ECSE 6961: Fundamentals of Wireless Broadband Networks
Homework Problem Set: 3
(Lecture Slide Set: Wireless Channel)
Due Date: March 4th 2007; [40 points]

1. **[dB, dBm, dBi]:** ***[5 pts]*** A transmitter produces 50 W. Express in units of (a) dBm and (b) dBW. If this is applied to a 12 dBi gain transmit antenna at 900 MHz, find the received power Pr in dBm at a distance of 100m. What is Pr (10 km) in dBm? Assume unity gain receive antenna. Hint: Recall the path loss formula:

$$\frac{P_r}{P_t} = \left[\frac{\sqrt{G_l}\lambda}{4\pi d}\right]^2.$$

   (a) dBm = 10 log [50/0.001] = 47 dBm
   (b) dbW = 10 log 50 = 17 dB
   (c) d = 100, λ = 0.33, P_t = 50, gain = 12 dBi
   P_r dBm = P_t dBm + 10 log G_l + 20 log λ – 20 log 4π – 20 log d
   = 47 + 12 + 20 (- 0.4771) – 20 (1.1) – 20 (2) = – 12.54 dBm
   (d) d = 10000m
   P_r dBm = P_t dBm + 10 log G_l + 20 log λ – 20 log 4π – 20 log d
   = 47 + 12 + 20 (- 0.4771) – 20 (1.1) – 20 (4) = – 52.54 dBm

2. **[Path Loss Effects, Small Cells, Cell Edge]:** ***[10 pts]*** Consider a user in the downlink of a cellular system, where the desired base station is at a distance of 900 meters, and there are numerous nearby interfering base stations transmitting at the same power level. If there are 3 interfering base stations at a distance of 1 km, 3 interfering base stations at a distance of 2 km, and 10 interfering base stations at a distance of 4 km, use the empirical path loss formula to find the signal-to-interference ratio (SIR, i.e. the noise is neglected) when α = 3, and then when α= 5. Explain the implications of your results.

Empirical Path Loss Formula:

$$P_r = P_t P_o \left(\frac{d_o}{d}\right)^\alpha$$

the path loss exponent α and the measured path loss Po at a reference distance of do. You don't need the values of Po and do to solve the problem.

SIR (α = 3) = $(0.9)^{-3}/[3(1)^{-3} + 3(2)^{-3} + 10(4)^{-3}]$ = 0.388 = – 4.1 dB
SIR (α = 5) = $(0.9)^{-5}/[3(1)^{-5} + 3(2)^{-5} + 10(4)^{-5}]$ = 0.5457 = – 2.63 dB

3. **[Shadowing, Outage, Modulation Choice:]** ***[15 pts]*** Consider a WiMAX base station (BS) communicating to a subscriber, with the channel parameters: path loss (α) = 3.5, Po = −35dB, d0 = 1m, shadowing parameter (σ_s) = 5dB. We assume a transmit power of P_t = 1 Watt (30 dBm), a bandwidth of B = 10 MHz. Due to rate 1/2 convolutional codes, a received SNR of 14.7 dB is required for 16QAM, while just 3 dB is required for BPSK. Finally, we consider only ambient noise with a typical power spectral density of No = −173dBm/Hz, with an additional receiver noise figure of N_f = 6 dB (noise from all other sources).

The question is this: At a distance of 400 meters from the base station, what is the likelihood that the BS can reliably send BPSK or 16 QAM? {Hints: Find receive power, receive noise, compare to thresholds for BPSK/16QAM. The effect of the shadowing will matter. Use the simple empirical shadowing model below}:

$$P_r = P_t P_o \chi \left(\frac{d_o}{d}\right)^{\alpha}$$

Pr dB = 30 dBm – 35 dB – 91 dB + X db = -96 dB + X dB
Noise Power = -173 + 6 dB + 70 = -97 dBm
SINR gamma = -96 dBm + X dB + 97 dBm = 1 dB + X dB
Without shadowing (X = 0) None works. Both fail all the time.
With shadowing sigmas = 5dB
BPSK: P[(X + 1)/5 > 3/5] = Q(0.4) = 0.346 ~34%
16QAM: P[(X+1)/5 > 14.7] = Q(2.74) = 0.0030 ~0.3%

4. [**Doppler spread**] ***[5 pts]*** Consider mobile nodes moving at speeds (v) of 60 km/h (car on a state road), 200 km/h (high speed train), 400 km/h (airplane taking off/landing). Consider two carrier frequencies fc = 900 Mhz (2G band) and 2.5 GHz (a WiMax band). What is the Doppler spread for each combination of v and fc ? What is the channel coherence time (Tc) in each case?

Doppler spread = 2 f v/c ; Channel coherence time = c /4 f v
Doppler spread for 900MHz : 100Hz (v = 60 Km/h), 333.33 Hz (v = 200km/h), 666.67 Hz(v = 400 km/h)
Doppler spread for 2.5 GHz : 277.78Hz (v = 60 Km/h), 925.92 Hz (v = 200km/h), 1851.87 Hz(v = 400 km/h)
Channel coherence time for 900 MHz = 5ms (v = 60 Km/h), 1.5ms (v = 200km/h), 0.75ms(v = 400 km/h)
Channel coherence time for 2.5 GHz = 1.8 ms (v = 60 Km/h), 0.55 ms (v = 200km/h), 0.27ms(v = 400 km/h)