ECSE-6961: Fundamentals of Wireless Broadband Networks
Spring 2007, Exam 2 - SOLUTIONS

Time: **75 min (strictly enforced)**

Points: **50**

YOUR NAME:

***Be brief*, but *DO NOT* omit necessary detail**

{Note: Simply copying text directly from the slides or notes will not earn (partial) credit. Brief, clear and consistent explanation will.}

1. [6 pts] Basics: Nyquist Sampling & Nyquist pulse

Explain why and how a sampling rate of 2W samples allows a band-limited analog signal to be perfectly reconstructed (4 pts). What is a Nyquist pulse and how is it approximated (2 pts) ?

The sampling rate of 2W samples allows a band-limited signal to be perfectly reconstructed because sinc() filters are used for reconstruction or interpolation. A band limited reason is finite dimensional (has 2W dimensions). For sampling rate f_s, as long as

$f_s - f_m >= f_m$, there is no aliasing => fs >= 2fm. Aliasing leads to corruption in the spectrum picked up by the reconstruction filter. Therefore, sampling rate of 2W leads to perfect reconstruction.

A Nyquist pulse is a pulse which has zero ISI at sampling instances. An excess bandwidth factor of 1 makes the sinc() function to fall off faster and reduces side lobes, which mitigates ISI. Nyquist pulse is approximated using pulse shaping e.g. Raised cosine and root-raised cosines. In raised cosine filter, the excess bandwidth factor leads to a tradeoff between the BW and the peak to average pulse ratio.

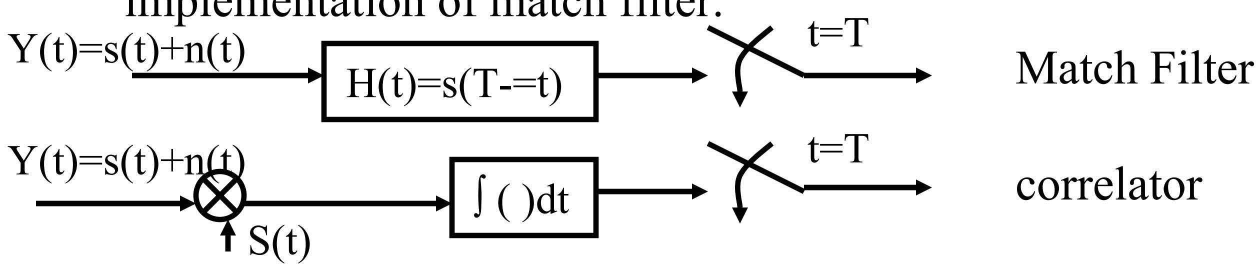
2. [9 pts] Matched Filter vs Equalizer, Correlator, MRC, Rake, Beamforming

What is a matched filter and why is noise outside signal space irrelevant for optimal detection (2 pts)? How is the matched filter different from an equalizer (2 pts)? How is the matched filter related to a correlator (1 pts)? Explain how it is essentially equivalent to Maximal-ratio-combining (MRC), Rake receiver and (transmit/receive) Beamforming (3 pts)? Under what circumstances is MRC significantly better than selection combining (1 pt) ?

Matched filter is a filter which projects the received signal vector onto signal space. Due to band limited channel, out of band noise gets thrown away hence maximizing the SNR at the sampling intervals. Matched filter rejects out of band noise, hence the noise outside signal space is irrelevant for optimal detection.

An equalizer removes the effect of ISI caused due to impairments in the channel whereas matched filter removes out of band noise and maximizes SNR at sampling intervals. Equalizer enhances deep fades in the received signal, i.e. if channel has fades, equalizer tries to invert it.

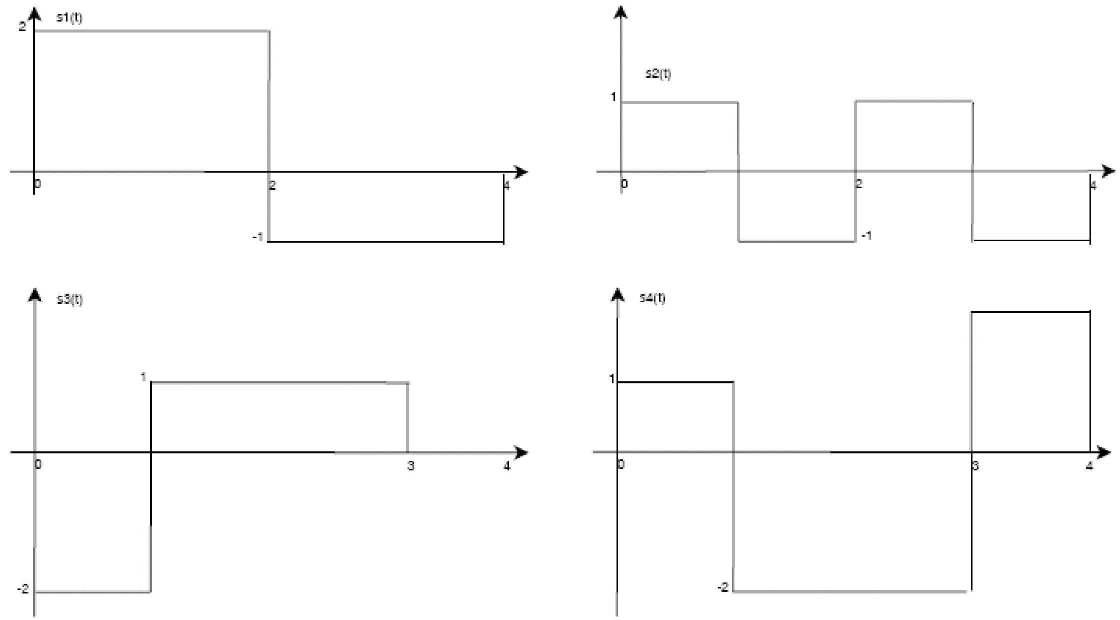
Matched filter projects the received signal onto signal space. Correlator is an alternative implementation of match filter.



The two figure above is equivalent. But correlator simplifies the filter operation (convolution with h(t)) as integrate-and-dump.

- Maximal ratio combining (MRC) weighs each branch with a complex factor q, which is the SNR on that branch, and then adds up all the branches. Matched filter taps frequencies in frequency domain, while MRC does a SNR weighted average. The complex factor helps cancel out phases, and results in gain proportional to SNR. Rake receiver in CDMA recovers source signal by getting power from each tap, adjusting time delays and adding them up, which is similar to MRC. Beam forming also takes the signal from multiple taps in the antenna, weighted by SNR and adds them up. It assigns more weightage to strongest signal and less to weaker signals, and is similar to MRC and Rake receiver.
- MRC is better than selection combining when there are more number of taps, eg. when the number of receiver antennas is large, since selection combining would pick only one of them. Also, in low-SNR regions MRC is better than selection combining since it combines each received signal and provides higher SNR.

3. [7 pts] <u>Orthogonality & Distance</u>: Show that these four signals are orthogonal (3 pts). What is the distance between the signal pairs (s1,s4) and the signal pairs (s2,s3)? {4 pts}



- <s1(t), s2(t)> = 0, <s1(t), s3(t)> = -3, <s1(t), s4(t)> = -2
- <s2(t), s4(t)> = -1, <s3(t), s4(t)> = -6, <s2(t), s3(t)> = -2
- Only s1 and s2 are orthogonal
- $\| s1 - s4\|^2 = \int_0^4 [(s1(t) - s4(t)]^2\, dt$
- Distance between s1 and s4: $\| s1 - s4\|^2 = 27$, Distance = sqrt(27)
- Distance between s2 and s3: $\| s2 - s3\|^2 = 14$, Distance = sqrt(14)

4. [6 pts] <u>Eb/No vs SNR & Shannon Limit</u>: What is the relationship between Eb/No vs SNR (2 pts)? Why is the former used in plots vs Pb (bit error rate) (1 pt)? How is Eb/No related to spectral efficiency (ρ) (1 pt) ?What is the minimum Eb/No possible (ultimate shannon limit) given a spectral efficiency (ρ) constraint of 4 bits/Hz? (2 pts)?

(a) SNR = Signal Power / Noise Power

Where No is the noise spectral density and Eb is the average energy per bit.

Eb/No = SNR/rho

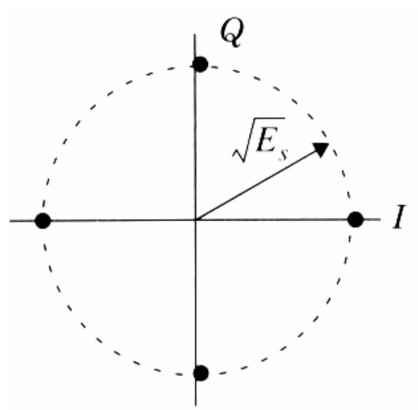
(b) Since Eb/No is independent of modulation schemes, it is used in the plots against bit error rate and helps compare schemes with different bits per symbol.

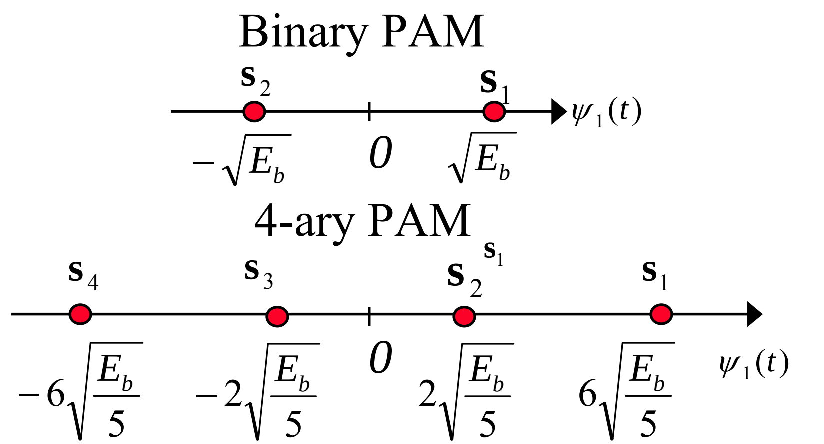
(c) The spectral efficiency rho = B/W = log (1 + SNR) = log (1 + rho Eb/No)

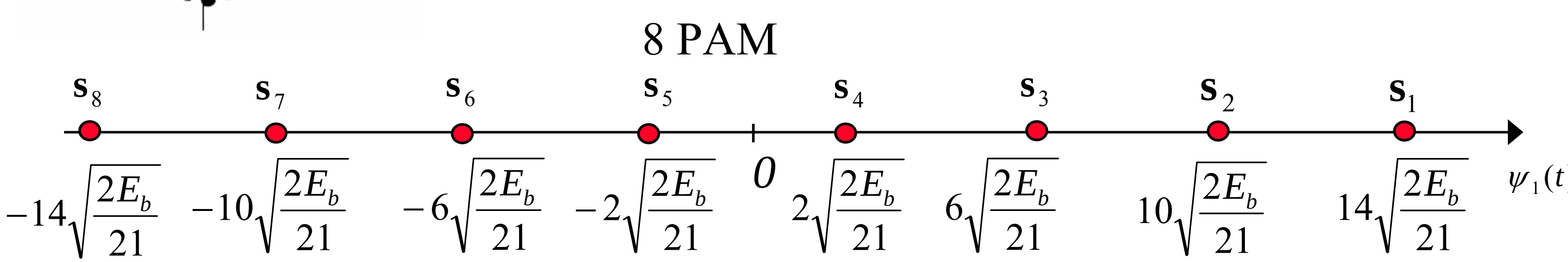
(d) Rho = 4 bits/Hz, (2^{rho} – 1)/rho = 15/4 = 3.75 = 5.74 dB

From Shannon's coding theorem, 5.74 dB is the minimum Eb/No possible.

5. [8 pts] Modulation & Signal Space: Below we have the figure of 2-PAM vs 4-PAM. Draw a rough figure for 8-PAM with the appropriate energy values at the constellation points (5 pts). 4-PSK provides a gain over 4-PAM by using the extra degree of freedom. How much is this gain (in dB) (3 pts)? Show your calculations.







$1/8[\ 1\ x^2 + 9\ x^2 + 25\ x^2 + 49\ x^2]\ *2 = 8\ Eb$

$\Rightarrow 21\ x^2 = 8\ Eb \Rightarrow x = 2\ \mathrm{sqrt}(2Eb/21)$

$1/8[\ 1\ x^2 + 9\ x^2 + 25\ x^2 + 49\ x^2]\ {*}2 = 3\ Eb$

(Since 1 symbol uses 3 bits=> $E_s=3E_b$)

⇒ $21\ x^2 = 3\ Eb \Rightarrow x = \mathrm{sqrt}(Eb/7)$

(Because the example for 4-psk is multiplied by a constant of sqrt(2), if your answer is x*sqrt(2)=sqrt(2Eb/7), this is acceptable answer)

(2) For 4-PSK, d_{min}=sqrt(2)*sqrt(Es)=2sqrt(Eb) (since Es=2Eb)

For 4-PAM, d_{min}=4sqrt(Eb/5)

To achieve the same BER=> d_{min} is the same

$E_{b,psk}=4/5E_{b,PAM}$

power gain=5/4=1.25

=10*log10(1.25)=0.97 dB

<u>6. [8 pts] Interference</u>: There are several sources of interference in wireless networks: ***inter-symbol interference*** (AWGN channels due to poor pulse-shaping; multi-path frequency-selective channels), ***inter-cell interference, intra-cell interference*** (CDMA), and ***inter-carrier interference*** (in OFDM). Briefly explain why each of these interference phenomena arise (4 pts)? How do techniques like pulse-shaping, spread spectrum/Rake, equalizers and OFDM deal with the ISI problem (4 pts)?

ISI arises due to band limited signals, because the pulse duration may be long and the signal is received from multiple paths at the receiver. Thus either the pulse duration could be changed (pulse shaping, Nyquist criteria) or we need to efficiently combine the multi-paths using spread spectrum (orthogonal codes) or rake receivers.

ISI could also be eliminated by boosting the deep fades in the channel using an equalizer (invert channel response to avoid ISI caused due to the channel).

OFDM maintains orthogonality between each symbols by using DFT, and ISI is eliminated by using cyclic prefixes.

Rake receivers consider same symbols on multiple taps, adjusts and adds them.

(Note: RAKE demodulator is optimum when there is neglible ISI, i.e., $T_b >> T_m$. When the condition is not satisfied, the RAKE demodulator output is corrupted by ISI. In such case, an equalizer is required to suppress the ISI. For example, assume BPSK signal and spread by a PN sequence (spread spectrum). The signal is transmitted through multi-path fading channel. At the receiver, after the signal is demodulated to baseband. It may be processed by the RAKE, which is math filter to the channel response, followed by an equalizer to suppress the ISI. (Proakis, “Digital Communications,” Chap 14.5.4, pp851-852))

Inter-cell interference is caused by signals from adjacent cells interfering due to frequency reuse.

Intra-cell interference is caused when signals from most subscribers use the same BW as in CDMA. To reduce this use Walsh codes, PN codes which are orthogonal to one another.

Inter-carrier interference is eliminated by making the sub carriers orthogonal to one another in OFDM. Also one should not have too many carriers.

7. [6 pts] Code Design & MIMO: Explain the product distance and determinant criteria for time-only and space-time code design (3 pts). Explain why the Alamouti code does not use the degrees of freedom of a MIMO channel, but is able to do so for a MISO channel (even without channel knowledge at the sender) (3 pts).

Product distance criteria describe the minimum distance among signal constellation which can be used to bound the coding gain. |d1| |d2| should be maximized for achieving maximum diversity along the coding gain. Here d1 and d2 are projections of symbols on BPSK. Choose the rotation angle to maximize the worst case product distance to all other code words. For space-time coding, coding gain is determined by maximizing the determinant of (xi – xj)(xi – xj)*

For a MISO channel, as only one symbol is received, Alamouti code is able to use the degrees of freedom (even without channel knowledge at the sender). It spreads information onto a single dimension (as opposed to two orthogonal dimensions for a MIMO channel) for MISO channel, and hence can use the degrees of freedom.