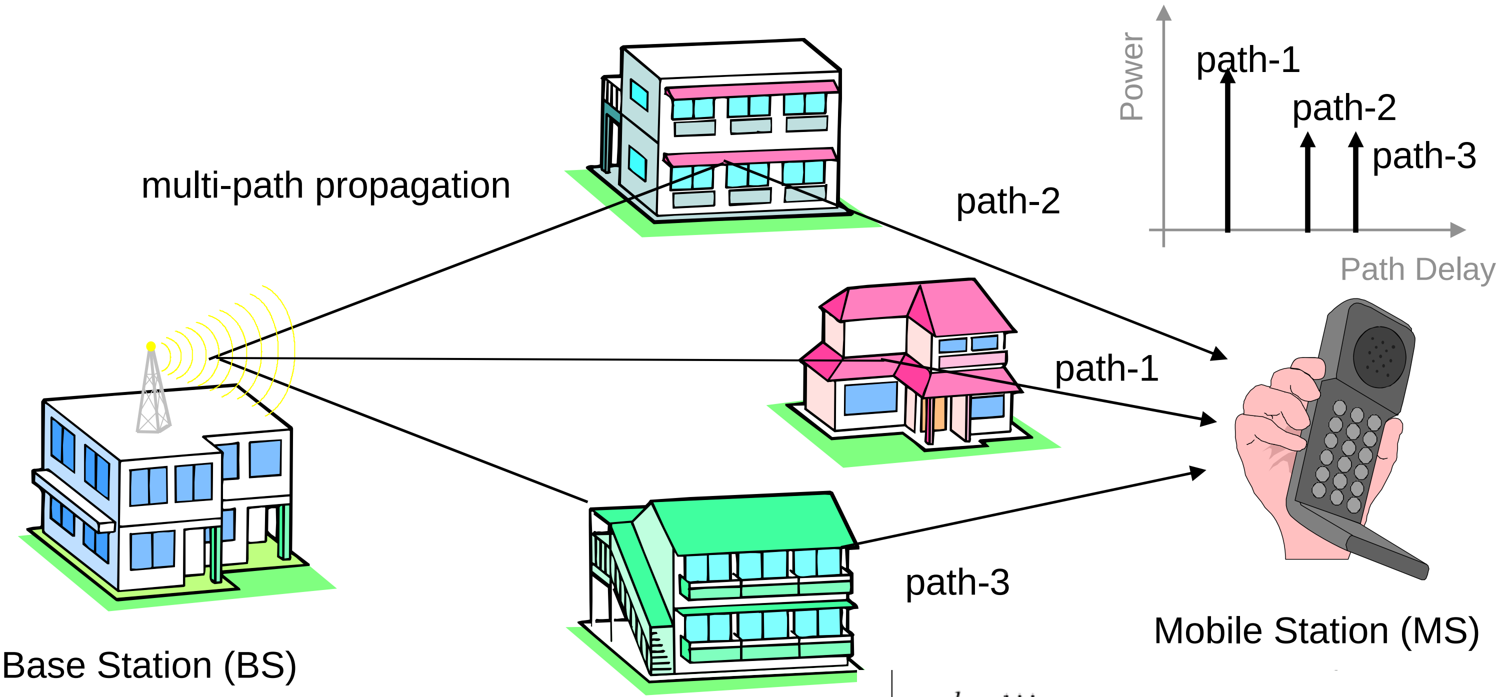
Fading: Small Scale vs Large Scale



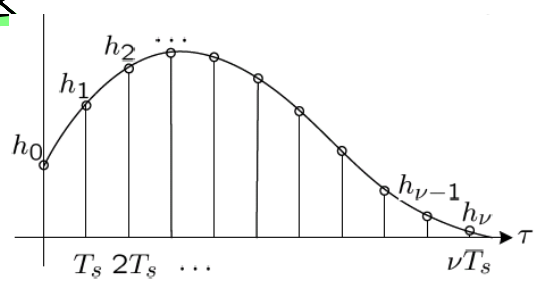
Source #1: Single-Tap Channel: Rayleigh Dist'n

- Path loss, shadowing => average signal power loss
  - Fading around this average.
  - Subtract out average => fading modeled as a zero-mean random process
- Narrowband Fading channel: Each symbol is long in time
  - The channel h(t) is assumed to be uncorrelated across symbols => single "tap" in time domain.
- Fading w/ many scatterers: Central Limit Theorem
  - In-phase (cosine) and quadrature (sine) components of the snapshot r(0), denoted as r_I (0) and r_Q(0) are independent Gaussian random variables.
  - Envelope Amplitude: $|r| = \sqrt{r_I^2 + r_Q^2}$ is Rayleigh.
  - Received Power: $|r|^2 = r_I^2 + r_Q^2$ is exponentially distributed.

Source #2: Multipaths: Power-Delay Profile



Channel Impulse Response:
Channel amplitude |h| correlated at delays τ.
Each "tap" value @ kTs Rayleigh distributed
(actually the sum of several sub-paths)



Eg: Power Delay Profile (WLAN/indoor)

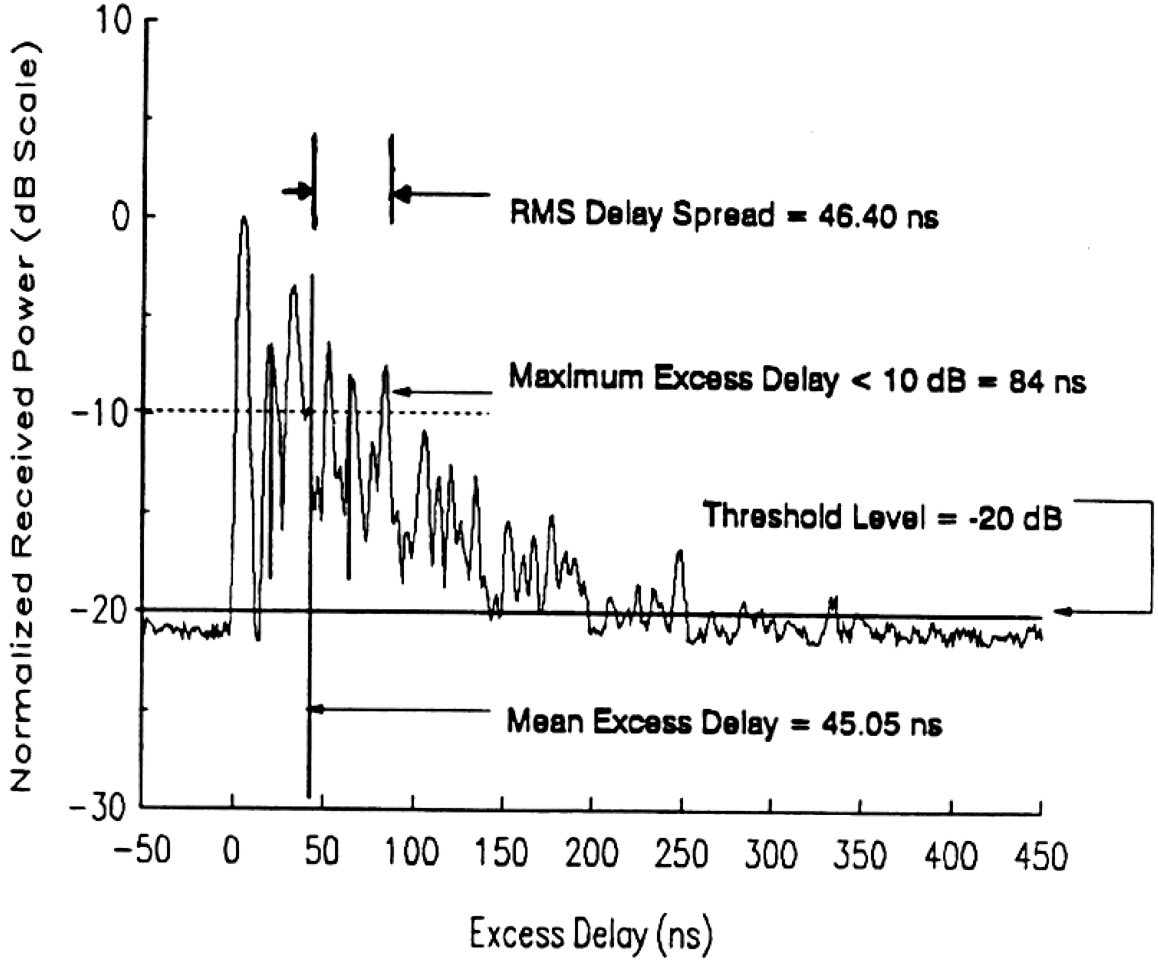
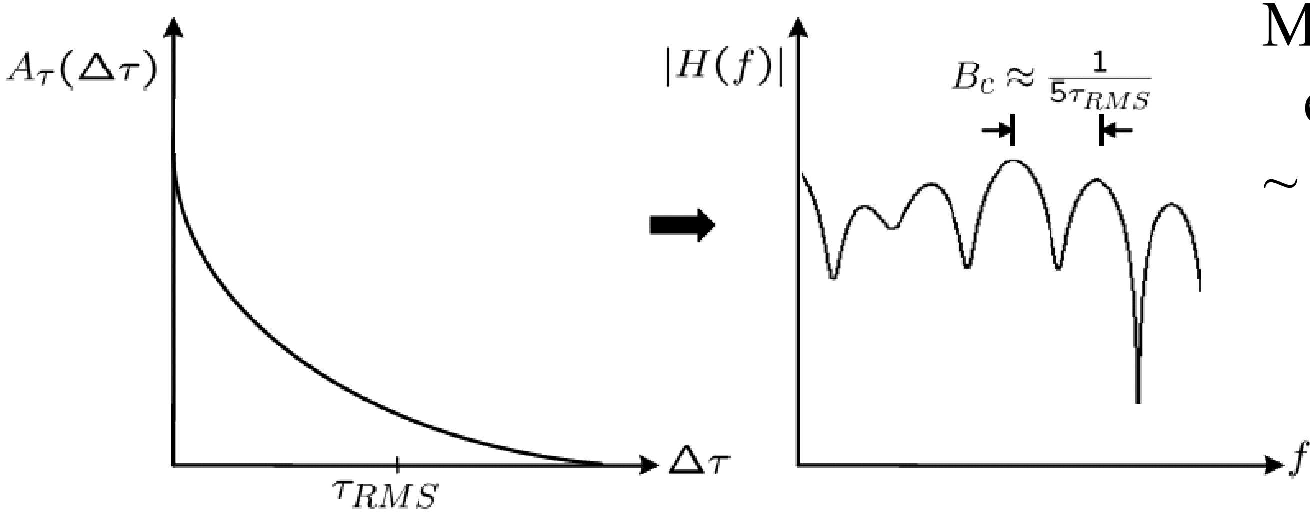


Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

<u>Multipath</u>: Time-Dispersion => Frequency Selectivity

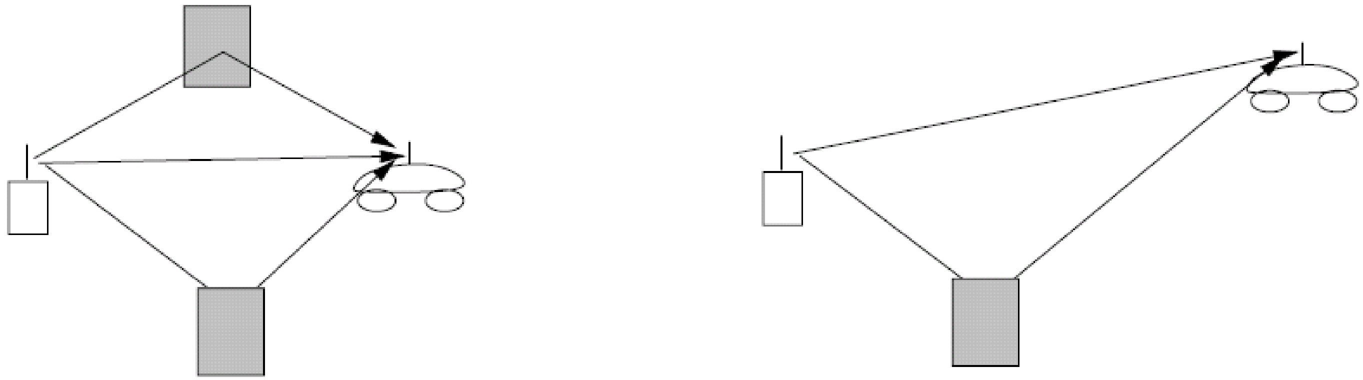
- The impulse response of the channel is correlated in the time-domain (sum of "echoes")
  - Manifests as a power-delay profile, dispersion in channel autocorrelation function A(Δτ)
- Equivalent to "selectivity" or "deep fades" in the frequency domain
- **<u>Delay spread</u>:** τ ~ *50ns (indoor) – 1μs (outdoor/cellular).*
- **<u>Coherence Bandwidth</u>**: Bc = *500kHz (outdoor/cellular) – 20MHz (indoor)*
- <u>Implications</u>: High data rate: symbol smears onto the adjacent ones (ISI).

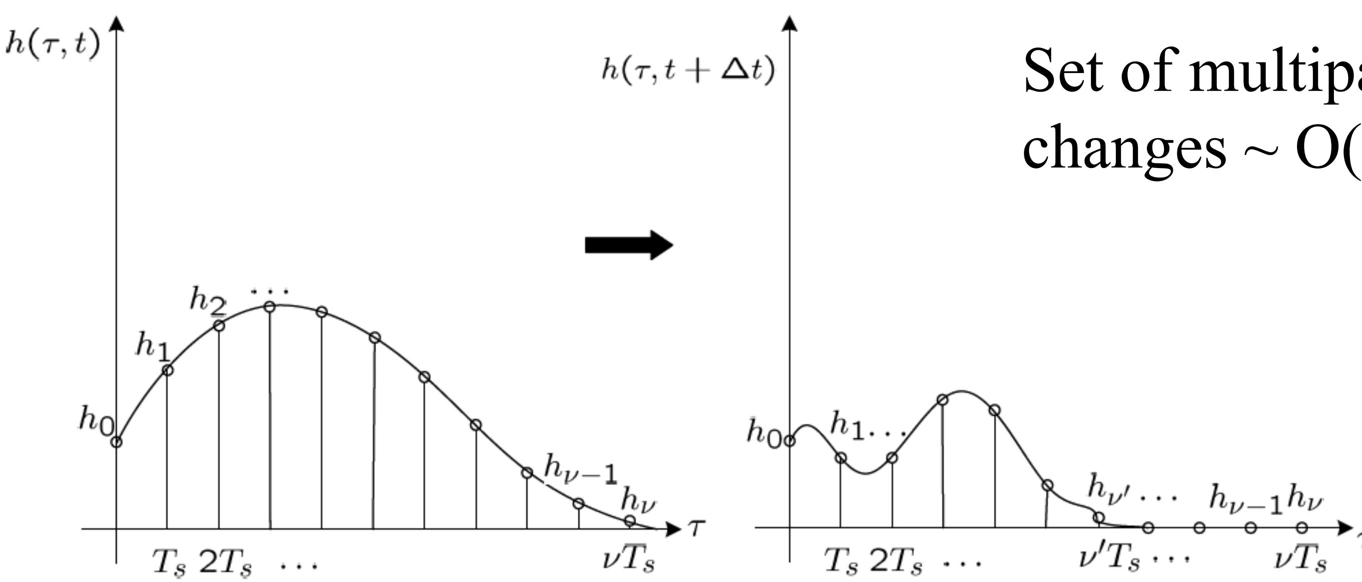


Multipath effects ~ O(1μs)

the shape of the multipath intensity profile $A_\tau(\Delta\tau)$ determines the correlation pattern of the channel frequency response (bottom)

Source #3: Doppler: Non-Stationary Impulse Response.





Set of multipaths changes ~ O(5 ms)

Doppler: Dispersion (Frequency) => Time-Selectivity

- The doppler power spectrum shows dispersion/flatness ~ doppler spread (100-200 Hz for vehicular speeds)
  - Equivalent to "selectivity" or "deep fades" in the time domain correlation envelope.
  - Each envelope point in time-domain is drawn from Rayleigh distribution. But because of Doppler, it is not IID, but correlated for a time period ~ Tc (correlation time).
- **Doppler Spread:** Ds ~ 100 Hz (vehicular speeds @ 1GHz)
- **Coherence Time**: Tc = 2.5-5ms.
- Implications: A deep fade on a tone can persist for 2.5-5 ms! Closed-loop estimation is valid only for 2.5-5 ms.

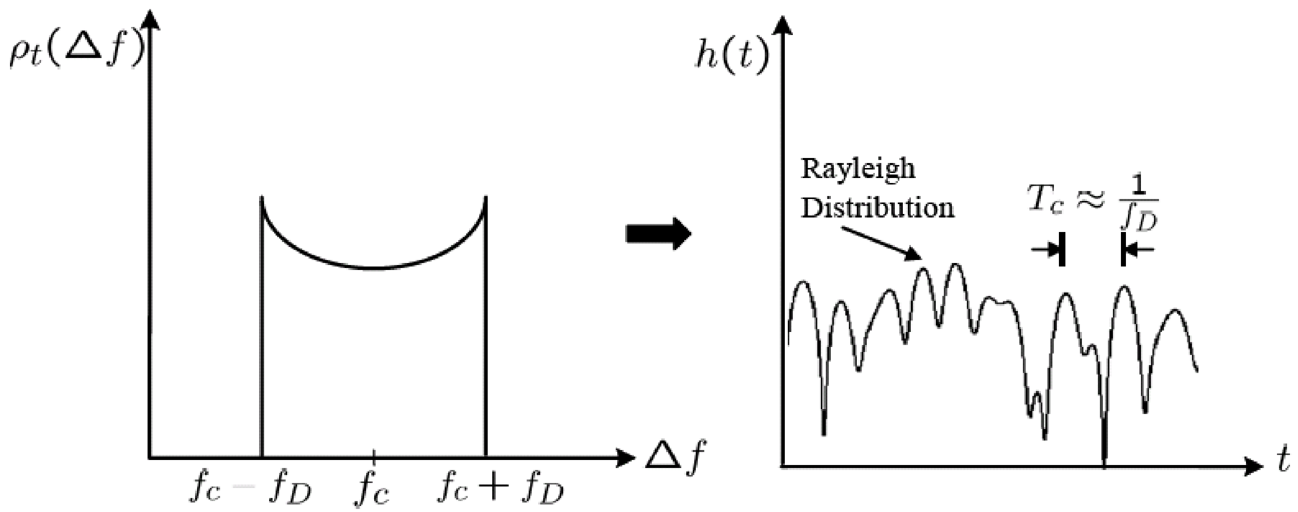


Figure 3.18: The shape of the Doppler power spectrum $\rho_t(\Delta f)$ determines the correlation envelope of the channel in time (top).

<u>Fading Summary</u>: Time-Varying Channel Impulse Response

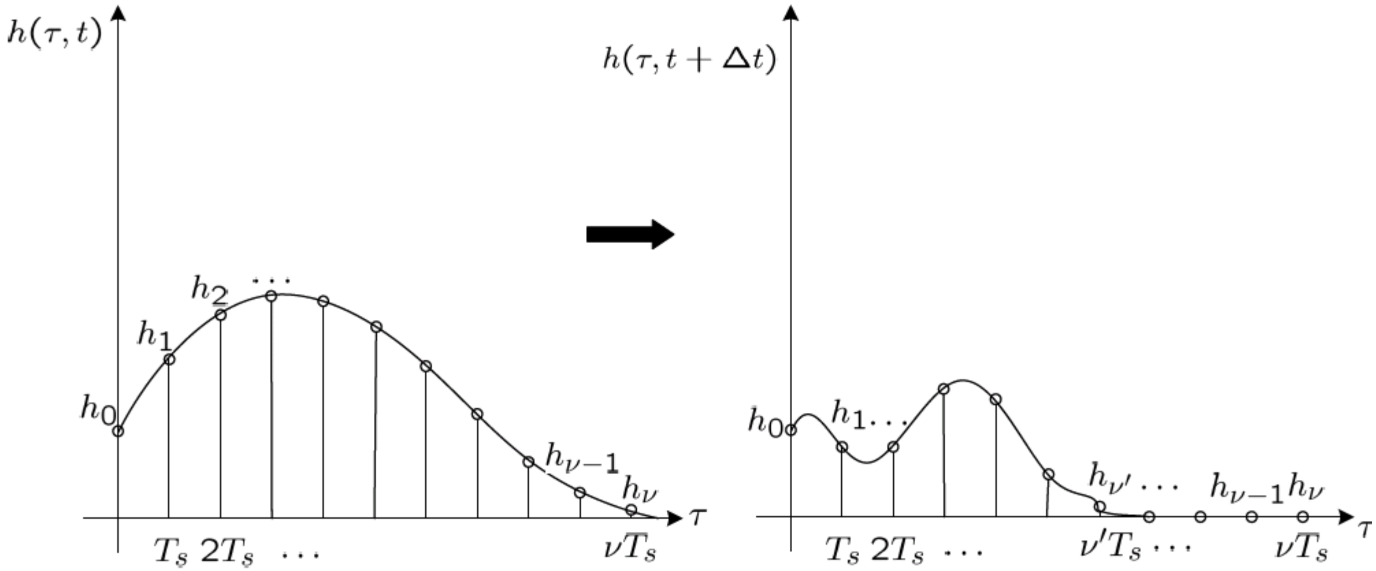


Figure 3.12: The delay τ corresponds to how *long* the channel impulse response lasts. The channel is time varying, so the channel impulse response is also a function of time, i.e. $h(\tau, t)$, and can be quite different at time $t + \Delta t$ than it was at time t.

- <u>**#1:**</u> At each tap, channel gain |h| is a Rayleigh distributed *r.v.*. The random *process* is <u>not</u> IID.
- <u>**#2:**</u> Response spreads out in the time-domain (τ), leading to inter-symbol interference and deep fades in the frequency domain: "***frequency-selectivity***" caused by <u>multi-path fading</u>
- <u>**#3:**</u> Response completely vanish (deep fade) for certain values of t: "***Time-selectivity***" caused by <u>doppler effects</u> (frequency-domain dispersion/spreading)

Dispersion-Selectivity Duality

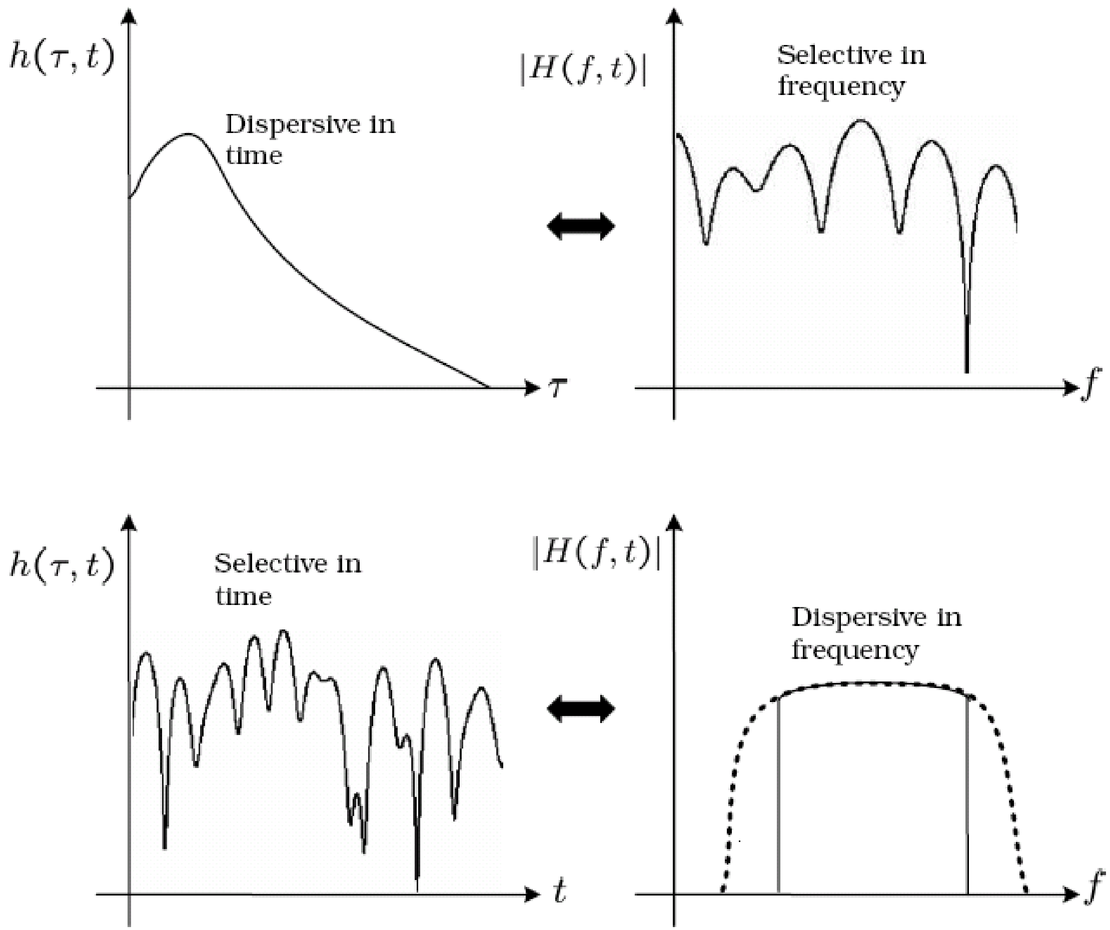


Figure 3.19: The dispersion–selectivity duality: Dispersion in time causes frequency selectivity, while dispersion in frequency causes time selectivity.

Dispersion-Selectivity Duality (Contd)

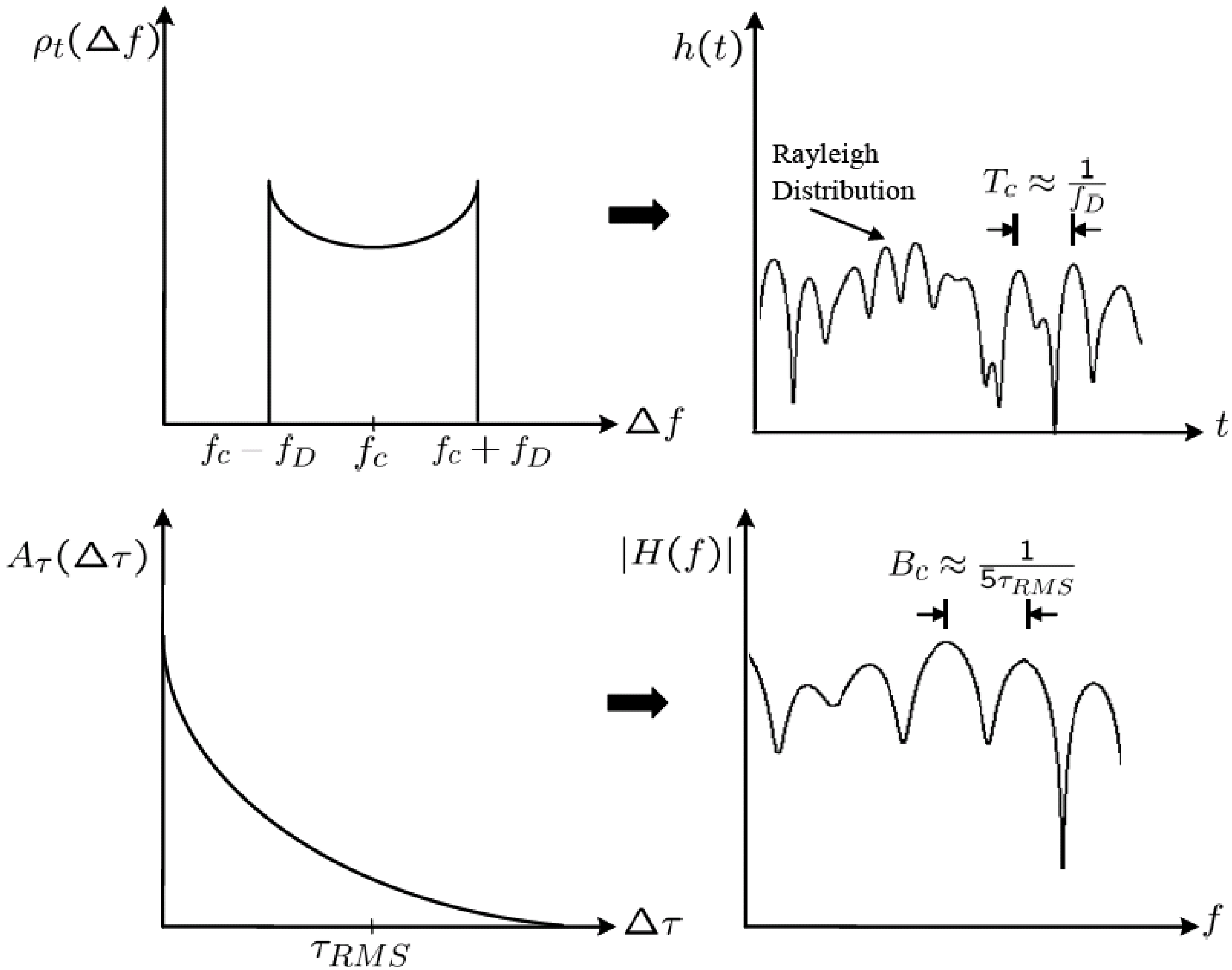


Figure 3.18: The shape of the Doppler power spectrum $\rho_t(\Delta f)$ determines the correlation envelope of the channel in time (top). Similarly, the shape of the multipath intensity profile $A_\tau(\Delta\tau)$ determines the correlation pattern of the channel frequency response (bottom)

Ren

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Fading: Jargon

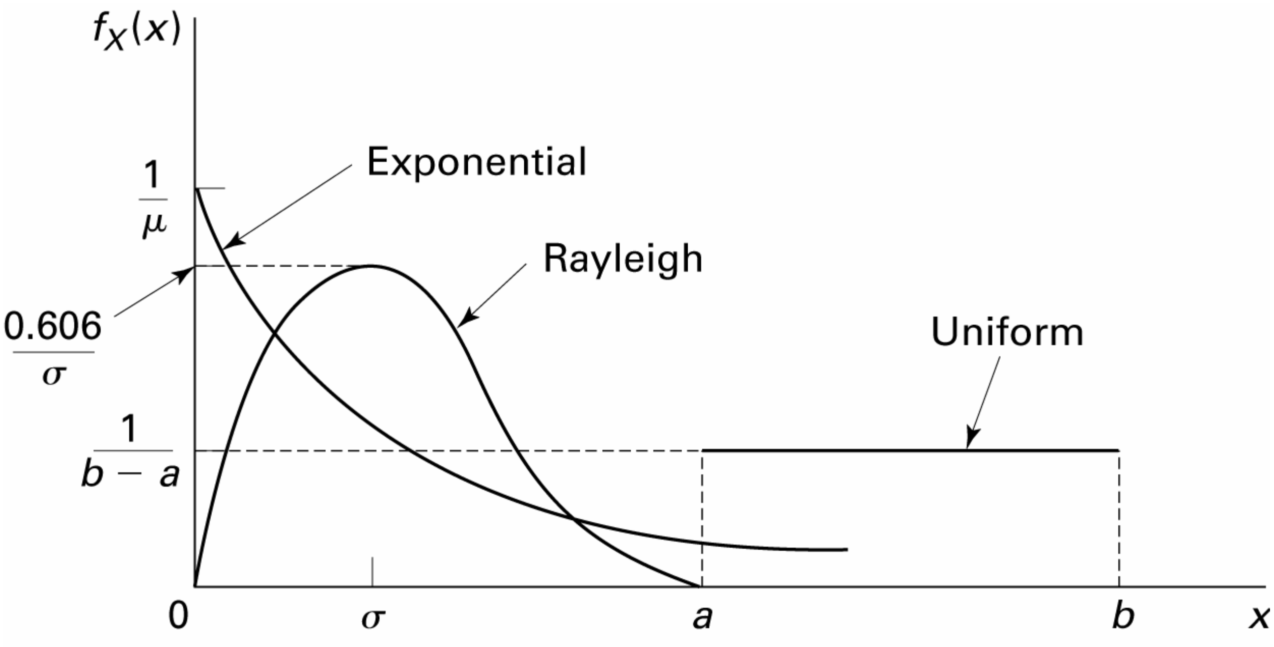
- **<u>Flat fading</u>**: no multipath ISI effects.
  - Eg: narrowband, indoors
- **<u>Frequency-selective fading:</u>** multipath ISI effects.
  - Eg: broadband, outdoor.

- **<u>Slow fading:</u>** no doppler effects.
  - Eg: indoor Wifi home networking
- **<u>Fast Fading:</u>** doppler effects, time-selective channel
  - Eg: cellular, vehicular

- Broadband cellular + vehicular => Fast + frequency-selective

Fading: Details
Single-Tap, Narrowband Flat Fading.

Normal Vector R.V, Rayleigh, Chi-Squared



The rayleigh, exponential, and uniform pdf 's.

X = [X_1, …, X_n] is ***Normal random** **vector***

||X|| is **Rayleigh** { eg: *magnitude* of a complex gaussian channel $X_1 + jX_2$ }

$||X||^2$ is **Chi-Squared w/ *n*-degrees of freedom**

When n = 2, chi-squared becomes **exponential**. {eg: *power* in complex gaussian channel: sum of squares…}

Rayleigh, Ricean, Nakagami-m fading

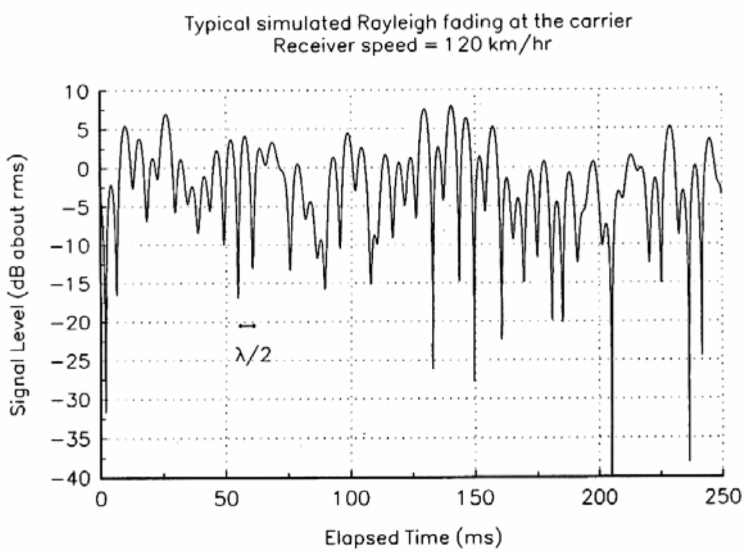


Figure 5.15 A typical Rayleigh fading envelope at 900 MHz [from [Fun93] © IEEE].

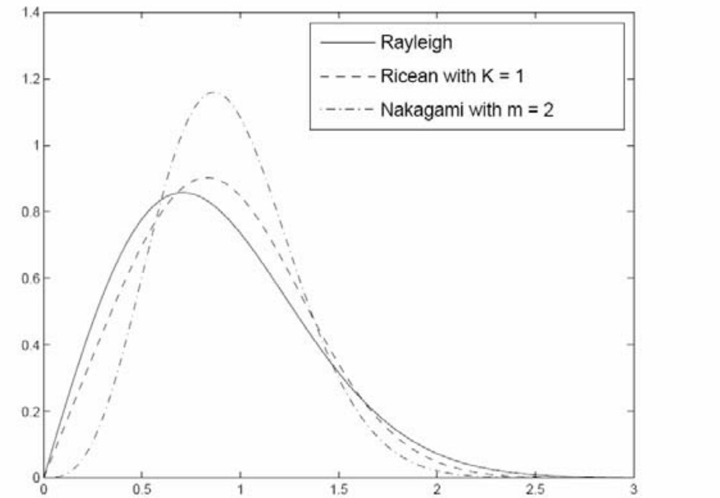


Figure 3.16: Probability distributions $f_{|r|}(x)$ for Rayleigh, Ricean w/ $K = 1$, and Nakagami with $m = 2$. All have average received power $P_r = 1$.

Ricean used when there is a dominant LOS path.
K parameter: strength of LOS to non-LOS. K = 0 => Rayleigh

Nakagami-m distribution can in many cases be used in tractable analysis of fading channel performance. More general than Rayleigh and Ricean.

Rayleigh Fading Example

Example 3.2: Consider a channel with Rayleigh fading and average received power $P_r = 20$ dBm. Find the probability that the received power is below 10 dBm.

Solution. We have $P_r = 20$ dBm =100 mW. We want to find the probability that $Z^2 < 10$ dBm =10 mW. Thus

$$p(Z^2 < 10) = \int_0^{10} \frac{1}{100} e^{-x/100} dx = .095.$$

- Non-trivial (1%) probability of *very* deep fades.

Rayleigh Fading (Fade Duration Example)

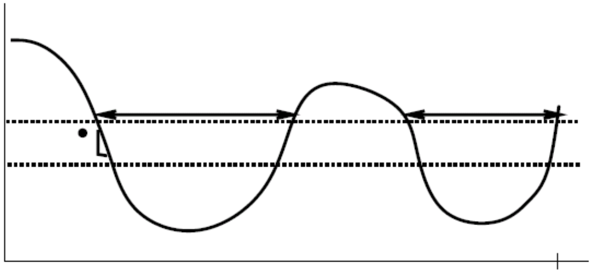
average fade duration is

$$\bar{t}_Z = \frac{1}{TL_Z}\sum_{i=1}^{L_Z T} t_i \approx \frac{p(z(t) < Z)}{L_Z}.$$

$$\boxed{\bar{t}_Z = \frac{e^{\rho^2} - 1}{\rho f_D \sqrt{2\pi}}}$$

L_Z: Level Crossing Rate

$$\boxed{L_Z = \sqrt{2\pi} f_D \rho e^{-\rho^2}}$$



Example 3.3:

Consider a voice system with acceptable BER when the received signal power is at or above half its averag value. If the BER is below its acceptable level for more than 120 ms, users will turn off their phone. Find the rang of Doppler values in a Rayleigh fading channel such that the average time duration when users have unacceptabl voice quality is less than $t = 60$ ms.

Solution: The target received signal value is half the average, so $P_0 = .5P_r$ and thus $\rho = \sqrt{.5}$. We require

$$\bar{t}_Z = \frac{e^{.5} - 1}{f_D\sqrt{\pi}} \leq t = .060$$

and thus $f_D \geq (e-1)/(.060\sqrt{2\pi}) = 6.1$ Hz. **<u>Faster motion & doppler better (get out of fades)!</u>**

Effect of Rayleigh Fading

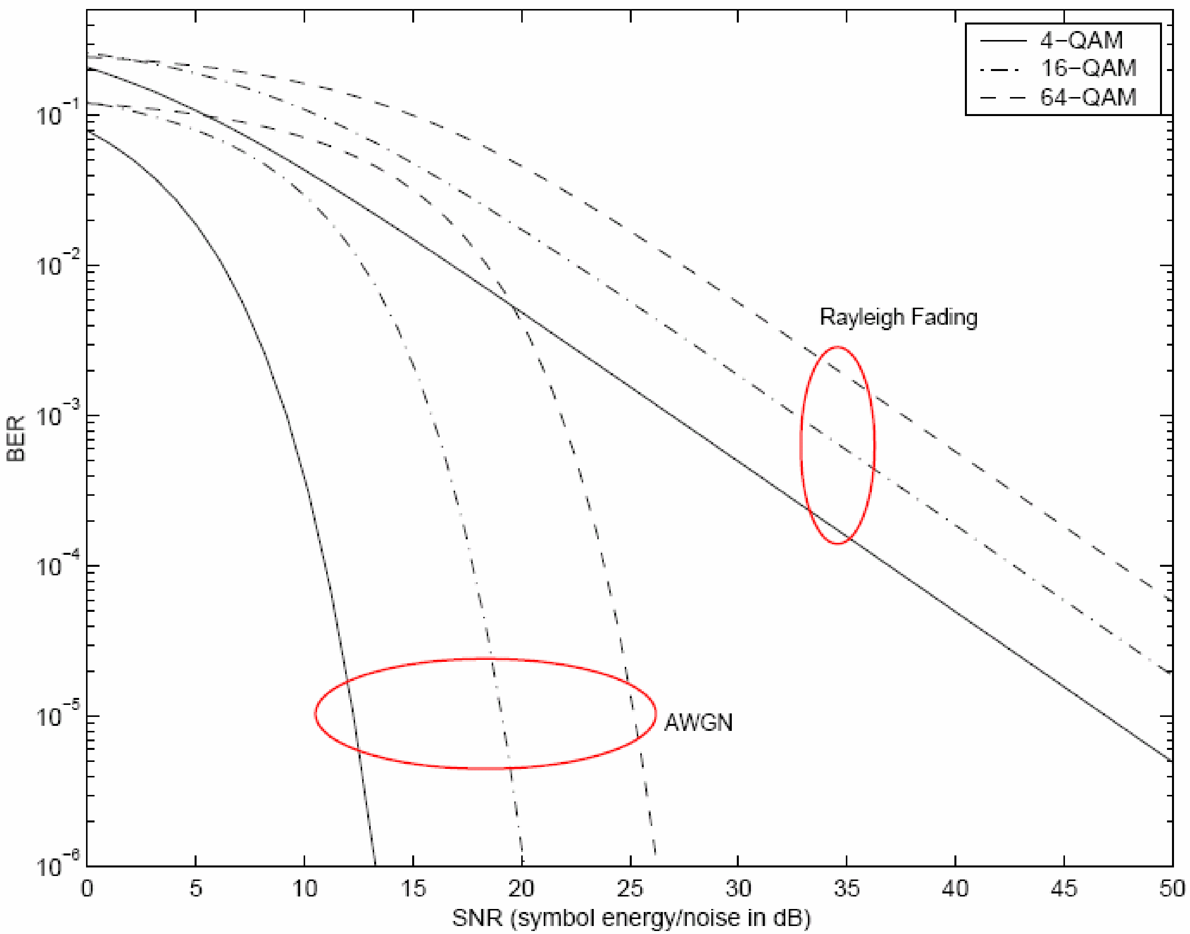
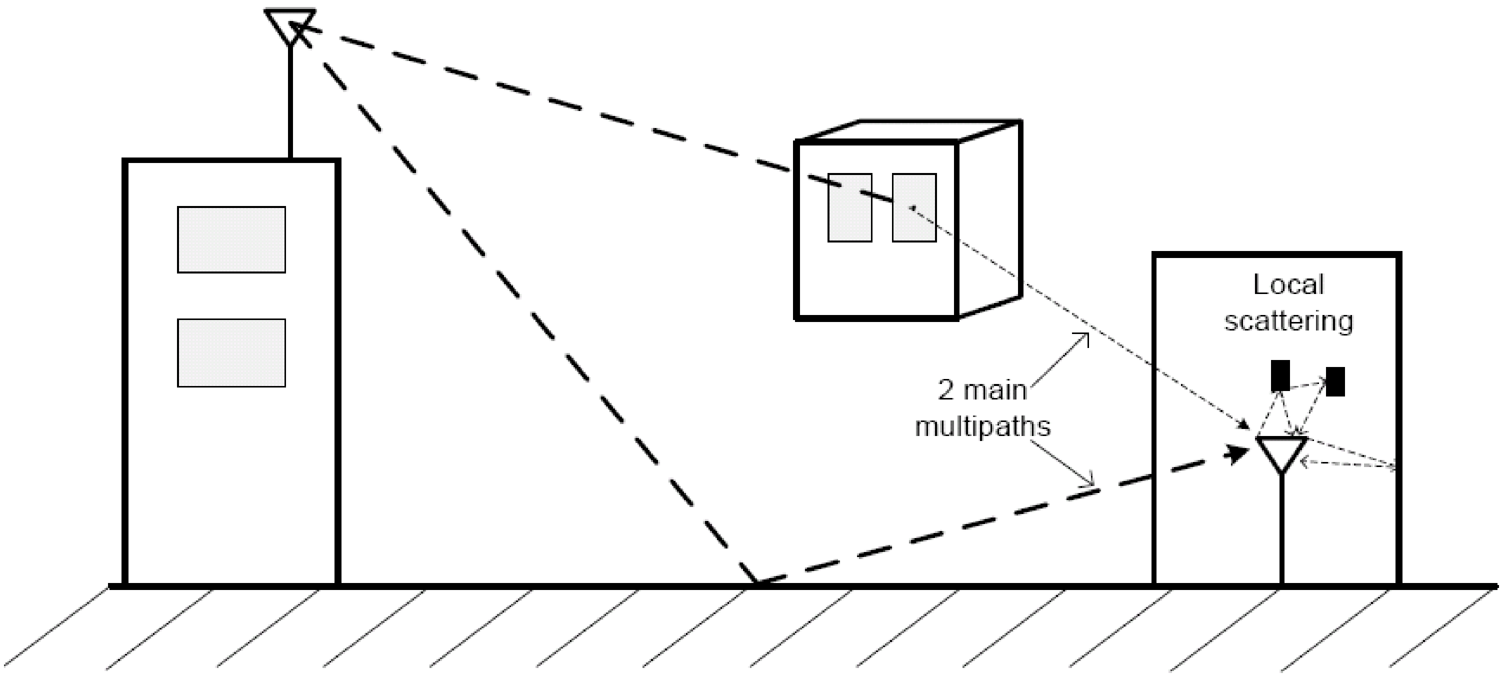


Figure 3.22: Flat fading causes a loss of at least 20-30 dB at reasonable BER values.

Fading: Details
Broadband, “Frequency-Selective” Fading. Multipath

Broadband Fading: Multipath Frequency Selectivity



- A few major multipaths, and lots of local scatterers => each channel sample "tap" can be modeled as Rayleigh
  - A "tap" period generally shorter than a symbol time.
- Correlation between tapped values.

Recall: Electric (Far) Field & Transfer Function

- Tx: a sinusoid: cos $2\pi ft$
- Electric Field: source antenna gain (α_s)

$$E(f,t,(r,\theta,\psi)) = \frac{\alpha_s(\theta,\psi,f)\cos 2\pi f(t-\frac{r}{c})}{r}.$$

- Product of antenna gains (α)

$$E_r(f,t,u) = \frac{\alpha(\theta,\psi,f)\cos 2\pi f(t-\frac{r}{c})}{r}$$

- Consider the function
  (transfer function)

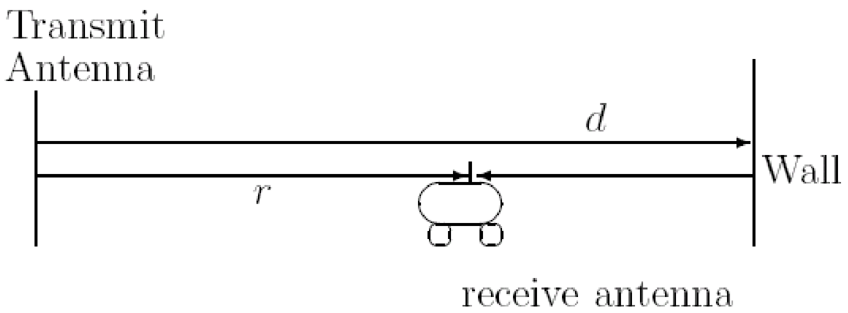
$$H(f) := \frac{\alpha(\theta,\psi,f)e^{-j2\pi fr/c}}{r}.$$

- The electric field is now:

$$\boxed{E_r(f,t,u) = \Re\left[H(f)e^{j2\pi ft}\right].}$$

Linearity is a good assumption, but time-invariance lost when Tx, Rx or environment in motion

Reflecting wall: Ray Tracing, Superposition

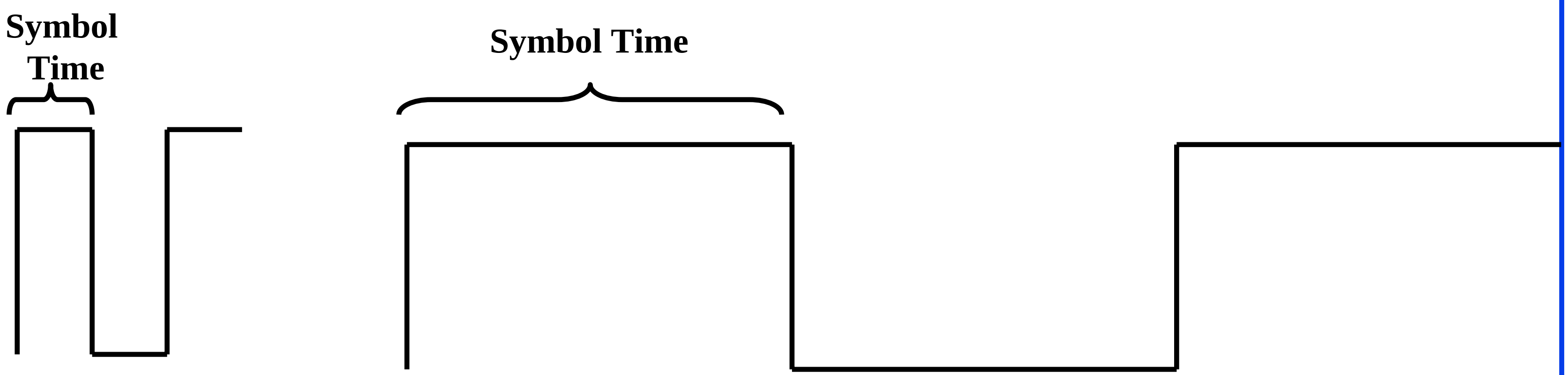


- Superposition of ***phase-shifted***, ***attenuated*** waves

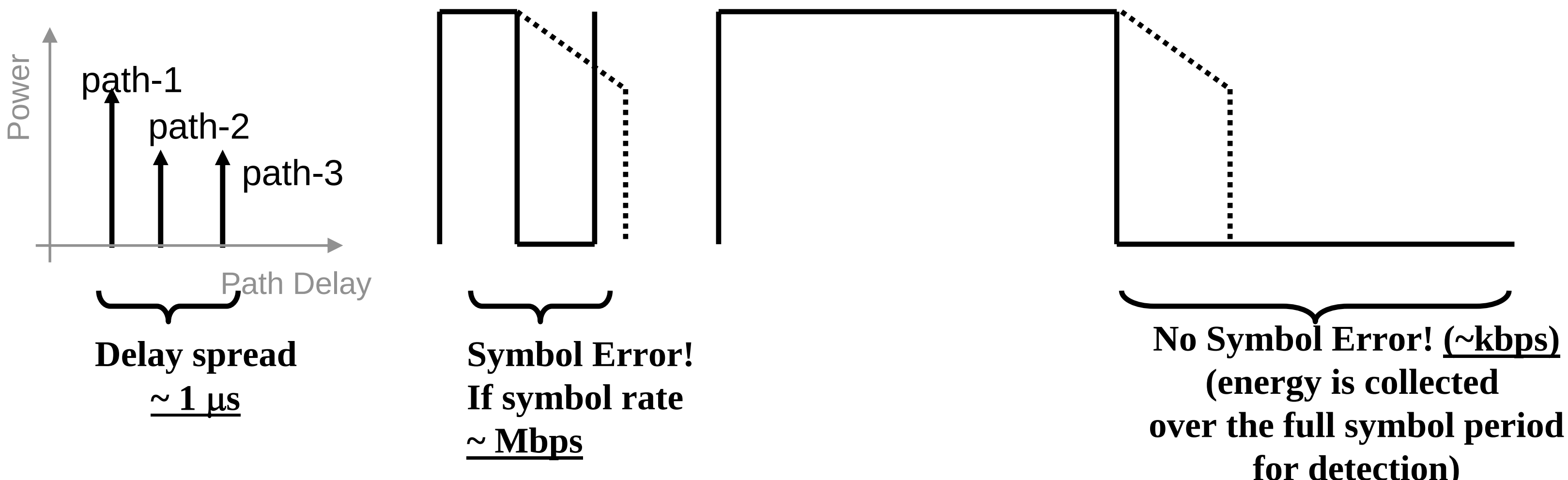
$$E_r(f,t) = \frac{\alpha \cos 2\pi f \left(t - \frac{r}{c}\right)}{r} - \frac{\alpha \cos 2\pi f \left(t - \frac{2d-r}{c}\right)}{2d - r}.$$

- Phase difference ($= \frac{4\pi f}{c}(d-r) + \pi.$): depends upon *f* & *r*
  - Constructive or destructive interference
  - Peak-to-valley: *coherence distance*: $\boxed{\Delta x_c := \frac{\lambda}{4}}$
  - *Delay spread*: $\boxed{T_d := \frac{2d-r}{c} - \frac{r}{c}}$
  - *Coherence bandwidth*: $\boxed{1/T_d}$
    - I/f pattern changes if frequency changes on the order of coherence bandwidth.

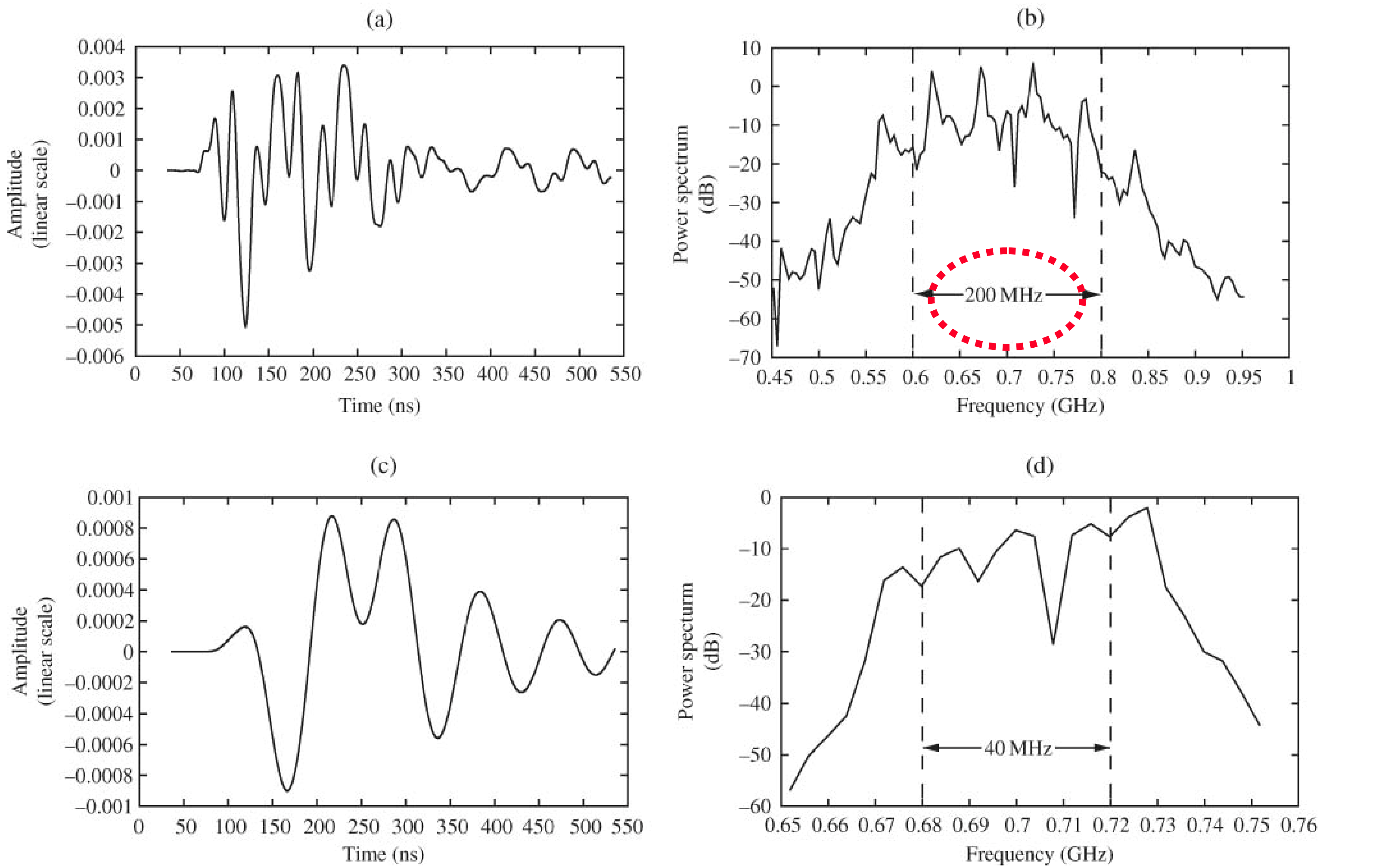
Power Delay Profile => Inter-Symbol interference



- Higher bandwidth => higher symbol rate, and smaller time per-symbol
- Lower symbol rate, more time, energy per-symbol
- If the delay spread is longer than the symbol-duration, symbols will “smear” onto adjacent symbols and cause symbol errors

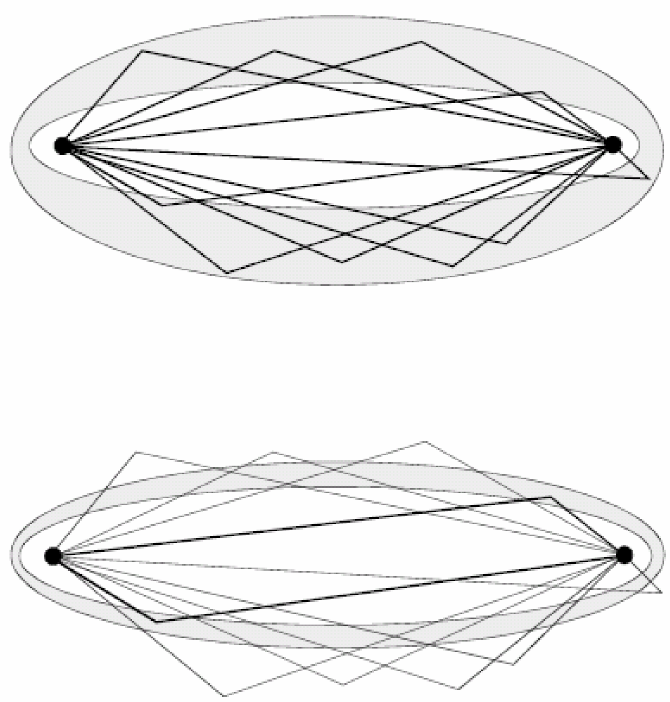


Effect of Bandwidth (# taps) on MultiPath Fading



Effective channel depends on both physical environment and bandwidth!

Multipaths & Bandwidth (Contd)

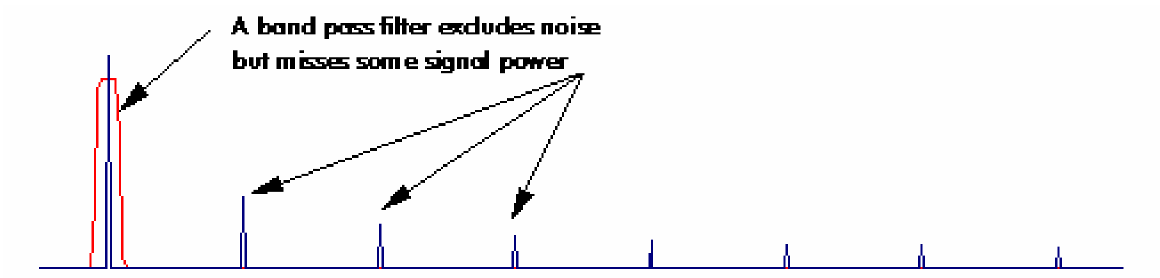


- Even though many paths with different delays exist (corresponding to finer-scale bumps in h(t))…
- Smaller bandwidth => fewer channel taps (remember Nyquist?)
- The receiver will simply not sample several multipaths, and interpolate what it does sample => smoother envelope h(t)
  - The power in these multipaths cannot be combined!
- In CDMA Rake (Equalization) Receiver, the power on multipath taps is received ("rake fingers"), gain adjusted and combined.
  - Similar to bandpass vs matched filtering (see next slide)

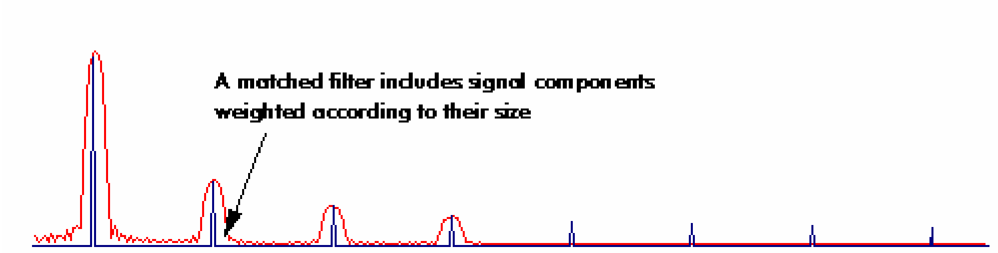
Rake Equalization Analogy: Bandpass vs Matched Filtering

Simple Bandpass (low bandwidth) Filter:

excludes noise, but misses some signal power in other mpath "taps"



Matched Filter: includes more signal power, weighted according to size
=> maximal noise rejection & signal power aggregation!



Power Delay Profile: Mean/RMS Delay Spreads

The **power delay profile** $A_c(\tau)$, also called the **multipath intensity profile**, is defined as the autocorrelation (3.52) with $\Delta t = 0$: $A_c(\tau) \triangleq A_c(\tau, 0)$. The power delay profile represents the average power associated with a given multipath delay, and is easily measured empirically. The average and rms delay spread are typically defined in terms of the power delay profile $A_c(\tau)$ as

$$\mu_{T_m} = \frac{\int_0^\infty \tau A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau}, \qquad \sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau - \mu_{T_m})^2 A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau}}. \qquad)$$

Table 3.2: Some typical RMS delay spread and approximate coherence bandwidths for different WiMAX applications.

Environment	f_c (GHz)	RMS delay, τ_{RMS} (ns)	Coherence bandwidth $B_c \approx \frac{1}{5\tau_{RMS}}$ (MHz)	Reference
Urban	9.1	1300	.15	[22]
Rural	9.1	1960	.1	[22]
Indoor	9.1	270	.7	[22]
Urban	5.3	44	4.5	[36]
Rural	5.3	66	3.0	[36]
Indoor	5.3	12.4	16.1	[36]

$$|f_1 - f_2| \le B_c \quad \Rightarrow \quad H(f_1) \approx H(f_2)$$

$$|f_1 - f_2| > B_c \quad \Rightarrow \quad H(f_1) \text{ and } H(f_2) \text{ are uncorrelated}$$

Multipath Fading Example

Example 3.5:

Consider a wideband channel with multipath intensity profile

$$A_c(\tau) = \begin{cases} e^{-\tau/.00001} & 0 \leq \tau \leq 20 \ \mu\text{sec.} \\ 0 & \text{else} \end{cases} .$$

Find the mean and rms delay spreads of the channel and find the maximum symbol rate such that a linearly-modulated signal transmitted through this channel does not experience ISI.

Solution: The average delay spread is

$$\mu_{T_m} = \frac{\int_0^{20*10^{-6}} \tau e^{-\tau/.00001} d\tau}{\int_0^{20*10^{-6}} e^{-\tau/.00001} d\tau} = 6.87 \ \mu\text{sec.}$$

The rms delay spread is

$$\sigma_{T_m} = \sqrt{\frac{\int_0^{20*10^{-6}} (\tau - \mu_{T_m})^2 e^{-\tau} d\tau}{\int_0^{20*10^{-6}} e^{-\tau} d\tau}} = 5.25 \ \mu\text{sec.}$$

We see in this example that the mean delay spread is roughly equal to its rms value. To avoid ISI we require linear modulation to have a symbol period T_s that is large relative to σ_{T_m}. Taking this to mean that $T_s > 10\sigma_{T_m}$ yields a symbol period of $T_s = 52.5$ μsec or a symbol rate of $R_s = 1/T_s = 19.04$ Kilosymbols per second. This is a highly constrained symbol rate for many wireless systems. Specifically, for binary modulations where the symbol rate equals the data rate (bits per second, or bps), high-quality voice requires on the order of 32 Kbps and high-speed data requires on the order of 10-100 Mbps.

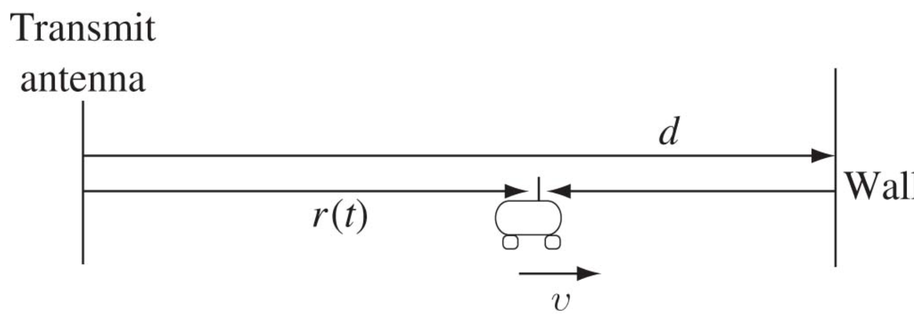
Fading: Details
Doppler “Fast” Fading: Time-selectivity

Doppler: Approximate LTI Modeling

- $r \rightarrow r_0 + vt$ $$E(f,t,(r_0+vt,\theta,\psi)) = \frac{\alpha_s(\theta,\psi,f)\cos 2\pi f(t-\frac{r_0}{c}-\frac{vt}{c})}{r_0+vt}.$$
- vt/c phase correction

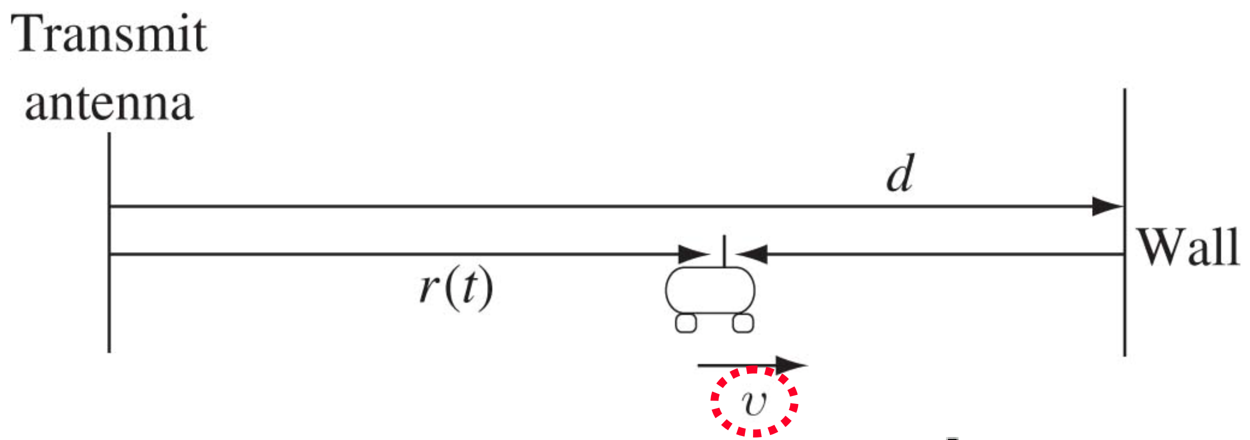
rewrite $f(t-r_0/c-vt/c)$ as $f(1-v/c)t - fr_0/c$. (Fixed phase & frequency shifts)

- <u>Doppler frequency shift of $-fv/c$</u> due to relative motion
- This is no longer LTI unlike wired channels
- We have to make LTI approximations assuming small-time-scales only (t small, $vt \approx 0$)



- If time-varying attenuation in denominator ignored ($vt \approx 0$), we can use the transfer function H(f) as earlier, but with doppler adjustment of $-fv/c$

Doppler: Reflecting Wall, Moving Antenna



$$E_r(f,t) = \frac{\alpha \cos 2\pi f[(1-\frac{v}{c})t - \frac{r_0}{c}]}{r_0 + vt} - \frac{\alpha \cos 2\pi f\left[(1+\frac{v}{c})t + \frac{r_0 - 2d}{c}\right]}{2d - r_0 - vt}.$$

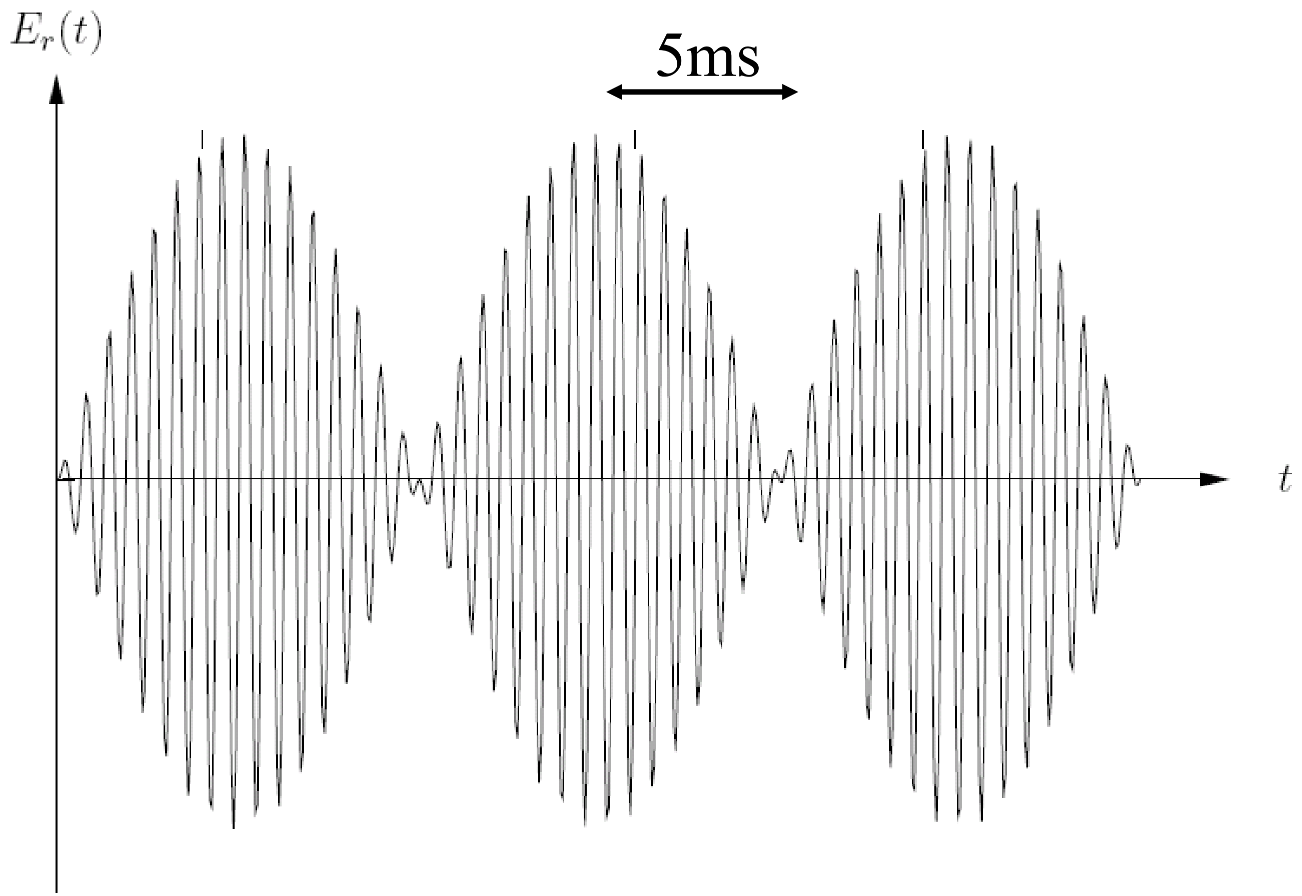
- *Doppler spread:* $\boxed{D_s := D_2 - D_1}$

$$D_1 := -fv/c.$$

$$D_2 := +fv/c$$

- Note: opposite sign for doppler shift for the two waves
- Effect is roughly like the *product of two sinusoids*

Doppler Spread: Effect



- Fast oscillations of the order of GHz
- Slow envelope oscillations order of 50 Hz => peak-to-zero every 5 ms
- A.k.a. **Channel coherence time (Tc)** = $c/4fv$

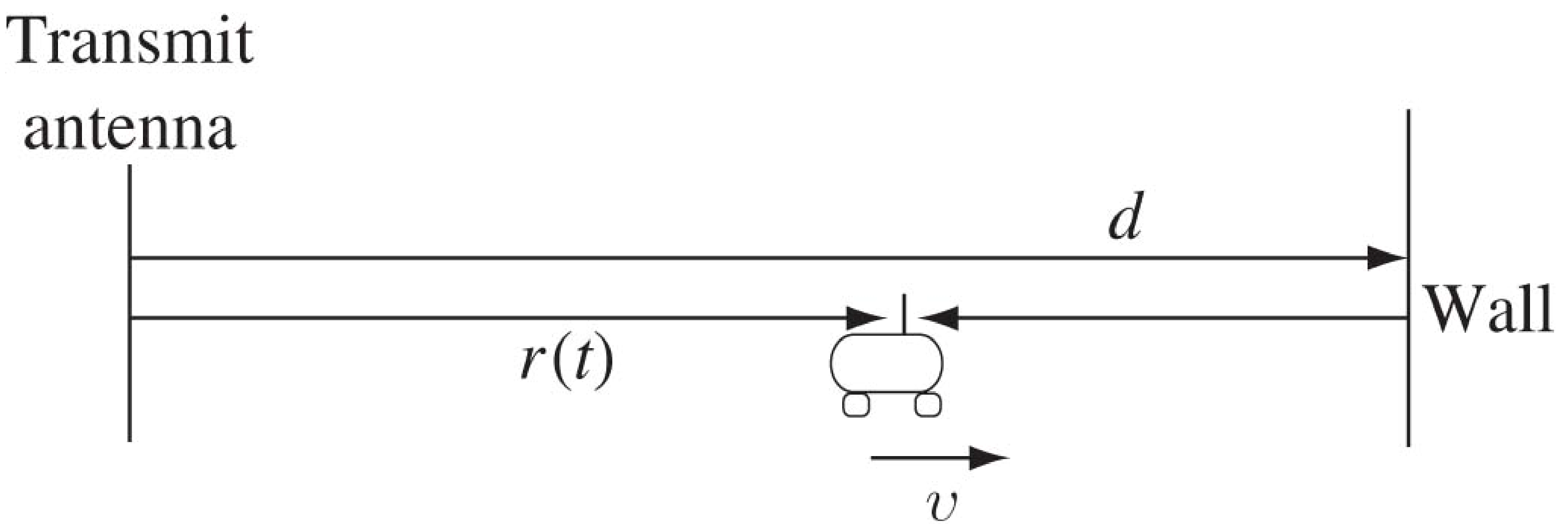
Two-path (mobile) Example

v= 60 km/hr, f_c = 900 MHz:

Direct path has Doppler shift of roughly -50 Hz = $-fv/c$

Reflected path has shift of +50 Hz

Doppler spread = 100 Hz



Doppler Spread: Effect

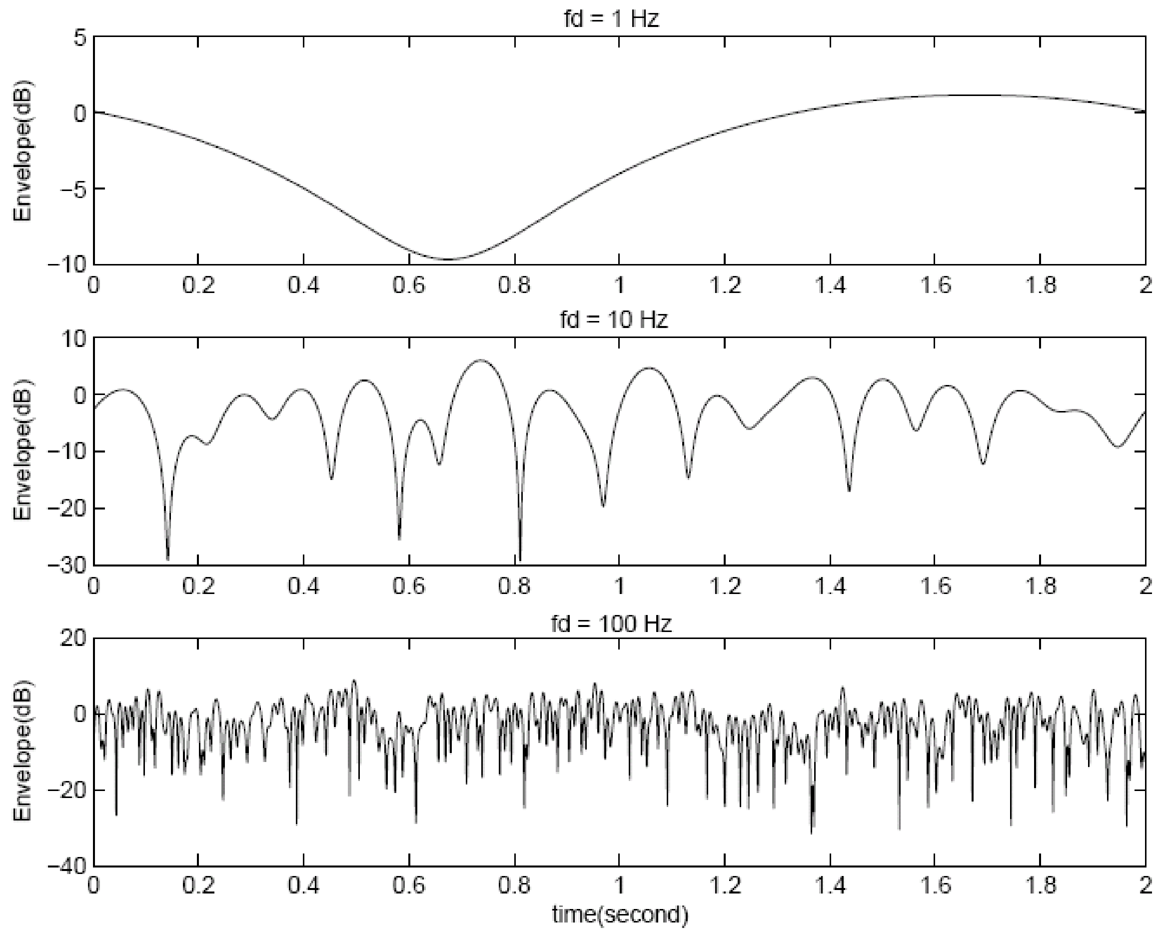


Figure 3.20: A sample output of the provide Rayleigh fading Matlab function for Doppler frequencies of $f_D = 1$, 10, and 100 Hz.

Angular Spread: Impact on Spatial Diversity

- Space-time channel models:
  - Mean/RMS angular spreads (similar to multipath delay spread)
  - The time-varying impulse response model can be extended to incorporate AOA (angle-of-arrival) for the array.
  - A(θ): average received signal power as a function of AoA θ.
  - Needs appropriate linear transformation to achieve full MIMO gains.

$$\overline{a}(\theta_n(t)) = [e^{-j\psi_{n,1}}, \ldots, e^{-j\psi_{n,M}}]^T$$

$$\mu_\theta = \frac{\int_{-\pi}^{\pi} \theta A(\theta) d\theta}{\int_{-\pi}^{\pi} A(\theta) d\theta},$$

$$\sigma_\theta = \sqrt{\frac{\int_{-\pi}^{\pi} (\theta - \mu_\theta)^2 A(\theta) d\theta}{\int_{-\pi}^{\pi} A(\theta) d\theta}},$$

Angular Spread and Coherence Distance

- **θ_{RMS}: RMS angular spread** of a channel
  - Refers to the statistical distribution of the angle of the arriving energy.
- Large θ_{RMS} => channel energy is coming in from many directions,
  - Lot of local scattering, and this results in more statistical diversity in the channel based upon AoA
- Small θ_{RMS} => received channel energy is more focused.
  - More focused energy arrival results in less statistical diversity.

- The dual of angular spread is **coherence distance, Dc**.
  - As the angular spread↑, the coherence distance ↓, and vice versa.
  - A coherence distance of d means that any physical positions separated by d have an essentially uncorrelated received signal amplitude and phase.
- An approximate rule of thumb is $\boxed{D_c \approx \frac{.2\lambda}{\theta_{\mathrm{RMS}}}.}$

↑freq =>
better angular diversity!

Rayleigh fading, which assumes a uniform angular spread, the well known relation is

$$D_c \approx \frac{9\lambda}{16\pi}.$$

Key Wireless Channel Parameters

Table 3.1: Key wireless channel parameters

Symbol	Parameter
α	path loss exponent
σ_s	Log normal shadowing standard deviation
f_D	Doppler spread (maximum Doppler frequency), $f_D = \frac{v f_c}{c}$
T_c	Channel coherence time, $T_c \approx f_D^{-1}$
$\tau_{\max}$	Channel delay spread (maximum)
τ_{RMS}	Channel delay spread (RMS)
B_c	Channel coherence bandwidth, $B_c \approx \tau^{-1}$
θ_{RMS}	Angular spread (RMS)

Fading Parameter Values

Key channel parameters and time-scales	Symbol	Representative values
Carrier frequency	f_c	1 GHz
Communication bandwidth	W	1 MHz
Distance between transmitter and receiver	d	1 km
Velocity of mobile	v	64 km/h
Doppler shift for a path	$D = f_c v/c$	50 Hz
Doppler spread of paths corresponding to a tap	D_s	100 Hz
Time-scale for change of path amplitude	d/v	1 minute
Time-scale for change of path phase	$1/(4D)$	5 ms
Time-scale for a path to move over a tap	$c/(vW)$	20 s
Coherence time	$T_c = 1/(4D_s)$	2.5 ms
Delay spread	T_d	1 μs
Coherence bandwidth	$W_c = 1/(2T_d)$	500 kHz

Small-Scale Fading Summary

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than base-band signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Figure 5.11 Types of small-scale fading.

Fading: Design Impacts (Eg: Wimax)

Table 3.3: Summary of Broadband Fading Parameters, with Rules of Thumb

Quantity	If "Large"?	If "Small"?	WiMAX Design Impact
Delay Spread, τ	If $\tau \gg T$, then frequency selective	If $\tau \ll T$, then frequency flat	The larger the delay spread relative to the symbol time, the more severe the ISI.
Coherence Bandwidth, B_c	If $\frac{1}{B_c} \ll T$, then frequency flat	If $\frac{1}{B_c} \gg T$, then frequency selective	Provides a guideline to subcarrier width $B_{sc} \approx B_c/10$, and hence number of subcarriers needed in OFDM: $L \geq 10B/B_c$.
Doppler spread, $f_D = \frac{f_c v}{c}$	If $f_c v \gg c$, then fast fading	If $f_c v \leq c$, then slow fading	As f_D/B_{sc} becomes nonnegligible, subcarrier orthogonality is compromised
Coherence Time, T_c	If $T_c \gg T$, then slow fading	If $T_c \leq T$, then fast fading	T_c small necessitates frequent channel estimation and limits the OFDM symbol duration, but provides greater time diversity.
Angular Spread, θ_{RMS}	Non LOS channel, lots of diversity	effectively LOS channel, not much diversity	Multi-antenna array design, beamforming vs. diversity
Coherence Distance, D_c	effectively LOS channel, not much diversity	Non LOS channel, lots of diversity	Determines antenna spacing

Mathematical Models

Physical Models

- Wireless channels can be modeled as linear time-varying systems:

$$y(t) = \sum_i a_i(t) x(t - \tau_i(t))$$

where $a_i(t)$ and $\tau_i(t)$ are the gain and delay of path i.

- The time-varying impulse response is:

$$h(t, \tau) = \sum_i a_i(t) \delta(\tau - \tau_i(t))$$

- Consider first the special case when the channel is ***time-invariant***:

$$h(\tau) = \sum_i a_i \delta(\tau - \tau_i)$$

Time-Invariance Assumption: Typical Channels are Underspread

- **Coherence time** T_c depends on carrier frequency and vehicular speed, of the order of *milliseconds* or more.
- **Delay spread** T_d depends on distance to scatterers, of the order of *nanoseconds (indoor) to microseconds (outdoor)*.
- Channel can be considered as **time-invariant** over a long time scale ("*underspread*").
  - Transfer function & frequency domain methods can still be applied to this approximately LTI model

Baseband Equivalence: $s(t) = \Re\left\{u(t)e^{j2\pi f_c t}\right\}$

- Easier to analyze complex numbers like (e^{jwt}), even though all baseband/passband are real signals involving sines and cosines.

$$\begin{aligned} s(t) &= \Re\left\{u(t)e^{j2\pi f_c t}\right\} \\ &= \Re\{u(t)\}\cos(2\pi f_c t) - \Im\{u(t)\}\sin(2\pi f_c t) \\ &= x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t), \end{aligned}$$

$$x(t) = \Re\{u(t)\} \qquad y(t) = \Im\{u(t)\} \qquad u(t) = x(t) + jy(t)$$

- Passband signal = baseband signal (*u(t)*) multiplying a complex carrier (e^{jwt}) signal, and extracting the real portion
  - *u(t):* complex envelope or complex lowpass equivalent signal
- <u>Quadrature concept:</u> Cosine and Sine oscillators modulated with *x(t)* and *-y(t)* respectively (the Real and Quadrature parts of u(t))

<u>Received Signal:</u> $r(t) = \Re\left\{v(t)e^{j2\pi f_c t}\right\}$

$v(t) = u(t) * c(t)$. *v(t)* and *c(t)* are baseband equivalents for received and channel

Block diagram

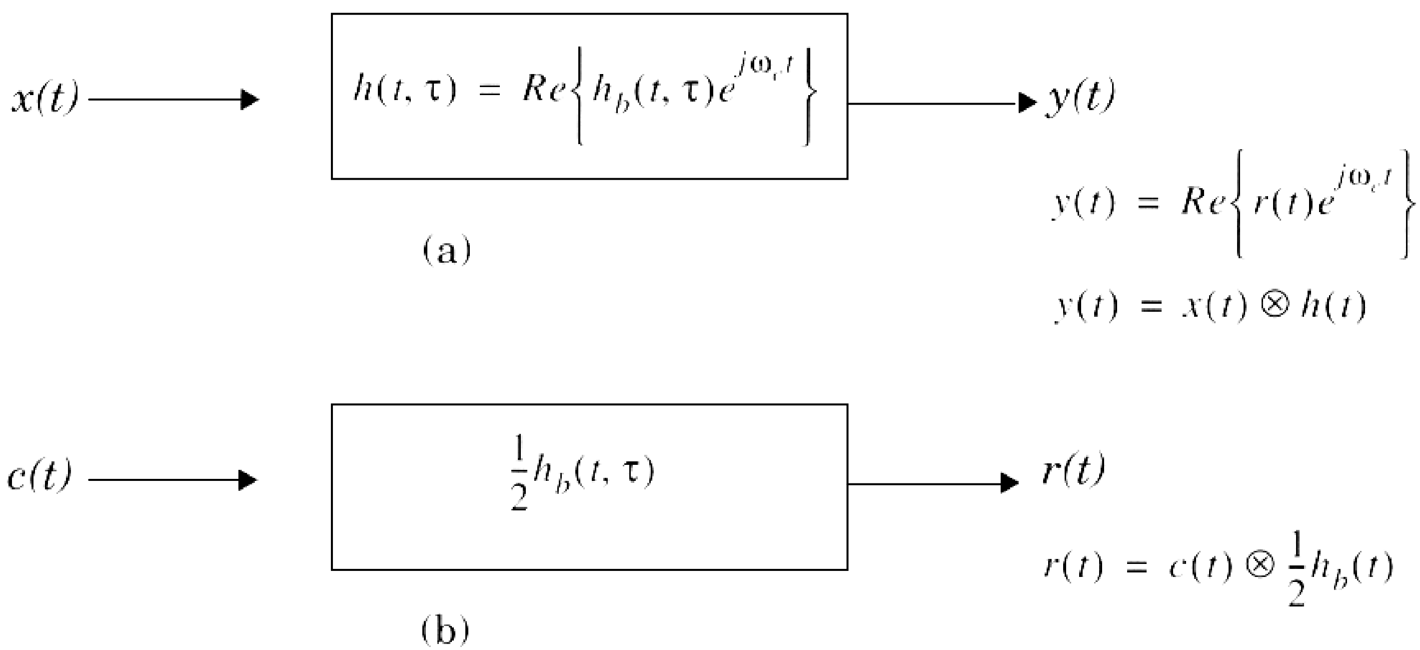
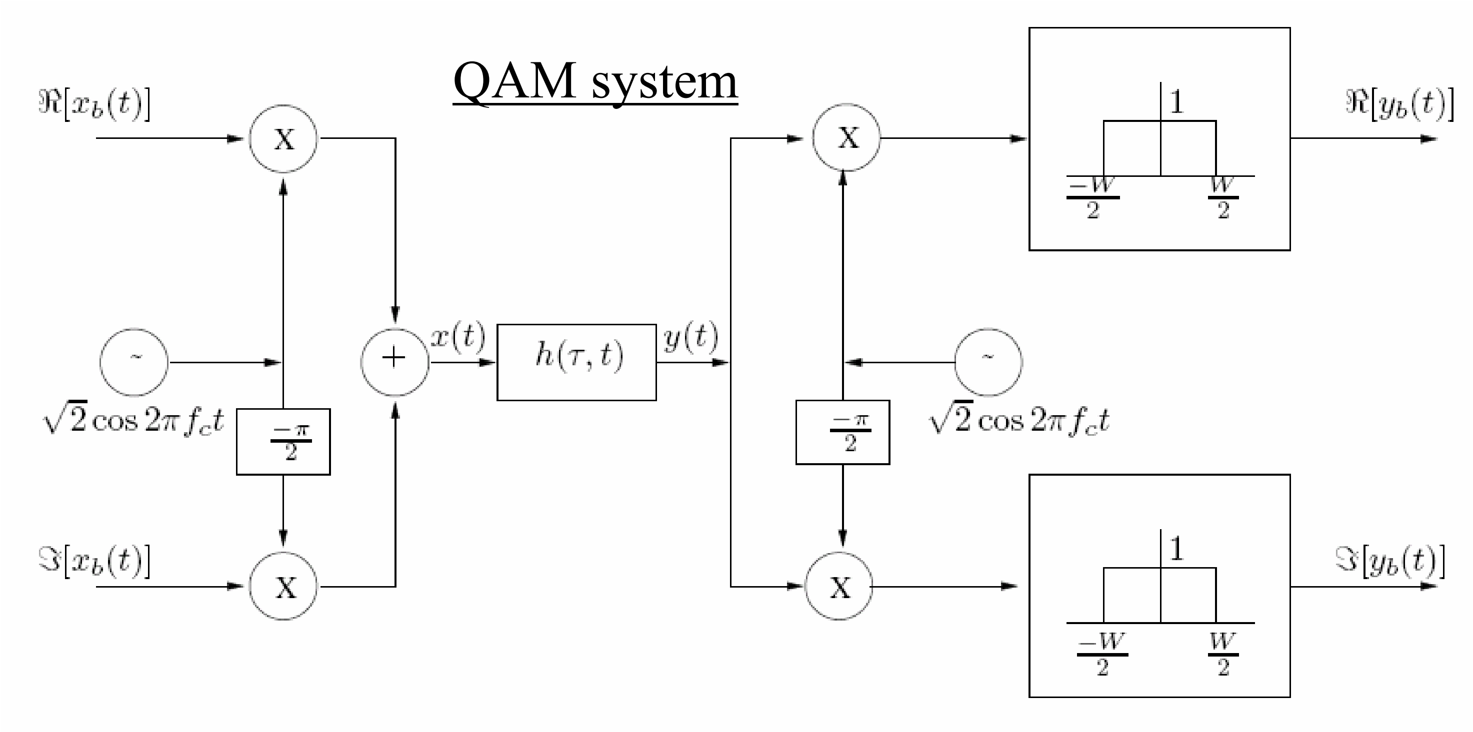


Figure 5.3 (a) Bandpass channel impulse response model; (b) baseband equivalent channel impulse response model.

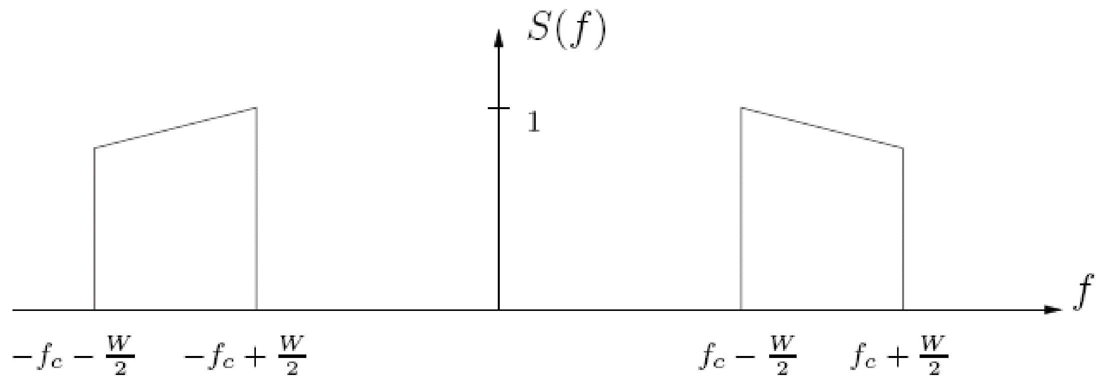
Passband-to-Baseband Conversion: Block Diagram

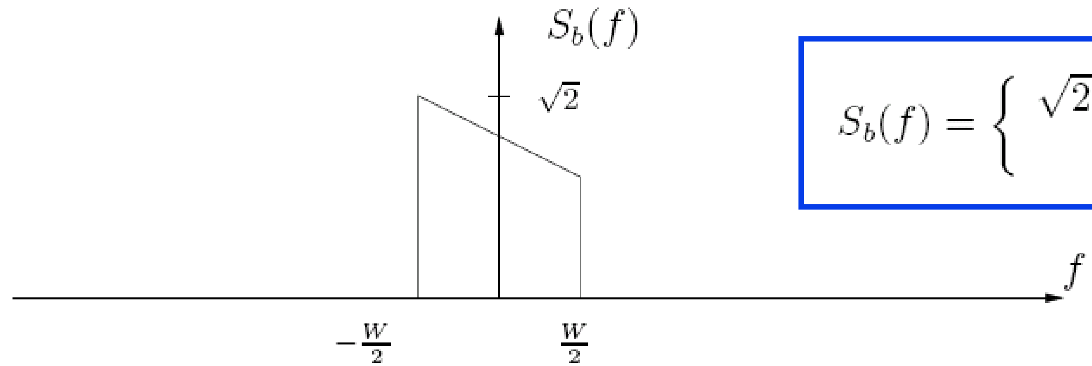
- Communication takes place at passband $[f_c - W/2, f_c + W/2]$
- Processing takes place at baseband $[-W/2, W/2]$



Note: transmitted power half of baseband power

Passband vs Baseband Equivalent Spectrum





$$\boxed{S_b(f) = \begin{cases} \sqrt{2}S(f+f_c) & f+f_c > 0 \\ 0 & f+f_c \leq 0 \end{cases}.}$$

- Communication at passband (allocated spectrum). Processing in baseband: modulation, coding etc. Upconvert/Downconvert.
- s_b contains same information as s: Fourier transform hermitian around 0 (“rotation”).
- If only one of the side bands are transmitted, the passband has half the power as the baseband equivalent

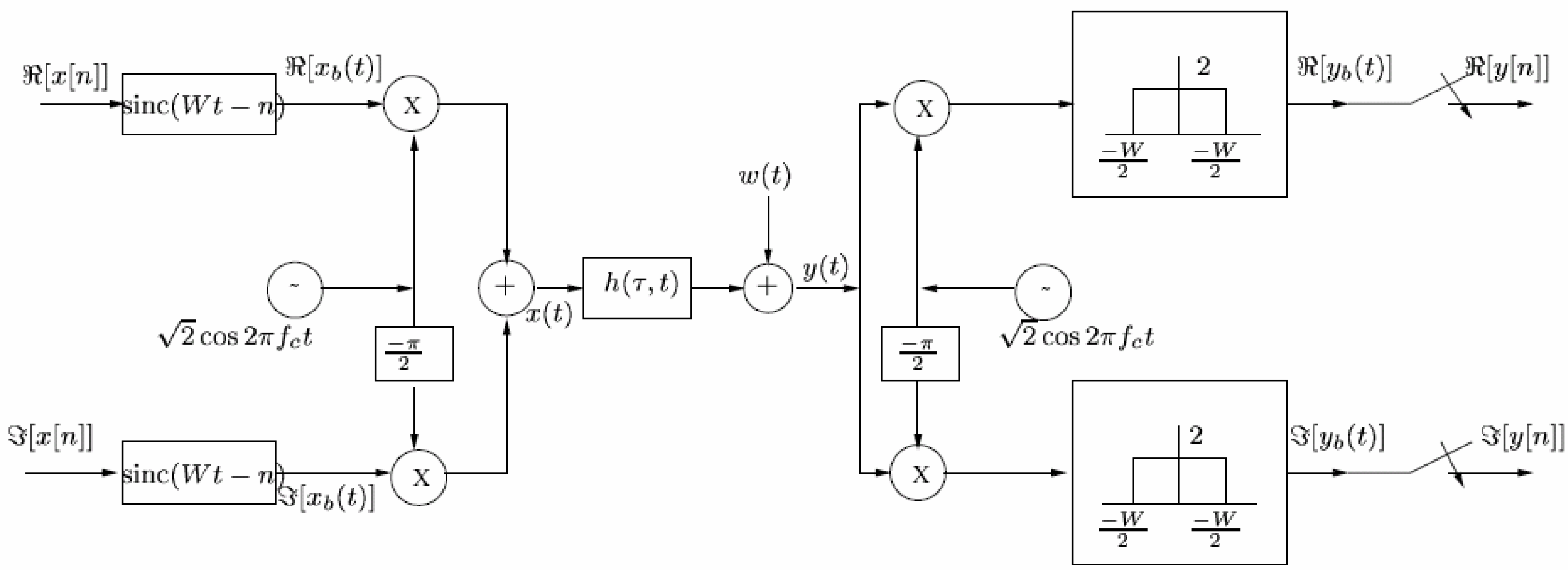
Per-path: Complex Baseband Equivalent Channel

- The frequency response of the system is shifted from the passband to the baseband.

$$\begin{aligned} H_b(f) &= H(f + f_c) \\ h_b(\tau) &= h(t)e^{-j2\pi f_c t} = \sum_i a_i^b \delta(\tau - \tau_i) \\ \text{where } a_i^b &= a_i e^{-j2\pi f_c \tau_i} \end{aligned}$$

- Each path i is associated with a delay (τ_i) and a complex gain (a_i).

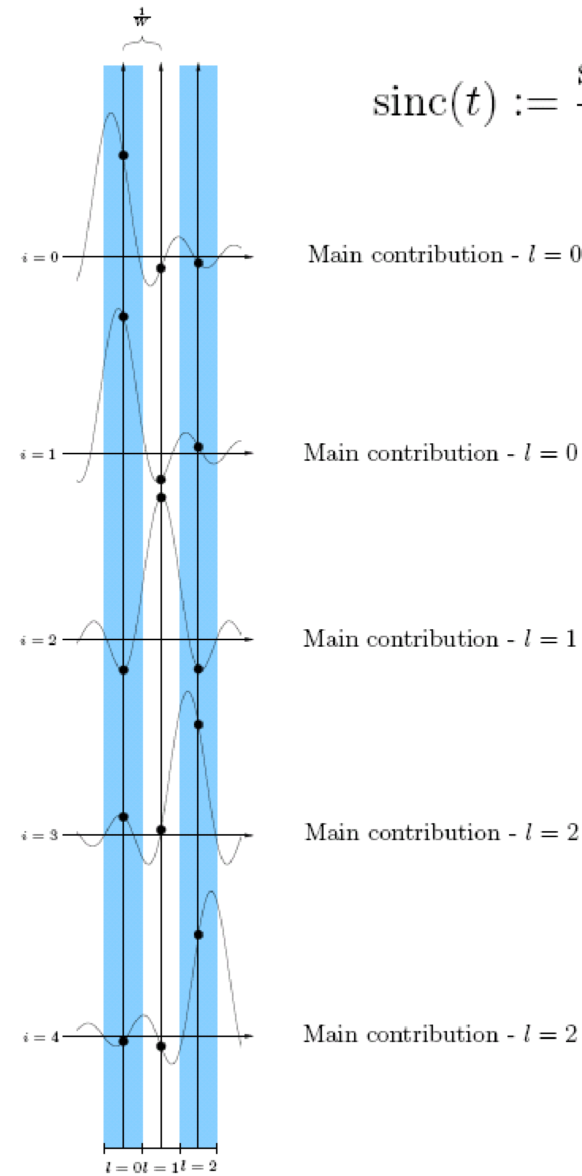
Discrete-Time Baseband Equivalence: With Modulation and Sampling



Sampling Interpretation

$$\mathrm{sinc}(t) := \frac{\sin(\pi t)}{\pi t}.$$

- Due to the decay of the sinc function, the *i*th path contributes most significantly to the *l*th tap if its delay falls in the window $[l/W - 1/(2W),\ l/W + 1/(2W)]$.



Discrete Time Baseband I/O relationship:

$$\boxed{y[m] = \sum_{\ell} h_\ell[m]\, x[m-\ell].}$$

where:

$h_\ell[m]$ as the ℓ^{th} (complex) channel filter tap at time m.

function of mainly the gains $a_i^b(t)$ of the paths, whose delays $\tau_i(t)$ are close to ℓ/W

Multipath Resolution: LTI Approximation

Sampled baseband-equivalent channel model:

$$y[m] = \sum_{\ell} h_{\ell} \, x[m-\ell]$$

where h_l is the l th complex channel tap.

$$h_{\ell} \approx \sum_{i} a_i e^{-j2\pi f_c \tau_i}$$

and the sum is over all paths that fall in the delay bin

$$\left[\frac{\ell}{W} - \frac{1}{2W}, \frac{\ell}{W} + \frac{1}{2W}\right]$$

System resolves the multipaths up to delays of 1/W .

$$h_{\ell} = (h_b * \mathrm{sinc})(\ell/W).$$

Baseband Equivalence: Summary

- Let $s(t)$ denote the input signal with **equivalent lowpass signal $u(t)$.**
- Let $h(t)$ denote the bandpass channel impulse response with **equivalent lowpass channel impulse response $h_l(t)$**
- The transmitted signal $s(t)$ and channel impulse response $h(t)$ are both real, so the channel output $r(t) = s(t) * h(t)$ is also real, with frequency response $R(f) = H(f)S(f)$
- $R(f)$ will also be a bandpass signal w/ complex lowpass representation:

$$r(t) = \Re\left\{v(t)e^{j2\pi f_c t}\right\}.$$

- It can be re-written (after manipulations as):

$$r(t) = \Re\left\{(u(t) * h_l(t))e^{j2\pi f_c t}\right\}$$

Summary: Equivalent lowpass models for s(t), h(t) and r(t) isolates the carrier terms (f_c) from the analysis. Sampled version allows discrete-time processing.

Multipaths in LTI Model: Flat/Frequency-Selective Fading

- Fading occurs when there is destructive interference of the multipaths that contribute to a tap.

$$h_\ell \approx \sum_i a_i e^{-j2\pi f_c \tau_i}$$

Delay spread $T_d := \max_{i,j} |\tau_i(t) - \tau_j(t)|$

Coherence bandwidth $W_c := \frac{1}{T_d}$

$T_d \ll \frac{1}{W}, W_c \gg W \Rightarrow$ single tap, flat fading

$T_d > \frac{1}{W}, W_c < W \Rightarrow$ multiple taps, frequency selective

Doppler: Time Variations in Model

$$y[m] = \sum_{\ell} h_\ell[m] x[m-\ell]$$

Time-varying delays

$$h_\ell[m] \approx \sum_i a_i(t) e^{-j2\pi f_c \tau_i(t)}, \qquad t = \frac{m}{W}$$

$f_c \tau_i'(t) =$ Doppler shift of the i th path

Doppler spread $D_s := \max_{i,j} |f_c \tau_i'(t) - f_c \tau_j'(t)|$

Coherence time $T_c := \frac{1}{D_s}$

Doppler Spread

$$D_s := \max_{i,j} |f_c \tau_i'(t) - f_c \tau_j'(t)|$$

Doppler spread is proportional to:

- the carrier frequency f_c;
- the angular spread of arriving paths.

$$\tau_i'(t) = \frac{v}{c} \cos \theta_i$$

where θ_i is the angle the direction of motion makes with the i th path.

Degrees of Freedom (Complex Dimensions)

- Discrete symbol x[m] is the m^{th} sample of the transmitted signal; there are W samples per second.
- Continuous time signal x(t), 1 s ≡ W discrete symbols
- Each discrete symbol is a complex number;
  - It represents *one* (*complex*) *dimension* or *degree of freedom*.
  - Bandlimited x(t) has W degrees of freedom per second.
  - Signal space of complex continuous time signals of duration T which have most of their energy within the frequency band [−W/2,W/2] has dimension approximately WT.
- Continuous time signal with bandwidth W can be represented by W complex dimensions per second.
- Degrees of freedom of the channel to be the <u>dimension</u> of the <u>*received*</u> signal space of y[m]

Statistical Models

- Design and performance analysis based on statistical ensemble of channels rather than specific physical channel.

$$h_\ell[m] \approx \sum_i a_i e^{-j2\pi f_c \tau_i}$$

- Rayleigh flat fading model: many small scattered paths

$$h[m] \sim \mathcal{N}(0, \frac{1}{2}) + j\mathcal{N}(0, \frac{1}{2}) \sim \mathcal{CN}(0, 1)$$

Complex circular symmetric Gaussian .

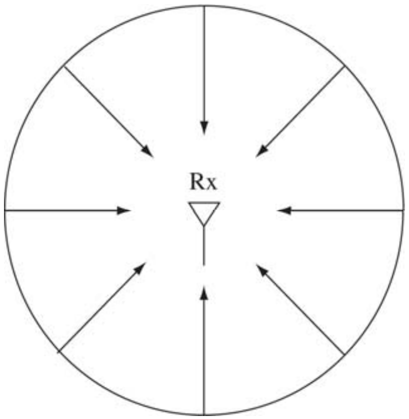
Squared magnitude is exponentially distributed.

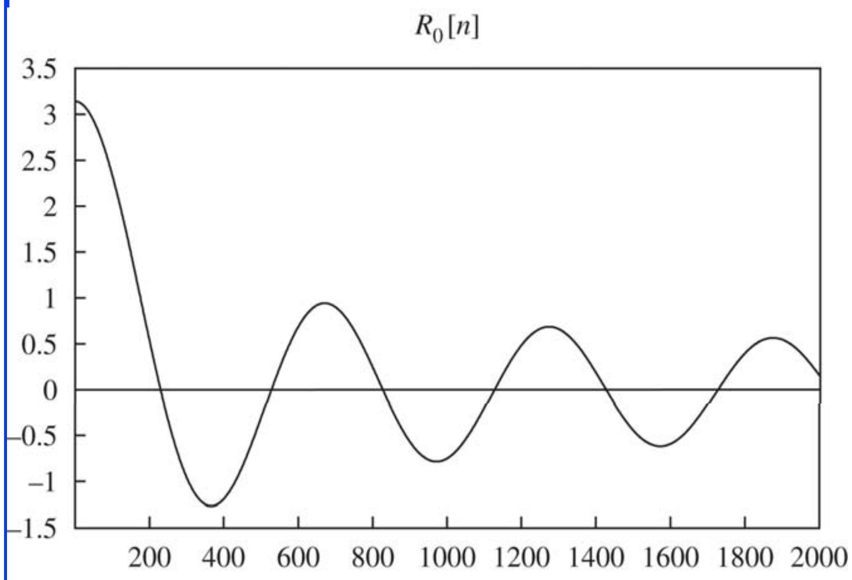
- Rician model: 1 line-of-sight plus scattered paths

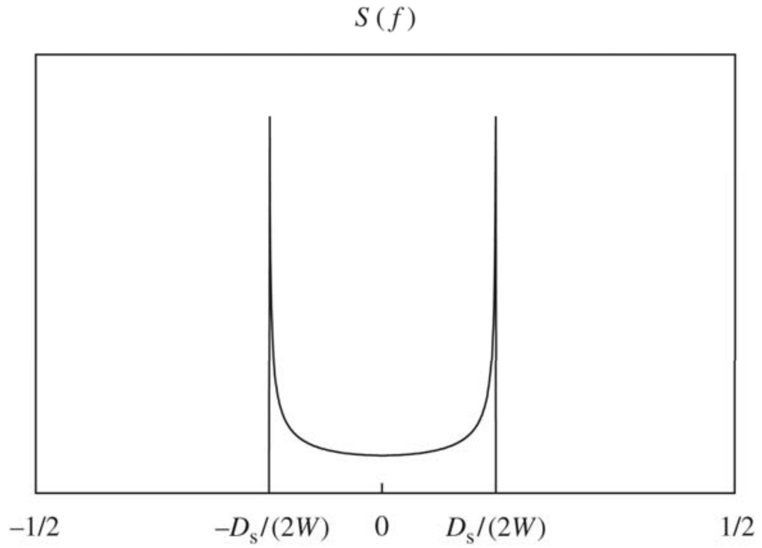
$$h[m] \sim \sqrt{\kappa} + \mathcal{CN}(0, 1)$$

Statistical Models: Correlation over Time

- Specified by autocorrelation function and power spectral density of fading process.
- Example: Clarke's (or Jake's) model.







Additive White Gaussian Noise (AWGN)

- Complete baseband-equivalent channel model:

$$y[m] = \sum_{\ell} h_{\ell}[m]x[m-\ell] + w[m]$$

$$w[m] \sim \mathcal{CN}(0, N_0)$$

- Special case: flat fading (one-tap):

$$y[m] = h[m]x[m] + w[m]$$

- Will use this throughout the course.

BER Effect of Fading: AWGN vs Fading

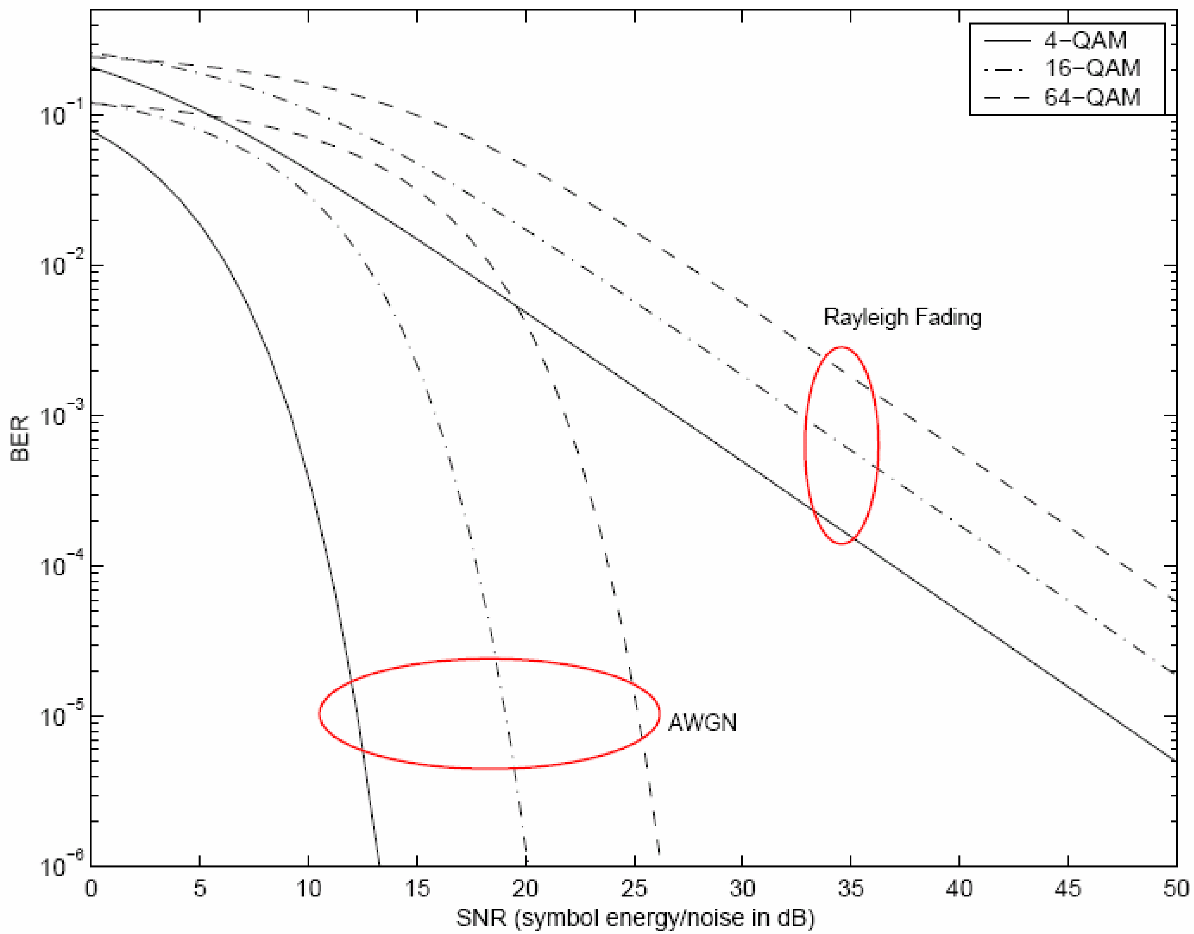


Figure 3.22: Flat fading causes a loss of at least 20-30 dB at reasonable BER values.

Types of Channels

Types of channel	Defining characteristic
Fast fading	$T_c \ll$ delay requirement
Slow fading	$T_c \gg$ delay requirement
Flat fading	$W \ll W_c$
Frequency-selective fading	$W \gg W_c$
Underspread	$T_d \ll T_c$

Summary

- We have understood both qualitatively and quantitatively the concepts of path loss, shadowing, fading (multi-path, doppler), and some of their design impacts.
- We have understood how time and frequency selectivity of wireless channels depend on key physical parameters.
- We have come up with linear, LTI and statistical channel models useful for analysis and design.