1. **[Constellations & Orthonormal bases]:** [10 pts]

Show using properties of orthonormal basis functions that if \( s_i(t) \) and \( s_j(t) \) have constellation points \( s_i \) and \( s_j \), respectively, then

\[
||s_i - s_j||^2 = \int_0^T (s_i(t) - s_j(t))^2 dt.
\]

2. **[Signal Space]:** [10 pts]

Consider the three signal waveforms \( \{\phi_1(t), \phi_2(t), \phi_3(t)\} \) shown below

(a) Show that these waveforms are orthonormal.

(b) Express the waveform \( x(t) \) as a linear combination of \( \{\phi_i(t)\} \) and find the coefficients, where \( x(t) \) is given as

\[
x(t) = \begin{cases} 
-1 & (0 \leq t \leq 1) \\
1 & (1 \leq t \leq 3) \\
3 & (3 \leq t \leq 4)
\end{cases}
\]

3. **[Matched Filter] [10 pts]** Find the matched filter for the following waveforms. (Optional: plot them by computer or roughly by hand to compare how it looks w.r.t. the original pulse)

(a) Rectangular pulse: \( g(t) = \sqrt{\frac{2}{T}} \)

(b) Sinc pulse: \( g(t) = \text{sinc}(t) \).

(c) Gaussian pulse: \( g(t) = \frac{\sqrt{\pi}}{\alpha^2} e^{-\pi^2 t^2 / \alpha^2} \)

4. **[Modulation perf.]:** [10 pts] We saw in class that:

\[
P_e(M) \leq \frac{1}{M} \sum_{i=0}^{M-1} P_i(s_i, s_i) \leq (M - 1) \Phi\left( \frac{d_{\min}^2}{\sqrt{N_0}} / 2 \right)
\]

Assume a target \( P_e \) of \( 10^{-5} \). What is the excess required SNR = Es/No (in dB) as we move from BPSK to 64PSK? Explain.

**(Hint:** Recall that in BPSK, the constellation points are at \( \pm \sqrt{\text{Es}} \) and \( \pm \sqrt{\text{Es}} \). \( d_{\min} \) will reduce. Note: MPSK is a circular constellation. You can also assume the exponential approximation for the Q function if necessary.)

5. **[Pulse Shaping]:** [10 pts] Consider the raised cosine filter (and formula) mentioned in the class slides. Explain the quantitatively the key tradeoffs (vs. the Nyquist Filter) for roll off factors of \( r = 0, 0.5 \) and 1.