Human-in-the-loop Thermal Management for Smart Buildings

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Abstract Most of the current studies and solutions developed for building thermal control have been designed independent of the occupant feedback. An acceptable temperature range for the occupancy level is estimated, and control input is designed to maintain temperature within that range during occupancy hours. Consider office floors with cubicles, conference rooms, student dorms, homes, and other multi-occupant spaces where temperature set-points on thermostats are chosen irrespective of the number of occupants and their individual preferences. This existing approach is not only non user-centric but also sub-optimal from both energy consumption and occupant satisfaction/productivity perspectives. It is thus highly desirable for such multi-occupant spaces to have a mechanism that would take into account each occupants individual comfort preference and the energy cost, to come up with optimal thermal setting. Individual occupants feedback and preference can be obtained through wearable sensors or smart phone applications. In this chapter we propose algorithms that take into account each occupants preferences along with the thermal correlations between different zones in a building, to arrive at optimal thermal settings for all zones of the building in a coordinated manner. First, we present a control algorithm that uses binary occupant feedback based on *singular perturbation theory* to minimize aggregate user discomfort and total energy cost. A consensus algorithm for attaining a common temperature set-point in a typical multi-occupant space is presented next that uses *Alternating Direction Method of Multipliers (ADMM)* to solve the consensus problem. We use our Watervliet, NY based test facility to simulate the performance of our algorithms.

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1 Introduction

Data suggests that nearly 40% of the total energy consumption in US, and 20% of the total energy consumption worldwide, is attributed to the residential and commercial building usage [1]. In 2012 this amount was estimated at $ 416 billions, much higher than the Industrial (at 33 %) and Transportation (at 27%) sectors. More than 75% of current electricity consumption is also due to the usage in buildings. However, even with the existing cost of HVAC systems, the occupant dissatisfaction associated with the prevailing indoor thermal conditions have been highlighted by several studies [2], [3]. A more recent news article [4] titled, “This cold war, too, has roots in the past,” highlights the problem with thermal comfort model that is being followed by commercial buildings. The occupants (employees) in turn have to face discomfort with regard to indoor thermal environment leading to direct loss of productivity and reduced overall satisfaction.

Occupant dissatisfaction associated with the indoor building environment (both residential and commercial) can be mainly attributed to the existing methods for indoor environment management employed commonly by the Building Management Systems (BMS). Most of the multi occupant space environment control systems today are based on set point scheme of thermal management. Either there is a centralized control by operation managers of the building, or the occupants set a specific temperature set point for the indoor environment agnostic of the changes in ambient conditions and occupancy patterns. However, a static set point does not seem to fit the dynamic lifestyle that we normally follow. This calls for the design of smart systems that can achieve desired human comfort levels without putting additional pressure on energy resources, which the existing set point scheme fails to achieve.

There have been studies based on weather prediction models that use the concept of preheating and cooling [5] to reduce energy consumption. However, this does not necessarily take into account occupant behavior and/or preferences. Some of the more recent studies based on human physiology [6] do propose an aggregate or average model for occupants but fails to provide a mechanism to incorporate individual differences in body metabolism and preferences. Smart and learning thermostats like Google-Nest [7] that claim to learn occupant preference dynamically, might however fail to perform in multi-occupant spaces.

The framework proposed in this chapter has the following novel components:

- Simultaneous minimization of energy cost and maximization of occupant comfort, with the flexibility to modify weights associated with each (say for instance to incorporate real-time energy pricing).
- Multi-zoning solution that takes into account the thermal correlation among different zones in a residential home or a commercial building.
- Consensus algorithm to enable deployment to multi-occupant spaces (corporate office buildings, university buildings, etc.). This algorithm drives co-located occupants to a logical consensus on thermal setting based on individual preferences.

Personalized comfort level expectations pose a conflicting situation in multi-occupant spaces such as residential homes, research laboratories, corporate office buildings, student dorms etc., where occupants have their own range of thermal comfort and other environmental settings. This range generally depends on individual occupant body type, external factors such as attire, physical and mental condition, and level of tolerance; and can also vary depending on other environmental factors such as time of the day, lighting conditions etc. [8], [9], [10]. This personalized thermal comfort range can be best captured by individual occupant feedback in real time. Further, in shared multi-occupant spaces personal comfort levels are affected both by the presence of co-occupants and the correlation between temperatures in different zones and rooms occupied [11], [12], [6]. Arriving at temperature set-points to minimize the aggregate discomfort among all occupants of different rooms or zones in a building is an important yet challenging problem. With rising energy cost and emphasis on energy conservation, the total energy cost also needs to be accounted for when trying to determine the optimal temperature set-points in different zones of a building.

We present a solution to optimally trade off the overall energy cost with the aggregate occupant discomfort. Our solution is occupant centric and focuses on designing an algorithm for efficient human interaction and utilizing the feedback. We start by depicting a high level work flow of the proposed solution for energy efficient operation of buildings in Figure 1.

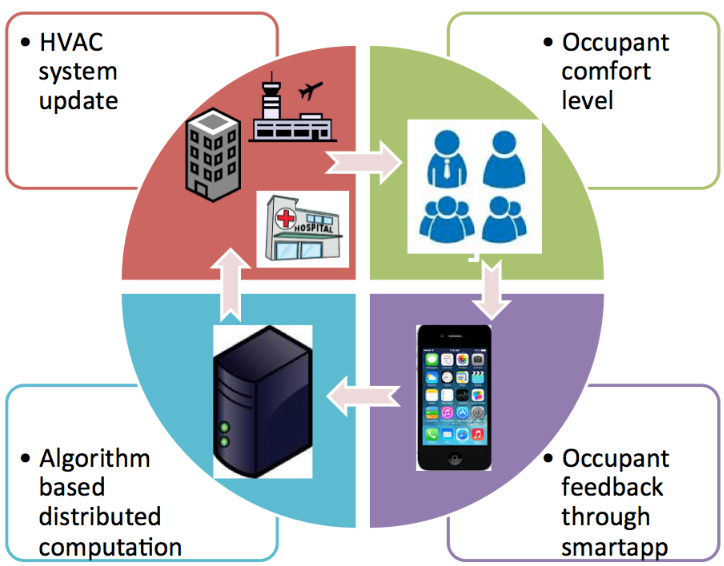

Fig. 1 High level process flow depiction of the proposed algorithm. The process runs in a loop incorporating real-time changes to occupancy pattern, occupant preferences and/or environmental changes.

Occupant comfort & sensor measurement: User experience is of utmost importance, and our solution also starts with the perception of thermal comfort level by the occupants to the current temperature settings. Simultaneously static sensors if present in the zones can feed real time zonal temperature (sensors can also be used to measure humidity and other related comfort parameters) to the central processor.

Occupant feedback through smart phone application: The occupants can use their smart phones to access our mobile application and provide their temperature preference feedback. It also shows the percentage distribution of the occupants based on their preferences for the given zone of the building. The displayed information gets auto updated at specific intervals. An update request can also be generated by the user manually.

Algorithm for Optimal temperature determination: The minimization objective is an aggregate of all the occupant discomfort functions and the total energy cost, subject to the constraint of a common temperature set-point for each zone. Pricing per unit temperature change serves as the feedback signal to the occupants, to drive them to declare their truthful temperature preference that optimizes the overall discomfort plus energy cost objective. The occupant feedback obtained through smart phone application are used by the central server under building management running the algorithm to determine the optimal zonal temperatures.

HVAC system update: The HVAC system is then set to the predicted optimal zonal temperature settings. For an integrated system the thermostat(s) can be automatically set to the suggested temperature settings. Our solution is designed to work for any multi-occupant space such as corporate building, airports, aircrafts, homes, apartments, etc.

1.1 Existing Solutions Incorporating Occupant Feedback

We briefly review the existing building thermal control studies that incorporate occupant feedback in some form. The PMV index has been used as the metric for user comfort integration in multiple studies [32], [33], [34]. Some studies proposed sensor network solutions to increase the accuracy of PMV calculation [35], [36]. Owing to the complexity of sensor network deployment, a number of studies have proposed utilizing occupant feedback for thermal comfort integration into the control logic of building systems. Through custom keyboards in each room, Guillemin and Morel made use of occupant preferences in the form of temperature set points [37]. Murakami et al. used binary preference of warmer and cooler along with a logic to build consensus for controlling the air-conditioning set point [38]. Daum et al. utilized too hot/too cold occupant complaint along with a probabilistic approach for determining user comfort profiles [39]. Thermovote [6] utilized a seven level occupant comfort voting to integrate with the building control logic. Purdon et al. developed a smart phone interface to receive 3-point scale comfort feedback from occupants and determine the direction for temperature drift with a system defined step-size [13]. Some more recent works have conducted experimental study for a group of occupants [30], and have presented thermal comfort model for a single person [31]. However, these are based on thermal complaint behavior using one-class classifier. Jazizadeh et al. used a fuzzy predictive model to learn occupant comfort profiles and a complementary control strategy for the HVAC control [40]. Zhao et al. conducted a simulation study tying occupant subjective thermal comfort feedback with MPC control algorithm for the active HVAC system against a baseline rule-based control algorithm [41]. Experimental facilities, such as Living Lab [28] at Purdue University are actively utilized by researchers to understand human behavior and productivity relationship to indoor environment and personalized control. Choi et al. have presented HVAC thermal models based on sensing of human vitals such as heart rate [29].

We start by presenting the overall system model in section 2 that includes the occupant discomfort function and building heat transfer model. In section 3, we present our novel algorithm that is developed and analyzed using *gradient optimization* and *singular perturbation theory*. We use simple feedback from users in the form of ‘heat up’ or ‘cool down’, which are further consolidated to estimate their comfort ranges or the discomfort functions. In section 4, we discuss the consensus algorithm that we developed through the use of *alternating direction method of multipliers*. In section 5, we share results from simulation and experimental study based on our Watervliet based test facility. Finally, we present some concluding remarks in section 6.

2 System Model

2.1 Occupant Discomfort Modeling

Occupant thermal comfort modeling has been extensively researched and can be summarized into the following three major approaches: (1) the chamber study model, based on mapping thermal comfort from environmental and personal factors to a 7-level comfort value scale, viz. the Predicted Mean Vote - Predicted Percent Dissatisfied (PMV-PPD) [12], [8]; (2) human body physiology based models such as Gagge’s core to skin model [23], Stolwijk’s comfort model for multi-human segments [24], and Zhang et al.’s sensation on human body segments [25]; and (3) adaptive comfort models developed in field study, viz. Humphreys [26] and [27]. Recent work based on thermal complaint behavior using one-class classifier [30], [31] have also been presented. However, existing work mainly focus on average thermal comfort models instead of individual comfort modeling. Such group comfort models only capture average behavior and are not particularly useful in maximizing aggregate comfort for multi-occupant spaces, with individual thermal preferences differing from each other.

In the current work, therefore, we take into consideration discomfort functions of the occupants individually - modeled as convex quadratic functions of temperature variation based on the PMV-PPD model. Our model captures the difference across occupants in their comfortable temperature range. For simulation and experimental study we adopt occupant discomfort function of the form:

$$D_i(y_j) = \begin{cases} (y_j - y_s^U)^2 & \text{if } y_j > y_s^U, \\ 0 & \text{if } y_s^L \leq y_j \leq y_s^U, \\ (y_j - y_s^L)^2 & \text{if } y_j < y_s^L, \end{cases} \tag{1}$$

where y_s^U and y_s^L are the upper and lower limit temperatures respectively of the user i located in zone j.

Note that the boundary values can be either explicitly indicated by the user, or can be conveyed implicitly to the system through user feedbacks in a simple binary form of whether he/she is feeling hot or cold in the current setting.

2.2 Optimization Objective

Consider a building with m zones, and let S_j represent the set of occupants located in zone j of the building. Let D_i represent the discomfort function of occupant i, and function E the overall energy cost. Then a reasonable objective is to attain (in steady state) the zonal temperature vector y that achieves the following objective:

$$\text{minimize } \sum_{j=1}^{m} \sum_{i \in S_j} D_i(y_j) + E(u). \tag{2}$$

where y_j is the temperature of zone j, and u is the heat input vector that is required to attain those zonal temperatures. Note that an occupant i located in zone j (i.e., $i \in S_j$) experiences temperature y_j, and therefore its discomfort can be represented as $D_i(y_j)$. We assume the discomfort function $D_i(y_j)$ as convex in its argument y_j. It is worth noting that the discomfort function *need not be* "strictly" convex. This allows for the occupants to be insensitive to temperature fluctuations over a certain range; or in other words, the discomfort function could be flat over the occupant's "comfort range".

In the above, $E(u)$ is assumed to be a convex function of the control input vector u. For the sake of definiteness, we use $E(u)$ to be of the following quadratic form (although other convex forms of the function $E(u)$ are also allowed by our framework):

$$E(u) = u^T \Gamma u, \tag{3}$$

where Γ is a positive definite matrix. The Γ matrix captures the weight of the energy cost relative to the total discomfort cost. In practice, it could be determined by the actual cost of energy, as well as additional input from the building operator to determine how much relative weight to associate with the energy cost as compared to the occupant discomfort cost.

Finally, since the optimization variable in the objective function (2) is only the zonal temperature vector y, the relationship between the heat input vector u and the zonal temperature vector y needs to be stated to make the formula meaningful. We can express $u = g(y)$ and using that write the energy cost $E(u)$ as $G(y) = E\big(g(y)\big)$, where the function $G(y)$ is convex in y. Taking the case of an RC model, we would express function $g(.)$ in terms of model parameters for the purpose of simulation later in the chapter.

2.3 Building Heat Transfer Model

Multiple building modeling strategies have been proposed in the literature, which include the finite element method based model [15], lumped mass and energy transfer model [16], and graph theoretic model based on electrical circuit analogy [17], [18], [19], [20], [21]. The system model selection entails a trade-off between computational efficiency and accuracy of representation of the temperature dynamics. For the purpose of this study we take the electrical circuit analogy approach, and combine it with the distributed consensus algorithm to achieve collaborative temperature control of buildings. A building is modeled as a collection of interconnected zones, with energy/temperature dynamics evolving according to a lumped heat transfer model. In the lumped heat transfer model, a single zone is modeled as a thermal capacitor and a wall is modeled as an RC network. This results in the standard lumped 3R2C wall model [22]. The heat flow modeling is based on temperature difference and thermal resistance: $Q = \Delta T/R$, where ΔT is the temperature difference, R is the thermal resistance and Q is the heat transferred across the resistance. This is analogous to the current due to voltage difference across a resistor. Also, note that the thermal capacitance denotes the ability of a space to store heat: $C\frac{\mathrm{d}\Delta T}{\mathrm{d}t} = Q$.

The heat flow and thermal capacitance model can be written for all the thermal capacitors in the system, with T_i as the temperature of the ith capacitor. Consider the system to have n thermal capacitors and l thermal resistors. Taking the ambient temperature (T_∞) into account, and neglecting any "thermal noise" in the system, we can write the overall heat transfer model of the system with m zones as [14]:

$$C\dot{T} = -DR^{-1}D^T T + B_0 T_\infty + Bu \tag{4}$$

where $T \in \mathbb{R}^n$ is the temperature vector (representing the temperature of the thermal capacitors in the model), $u \in R^m$ is the vector of heat inputs into the different zones of the building, and $B \in \mathbb{R}^{n\times m}$ is the corresponding input matrix. Also, note that (T,u) are functions of time $(T(t),u(t))$ and accordingly $\dot{T} = \frac{dT}{dt}$. Note that positive values of u correspond to heating the system while negative values of u correspond to cooling. In the above equation, $C \in \mathbb{R}^{n\times n}$ consists of the wall capacitances and is a diagonal positive definite matrix; $R \in \mathbb{R}^{l\times l}$ consists of the thermal resistors in the system and is a diagonal positive definite matrix as well. Also, $D \in \mathbb{R}^{n\times l}$ is the incidence matrix, mapping the system capacitances to the resistors, and is of full row rank [19], and $B_0 = -DR^{-1}d_0^T \in \mathbb{R}^n$ is a column vector with non-zero elements denoting the thermal conductances of nodes connected to the ambient.

In our model, the zones are picked such that each of them has a heating/cooling unit, which in turn implies that B is of full row rank. Also, since matrix D is of full row rank the product $DR^{-1}D^T$ is a positive definite matrix. The vector of zone temperatures, denoted by y (which is a function of T) can be expressed as,

$$y = B^T T. \tag{5}$$

3 Singular Perturbation Method

In this section we present a novel algorithm using gradient optimization and analyzed with *singular perturbation theory*. The user input when combined with the resulting energy cost, determines the direction in which the energy control input is adjusted. We consider a multi-zone building, and use a lumped heat transfer model based on thermal resistance and capacitance for system analysis. Collecting user feedback before the system has settled down sufficiently close to the equilibrium value (for a given control input) is problematic however, as such user feedback do not correctly reflect the effect of the current control input on user discomfort. This implies that for convergence of our control algorithm to the optimal, sufficient separation between the user feedback frequency and the dynamics of the system is necessary. We use singular perturbation theory to analyze the system, with temperature evolution on a faster time scale and user input on a relatively slower time scale. With such time scale separation, we establish the stability condition under which the proposed control algorithm achieves convergence to a desired temperature that minimizes the sum of total energy cost and the aggregate occupant discomfort. We run simulations (using parameters of our smart building testbed) as well as conduct experimental study in the testbed to establish viability and evaluate the performance of our proposed algorithm.

3.1 Solution Approach

Using equilibrium condition (setting $\dot{T} = 0$ in (4)) we obtain:

$$T = h(u) = (DR^{-1}D^T)^{-1}(B_0 T_\infty + Bu). \tag{6}$$

$$\text{Define, } J(u) = U(u, h(u)), \tag{7}$$

i.e., $J(u)$ is obtained by plugging in $T = h(u)$ from (6) into (2). Note that energy cost term in (2) is strictly convex in u; and the aggregate occupant term is convex in T, and therefore convex in u when T is set to $h(u)$, since $h(u)$ is affine in u.

This implies that $J(u)$ is strictly convex in u. Therefore $J(u)$ has a unique optimal solution u^*. Define

$$T^* = h(u^*), \tag{8}$$

which is also unique by definition.

With the goal of driving the system to (u^*, T^*), we propose the control input u be updated once every Δ time units as

$$u_{k+1} = u_k - \eta\left(\Gamma u + Y\Lambda F(y)\right), \tag{9}$$

where η is a scalar that can be loosely interpreted as the "feedback gain" of the system.

Furthermore, $Y \in \mathbb{R}^{m\times m}$ in the above is the Jacobian obtained using (5) and the equilibrium condition (6), expressed as

$$Y = (\frac{\partial y}{\partial u}) = B^T(DR^{-1}D^T)^{-1}B. \tag{10}$$

Also, $\Lambda \in \mathbb{R}^{m\times\rho}$ is the zone-occupant matrix that indicates which occupants are present in a zone ($\Lambda_{js} = 1$ if $s \in S_j$, and 0 otherwise), and $F(y) \in \mathbb{R}^{\rho\times 1}$ is the "marginal discomfort" vector of the occupants, obtained by taking partial derivative of the occupant discomfort functions with respect to y.

In other words, the s^{th} element of $F(y)$, where $s \in S_j$, is obtained as

$$F_s(y_j) = \frac{\mathrm{d}D_i(y_j)}{\mathrm{d}y_j}, \ \ s \in S_j. \tag{11}$$

Comparing (9) with (2) provides the motivation of our control algorithm: roughly speaking, (9) updates u in the gradient direction of $U(u,T)$, while taking in account the relationship between T and u at equilibrium, as given by (6). In other words, it attempts to update u is the direction of $-\nabla J(u)$, where $J(u)$ is defined by (7). In this interpretation, η represents the constant "step size" associated with the gradient descent.

Note however that using (2) - (7), $\nabla J(u)$ is expressed as:

$$\nabla J(u) = \Gamma u + Y\Lambda F(B^T h(u)). \tag{12}$$

From (12) we note that update of u in the gradient direction of $J(u)$ requires user discomfort feedback at $y = B^T h(u)$, the equilibrated zone temperatures corresponding to u. In practice, however, a user $s \in S_j$ will provide a comfort feedback at the current temperature it experiences, $y_j = [B^T T]_j$ (different in general from the equilibrated temperature $[B^T h(u)]_j$), which is what we incorporate into our control algorithm as stated in (9). This implies that our control algorithm as described in (9) does not exactly move u in the gradient direction ($-\nabla J(u)$).

The effect of this difference (error) can be analyzed using *singular perturbation theory* [43], [44], which in our case requires (for convergence to optimality) that the occupant feedback be collected after long intervals (i.e. Δ is large), allowing the temperature T to settle down close to $h(u)$ before the next occupant feedback collection.

Towards developing a singular perturbation model of our system, we first consider a continuous approximation to the evolution of the control input u:

$$\dot{u} \approx \frac{u_{k+1} - u_k}{\Delta} = -\frac{\eta}{\Delta}\Big(\Gamma u + Y\Lambda F(y)\Big). \tag{13}$$

Note that time step Δ is the interval at which user feedback is solicited and the control input u is updated.

A larger Δ implies a slower evolution of u.

We next express the system evolution in the time scale of the evolution of u (slower time scale as compared to the time scale at which T evolves).

Define $\varepsilon = \frac{1}{\Delta}$ as the perturbation parameter; then $\tau = \frac{t}{\Delta} = \varepsilon t$ is the slower time scale. Then

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \varepsilon \implies \dot{u} = \frac{\mathrm{d}u}{\mathrm{d}t} = \varepsilon\big(\frac{\mathrm{d}u}{\mathrm{d}\tau}\big);\;\; \dot{T} = \frac{\mathrm{d}T}{\mathrm{d}t} = \varepsilon\big(\frac{\mathrm{d}T}{\mathrm{d}\tau}\big). \tag{14}$$

Control input equation (13) can now be expressed in terms of τ as follows, using the fact that $y = B^T T$:

$$\frac{\mathrm{d}u}{\mathrm{d}\tau} = -\eta\Big(\Gamma u + Y\Lambda F(B^T T)\Big). \tag{15}$$

Similarly, equation (4) modeling the temperature evolution of the building can now be expressed as:

$$C\varepsilon\frac{\mathrm{d}T}{\mathrm{d}\tau} = -DR^{-1}D^T T + B_0 T_\infty + Bu. \tag{16}$$

Equations (15) and (16) represent a singularly perturbed system. Note, $\Delta \uparrow \implies \varepsilon \downarrow$ leading to steady state condition for temperature evolution. In the next section as we establish the global asymptotic stability of our system as given by equations (15) and (16).

Finally, note that implementation of our control algorithm would require that $F_s(y_j)$, the "marginal discomfort" value of user s in zone j at the current zonal temperature $y_j = [B^T T]_j$, be reasonably estimated from the discomfort feedback of s at any time. In practice, the occupants may provide the feedback in some simple form describing their actual level of discomfort ("I am feeling hot", "I am feeling very cold" etc.). This feedback must be processed to estimate the marginal discomfort (derivative of the actual discomfort function), as we do in our simulation study described in Section 5.

3.2 System Analysis

The system evolution is governed by the set of equations (15) and (16). In equation (16) the coefficient $DR^{-1}D^T$ is positive definite which makes the unforced system (with $u = 0$) exponentially stable. We use singular perturbation analysis [44] to establish the condition for stability of the system.

Theorem 1 *There exists an $\varepsilon^* > 0$ such that (u^*, T^*) is a globally asymptotically stable equilibrium of the system given by (15) and (16) for all $\varepsilon < \varepsilon^*$.*

We just introduce the Lyapunov functions $V(u)$ and $W(u,T)$ that is used in the stability analysis:

$$V(u) = J(u) - J(u^*), \text{ and} \tag{17}$$

$$W(u,T) = (T - h(u))^T P(T - h(u)), \tag{18}$$

where P in the above equation is a symmetric positive definite matrix (the exact choice of matrix P will be determined at a later stage). We now define a combined Lyapunov function $L(u,T)$:

$$L(u,T) = (1-\alpha)V(u) + \alpha W(u,T), \tag{19}$$

where α satisfies $0 < \alpha < 1$.

Detailed analysis involves evaluating the conditions to establish stability using Theorem 2.1 and Corollary 2.1 from chapter 7 of [44], and can be referred to in our work [42]. Finally, note that $L(u,T)$ is minimized uniquely at (T^*, u^*). It follows therefore that (T^*, u^*) is a globally asymptotically stable equilibrium point for all $\varepsilon < \varepsilon^*$, or all $\Delta > \Delta^*$.

4 Consensus Algorithms

In the singular perturbation method, the occupants have no incentive to provide truthful feedback. A malicious user can gain the system by providing irrational feedback. Moreover, users or their agents have to provide constant feedback to maintain the temperature at their desired levels. We address these drawbacks through our consensus algorithm developed in this section.

Achieving a common temperature set-point in a distributed framework, where the exact discomfort functions are held privately by each occupant, remains an open question which we address in this section. Pricing per unit temperature change serves as the feedback signal to the occupants, to drive them to a consensus on zonal temperatures that optimize the overall discomfort plus energy cost. The consensus algorithm that we develop, through the use of the alternating direction method of multipliers (ADMM), is amenable to distributed implementation and has the following appealing properties. Firstly, occupants (or their agents) are only assumed to be rational, in that they choose their preferred temperature set- points so as to minimize their personal discomfort plus energy cost, given the pricing signals. In other words, the occupants are not required to explicitly declare their discomfort functions (which can be held privately), but only react rationally to the pricing signals by choosing their preferred temperature set- point. On the other hand, the building thermal management system (BTMS) chooses the zonal temperature set-points to maximize the overall profit of the building operator (for the current prices); the price signals are then updated so as to attain consensus among the occupants, and with the building operator, on the zonal temperatures. Finally, as we formally show, the algorithm converges to the optimal zonal temperatures, from which rational occupants would not have any incentive to deviate. In terms of practical implementation, occupant feedback could be obtained through a smart phone application, and the zonal temperature set-points could be calculated by the building operator on a central server.

4.1 Solution Approach

If the individual occupant discomfort functions are assumed to be known to the building operator, the optimal zonal temperature vector y^* could be computed directly. Such a centralized approach suffers from several practical limitations, however. Firstly, reporting the entire discomfort function to the building operator is complex, and the occupant may not even be able to correctly estimate its discomfort function. Secondly, even if we assume that the occupant knows its discomfort function exactly, there is no incentive for it to report the same truthfully. In practice, therefore, it may be more desirable to have a mechanism through which the building operator indirectly learns about the true discomfort functions of the occupants, who are providing their temperature preference feedback in a simple and convenient format, acting in best response to some price signals provided by the building operator. Furthermore, the price signals should be such that it guides the occupants towards a *consensus*, i.e., rational users (acting in self-interest) in a zone will end up agreeing on their temperature choice for each zone. The distributed consensus algorithm that we describe in the next subsection works according to the above principles.

To provide an overview of our approach, we first introduce new notations to denote the choice of zonal temperatures by the occupants and the building thermal management system (BTMS); these temperature choices will in general be different from the actual (current) zonal temperatures. Let x_{ij} denote the desired temperature of occupant $i \in S_j$ located in zone j. Let z_j denote the target temperature of zone j as set by the BTMS. Then vector z represents the target temperature of the entire building consisting of m zones. In general, x_{ij} for any occupant $i \in S_j$ can differ from z_j; the actual zonal temperature y_j could also differ from these temperatures. On convergence however, the consensus algorithm ensures that x_{ij} for all occupants $i \in S_j$ equals z_j, which optimizes the objective function in (2). The zonal temperatures obtained through consensus is then attained in the building by utilizing some temperature set-point based HVAC control system.

In the distributed consensus algorithm, the BTMS provides pricing signal p_{ij} and target zonal temperature z_j to each occupant i located in zone j. Based on that, occupant i chooses its desired temperature x_{ij} and sends it back to the BTMS, which is then used to recompute the new target zonal temperatures and pricing signals. Fig-

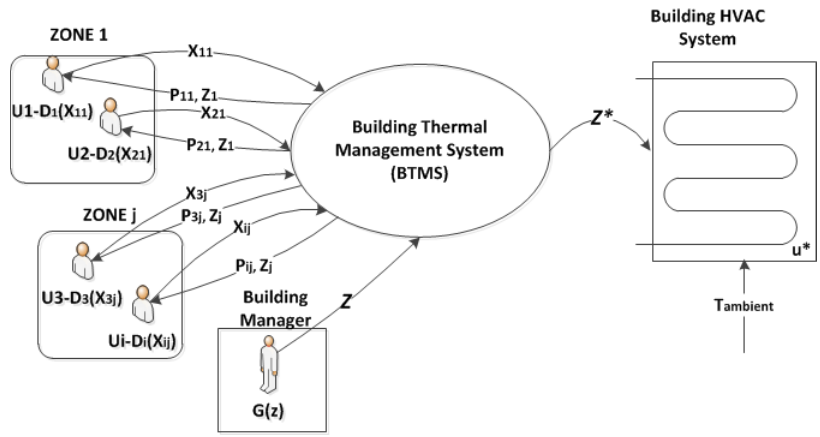

Fig. 2 Distributed consensus algorithm flow between the building occupants and the central building management system.

ure 2 provides an overview of how the distributed consensus algorithm works. Upon convergence of the distributed consensus algorithm, let x^*, z^* and p^* respectively denote the (vectors of) occupant temperature choices, zonal target temperatures, and pricing signals. Then we desire that x^*, z^*, p^* satisfy the following properties:

- *(Individual Rationality)* Each occupant chooses its desired temperature so as to minimize its total cost function, represented as the sum of its discomfort plus the energy price paid to the building operator:

$$x_{ij}^* = \arg\min_{x_{ij}} \left\{ D_i(x_{ij}) + p_{ij}(x_{ij} - z_j) \right\}.$$

- *(Consensus)* For each zone, the temperature choices of the occupants of the zone agree with each other, and with the target zone temperature set by the BTMS:

$$x_{ij}^* = z_j^*, \ \forall i \in S_j.$$

- *(Optimality)* The target zone temperatures minimize the aggregate occupant discomfort plus the building energy cost, given by (2):

$$z^* = \arg\min_{z} \sum_{j=1}^{m} \sum_{i \in S_j} D_i(z_j) + E(g(z)).$$

4.1.1 Distributed Consensus Algorithm

To develop the consensus algorithm, we re-write the minimization objective in (2) in terms of the zonal temperature choices of the occupants and the BTMS, as:

$$\text{minimize} \sum_{j=1}^{m} \sum_{i \in S_j} D_i(x_{ij}) + G(z)$$
$$\text{subject to } x_{ij} = z_j, i \in S_j, \tag{20}$$

where function $G(z) = E\big(g(z)\big)$ represents the total energy cost in terms of the target zonal temperature vector z.

We can now solve (20) through the ADMM approach as described in [48]. The ADMM approach blends the decomposability of dual ascent with the superior convergence properties of the method of multipliers, to develop an algorithm that is amenable to distributed implementation, and also has good convergence properties.

To motivate the ADMM based consensus algorithm, let us consider the augmented Lagrangian:

$$L_\rho(x,z,p,\rho) = \sum_{j=1}^{m} \sum_{i \in S_j} \Big(D_i(x_{ij}) + p_{ij}(x_{ij} - z_j) + (\rho/2)|x_{ij} - z_j|^2 \Big) + G(z) \tag{21}$$

where p_{ij} is the dual variable, $\rho > 0$ is a constant.

The ADMM based consensus algorithm can then be derived as iterations of coordinate-wise optimization of this augmented Lagrangian along each x_{ij} and z directions, followed by update of the dual variable in a gradient direction. More precisely, in our consensus algorithm, in iteration $k = 1, 2, \ldots$, the variable vector z, and the variables x_{ij}, p_{ij} for all $i \in S_j,\ j = 1, \ldots m$, are updated as follows:

$$x_{ij}^{k+1} := \underset{x_{ij}}{\operatorname{argmin}} \Big(D_i(x_{ij}) + p_{ij}^k x_{ij} + (\rho/2)|x_{ij} - z_j^k|^2 \Big), \tag{22}$$

$$z^{k+1} := \underset{z}{\operatorname{argmin}} \Big(G(z) + \sum_{j=1}^{m} \big(- \sum_{i \in S_j} p_{ij}^k z_j + \sum_{i \in S_j} (\rho/2)|x_{ij}^{k+1} - z_j|^2 \big) \Big), \tag{23}$$

$$p_{ij}^{k+1} := p_{ij}^k + \rho(x_{ij}^{k+1} - z_j^{k+1}). \tag{24}$$

The above set of update equations has a nice game theoretic (price-driven rational-response) interpretation, as follows. The BTMS iteratively communicates to each occupant i in any zone j two parameters, p_{ij} and z_j, based on which the occupant's cost (price paid) for a chosen temperature set-point x_{ij} would be computed as $p_{ij}x_{ij} + (\rho/2)|x_{ij} - z_j|^2$. A rational occupant then chooses its personal temperature preference x_{ij} to minimize their individual cost function:

$$\text{minimize } D_i(x_{ij}) + p_{ij}x_{ij} + (\rho/2)|x_{ij} - z_j|^2. \tag{25}$$

The BTMS, acting on behalf of the building operator, would choose the target building temperature vector z so as to minimize

$$\text{minimize } G(z) - \langle p, z \rangle + (\rho/2)|x - z|^2, \tag{26}$$

which on convergence (when consensus is attained) would equate to the total energy cost incurred by the building operator, when the payments made by the occupants are taken into account. Finally, the per-unit prices (p_{ij}) are updated in a way that helps in the consensus, i.e., in bringing x_{ij} and z_j close to each other in each zone j, for each occupant $i \in S_j$.

4.2 System Analysis

The convergence proof presented in this section assumes that the functions $D(.)$ and $G(.)$ are closed, proper, and convex, and the un-augmented Lagrangian L_o in (27) below has a saddle point.

$$L_o(x, z, p) = \sum_{j=1}^{m} \sum_{i \in S_j} \Big(D_i(x_{ij}) + p_{ij}(x_{ij} - z_j) \Big) + G(z). \tag{27}$$

Based on these assumptions we establish the objective convergence, the residual convergence, and the convergence of the dual variables, for our consensus algorithm as described in Section 4.1.1. In doing so, we utilize the convergence analysis of the ADMM approach as described in [21], suitably adapted to our model. Consider the objective,

$$\begin{aligned} O^* &= \text{minimum } \sum_{j=1}^{m} \sum_{i \in S_j} D_i(x_{ij}) + G(z) \\ &= \sum_{j=1}^{m} \sum_{i \in S_j} D_i(x^*_{ij}) + G(z^*), \end{aligned} \tag{28}$$

where x^*_{ij} and z^* denote the corresponding optimal values of temperature choices. Note that for any zone j, $x^*_{ij} = z^*_j$ for all $i \in S_j$. Also, define residual for zone j as:

$$r_{ij} = x_{ij} - z_j \tag{29}$$

We prove our result through a sequence of lemmas, each involving an inequality (refer to [47] for complete proof of the lemmas).

Lemma 1.

$$O^* - O^{k+1} \leq \sum_{j=1}^{m} \sum_{i \in S_j} p^*_{ij} r^{k+1}_{ij}. \tag{30}$$

Lemma 2.

$$O^{k+1} - O^* \le -\sum_{j=1}^{m}\sum_{i\in S_j}\Big(p_{ij}^{k+1} r_{ij}^{k+1} \\ +\rho(z_j^{k+1}-z_j^k)(-r_{ij}^{k+1}-(z_j^{k+1}-z_j^*))\Big). \tag{31}$$

Next, define Lyapunov function V for the ADMM algorithm as:

$$V^k = (1/\rho)\sum_{j=1}^{m}\sum_{i\in S_j}|p_{ij}^k - p_{ij}^*|^2 + \rho\sum_{j=1}^{m}|z_j^k - z_j^*|^2 \tag{32}$$

This Lyapunov function satisfies the inequality as stated in the lemma below.

Lemma 3.

$$V^{k+1} \le V^k - \rho\sum_{j=1}^{m}\sum_{i\in S_j}|r_{ij}^{k+1}|^2 - \rho\sum_{j=1}^{m}|z_j^{k+1} - z_j^k|^2 \tag{33}$$

Now, since $V^k \le V^0$, p_{ij}^k and z_j^k are bounded. Iterating (33) gives:

$$\rho\sum_{k=0}^{\infty}\big((r_{ij}^{k+1})^2 + |z_j^{k+1} - z_j^k|^2\big) \le V^0, \tag{34}$$

which implies $r_{ij}^k \to 0$ and $|z_j^{k+1} - z_j^k| \to 0$ as $k \to \infty$. Further, from inequalities (30) and (31) we have $\lim_{k\to\infty} O^k = O^*$ or the objective convergence.

Hence, the inequalities (30), (31) and (33) implies the convergence of our algorithm.

4.2.1 Profit Analysis for the Building Operator

Let the vectors y^* and p^* respectively denote the optimal zonal temperatures and the optimal prices. It is easy to establish that the optimal pricing feedback signal p_{ij}^* satisfies:

$$p_{ij}^* = -D_i'(y_j^*). \tag{35}$$

We can further show that,

$$\sum_{i\in S_j} p_{ij}^* = \frac{\partial G(y^*)}{\partial y_j}. \tag{36}$$

Now, we can obtain an expression for the difference of the net revenue from occupants and the operational cost of maintaining the building at the consensus temperature set-point. Note, that this expression represents the operational profit of the building operator and is given by:

$$\sum_j \sum_{i\in S_j} p_{ij}^*(y_j^* - T_\infty) - (G(y^*) - G(T_\infty)). \tag{37}$$

Using Taylor series:

$$\begin{aligned} G(T_\infty) &= G(y^*) + (T_\infty - y^*)^T \nabla G(y^*) \\ &\quad + \frac{1}{2}(T_\infty - y^*)^T \nabla^2 G(y^*)(T_\infty - y^*), \end{aligned} \tag{38}$$

the building operator's profit can be further expressed as:

$$\frac{1}{2}(T_\infty - y^*)^T \nabla^2 G(y^*)(T_\infty - y^*). \tag{39}$$

Since $G(y)$ is convex in y, $\nabla^2 G(y^*) \geq 0$, and (39) is upper bounded by $\leq \frac{\lambda_{max}}{2}||y^* - T_\infty||^2$, where λ_{max} is the maximum eigenvalue of $\nabla^2 G(y^*)$. From (39) we can assert that if the pricing signals were to be translated to real money (or equivalent credit) transaction between the building operator and the occupants, he building operator does not lose money, and instead may end up making a small profit that is bounded by the convexity of the energy cost function $G(y)$. The expression in (39) equals zero ($= 0$) when $G(y)$ is affine in y. Therefore, when $G(y)$ is affine in y, perfect budget balance is attained, i.e. payments (credits) of the users are just redistributed between themselves, and the building operator does not make any profit or loss.

5 Simulation Study

5.1 Testbed Layout

We consider our six zone physical testbed of an intelligent building located in Watervliet, NY for simulation and experimental study of our proposed algorithm and validating performance of the same. Figure 3 represents the dimensions of the facility as generated using the BRCM toolbox [45]. BRCM toolbox is used to generate the RC model of the six zone test facility, mapping it to 31 building elements resulting in a total of 93 capacitive elements.

Each zone of the testbed (except for zone 2, which is the hallway) is actuated with thermo electric coolers. Real time temperature sensing is enabled through J-type thermocouples spread across the test facility. Sensor data is acquired through wireless communication in real time to a central server, which also runs the control loops to operate the coolers and achieve the desired ambient condition. Further details of the test bed layout, instrumentation, and software architecture can be referred to in [46].

The simulated occupancy of the building is represented in Figure 4. Zones 1 and 6 are occupied by two occupants each and the other zones 3, 4 and 5 have one occupant each. Occupants U1 and U2 are in Zone 1, U3 in Zone 3, U4 in Zone 4, U5 in Zone 5, and finally U6 and U7 in Zone 6.

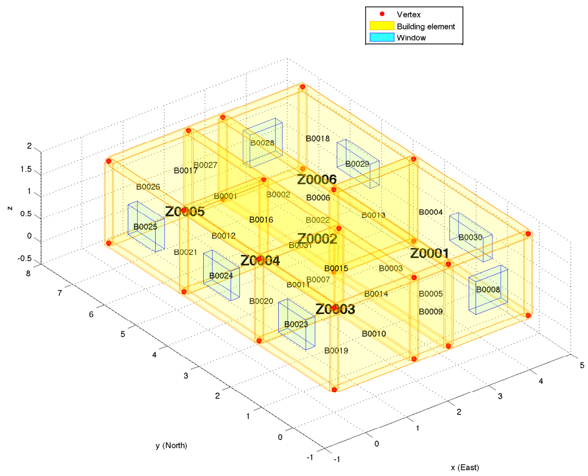

Fig. 3 Watervliet based smart building test bed with the building elements as generated by the BRCM Toolbox. This set up is used for the simulation and experimental study of the solution framework.

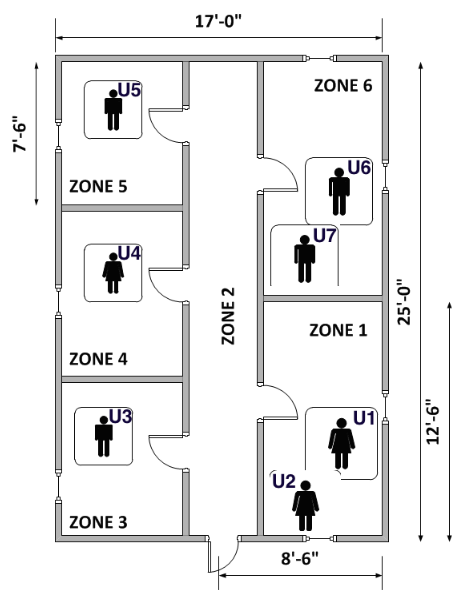

Fig. 4 Layout of the Watervliet test facility with the simulated occupancy pattern of each zone used for simulation and experimental study.

All the occupants have their own specific temperature preference as depicted in Figure 5. Note that the occupants U1 and U2 co-located in Zone 1 have no common range of comfort preferences, whereas the occupants U6 and U7 co-located in Zone 6 have an overlapping region of comfort preference. This distribution enables us to capture all possible scenarios in terms of conflicting and common preferences among co-located occupants.

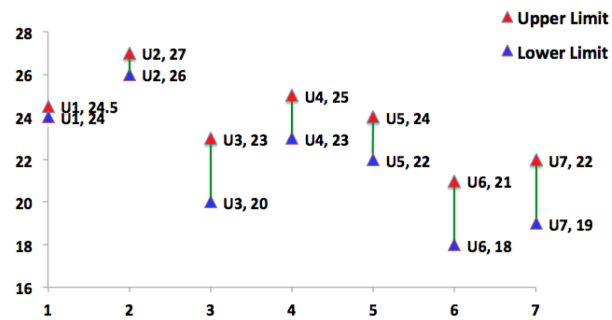

Fig. 5 Preferred temperature comfort range of each occupant in ° C. U6, U7 in Zone 6 have an overlapping comfort preference, whereas U1, U2 in Zone 1 have no common range.

5.2 Simulation Results

For simulation purpose we consider both heating and cooling actuation in each zone with a typical power rating of 1000 W. The system dynamics and the control algorithm are simulated using MATLAB and SimuLink. Using the occupancy distribution as per Figure 4, occupant preference from Figure 5 and the model parameters for the Jacobian generated using BRCM toolbox, we first simulate temperature dynamics for a 48 hour period with a fixed ambient condition of 30°C. Figure 6 represents the temperature dynamics for the 48 hour period simulation run.

The temperature of zone 6 settles at 21°C, which is acceptable to both the occupants U6 and U7, and simultaneously energy optimal being closer to the ambient temperature. Note that anything between 19°C to 21°C would have been comfortable for both U1 and U2 based on their comfort preferences, with 21°C being optimal for the given ambient condition. Zone 1 settles around 25.5°C, which tends to minimize the aggregate discomfort of both users U1 and U2 and simultaneously minimizes energy consumption considering the thermal correlation among all the zones. Similarly, the temperature of other zones also settle at a point to minimize aggregate occupant discomfort and the energy cost. In this simulation we also set the initial condition of each zone to the ambient temperature as that represents the

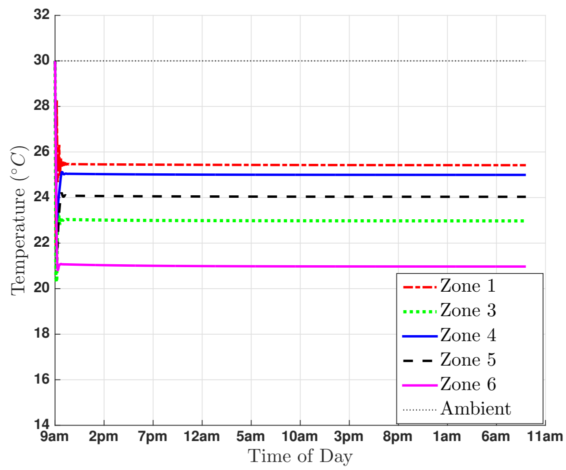

Fig. 6 Temperature dynamics for a 48 hour period simulation with ambient condition higher than the occupant preferences with occupancy pattern as per Figure 4.

extreme scenario, thus providing a good performance evaluation of the proposed algorithm. In most practical condition the initial zonal temperatures would be much closer to the corresponding desired temperatures.

To demonstrate the energy saving in this scenario, we compare it to the prevalent set point based method of temperature control in buildings. With the given occupant preferences, set point method would consider the mid point of each occupants comfort range as the zonal set point. In case of multiple occupants an average of the occupant set point can be used. Using this approach the corresponding zonal set points are presented in Table 1. Compared to the set point based approach, our algorithm achieves energy optimal temperature for the zones and in this particular case results in an energy saving of 12.1%.

Table 1 Temperature set point for each zone when using the set point based method of building temperature control.

Location	Occupant set points (° C)	Zonal set points (° C)
Zone 1	24.25, 26.5	25.4
Zone 3	21.5	21.5
Zone 4	24	24
Zone 5	23	23
Zone 6	20.5, 19.5	20

Next we simulate another case with fixed ambient temperature at 15°C. The corresponding temperature dynamics are represented in Figure 7. Note that in this case the temperature of Zone 6 settles at 19°C, compared to 21°C in Figure 6 as 19°C is more energy optimal for the ambient condition of 15°C.

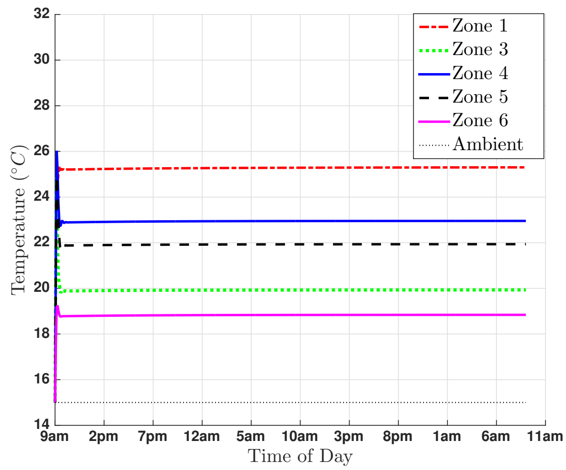

Fig. 7 Temperature dynamics for a 48 hour period simulation with ambient condition lower than the occupant preferences with occupancy pattern as per Figure 4.

Fixed ambient temperature is an over simplification and does not represent true variations in ambient conditions. Next, we consider a sinusoidal variation of the ambient with T_∞ taking the form: $T_\infty = 20°C + 5°C\sin(2\pi t/t')$, with $t' = 24$ hr. The corresponding temperature dynamics are presented in Figure 8. The control algorithm lets the zonal temperature vary with ambient till it hits the comfort limit, at which point appropriate heating/cooling input is applied. This approach is more optimal than maintaining a fixed set point as it harnesses the ambient variation without the need to constantly re-adjust the fixed set point. For our particular setup we observed a relative energy saving of 5.3%.

Another simplification that has been applied to our study so far has been the uninterrupted occupancy of the occupants throughout the period of 48 hours. We next consider a typical work environment occupancy schedule of the occupants entering their respective zones at 9 am in the morning and departing at 5 pm in the evening. Further, we also consider lunch time and simulate non occupancy during 12 to 1 pm. In the evening after 5 pm we increase the heat cost factor Γ. For the lunch break heat cost factor remains unchanged but due to non-occupancy no occupant feedback is generated. The results are presented in Figure 9.

Due to increase in energy cost factor after 5 pm, the zonal temperatures tend to follow ambient variation resulting in immense energy savings. The kink in Zone 1

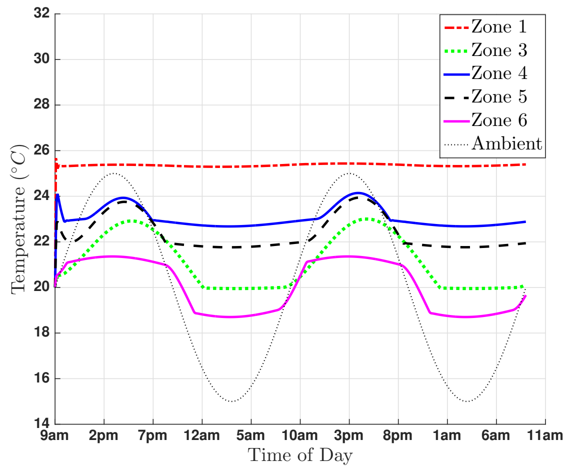

Fig. 8 Temperature dynamics for a 48 hour period simulation where ambient condition follows a sinusoidal variation with period of 24 hours with occupancy pattern as per Figure 4.

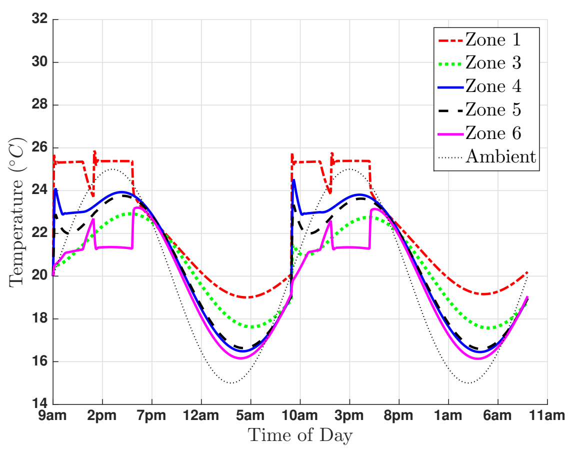

Fig. 9 Temperature dynamics corresponding to 48 hour simulation of occupancy schedule along with sinusoidally varying ambient with occupancy pattern as per Figure 4. Energy cost is increased between 5 pm to 9 am with no occupant feedback during lunch time.

and Zone 6 temperatures during lunch time is attributed to thermal correlation as no occupant feedback is generated resulting in temperature variation as per prevailing thermal conditions. Compared to a set point based approach (with predetermined energy saving set point after 5pm) this approach can result in energy savings of 6.1%. Note that if the set point based approach does not implement energy saving mode after 5pm then the relative saving through our approach would be even higher. Compared to scenario in Figure 8 additional energy saving can be attributed to efficiency during lunch break.

Table 2 Ideal temperature setting in $^{\circ}C$ of each zone as per its occupant and the building operator

Zone	**Occupant(s) pref**	**Building Operator pref**
Zone 1	18.5 & 19$^{\circ}C$	15$^{\circ}C$
Zone 3	20$^{\circ}C$	15$^{\circ}C$
Zone 4	21$^{\circ}C$	15$^{\circ}C$
Zone 5	22$^{\circ}C$	15$^{\circ}C$
Zone 6	22.5 & 23$^{\circ}C$	15$^{\circ}C$

In Figure 10 we present the result of the distributed consensus algorithm using ADMM approach. Each zone (room) occupant agent starts with the ideally preferred temperature set-point of the corresponding occupant as per Table 2 and the BTMS with the preferred set-point of the building operator for the corresponding zones.

Each iteration in Figure 10 represents one round of communication (computation) between (at) the smart applications and the BTMS. So 100 iterations would represent 100 round-trip communication (100 computation) rounds, and typically evaluate to a few seconds. The occupant at their end would only see the final (converged) temperature and penalty factor signal, the intermediate values being internal to the algorithm would not be visible to the occupants.

With each iteration of the algorithm, the difference between the corresponding zonal temperature preference of the occupant and that of the BTMS narrows and finally compromise is attained in all the zones. Note that in Figure 10 the consensus temperature of zone 6 comes down to that of zone 5, irrespective of the zone 6 occupants preferring much higher temperature than the occupant of zone 5. This is in accordance with the energy cost attributed to maintaining zone 6 relative to zone 5. The trend can also be reasoned from the penalty factor curve in Figure 11, as the penalty factor feedback to zone 6 occupants is much higher compared to the occupant of zone 5. Further, the consensus temperature for both the occupants of zone 1 and zone 6 converge to the respective consensus zonal temperature.

The penalty factor for unit change in temperature varies with each iteration, as shown in Figure 11. The penalty factor increases for the zone occupant if the temperature choice is away from the BTMS' preference and the ambient temperature. In Figure 11 the per-unit penalty factor for occupant 1 (located in zone 1) turns negative. This can be attributed to the fact that on consensus, the temperature for that zone moves away from the ambient and building operator's preferred temperature for the zone, even beyond the occupant's preferred value.

The penalty factor signals in the form of notifications/information can serve as a means for the building operator to communicate with rational occupants into selecting a lower/higher preferred temperature. An occupant of a particular zone might not be aware of their zone's thermal correlation and the temperature preference of

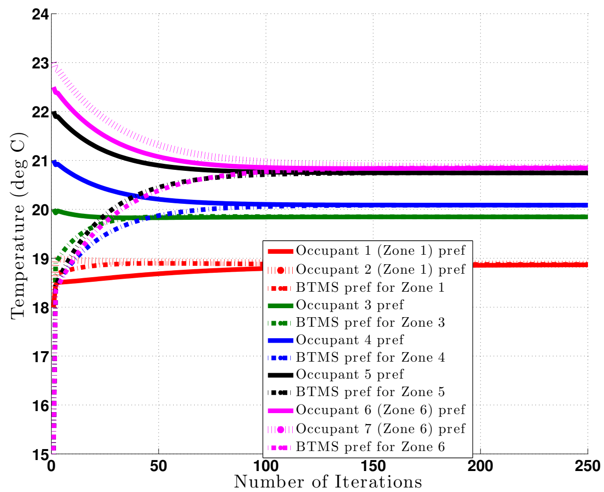

Fig. 10 Convergence of temperature set-point preferences in each zone, for the occupants and the BTMS. The solid lines depict the occupant temperature preferences, and the dashed ones the BTMS' corresponding preferences.

the occupants of their neighboring zones. However, the building operator based on the energy cost data can share that picture with the occupants, through penalty factor signal, without disclosing any private information. Using this information, rational occupants and the building operator can work together to modify their preference and accommodate users with different and at times extreme (which in general incurs greater overall operating energy cost to the building) thermal preferences.

6 Concluding Remarks

In this chapter we have demonstrated that the building temperature and energy usage can be controlled successfully and efficiently through dynamic feedback from the occupants based on their comfort levels. The collaborative temperature control in multi-occupant spaces based on the consensus algorithm attains temperature set-points that minimizes the sum of the aggregate discomfort of the occupants and the total energy cost in the building. Through simulations we demonstrated the energy efficiency achievable through our algorithms.

Key to real world implementation would be an end-to-end framework designed for enabling occupant feedback collection and incorporating the feedback data towards energy efficient operation of a building. A mobile application that occupants can use on their smart phones and other wearables to provide their thermal prefer-

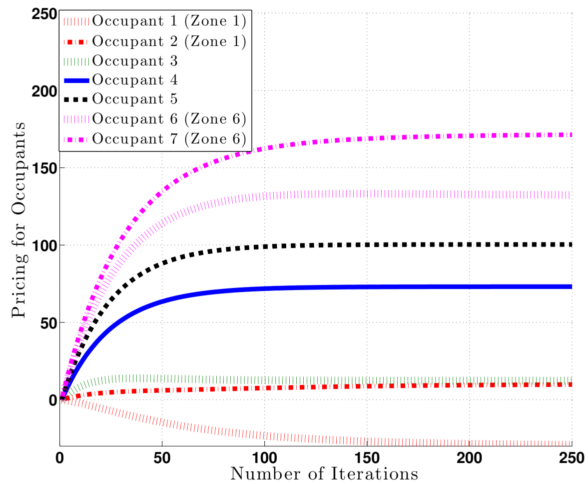

Fig. 11 Variation in penalty factor for the zone occupants for desired change in the zonal temperatures. A negative penalty factor indicates the corresponding occupant receiving reward from the building operator.

ence feedback would be needed, that can also use indoor location techniques to tie the occupant preference to their current thermal zone when relaying the feedback to central server. Using sensors for real time zonal temperature readings, and based on the information from mobile application the central server can use real-time learning algorithm to learn the environment and using occupant feedback calculates the optimal temperature set point. The entire process needs to be triggered upon change of occupancy, environmental conditions, and/or occupant preference. The learning algorithm can be scheduled to run at regular intervals to respond dynamically to environmental and occupancy changes.

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