WBN : Homework #1
Linear Algebra

1. Let $u = (1, 1, 2)$, $v = (2, 3, 1)$, $w = (4, 5, 5)$ in R^3

(i) Show that the vectors are linearly dependent. ($2u + v = w$)

(ii) Find a vector q in R^3 which can not be represented as a linear combination of u, v and w. ($q = (-5\ \ 3\ \ 1)^T$)

(iii) Find a solution to $Ax = 0$, where $A = [u\ \ v\ \ w]$, $x = (x_1, x_2, x_3)^T$.

($x = (2\ \ 1\ \ -1)^T$)

Also comment on the solution to $Ax = (1\ \ 1\ \ 1)^T$.

(No solution since A is not of full rank and is not invertible)

2. Let $A = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$

(i) Find determinant, transpose and inverse of A. (det A = -1, $A^T = A$, $A^{-1} = A = A^T$)

(ii) Find rank of A. (Full rank = 3)

(iii) Show that the columns of A form an orthonormal basis for R^3. (columns are linearly independent, orthogonal and of magnitude 1)

(iv) What kind of matrix is A. (Orthogonal)

3. Find the projection vector p of $b = (3\ \ -5)^T$ onto $a = (1\ \ 1)^T$. ($p = (-1\ \ -1)^T$)

(i) Find the error vector (b – p) and show that it is orthogonal to p. $(4\ \ -4)^T$

4. Show that for a square symmetric matrix A, $A^i = Q\Lambda^i Q^T$.

Find Q, Λ for $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Find A^3 using the above and verify.

Q = [1/sqrt(2) 1/sqrt(2); -1/sqrt(2) 1/sqrt(2)], Λ = [2 0; 0 4], A^3 = [36 28; 28 36]

5. Let $A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$.

(i) Find all eigenvalues and corresponding eigenvectors. (2, -5, (4, 1), (1, 2))

(ii) Find Λ using eigen decomposition. ([2 0; 0 -5])

(iii) Is it possible to represent A as $Q\,\Lambda\,Q^T$ where Q is a orthogonal matrix ($Q^{-1} = Q^T$) ? Why or why not? (No. A is not symmetric)

(iv) Perform SVD for A and interpret the result.

U = [-0.6154 -0.7882; -0.7882 0.6154]; V = [-0.4298 -0.9029; 0.9029 -0.4298]; S = [7.9639 0; 0 1.2557];