WBN : Homework #1
Linear Algebra
(45 points)

1. 10 pts. Let $u = (1,\ 1,\ 2)$, $v = (2,\ 3,\ 1)$, $w = (4,\ 5,\ 5)$ in R^3

(i) **(2 pts)** Show that the vectors are linearly dependent.

(ii) **(2 pts)** Find a vector q in R^3 which can not be represented as a linear combination of u, v and w.

(iii) **(6 pts)** Solve $Ax = (1\ \ 1\ \ 1)^T$. $A = [u\ \ v\ \ w]$, $x = (x_1, x_2, x_3)^T$. Note: here the vectors u, v, w are columns of A. Hint: (this is a least-squares problem) multiply both sides by A^T and then invert A^TA

2. (10 pts) Let A =

$$A = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

(i) (6 pts) Find determinant, transpose and inverse of A.

(ii) (1 pt) Find rank of A.

(iii) (2 pts) Show that the columns of A form an orthonormal basis for R^3.

(iv) (1 pt) What kind of matrix is A.

3. (5 pts): Projection, Orthogonality

(i) (3 pts) Find the projection vector p of b = $(3\ \ -5)^T$ onto a = $(1\ \ 1)^T$.

(ii) (2 pts) Find the error vector (b – p) and show that it is orthogonal to p.

4. (10 points) Eigendecomposition:

(4 pts) Show that for a square symmetric matrix A, the *n*-th power: $A^n = Q\Lambda^n Q^T$.

(4 pts) Find Q, Λ for: $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. ***(2 pts)*** Find A^3 using the above and verify.

5. (10 pts) Let $A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$.

(i) (3 pts) Find all eigenvalues and corresponding eigenvectors.

(ii) (2 pts) Find Λ using eigen decomposition.

(iii) (1 pts) Is it possible to represent A as $Q \Lambda Q^T$ where Q is a orthogonal matrix ($Q^{-1} = Q^T$) ? Why or why not?

(iv) (4 pts) Perform SVD for A and interpret the result.