1. 10 pts. Let \( u = (1, 1, 2), \ v = (2, 3, 1), \ w = (4, 5, 5) \) in \( \mathbb{R}^3 \)

(i) \((2 \text{ pts})\) Show that the vectors are linearly dependent.

(ii) \((2 \text{ pts})\) Find a vector \( q \) in \( \mathbb{R}^3 \) which cannot be represented as a linear combination of \( u, v \) and \( w \).

(iii) \((6 \text{ pts})\) Solve \( Ax = (1 \ 1 \ 1)^T \). \( A = [u \ v \ w] \), \( x = (x_1, x_2, x_3)^T \). Note: here the vectors \( u, v, w \) are columns of \( A \). \textbf{Hint:} (this is a least-squares problem) multiply both sides by \( A^T \) and then invert \( A^T A \).
2. (10 pts) Let \( A = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \)

(i) (6 pts) Find determinant, transpose and inverse of \( A \).

(ii) (1 pt) Find rank of \( A \).

(iii) (2 pts) Show that the columns of \( A \) form an orthonormal basis for \( \mathbb{R}^3 \).

(iv) (1 pt) What kind of matrix is \( A \).

3. (5 pts): Projection, Orthogonality

(i) (3 pts) Find the projection vector \( p \) of \( b = (3 \ -5)^T \) onto \( a = (1 \ 1)^T \).

(ii) (2 pts) Find the error vector \((b - p)\) and show that it is orthogonal to \( p \).
4. (10 points) Eigendecomposition:

(4 pts) Show that for a square symmetric matrix $A$, the $n$-th power: $A^n = Q\Lambda^n Q^T$.

(4 pts) Find $Q$, $\Lambda$ for: $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. (2 pts) Find $A^3$ using the above and verify.

5. (10 pts) Let $A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$.

(i) (3 pts) Find all eigenvalues and corresponding eigenvectors.

(ii) (2 pts) Find $\Lambda$ using eigen decomposition.

(iii) (1 pts) Is it possible to represent $A$ as $Q\Lambda Q^T$ where $Q$ is an orthogonal matrix ($Q^{-1} = Q^T$) ? Why or why not?

(iv) (4 pts) Perform SVD for $A$ and interpret the result.