ECSE 6961: Fundamentals of Wireless Broadband Networks
Homework Problem Set: 4
Due Date: April 1st 2007; [50 points]

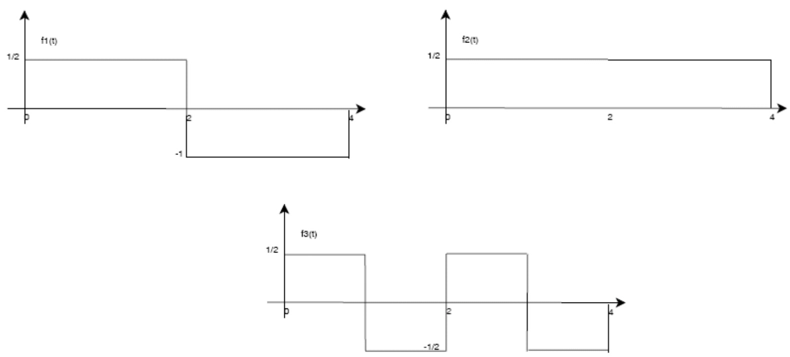
1. **[Constellations & Orthonormal bases]:** ***[10 pts]***

Show using properties of orthonormal basis functions that if $s_i(t)$ and $s_j(t)$ have constellation points $\mathbf{s}_i$ and $\mathbf{s}_j$, respectively, then

$$||\mathbf{s}_i - \mathbf{s}_j||^2 = \int_0^T (s_i(t) - s_j(t))^2 dt.$$

2. **[Signal Space]:** ***[10 pts]***

Consider the three signal waveforms $\{\phi_1(t), \phi_2(t), \phi_3(t)\}$ shown below



(a) Show that these waveforms are orthonormal.

(b) Express the waveform $x(t)$ as a linear combination of $\{\phi_i(t)\}$ and find the coefficients, where $x(t)$ is given as

$$x(t) = \begin{cases} -1 & (0 \le t \le 1) \\ 1 & (1 \le t \le 3) \\ 3 & (3 \le t \le 4) \end{cases}$$

3. **[Matched Filter]** [***10 pts***] Find the matched filter for the following waveforms. (Optional: plot them by computer or roughly by hand to compare how it looks w.r.t. the original pulse)

(a) Rectangular pulse: $g(t) = \sqrt{\frac{2}{T}}$

(b) Sinc pulse: $g(t) = \mathrm{sinc}(t)$.

(c) Gaussian pulse: $g(t) = \frac{\sqrt{\pi}}{\alpha} e^{-\pi^2 t^2/\alpha^2}$

4. [**Modulation perf.**]: ***[10 pts]*** We saw in class that:

$$\boxed{P_E(M) \le \frac{1}{M} \sum_{i=1}^{M} \sum_{\substack{k=1 \\ k \ne i}}^{M} P_2(\mathbf{s}_k, \mathbf{s}_i) \le (M-1) Q\left(\frac{d_{\min}/2}{\sqrt{N_0/2}}\right)}$$

Assume a target P_E of 10^{-5}. What is the excess required SNR = Es/No (in dB) as we move from BPSK to 64PSK? Explain.
(**Hint**: Recall that in BPSK, the constellation points are at +sqrt(Es) and –sqrt(Es). dmin will reduce. Note: MPSK is a circular constellation. You can also assume the exponential approximation for the Q function if necessary.)

5. **[Pulse Shaping:]** ***[10 pts]*** Consider the raised cosine filter (and formula) mentioned in the class slides. Explain the quantitatively the key tradeoffs (vs. the Nyquist Filter) for roll off factors of r = 0, 0.5 and 1.