ECSE 6961: Multi-User Capacity and Opportunistic Communication

Shiv Kalyanaraman Google: "Shiv RPI" shivkuma@ecse.rpi.edu

Based upon slides of Viswanath/Tse,

& textbooks by Tse/Viswanath & A. Goldsmith

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Outline

- □ Reference: Chapter 6 (and 5): Tse/Viswanath
- Multiple access (or multi-user) channels are different from ptpt channels!
- New concepts/techniques: successive interference cancellation (SIC), superposition coding, multi-user diversity.
- □ **<u>AWGN multiuser uplink</u>**: CDMA + SIC
- AWGN multiuser downlink: superposition-coding (CDMAlike) + SIC
- **Fast Fading:** ability to track channel at sender (CSI) + opportunistic more important due to multi-user diversity
 - Gains over CSIR for *full range of SNR* (not just low SNR)
- □ Opportunistic beamforming, IS-856 (1x EV-DO) etc...

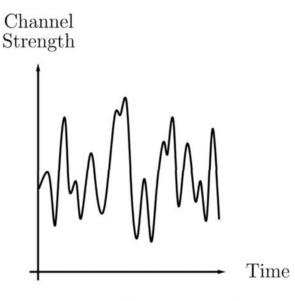
Pt-pt channel Capacity

- □ A slow fading channel is a source of unreliability: very poor outage capacity. Diversity is needed.
- A fast fading channel with only receiver CSI has a capacity close to that of the AWGN channel. Delay is long compared to channel coherence time.
- A fast fading channel with full CSI can have a capacity *greater* than that of the AWGN channel: fading now provides more *opportunities* for performance boost.
- □ The idea of *opportunistic communication* is even more powerful in multiuser situations.

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Fundamental Feature of Wireless Channels: Time Variation



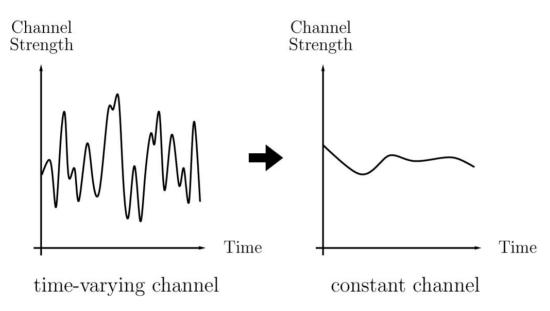
time-varying channel

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- multipath fading
- large-scale channel variations
- □ time-varying interference

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Traditional Approach to (Multi-user) Wireless System Design



Compensates for channel fluctuations.

I.e. treats a multi-user channel like a set of disjoint single-user (or pt-pt) channels.

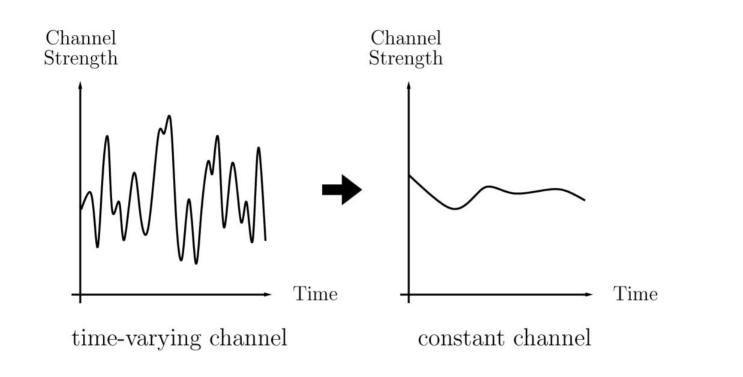
Examples: interference averaging; near-far power control, fixed coding/modulation rates

Example: CDMA Systems

Two main compensating mechanisms:

- 1. Channel diversity:
 - □ frequency diversity via Rake combining
 - macro-diversity via soft handoff
 - □ transmit/receive antenna diversity
- 2. Interference management:
 - power control
 - □ interference averaging

What Drives this Approach?



Main application is voice, with very tight <u>latency</u> requirements. Needs a consistent channel.

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Opportunistic Communication: A Different View

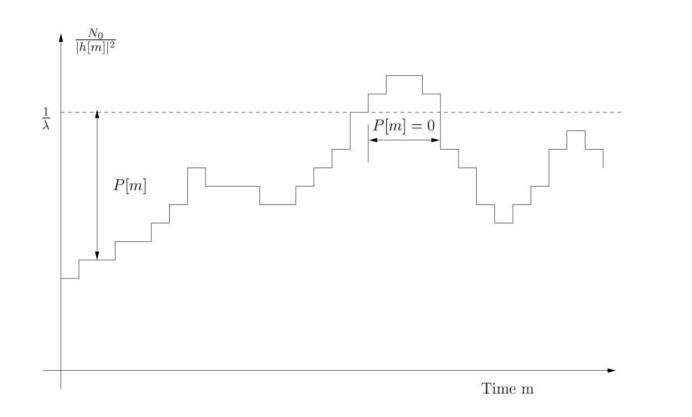
Transmit more when and where the channel is good.

Exploits fading to achieve higher long-term throughput, but no guarantee that the "channel is always there".

Appropriate for data with non-real-time latency requirements (file downloads, video streaming).

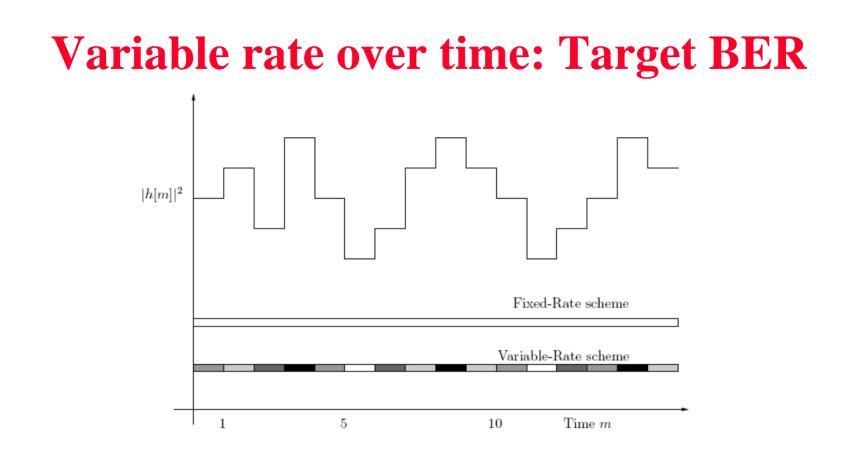
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Recall: Point-to-Point Fading Channels



Capacity-achieving strategy is waterfilling over time.

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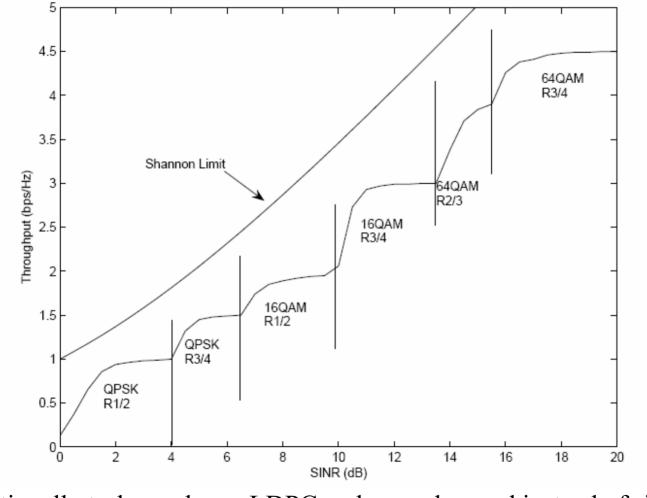


- □ In the fixed-rate scheme, there is only one code spanning across many coherence periods.
- □ In the variable-rate scheme, different codes (distinguished by difference shades) are used depending on the channel quality at that time.
- □ For example, the code in white is a low-rate code used only when the channel is weak.

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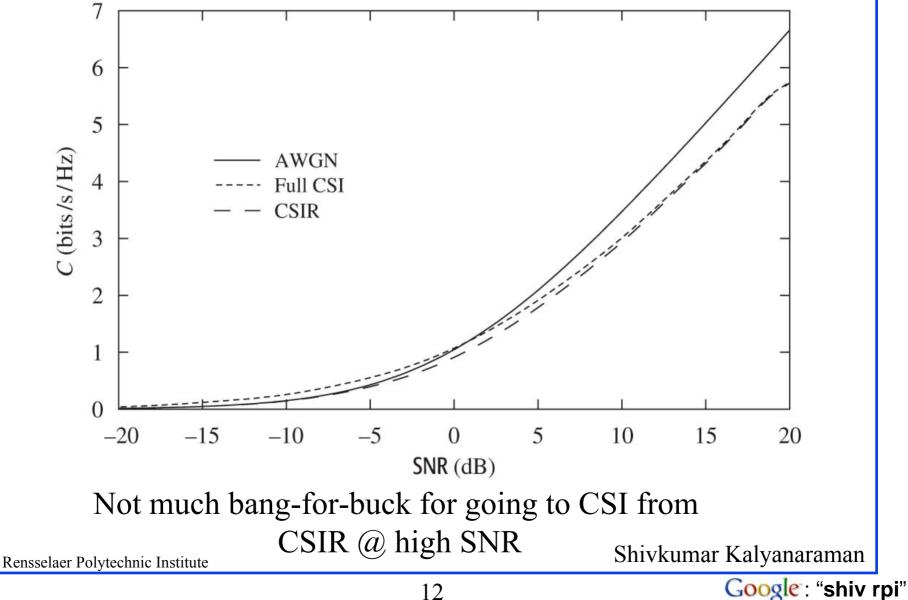
Adaptive Modln/Coding vs Shannon Limit

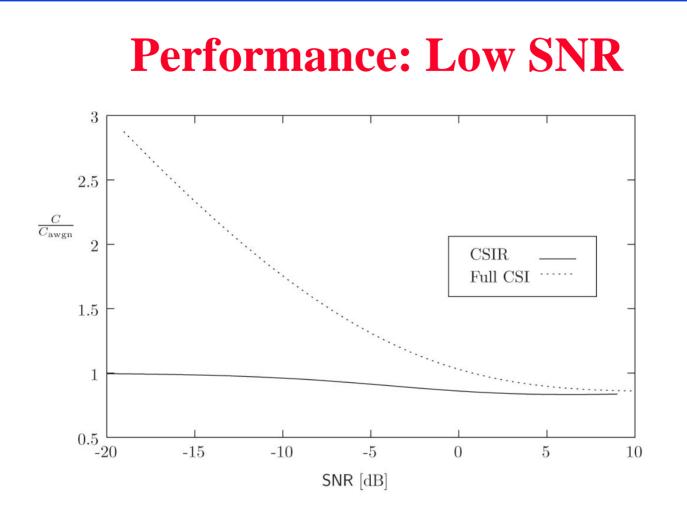


Optionally turbo-codes or LDPC codes can be used instead of simple block/convolutional codes in these schemes Rensselaer Polytechnic Institute

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Performance over Pt-Pt Rayleigh Channel



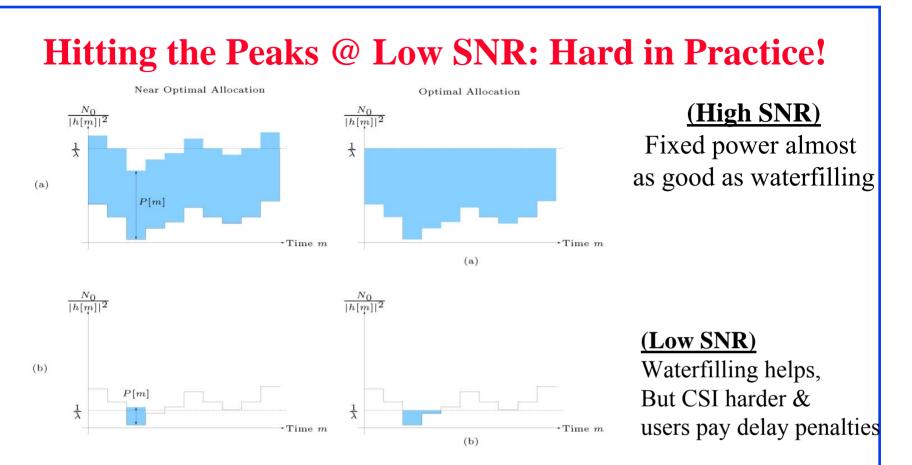


At low SNR, capacity can be greater (w/ CSI) when there is fading.

<u>Flip side:</u> harder to get CSI at low SNR \otimes

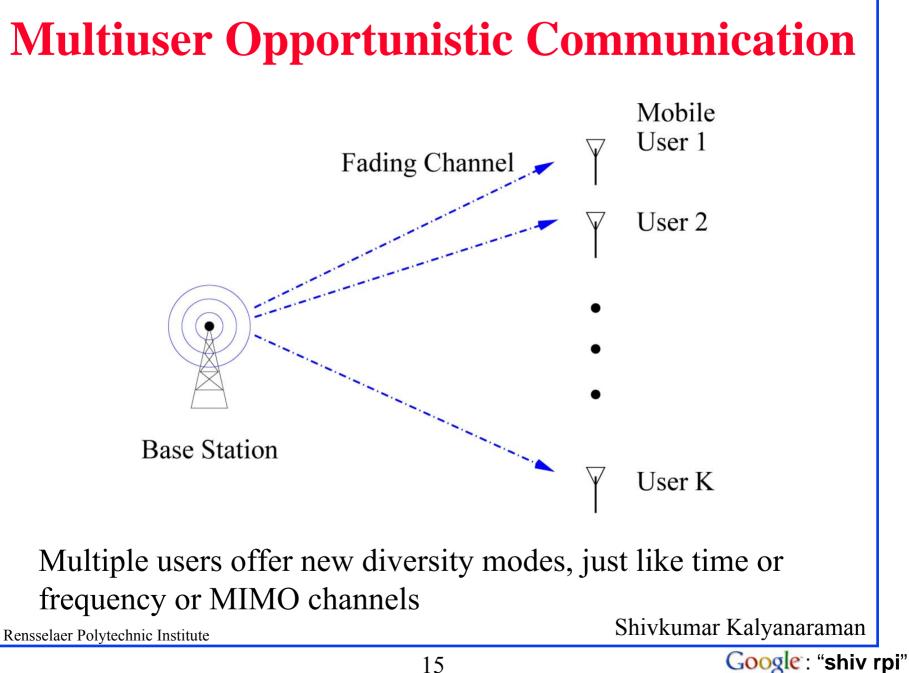
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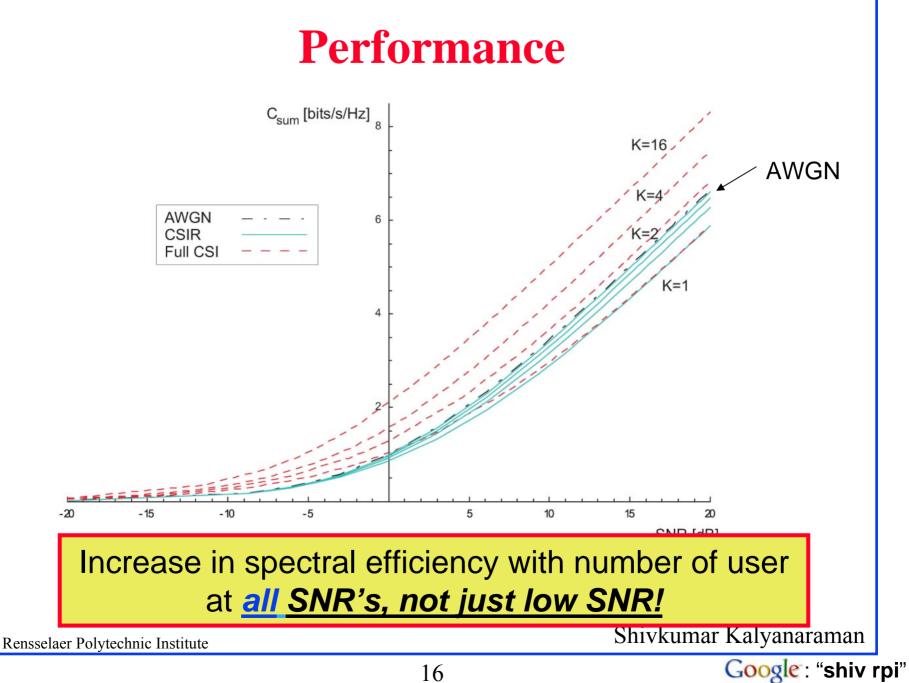
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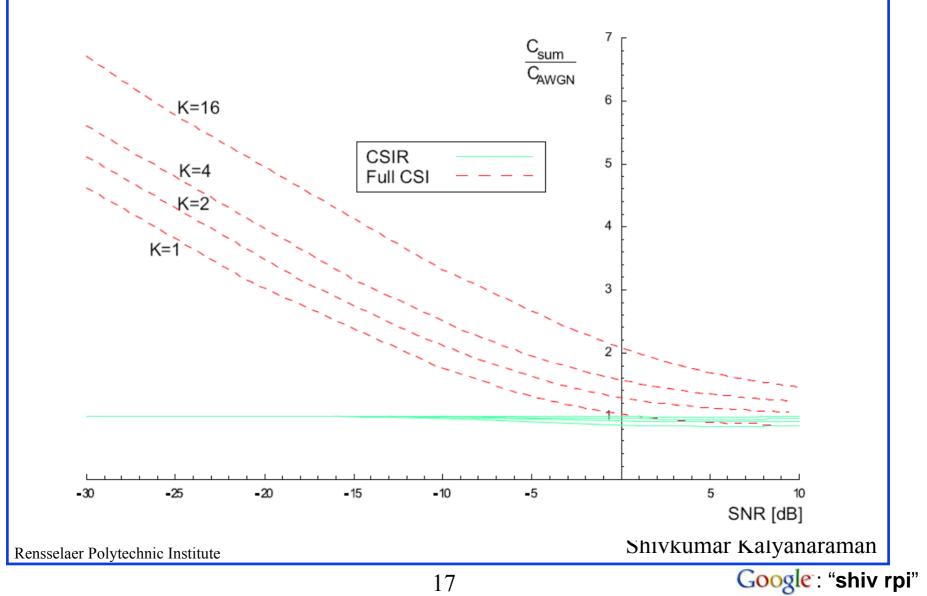
At low SNR, one can transmit <u>only</u> when the channel is at its <u>peak</u>. Primarily a *power gain*.

In practice, hard to realize such gains due to *difficulty in tracking the channel when transmitting so infrequently*.



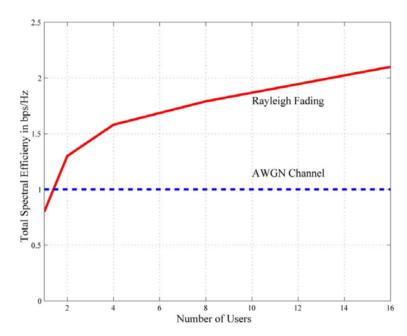


Multi-user w/ CSI: Low SNR case



Multiuser Diversity

Total average SNR = 0 dB.

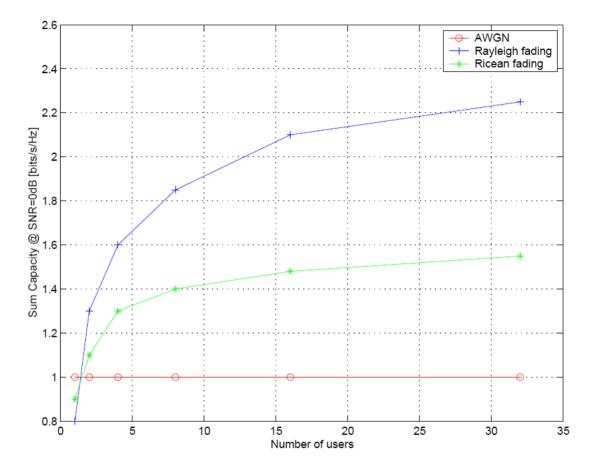


- □ In a large system with users fading independently, there is likely to be a user with a very good channel at any time.
- Long-term total throughput can be maximized by always serving the user with the strongest channel.

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Sum Capacity: AWGN vs Ricean vs Rayleigh

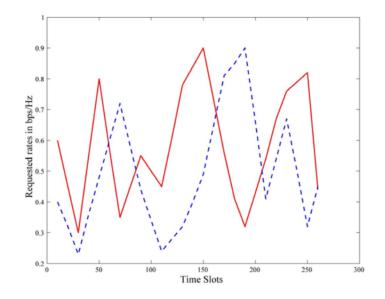


Multiuser diversity gain for Rayleigh and Ricean channels (= 5); KP/N0 = 0 dB.
 Note: Ricean is less random than Rayleigh and has lesser sum capacity!

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Multiuser Diversity: A More Insightful Look

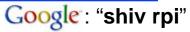


- □ Independent fading makes it likely that users peak at different times.
- □ In a wideband system with many users, each user operates at low average SNR, effectively accessing the channel only when it is near its peak.
- □ In the downlink, channel tracking can be done via a strong pilot amortized between all users.

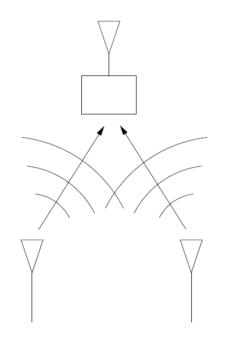
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Theory

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2-user uplink AWGN: Capacity Region



 $y[m] = x_1[m] + x_2[m] + w[m]$

User k has an average power constraint of P_k Joules/symbol (with k = 1, 2)

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- Capacity region C: is the set of all pairs (R1,R2) such that simultaneously user 1 and 2 can reliably communicate at rate R1 and R2.
- Tradeoff: if user 1 wants to communicate at higher rate: user 2 may need to lower rate
 - Eg: OFDM: vary allocation of subcarriers or slots per user
- Capacity region: <u>optimal</u> tradeoff for <u>any</u> MAC scheme
- □ Performance measures:
 - $\Box Symmetric capacity C_{sy}$

$$_{\text{ym}} := \max_{(R,R)\in\mathcal{C}} R$$

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Sum capacity:

 $C_{\text{sum}} := \max_{(R_1, R_2) \in \mathcal{C}} R_1 + R_2$

Uplink AWGN Channel Capacity Region

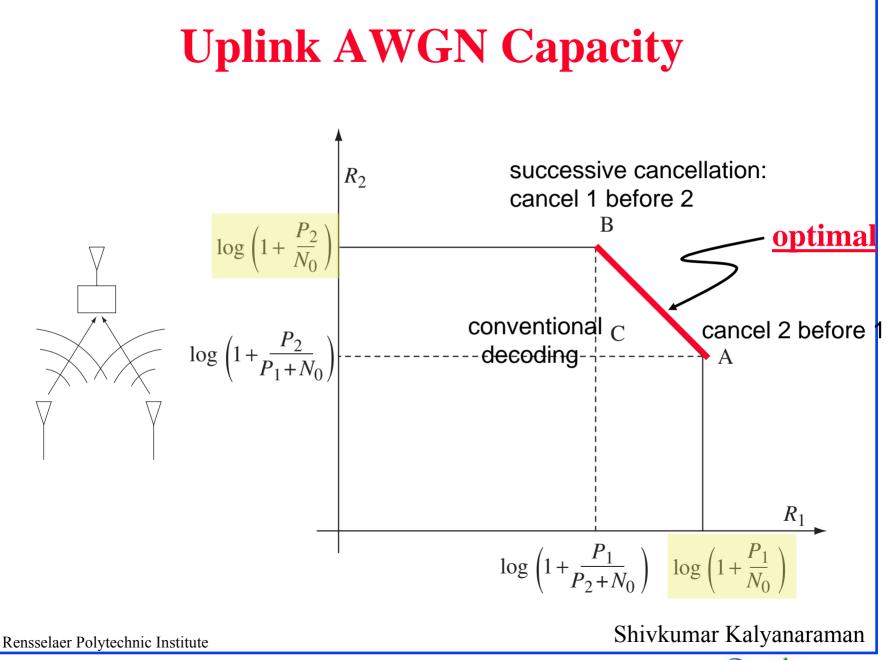
□ Satisfies three constraints: R1, R2, and (R1+R2)

$$R_1 < \log\left(1 + \frac{P_1}{N_0}\right),$$

$$R_2 < \log\left(1 + \frac{P_2}{N_0}\right),$$

$$R_1 + R_2 < \log\left(1 + \frac{P_1 + P_2}{N_0}\right)$$

- □ Without the third constraint, the capacity region would have been a rectangle, ...
- In and both users could simultaneously transmit at the point-topoint capacity as if the other user did not exist.



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AWGN Multiuser Capacity & SIC Decoder

User 1 can <u>achieve its single-user bound</u> while at the same time user 2 can get a <u>non-zero rate:</u>

$$R_2^* = \log\left(1 + \frac{P_1 + P_2}{N_0}\right) - \log\left(1 + \frac{P_1}{N_0}\right) = \log\left(1 + \frac{P_2}{P_1 + N_0}\right)$$

□ Each user encodes its data using a capacity-achieving AWGN channel code.

<u>2-stage decoding:</u>

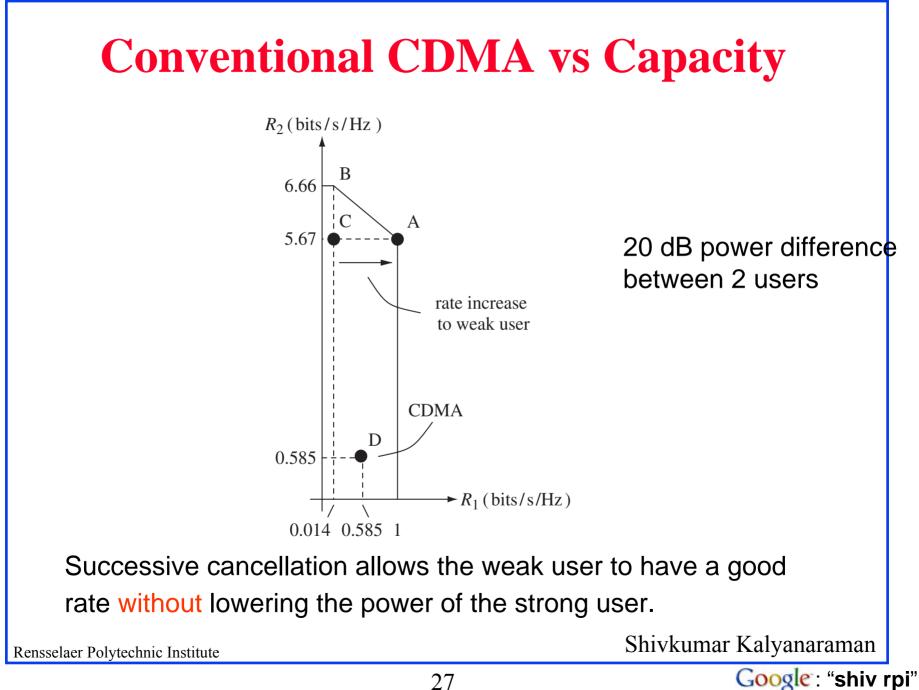
- □ 1. Decodes the data of user 2, treating the signal from user 1 as Gaussian interference.
- □ 2. Once the receiver decodes the data of user 2, it can reconstruct user 2's signal and subtract it from the aggregate received signal.
 - □ Then decode the data of user 1.
 - Only the background Gaussian noise left in the system, the maximum rate user 1 can transmit at is its single-user bound log $(1 + P_1/N_0)$.
- □ This receiver is called a *successive interference cancellation (SIC)*

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SIC vs Conventional CDMA/Orthogonal Schemes

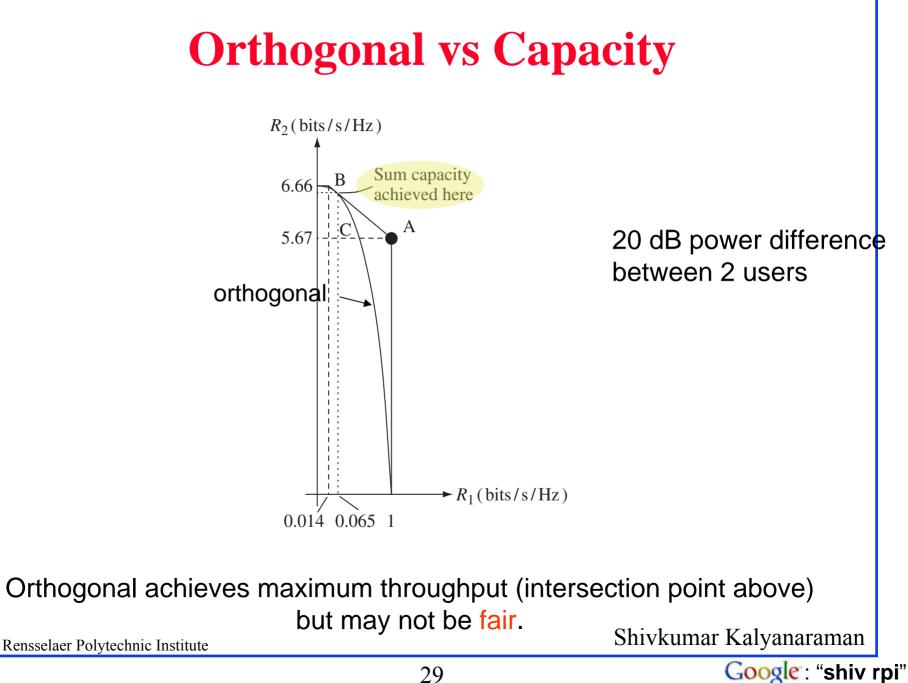
- Minimizes transmit power to achieve target rates of two users
- □ In interference limited scenarios, increases system capacity!
- Conventional CDMA is suboptimal because it controls power of strong users downwards to handle the near-far problem
 - □ => such high SNR users cannot transmit at high rates
 - They have to depress their SNRs and transmit at lower rates!
- With SIC: near-far is not a *problem*, but an *advantage*!
 Less apparent for voice, but definitely for data
- Orthogonal: allocates a fraction α of the degrees of freedom to user 1 and the rest (1α) to user 2

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$$\alpha W \log \left(1 + \frac{P_1}{\alpha N_0}\right)$$
 $(1 - \alpha) W \log \left(1 + \frac{P_2}{(1 - \alpha)N_0}\right)$ bits/s mar Kalyanaraman **Google**: "shiv rpi"



Waterfilling vs Channel Inversion

- Waterfilling and rate adaptation (across users) maximize longterm throughput but incur significant delay.
- Channel inversion in <u>downlink</u> ("perfect" power control in CDMA jargon) is power-inefficient but maintains the same data rate (received SNR) at all channel states.
 - Huge power penalty during deep fades. Peak power constraints => method cannot work.
- □ Channel inversion achieves a delay-limited capacity.



General K-user Uplink AWGN Capacity

□ K-user capacity region is described by 2^K − 1 constraints, one for each possible non-empty subset S of users:

 Eg: OFDMA w/ allocation of 1/K degrees of freedom per user better than CDMA w/ conventional receivers. (see CDMA limits next slide)
 Sum capacity is <u>unbounded</u> as the number of users grow. Rensselaer Polytechnic Institute

Example: CDMA Uplink Capacity (I/f limited)

- □ Single cell with K users (conventional, i.e. non-SIC receiver):
 - □ Treat interference as additive noise

SINR =
$$\frac{P}{N_0 + (K-1)P} \approx \frac{1}{K}$$
 (-15 dB for $K = 32$)

Capacity <u>per user</u>

 $= \log (1 + SINR) \approx SINR \log_2 e$ bits/s/Hz.

Cell capacity (interference-limited) $\approx K \cdot \text{SINR} \log_2 e \approx 1.442 \text{bits/s/Hz}$

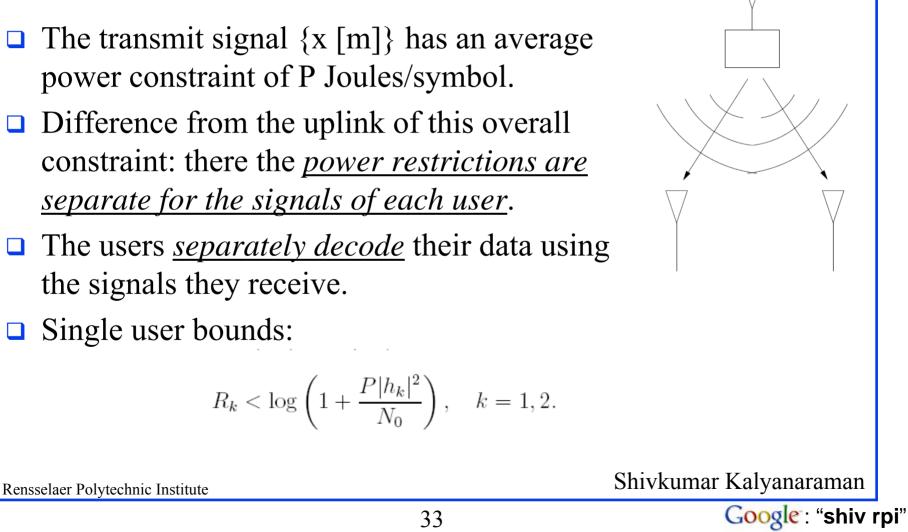
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CDMA Uplink Capacity Example (continued)

□ If out-of-cell interference is a fraction f of in-cell interference:

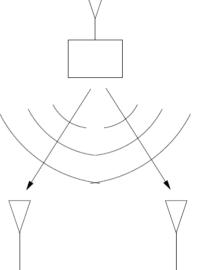
$$C pprox rac{1.442}{1+f}$$
 bits/s/Hz



Downlink AWGN Channel: 2-users

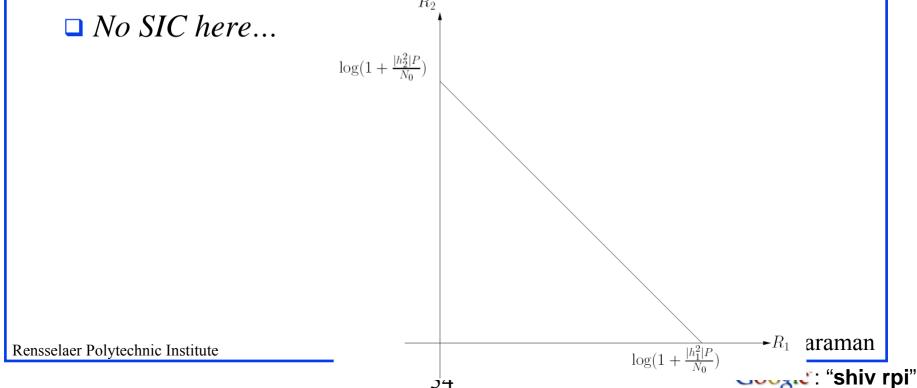
 $y_k[m] = h_k x[m] + w_k[m], \quad k = 1, 2,$

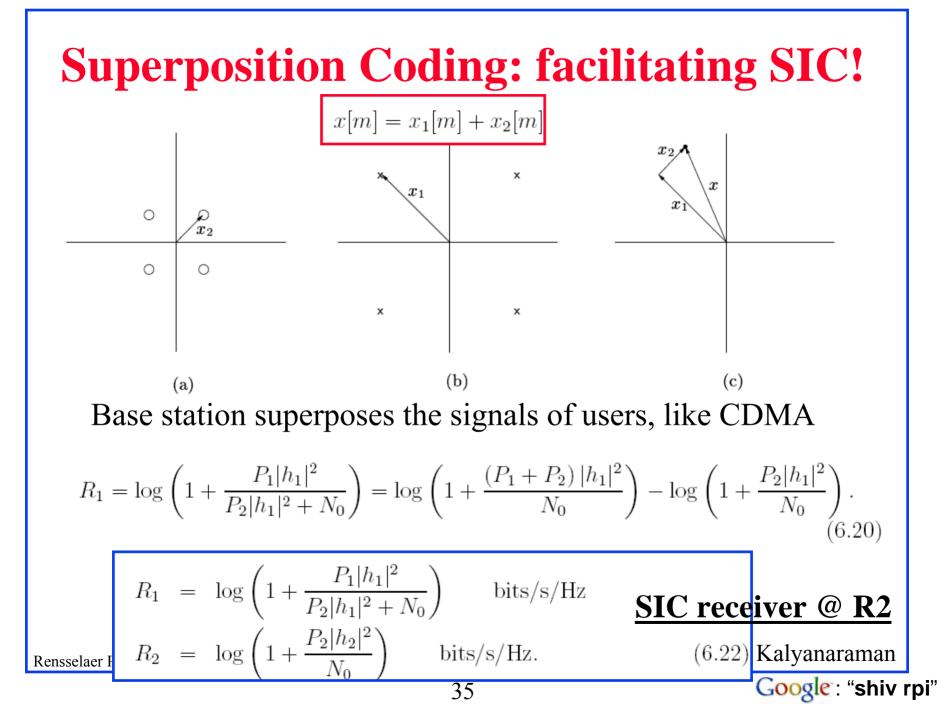
- The transmit signal $\{x [m]\}\$ has an average power constraint of P Joules/symbol.
- Difference from the uplink of this overall constraint: there the *power restrictions are* separate for the signals of each user.
- The users *separately decode* their data using the signals they receive.
- Single user bounds:

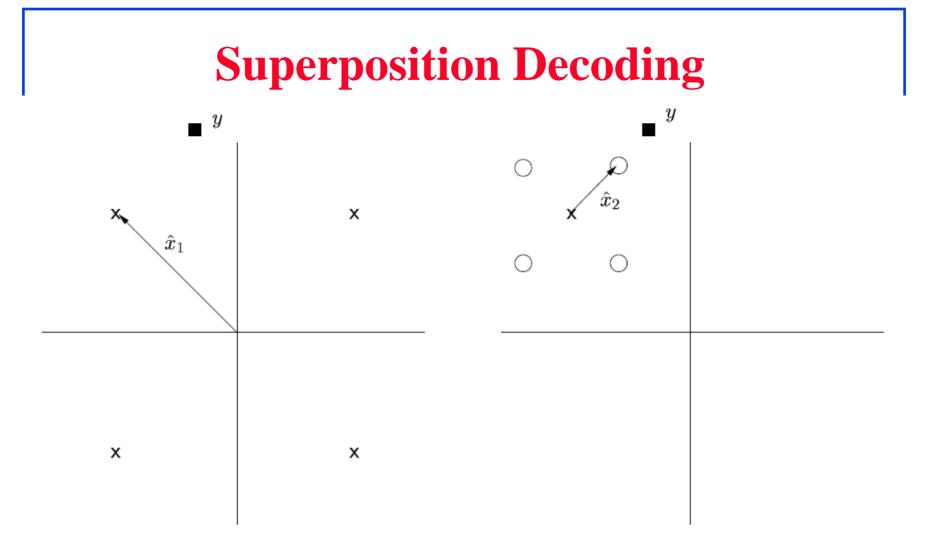


Symmetric 2-user downlink AWGN case

- The capacity region of the downlink with two users having symmetric AWGN channels, i.e., |h1| = |h2|.
- This upper bound on R_k can be attained by using all the power and degrees of freedom to communicate to user k (with the other user getting zero rate).



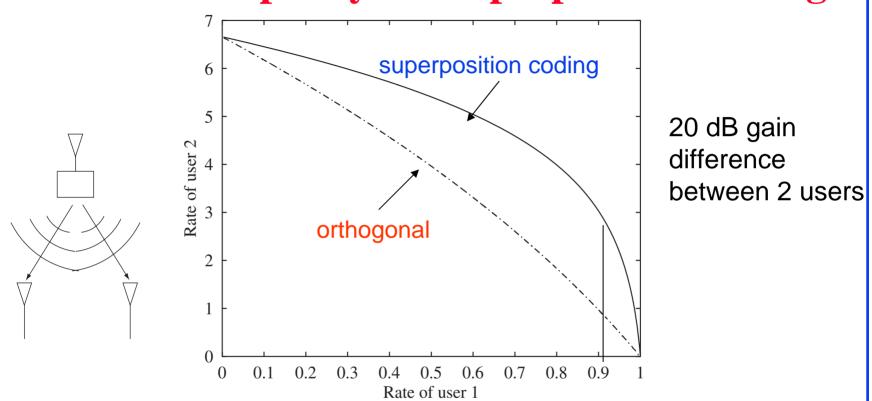




 Superposition decoding example. The transmitted constellation point of user 1 is decoded first, followed by decoding of the constellation point of user 2.
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Downlink Capacity: w/ superposition coding



The boundary of rate pairs (in bits/s/Hz) achievable by superposition coding (solid line) and orthogonal schemes (dashed line) for the two user asymmetric downlink AWGN channel with the user SNRs equal to 0 and 20 dB (i.e., $P|h_1|^2/N_0 = 1$ and $P|h_2|^2/N_0 = 100$). Eg: at R1 = 0.9 b/s/Hz, superposition coding gives R2 = 3b/s/Hz vs orthogonal of 1 b/s/Hz.

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Uplink AWGN Capacity: Summary

Summary 6.1 Uplink and Downlink AWGN Capacity

Uplink:

$$y[m] = \sum_{k=1}^{K} x_k[m] + w[m]$$
(6.26)

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with user k having power constraint P_k .

Achievable rates satisfy:

$$\sum_{k \in \mathcal{S}} R_k \le \log\left(1 + \frac{\sum_{k \in \mathcal{S}} P_k}{N_0}\right) \quad \text{for all } \mathcal{S} \subset \{1, \dots, K\} \quad (6.27)$$

The K! corner points are achieved by SIC, one corner point for each cancellation order. They all achieve the same optimal sum rate.

A natural ordering would be to decode starting from the strongest user first and move towards the weakest user.

Downlink AWGN: Summary

Downlink:

$$y_k[m] = h_k x[m] + w_k[m], \qquad k = 1, \dots K$$
 (6.28)

with $|h_1| \leq |h_2| \leq \ldots \leq |h_K|$.

The boundary of the capacity region is given by the rate tuples:

$$R_k = \log\left(1 + \frac{P_k |h_k|^2}{N_0 + \left(\sum_{j=k+1}^K P_j\right) |h_k|^2}\right), \quad k = 1 \dots K, \quad (6.29)$$

for all possible splits $P = \sum_k P_k$ of the total power at the base station.

The optimal points are achieved by superposition coding at the transmitter and SIC at each of the receivers.

The cancellation order at every receiver is *always* to decode the weaker users before decoding its own data.

SIC Implementation Issues

Complexity scaling with the number of users:

- □ At mobile node complexity scales if more users!
- Can group users by SNR bands and do superposition coding within the group
- Error propagation: degrades error prob by at most K (# users). Compensate w/ stronger code.

Imperfect channel estimates:

Stronger user: better channel estimates. Effect does not grow...

□ Analog-to-digital quantization error:

□ Implementation constraint with asymmetric signals

Uplink Fading Channel: Summary

Summary 6.2 Uplink Fading Channel

Slow Rayleigh Fading: At low SNRs, the symmetric outage capacity is equal to the outage capacity of the point-to-point channel, but scaled down by the number of users. At high SNRs, the symmetric outage capacity for moderate number of users is approximately equal to the outage capacity of the point-to-point channel. Orthogonal multiple access is close to optimal at low SNRs.

Fast Fading, receiver CSI: With a large number of users, each user gets the same performance as in an uplink AWGN channel with the same average SNR. Orthogonal multiple access is strictly suboptimal.

Fast Fading, full CSI: Orthogonal multiple access can still achieve the sum capacity. In a symmetric uplink, the policy of allowing only the best user to transmit at each time achieves the sum capacity.

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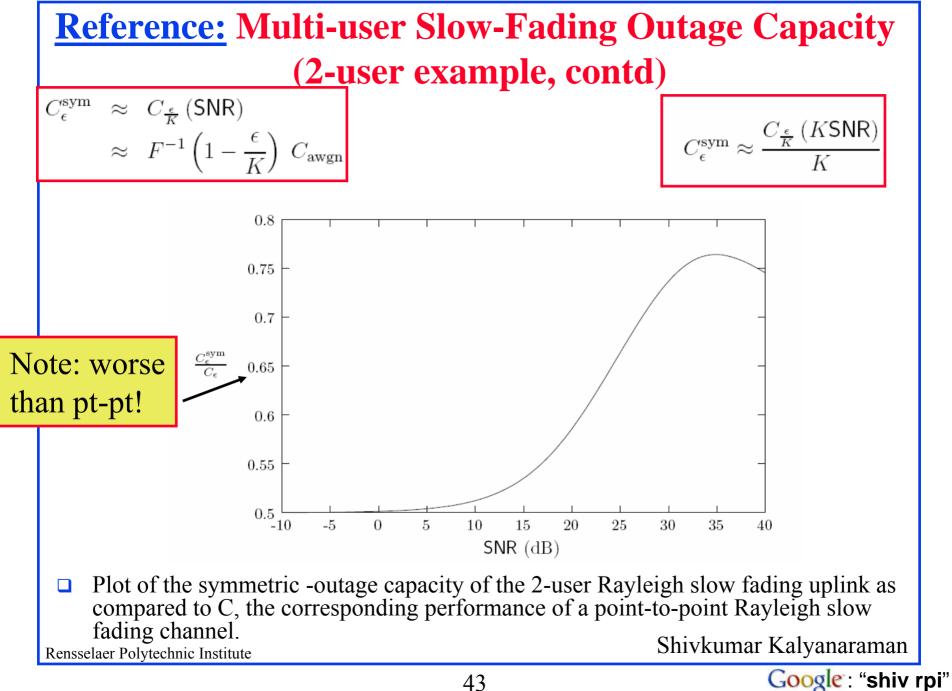
Reference: Uplink Fading Channel

users:
$$y[m] = \sum_{k=1}^{K} h_k[m] x_k[m] + w[m]$$

- Outage Probability: $p_{\text{out}}^{\text{ul}} := \mathbb{P}\left\{ \log\left(1 + \mathsf{SNR}\sum_{k=1}^{\infty} |h_k|^2\right) < |\mathcal{S}|R, \text{ for some } \mathcal{S} \subset \{1, \dots, K\} \right\}.$ (6.32)
- Individual outage: $\varepsilon \Rightarrow$ overall outage prob (orthogonal):

$$1 - (1 - \epsilon)^{K} \approx K\epsilon \qquad \qquad \frac{C_{\frac{\epsilon}{K}}(K\mathsf{SNR})}{K}$$

In general: $p_{\text{out}}^{\text{ul}} \approx \mathbb{P}\left\{\frac{|h_k|^2 P}{N_0} < R \log_e 2, \text{ for some } k \in \{1, \dots, K\}\right\}$ $\approx K p_{\text{out}}$. ·Kalyanaraman Rensselaer Polvte Google: "shiv rpi"



<u>Reference:</u> Uplink: <u>Fast</u> Fading, CSIR

 ${h_k[m]}_m$ is modelled as a time-varying ergodic process.

$$C_{\text{sum}} = \mathbb{E}\left[\log\left(1 + \frac{\sum_{k=1}^{K} |h_k|^2 P}{N_0}\right)\right].$$
(6.37)

$$\mathbb{E}\left[\log\left(1 + \frac{\sum_{k=1}^{K} |h_k|^2 P}{N_0}\right)\right] \leq \log\left(1 + \frac{\mathbb{E}\left[\sum_{k=1}^{K} |h_k|^2\right] P}{N_0}\right)$$
$$= \log\left(1 + \frac{KP}{N_0}\right).$$

$$R_k \approx \mathbb{E}\left[\frac{|h_k|^2 P}{\sum_{i=k+1}^K |h_i|^2 P + N_0}\right] \log_2 e \approx \frac{P}{(K-k)P + N_0} \log_2 e$$

Without CSI (i.e_channel state information at Tx), <u>fading always hurts</u> as in point-to-point case...

With large number of users, the penalty vanishes, but no improvement over pt-pt Rensselaer Polytechnic Institute

<u>Reference:</u> Fast Fading uplink, Orthogonal

$$\sum_{k=1}^{K} \frac{1}{K} \mathbb{E} \left[\log \left(1 + \frac{K|h_k|^2 P}{N_0} \right) \right] = \mathbb{E} \left[\log \left(1 + \frac{K|h_k|^2 P}{N_0} \right) \right]$$

- □ ... which is strictly less than the sum capacity of the uplink fading channel for $K \ge 2$.
- □ In particular, the penalty due to fading persists even when there is a large number of users.

Multi-user Fast-Fading <u>w/ CSI</u>

- □ Central interest case!
- Dynamically allocate powers to users as a function of CSI
- □ To achieve the maximum sum rate, we can use orthogonal multiple access...
 - □ this means that the codes designed for the point-to-point AWGN channel can be used w/ variable rate coding...
- Contrast this with the case when only the receiver has CSI (i.e. CSIR), where orthogonal multiple access is strictly suboptimal for fading channels. (see previous slide)
 - Note that, this argument on the optimality of orthogonal multiple access holds regardless of whether the users have symmetric fading statistics.

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Multi-users: <u>diversity</u> gain, <u>not</u> d.f gain!

- Having multiple users does not provide additional degrees of freedom in the system:
 - the users are just sharing the time/frequency degrees of freedom already existing in the channel.
- Thus, the optimal power allocation problem should really be thought of as how to <u>partition</u> the total resource (power) across the time/frequency degrees of freedom ...
 - □ ... and how to share the resource across the users in each of those degrees of freedom.
- □ The above solution says that from the point of view of maximizing the sum capacity, ..
 - □ ... the optimal sharing is just to allocate all the power to the user with the strongest channel on that degree of freedom.

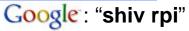
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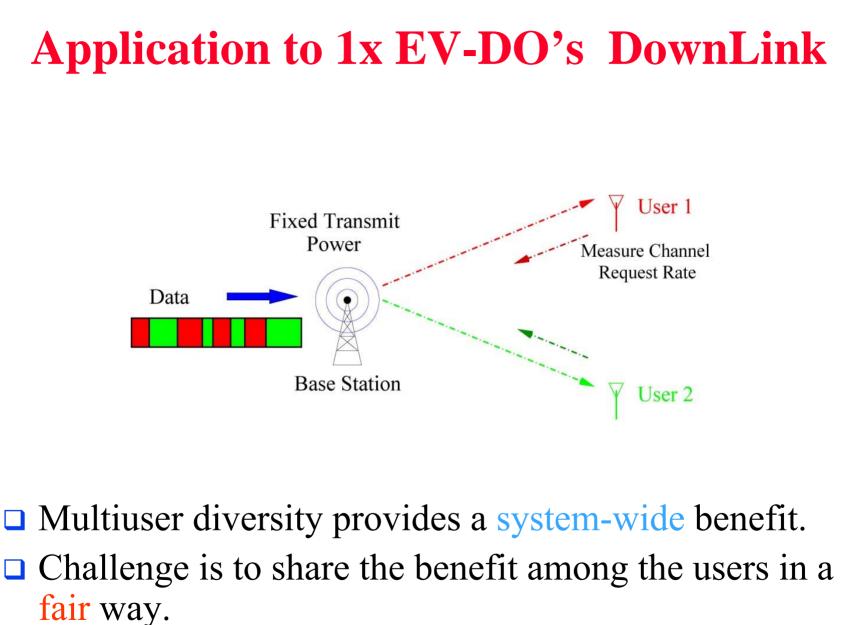
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Applications & Fairness/Scheduling

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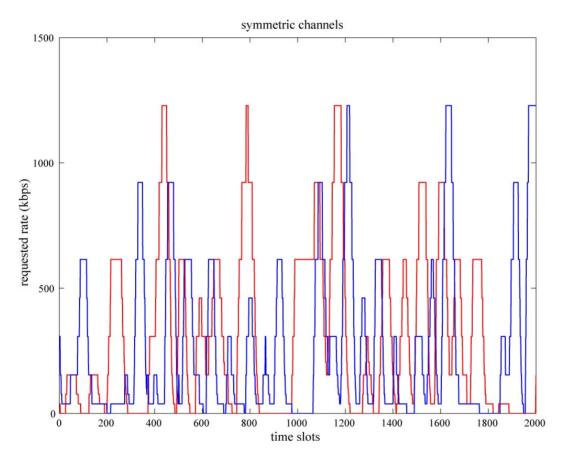




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Symmetric Users

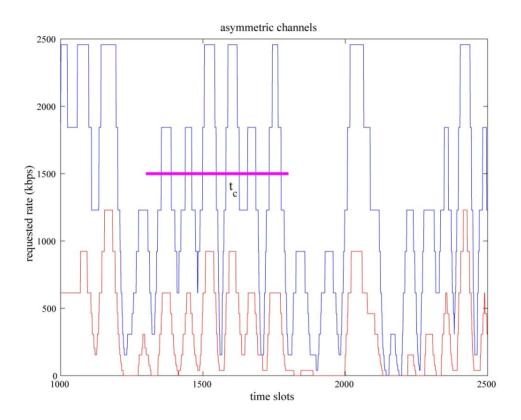


Serving the best user at each time is also fair in terms of long term throughputs.

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Asymmetric Users: Hitting the Peaks



Want to serve each user when it is at its peak.

A peak should be defined with respect to the latency time-scale t_c of the application to provide short-term <u>fairness</u>. Rensselaer Polytechnic Institute

Proportional Fair Scheduler

Schedule the user with the highest ratio

 $\frac{R_k}{T_k}$

 R_k = current requested rate of user k

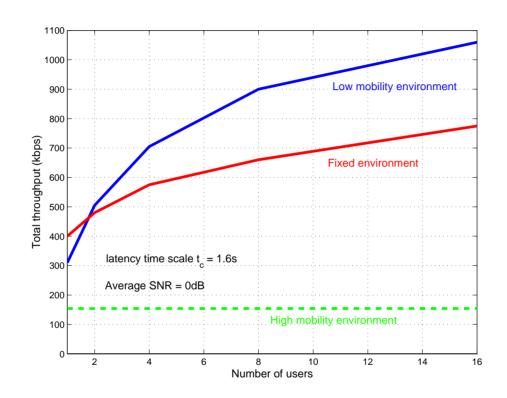
 T_k = average thruput of user k in the past t_c time slots.

Like a dynamic priority scheme

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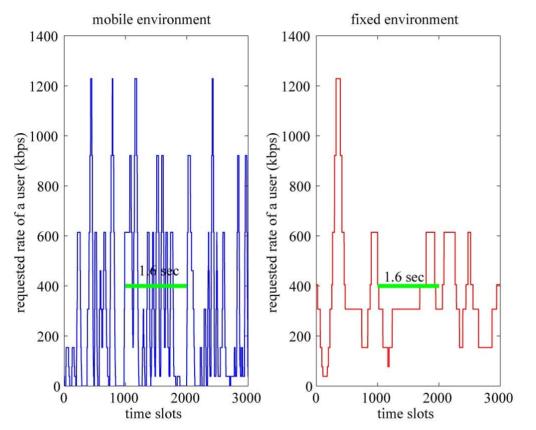
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Performance



Fixed environment: 2Hz Rician fading with $E_{fixed}/E_{scattered} = 5$. Low mobility environment: 3 km/hr, Rayleigh fading High mobility environment: 120 km/hr, Rayleigh fading Rensselaer Polytechnic Institute

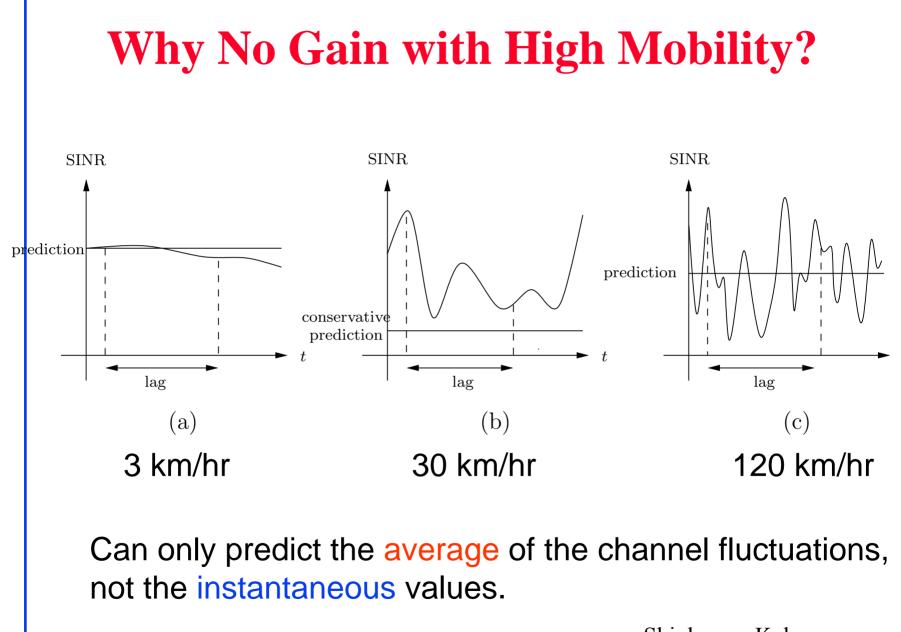
Channel Dynamics



Channel varies faster and has more dynamic range in mobile environments.

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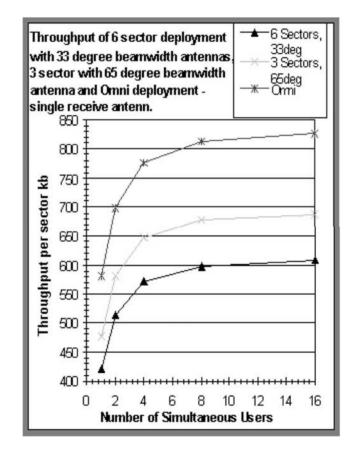
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Throughput of Scheduler: Asymmetric Users



(Jalali, Padovani and Pankaj 2000)

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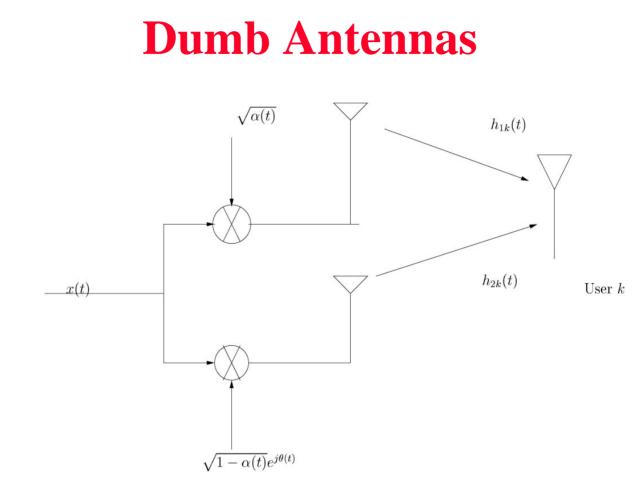
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Inducing Randomness

Scheduling algorithm exploits the nature-given channel fluctuations by hitting the peaks.

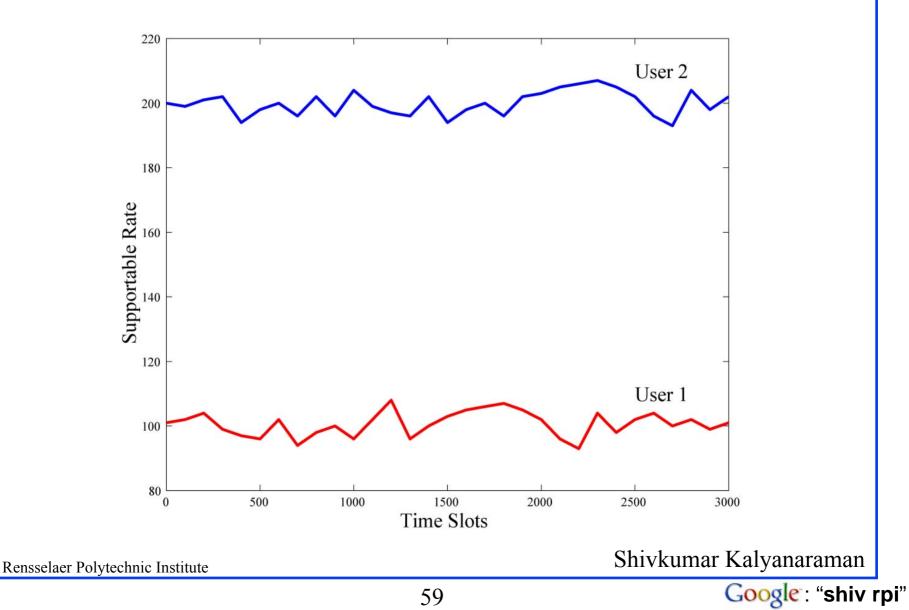
□ If there are not enough fluctuations, why not purposely induce them? (eg: in fixed situation!)

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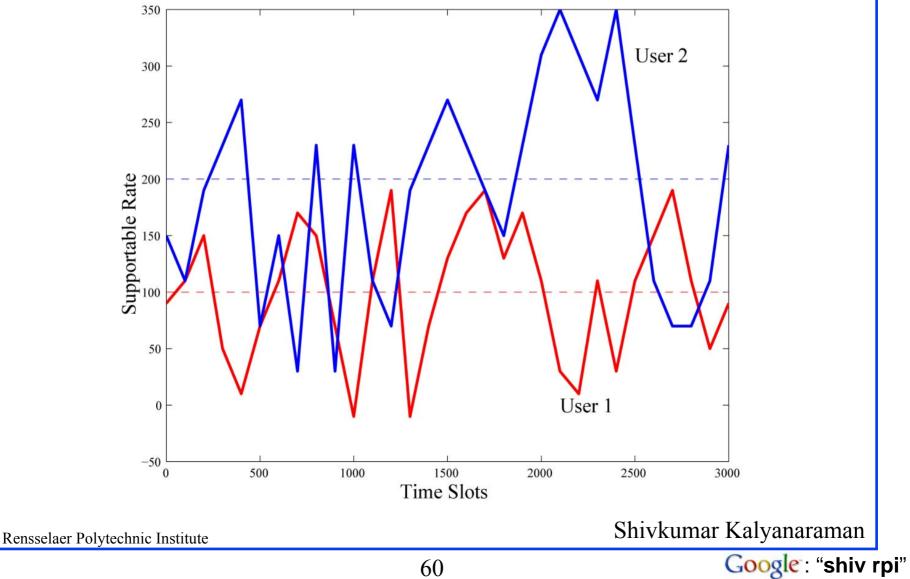


The information bearing signal at each of the transmit antenna is multiplied by a random complex gains.

Slow Fading Environment: Before

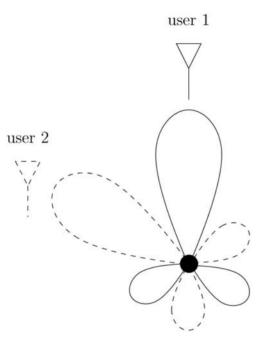


After



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Slow Fading: Opportunistic Beamforming

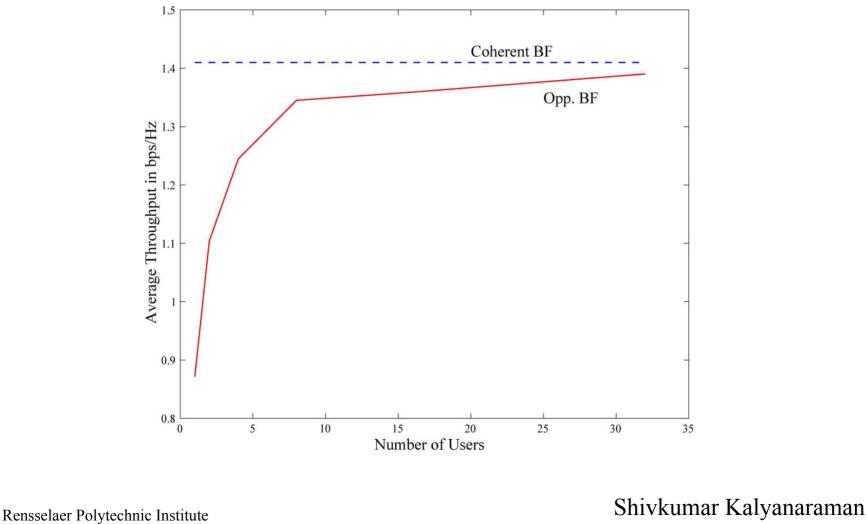


- Dumb antennas create a beam in random time-varying direction.
- □ In a large system, there is likely to be a user near the beam at any one time.
- By transmitting to that user, close to true beamforming performance is achieved.

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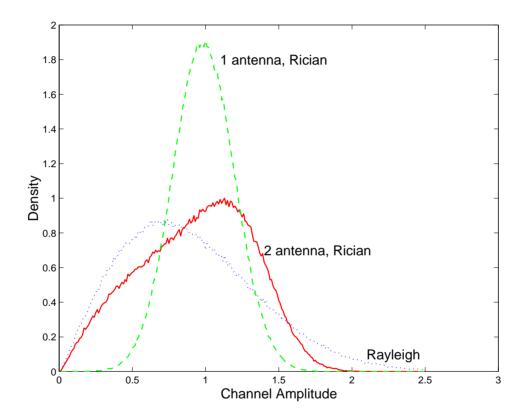
Opportunistic Beamforming: Slow Fading



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Opportunistic Beamforming: Fast Fading



Improves performance in fast fading Rician environments by *spreading the fading distribution*.

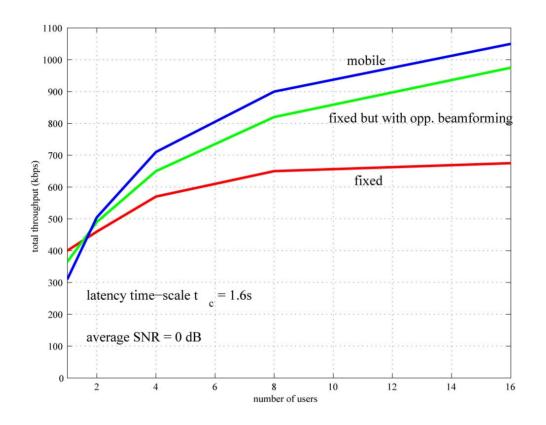
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Overall Performance Improvement



Mobile environment: 3 km/hr, Rayleigh fading Fixed environment: 2Hz Rician fading with $E_{fixed}/E_{scattered} = 5$.

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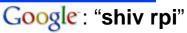
Smart vs Dumb Antennas

Space-time codes improve reliability of point-to-point links but reduce multiuser diversity gain.

Dumb (random beamforming) antennas add fluctuations to point-to-point links but increases multiuser diversity gains.

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Cellular System: Opportunistic Nulling

- In a cellular systems, users are scheduled when their channel is strong and the interference from adjacent base-stations is weak.
- □ Multiuser diversity allows interference avoidance.
- Dumb antennas provides opportunistic nulling for users in other cells (a.k.a interference diversity).
- Particularly important in interference-limited systems with no soft handoff.

Conventional vs Opportunistic Communication

	Conventional Multiple Access	Opportunistic Communication	
Guiding principle	averaging out fast	$\operatorname{exploiting}$	
	Channel fluctuations	channel fluctuations	
Knowledge at Tx	track slow fluctuations	track as much	
	no need to track fast ones	fluctuations as possible	
Control	power control the	rate control to	
	slow fluctuations	all fluctuations	
Delay requirement	can support tight delay	needs some laxity	
Role of Tx antennas	point-to-point diversity	increase fluctuations	
Power gain in downlink	multiple Rx antennas	opportunistic beamform	
		via multiple Tx antennas	
Interference management	averaged	opportunistically avoided	

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Chapter 6: The Main Plot

This chapter looked at the capacities of uplink and downlink channels. Two important sets of concepts emerged:

- successive interference cancellation (SIC) and superposition coding;
- multiuser opportunistic communication and multiuser diversity.

SIC and Superposition Coding

Uplink:

Capacity is achieved by allowing users to simultaneously transmit on the full bandwidth and the use of SIC to decode the users.

SIC has a significant performance gain over conventional multiple access techniques in near-far situations. It takes advantage of the strong channel of the nearby user to give it high rate while providing the weak user with the best possible performance.

Downlink:

Capacity is achieved by superimposing users' signals and the use of SIC at the receivers. The strong user decodes the weak user's signal first and then decode its own.

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Superposition coding/SIC has a significant gain over orthogonal techniques. Only small amount of power has to be allocated to the strong user to give it a high rate, while delivering near-optimal performance to the weak user.

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Opportunistic Communication

Symmetric uplink fading channel:

$$y[m] = \sum_{k=1}^{K} h_k[m] x_k[m] + w[m].$$
(6.64)

Sum capacity with CSI at receiver only:

$$C_{\text{sum}} = \mathbb{E}\left[\log\left(1 + \frac{\sum_{k=1}^{K} |h_k|^2 P}{N_0}\right)\right] \tag{6.65}$$

Very close to AWGN capacity for large number of users. Orthogonal multiple access is strictly sub-optimal.

Sum capacity with full CSI:

$$C_{\text{sum}} = \mathbb{E}\left[\log\left(1 + \frac{P_{k^*}(\mathbf{h})|h_{k^*}|^2}{N_0}\right)\right],\tag{6.66}$$

where k^* is the user with the strongest channel at joint channel state **h**. This is achieved by transmitting only to the user with the best channel and a waterfilling power allocation $P_{k^*}(\mathbf{h})$ over the fading state.

Symmetric downlink fading channel:

$$y_k[m] = h_k[m]x[m] + w_k[m], \qquad k = 1, \dots, K.$$
 (6.67)

Sum capacity with CSI at receiver only:

$$C_{\rm sum} = \mathbb{E}\left[\log\left(1 + \frac{|h_k|^2 P}{N_0}\right)\right] \tag{6.68}$$

Can be achieved by orthogonal multiple access.

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Sum capacity with full CSI: same as uplink.

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Multiuser Diversity

Multiuser diversity gain: under full CSI, capacity increases with the number of users: in a large system with high probability there is always a user with a very strong channel.

System issues in implementing multiuser diversity:

- fairness: fair access to the channel when some users are statistically stronger than others.
- delay: cannot wait too long for a good channel.
- **channel tracking**: channel has to be measured and fed back fast enough.
- small and slow channel fluctuations: multiuser diversity gain is limited when channel varies too slowly and/or has a small dynamic range.
- Proportional fair scheduler transmits to a user when its channel is near its peak within the delay constraint. Every user has access to the channel for roughly the same amount of time.
- Channel feedback delay can be reduced by having shorter time slots and feeding back more often. Aggregate feedback can be reduced by each user selectively feeding channel state back only when its channel is near its peak.
- Channel fluctuations can be sped up and their dynamic range increased by the use of multiple transmit antennas to perform opportunistic beamforming. The scheme sweeps a random beam and schedules transmissions to users when they are beamformed.

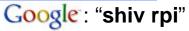
In a cellular system, multiuser diversity scheduling performs interference avoidance as well: a user is scheduled transmission when its channel is strong *and* the out-of-cell interference is weak. **raman : "shiv rpi**"

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Extra Slides: not covered...

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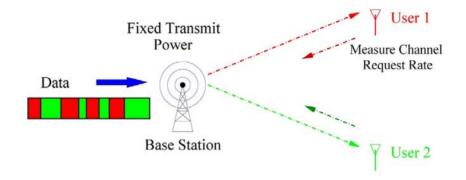


Uplink and Downlink Capacity

CDMA and OFDM are specific multiple access schemes.

But information theory tells us what is the capacity of the uplink and downlink channels and the optimal multiple access schemes.

Example of Rate Adaptation: 1xEV-DO Downlink



Multiple access is TDMA via scheduling.

Each user is rate-controlled rather than power-controlled. (But no waterfilling: fixed transmit power, different code/modulation rates.)

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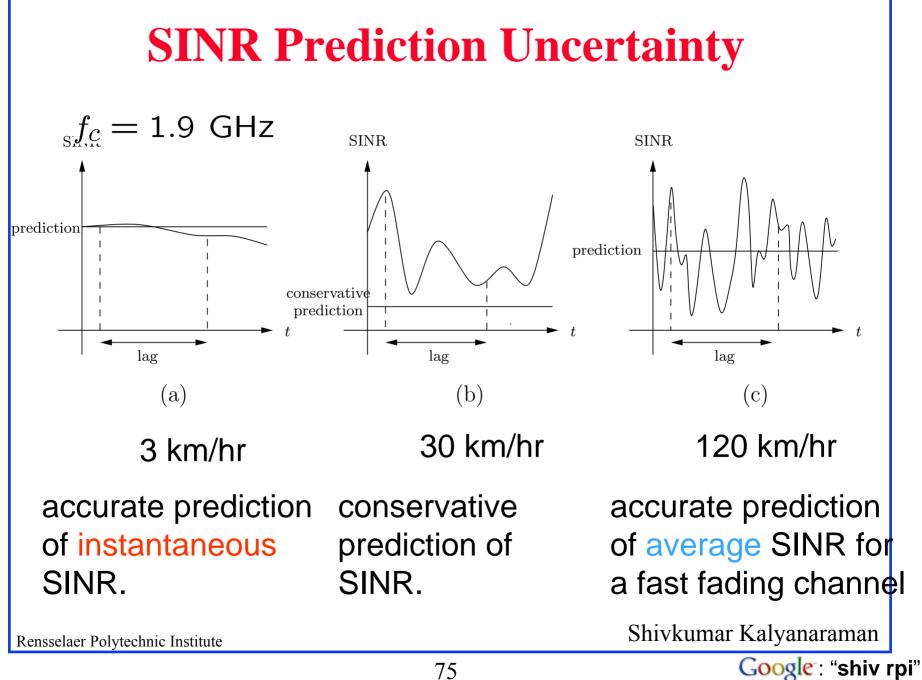
Rate Control: Adaptive Modulation/Coding

Mobile measures the channel based on the pilot and predicts the SINR to request a rate.

Requested rate (kbits/s)	SINR threshold (dB)	Modulation	Number of slots
38.4	-11.5	QPSK	16
76.8	-9.2	QPSK	8
153.6	-6.5	QPSK	4
307.2	-3.5	QPSK	2 or 4
614.4	-0.5	QPSK	1 or 2
921.6	2.2	8-PSK	2
1228.8	3.9	QPSK or 16-QAM	1 or 2
1843.2	8.0	8-PSK	1
2457.6	10.3	16-QAM	1

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Incremental ARQ

- □ A conservative prediction leads to a lower requested rate.
- □ At such rates, data is repeated over multiple slots.
- □ If channel is better than predicted, the number of repeated slots may be an overkill.
- This inefficiency can be reduced by an incremental ARQ protocol.
- The receiver can stop transmission when it has enough information to decode.
- Incremental ARQ also reduces the power control accuracy requirement in the reverse link in Rev A.

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