Proportional Pricing for Efficient Traffic Equilibrium at Internet Exchange Points

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***Abstract*—We analyze traffic exchange between Internet Service Providers (ISPs) at an Internet Exchange Point (IXP) as a non-cooperative game with ISPs as self-interested agents. Each ISP has the choice of exchanging traffic either using the shared IXP facilities, or outside the IXP – through their transit providers or private peering. We analyze the efficiency (social cost optimality) of the traffic exchange equilibrium at the IXP taking into consideration the congestion cost experienced by the ISPs at the IXP, under a proportional pricing model where the per-unit price charged to ISPs is proportional to the aggregate level of congestion at the IXP. We obtain worst case bounds on the efficiency at traffic exchange equilibrium under two different models of the congestion cost (delay) functions. Simulations conducted using data for actual IXPs obtained from PeeringDB demonstrate that the theoretical bounds derived for social cost optimality at equilibrium (measured as the Price of Anarchy) are fairly tight, and correctly capture the performance trends against the variation of key model parameters. Further, the results show that for a certain range of the proportionality constant, proportional pricing not only results in significantly better efficiency compared to zero pricing, but also attains near-optimal social cost and near-optimal IXP revenue simultaneously.**

I. Introduction

A. Background and Motivation

ISPs typically connect (mostly peer) with each other at Internet eXchange Points (IXPs). In most basic terms, an IXP is a data center with network switches through which ISPs form connections (peering relationships) to exchange traffic [1], [2]. In return, IXPs recover their operating costs by charging fees to each member/client ISP. A number of IXPs, especially in Europe, operate on a non-profit basis [3], and the operating costs of an IXP is largely determined by the cost of the infrastructure needed for traffic exchange [4]. Other IXPs, both in Europe and particularly US, operate for profit, e.g., Equinix [5]. In both cases, while IXPs provide the platform for ISPs to connect (peer) with each other, they play a passive role focused on infrastructure cost recovery or profit-making, and the peering decisions are determined bilaterally by the ISPs themselves. Nevertheless, ISPs at an IXP make these peering decisions taking into account the potential quality of service improvements due to peering, the prices charged by the IXP, and comparing those with alternatives such as sending the traffic through their transit providers.

In recent years, transit prices per unit bandwidth have been steadily declining [6]. Despite falling transit costs, peering between ISPs has been on the rise, and content and access ISPs are increasingly getting into peering relationships with each other [7], [8], a phenomenon known as the flattening of the Internet [2], [9]–[11]. It has been shown [12] that almost 80% of the IP addresses can be reached via public peering, and 20% of all the traffic traces go through IXPs. Peering between ISPs, which is typically settlement-free, can help bring content closer to customers, resulting in lower delays and losses, and thus better Quality-of-Experience (QoE) for the end users (content consumers). Some of the recent literature has therefore argued that *paid peering* is necessary for overall stability and efficiency of inter-domain traffic flows [13], though its interplay with traditional settlement-free peering needs careful treatment [14]. Even if the IXPs play a passive role by providing peering facilities for a price, the pricing policy applied by the IXP to facilitate this traffic exchange needs to be designed carefully for making the peering relationships and traffic flows between the ISPs stable and efficient.

B. Contribution of this Paper

This paper investigates how the pricing policy at an IXP impacts the efficiency of the peering relationships that form between the ISPs as a result of that policy. We define the traffic exchange problem between ISPs (at an IXP) as a non-cooperative game between ISPs (selfish agents), where each pair of ISPs have a certain pre-determined amount of traffic to exchange, and the strategic decision involves determining whether to send this traffic through the IXP or through the external routing option. We consider a *proportional pricing* policy, where the price charged by the IXP per unit traffic is proportional to the aggregate level of congestion at the IXP (shared switch). The benefit of proportional pricing – as discussed later in more detail – is that it can be implemented with only aggregate load information at the shared switch used for public peering. In other words, choosing a good pricing does not require the IXP to know the transit options or other details about the participating ISPs. We also analyze the performance benefits of proportional pricing over zero pricing where the IXP does not charge any price at all.[1]

[1]Even with zero pricing, ISPs still face congestion costs at the shared switch and must determine its routing/peering choice taking into account how that cost compares with its alternate routing option. The zero pricing model is applicable to non-profit IXPs.

More specifically, this work makes the following contributions. First, we characterize the pricing policy that is economically efficient, i.e., attains the socially optimal traffic exchange solution (Section II). Then in Section III, we characterize the traffic flow efficiency (i.e., social cost optimality) of the equilibrium solutions – measured by the Price of Anarchy (PoA) of the system – under *proportional pricing*, where IXPs charge traffic a per-unit price that is proportional to the average level of congestion experienced by traffic in the public switch operated by the IXP. The quality of service experienced by the ISPs also suffers due to this congestion, and is taken into account as well. The PoA for *zero pricing*, where the traffic through the IXP only experiences a congestion cost, but no additional price is charged by the IXP, also follows from this result as a special case. We quantify the PoA under these pricing systems using two broad classes of delay functions (polynomial delay and queuing delay functions), and also discuss how the PoA results generalize when the external routing costs between ISPs are asymmetric. Finally, in Section IV, we simulate real-world scenarios of public peering in IXPs, and compare the actual performance values obtained against their respective theoretical bounds for a wide range of model parameters. Proofs of all theoretical results are omitted due to space limitations, but can be found in [15].

C. Comparison with Prior Work

Our game-theoretic model and analysis is inspired by a prior line of work on network formation games (introduced in [16]), where the stability of networks was modeled and analyzed when two nodes can build links mutually but can sever links individually. These types of network formation games and their extensions have been studied extensively for different settings (e.g., [17]–[26]). Unlike these prior studies, in our model the cost of forming these (peering) connections is not fixed, but depends on both the congestion (measured by the total number of connections already formed), and the prices charged by the planner (IXP). In this sense, our work is a generalization of the previous work on network formation games, as it includes a central planner (the IXP), who can greatly affect the quality of outcomes by choosing different pricing schemes. There are many prior works on pricing network services and traffic ([27]–[32], to list a few), but these models do not consider an IXP setting and hence are not directly related to ours.

Our work is most related to the model in [33], but differs from this existing work in several important aspects. First, while [33] considers the question of how the operational cost of the IXP should be shared among the ISPs (which is more representative of non-profit IXP operations), in our model the IXP directly charges the ISPs for their traffic. Secondly, we also analyze the revenue earned by IXPs at equilibrium, and the trade-offs between social cost and IXP revenue. Finally, our work also models congestion cost at the IXP, considers asymmetric external routing costs and paid peering, and evaluates social optimality and IXP revenue for the pricing policies through extensive simulations.

TABLE I
SUMMARY OF COMMONLY USED NOTATION.

Term	Description
y_{ij}	Traffic of ISP pair (i,j) sent publicly through the IXP.
y_i	$\sum_j y_{ij}$, total traffic of ISP i going through the IXP.
y	$\frac{1}{2}\sum_i \sum_j y_{ij}$, total traffic flowing through the IXP.
$\overrightarrow{y}$	Total traffic allocation vector (vector of values y_{ij})
λ_{ij}	Per-unit cost incurred by (i,j) for routing traffic externally
$d(y)$	Congestion cost per unit traffic incurred at the IXP
$p(y)$	Price per unit traffic set by the IXP

II. System Model and Properties

A. Game-Theoretic Model

We consider an IXP, and a set N of ISPs (agents in our game-theoretic model) that are involved in traffic exchange through a public switch offered by the IXP. An ISP pair (i,j) has a total traffic demand of B_{ij} between themselves; part of this traffic, y_{ij}, is routed through the public switch, while the rest is sent *externally*. The traffic sent externally (i.e., outside of the public switch) is typically done in one of two ways: (a) through private peering between ISPs i and j;[2] (b) through the use of the ISPs' transit service providers. The traffic that is exchanged through the public switch incurs a congestion cost of $d(y)$ per unit traffic, which depends on the total traffic y sent through the switch. This congestion cost will typically be reflected in terms of average delay experienced by the traffic (and therefore we will sometimes use the terms 'congestion cost' and 'delay' interchangeably); however, $d(y)$ could also represent other Quality-of-Service (QoS) parameters (or a combination of them) that are affected by the overall load at the public switch. Additionally, each ISP has to pay a price of $p(y)$ to the IXP per unit traffic, for the use of the public switch. We assume that $d(y)$ and $p(y)$ are given functions (i.e., not part of the strategy); however, we will explore the efficiency of the equilibrium for different forms of the functions $p(y)$ and $d(y)$. For the traffic sent externally, ISP i encounters a per-unit cost of λ_{ij} for traffic exchange with ISP j. This cost may be in terms of additional traffic delays due to longer routes, transit price paid to the ISP's provider, or the cost ISP i incurs for private peering with ISP j. The strategy of each agent (ISP) involves deciding how much of its traffic it should send through the public switch, as opposed to sending externally. In making this decision, we assume that each ISP acts selfishly, focusing on minimizing its own cost. The decisions of ISPs i and j are coupled, and they must agree upon the amount of traffic y_{ij} of the B_{ij} units that is sent through the IXP. Table I summarizes some of the most commonly used terms and notations in our model.

Remarks on the model: In the following, by 'traffic sent through the IXP', we refer to the traffic sent through the public switch at the IXP. Thus any traffic that is sent through private peering (even if the private peering happens at the same IXP under consideration) is considered a part of the externally

[2]The private peering can happen at an IXP (if the IXP offers private peering services), or separately.

routed traffic, i.e., included in $B_{ij} - y_{ij}$ for ISP pair (i, j). Finally, we do not distinguish between the traffic sent from i to j and traffic sent from j to i. In general, an ISP (or the customers of the ISP) benefits from the traffic in both directions, and the two ISPs involved in an exchange must jointly decide whether to exchange this traffic via the IXP or outside of it. The quantity y_{ij} can be interpreted as the amount of port capacity that ISP i (ISP j) must provision for traffic exchange with ISP j (ISP i) through the IXP[3]. Since the pricing policies at IXPs heavily depend on port capacities [6], we assume that the price paid by both ISPs i and j for their traffic exchange is $p(y)y_{ij}$. For easy exposition, in the following, both terms y_{ij} and y_{ji} are utilized, but with the understanding that they represent the same quantity. If the remaining traffic, $B_{ij} - y_{ij}$, is routed through private peering, it is reasonable to assume that the cost of purchasing or leasing any links, ports, etc., to enable this exchange will be proportional to $B_{ij} - y_{ij}$ for both ISPs i and j. Similarly, if this remaining traffic is routed through the ISPs' transit providers, the cost each ISP needs to pay its transit provider, λ_{ij} and λ_{ji}, can be assumed to be proportional to $B_{ij} - y_{ij}$. We first assume that $\lambda_{ij} = \lambda_{ji}$; however, in Section III-B, we consider a more general model which allows for asymmetric per-unit external routing costs; thus λ_{ij} can be different from λ_{ji}. This allows for possible differences between the two ISPs' transit costs, or their individual costs to privately peer with each other.[4]

Typically in practice, an ISP pair (i, j) will either send all of their traffic through peering at an IXP, or use the external routing option for all of their mutual traffic. However, our model is more general in that it allows the ISP pair to split their traffic between the two options. This relaxation eases our mathematical discourse and enables us to explore regimes beyond the current practice in traffic exchange between ISPs. Interestingly, from our model it turns out that at equilibrium almost all ISP pairs use only one of the two options (the IXP or external routing), verifying the current practice.

Some definitions: Given the above model setup, we next define the Social Cost (SC) in order to gain insight into pricing efficiency of the IXP. Overall, SC can be split into the costs incurred by the ISPs at the IXP, and the total payments received by the IXP. The cost for an ISP i, denoted by $P_i(\vec{y}, p(y), d(y))$, is calculated as

$$p(y) \sum_{(ij)\ni i} y_{ij} + d(y) \sum_{(ij)\ni i} y_{ij} + \sum_{(ij)\ni i} (B_{ij} - y_{ij})\lambda_{ij}, \quad (1)$$

where the first and second terms are the costs of sending peering traffic through the public switch – the first is the amount paid to the IXP, and the second is the (implicit) loss of the ISP's revenue caused by the congestion at the switch. The third term is the cost of sending the remaining traffic externally. Denoting $c(y) = p(y) + d(y)$ and $L_i(\vec{y}) = \sum_{(ij)\ni i} \lambda_{ij}(B_{ij} - y_{ij})$, the cost of ISP i is expressed as

$$P_i(\vec{y}, c(y)) \quad = \quad c(y)y_i + L_i(\vec{y}). \quad (2)$$

Note that $c(y)$ can be viewed as the aggregate cost seen by the ISPs per unit traffic, and therefore equals the sum of the per-unit price charged by the IXP ($p(y)$) and the congestion (delay) cost ($d(y)$). The total cost for all the ISPs is just the summation of P_i for all i. If we denote $\sum_i L_i(\vec{y}) = 2L(\vec{y})$, then the total cost of ISPs becomes

$$P(\vec{y}, c(y)) \quad = \quad 2(c(y)y + L(\vec{y})), \quad (3)$$

where the multiplier of 2 comes from the fact that y_i and y_j both include y_{ij}, i.e., y_{ij} is counted twice.

The IXP receives ISP payments, so the cost $Q(\vec{y}, p(y))$ incurred by the IXP is:

$$Q(\vec{y}, p(y)) = -p(y) \sum_i \sum_{(ij)\ni i} y_{ij} = -2p(y)y. \quad (4)$$

Thus, we define the social cost (SC) for the given network model as the total cost of the ISPs and IXP cumulatively, which is

$$\begin{aligned} SC(\vec{y}) &= P(\vec{y}, c(y)) + Q(\vec{y}, p(y)), \\ &= 2d(y)y + 2L(\vec{y}) = 2E(y) + 2L(\vec{y}), \end{aligned} \quad (5)$$

where $E(y) = d(y)y$. The first term of this SC is the cost of the congestion at the shared switch in the IXP, and the second is the cost of sending the traffic via external means. Intuitively, both of these components are detrimental to the efficiency of the IXP, and should be minimized. For the rest of the paper, unless otherwise stated, we will assume $E(y)$ to be a continuous, piece-wise differentiable function with $E(0) = 0$ and $E'(y)$ to be a non-decreasing function with $E'(0) = 0$. Note that SC does not consist of $p(y)$ which is the price of per-unit traffic charged by the IXP to the ISPs. However, any change of $p(y)$ will in general affect the traffic flows through the IXP, thereby changing SC($\vec{y}$).

B. Equilibrium Properties

In this paper we assume *price-taking* ISPs, i.e., they see the current cost per unit traffic is $c(y)$, and will only send traffic which is worth paying that cost. Thus, ISP i will send all the traffic through the IXP as long as $\lambda_{ij} \geq c(y)$, and will route the rest of its traffic externally. Of course, sending more traffic changes the "price" that the ISPs see per unit of traffic, $c(y)$, since it changes y. This leads to the notion of of equilibrium traffic flow, defined as follows:

Definition II.1. *A traffic flow $\vec{y}_e$ with $y_e = |\vec{y}_e|$ is said to be an equilibrium flow if and only if all the traffic with $\lambda_{ij} > c(y_e)$ is sent and the traffic with $\lambda_{ij} < c(y_e)$ is not sent.*

Based on this definition, we simplify our terminology and use the term 'equilibrium traffic flow' to refer to the *total* flow through the IXP at equilibrium (a scalar), and denote it by y_e.

Next, we state two important properties of equilibrium traffic flows that will be useful in our PoA analysis. We first

[3]Although not necessary for the analysis, for the sake of concreteness, y_{ij} can be considered to be either the max or the sum of the traffic in the two directions.

[4]While we usually assume a fixed per-unit cost λ_{ij}, all our findings also extend to λ_{ij} being a distribution, with different traffic units between i and j having different costs. We omit this model generalization for ease of exposition.

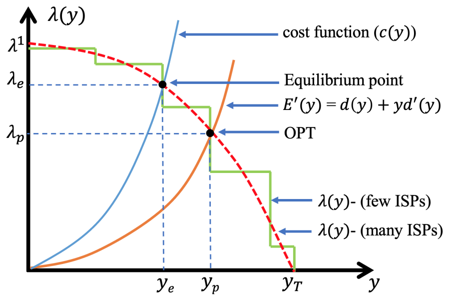

Fig. 1. $\lambda(y)$ curve with $E'(y)$ and $c(y)$.

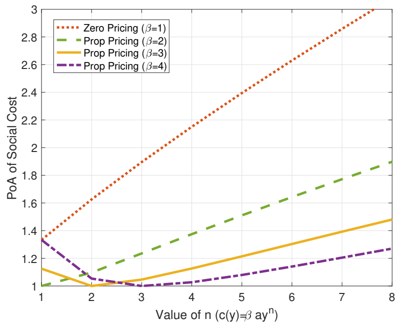

Fig. 2. PoA bounds for SC (polynomial delay function).

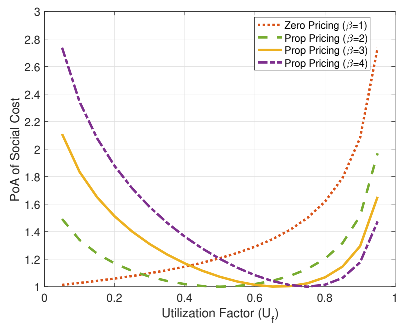

Fig. 3. PoA bounds for SC (queuing delay function).

define the notion of the *inverse demand curve*, denoted as $\lambda(y)$. Loosely speaking, $\lambda(y)$ shows the *inverse of* the total traffic of the IXP as a function of the cost of external traffic. Illustrated by Figure 1, this curve is constructed as follows. First, the λ_{ij} values are arranged in a decreasing order (ties broken arbitrarily); let λ^k be the k^{th} highest value, and B^k be the corresponding traffic demand. Then, the $\lambda(y)$ curve is a non-increasing step-function, with the step of height λ^k having a width of B^k. Let $\lambda(y^-)$ denote the limit of $\lambda(x)$ as x approaches y from below, and similarly $\lambda(y^+)$ if it approaches y from above. We then have the following property:

Theorem II.1. *y_e is an equilibrium traffic flow if and only if $\lambda(y_e^-) \geq c(y_e) \geq \lambda(y_e^+)$. Moreover, such a flow always exists.*

Notice also that multiple equilibria may exist. First, there could be several flow amounts y_e with $\lambda(y_e^-) \geq c(y_e) \geq \lambda(y_e^+)$ if the function $c(y)$ is not strictly increasing. Second, even for a fixed total flow y_e, there could be several different traffic pairs with equal $\lambda_{ij} = c(y_e)$ values, and sending any subset of them (as long as the total flow amount equals y_e) will yield an equilibrium flow.

Finally, we derive an important property of the optimal traffic vector (OPT), one that minimizes the total Social Cost, with which the equilibrium solution will be compared. Again, with slight abuse of terminology, by 'optimal traffic flow' we refer to the *total* traffic flow at the social optimum, denoted by y_p, since we know that in such a traffic vector the traffic with largest λ_{ij} will be sent in order to minimize social cost.

Theorem II.2. *At social optimality, all the traffic with $\lambda_{ij} > E'(y_p)$ flows through the IXP and all traffic with $\lambda_{ij} < E'(y_p)$ does not. Also, $\lambda(y_p^-) \geq E'(y_p) \geq \lambda(y_p^+)$).*

III. Equilibrium Social Cost Analysis

In this section, we analyze the Price of Anarchy (PoA) for the traffic exchange game defined in Section II, calculated as the ratio of Social Cost (SC) at the worst equilibrium to the SC at the optimal solution (OPT) that minimizes social cost. We first show that under an "optimal" pricing scheme, the PoA equals unity, i.e., all equilibria of the traffic exchange game attain social optimality. We then analyze the PoA attained by two other natural pricing policies, under two broad classes of delay functions.

Theorem III.1. *The pricing policy $p(y) = d'(y)y$ attains a PoA of 1.*

A. PoA Analysis for Social Cost under Proportional Pricing

Definition III.1. *Proportional Pricing with a proportionality factor $\beta \geq 1$ has a per-unit price $p(y)$ defined as $p(y) = (\beta - 1)d(y)$. In other words, the effective cost seen by the ISPs sharing the IXP per unit traffic is $c(y) = p(y) + d(y) = \beta d(y)$.*

Definition III.2. *Zero Pricing has a pricing function $p(y) = 0$, thus making the effective per-unit cost for ISPs consist only of congestion cost: $c(y) = d(y)$.*

Clearly, zero pricing can be viewed as a special case of proportional pricing with $\beta = 1$.

In the PoA analysis for social cost that follows next, we consider two broad classes of congestion cost (delay) functions: 1) polynomial delay functions, 2) queuing delay functions.[5]

1) PoA for Social Cost under Polynomial Delay Functions: The PoA for social cost in our model crucially depends on the properties and convexity of the congestion cost function $d(y)$. We make no assumptions about the λ_{ij} distribution, but only about the congestion cost functions. We begin by considering congestion cost (delay) functions which exhibit polynomial growth rates.

Theorem III.2. *For Proportional Pricing (i.e., $c(y) = \beta d(y)$), if congestion cost (delay) function $d(y) = ay^n$ with $a > 0, n \geq 1$, and*
i) $\beta \leq n+1$, then PoA is bounded by $[\beta - n(\frac{\beta}{n+1})^{(n+1)/n}]^{-1} \leq \frac{n+1}{\beta}$;

[5]Our PoA results on social cost hold even if there is some internal operational cost $r(y)$ which the IXP has, and passes it on to its ISPs by charging each ISP i a value $r(y)y_i/y$. To obtain the same results, we redefine $d(y)$ to be the total of the congestion (delay) cost to the ISPs and the price they are paying to the IXP, and $p(y)$ to be the additional profit that the IXP demands from the ISPs in addition to recovering its operational cost. In other words, we redefine $d(y)$ to be $d(y) + r(y)/y$, and then all the same results hold.

ii) $\beta > n+1$, *then PoA is bounded by* $\frac{\beta}{n+1}[\frac{\beta n}{(\beta-1)(n+1)}]^n \leq \frac{\beta}{n+1}$.

Corollary III.2.1. *For Zero Pricing (i.e.,* $c(y) = d(y)$*), if congestion cost (delay) function* $d(y) = ay^n$ *for some constant* $a > 0$*, then the PoA is bounded by* $(1 - n(1+n)^{-(n+1)/n})^{-1}$.

Corollary III.2.2. *If the delay cost,* $d(y)$*, satisfies* $\frac{d}{dy}(by^n) \leq d'(y) \leq \frac{d}{dy}(ay^n)$ *for some positive constant b, then the PoA bounds of Theorem III.2.1 and Corollary III.2 hold with an additional multiplicative factor of* $\gamma = \frac{a}{b}$.

Corollary III.2.1 follows directly from part i) of Theorem III.2, and taking $\beta = 1$. Corollary III.2.2 shows how our results generalize when the congestion cost (delay) function can be sandwiched between two polynomial functions with the same exponent n.

For convex congestion cost (delay) functions, we can derive an additional bound on the PoA for social cost, as follows.

Lemma III.1. *For Proportional Pricing (i.e.,* $c(y) = \beta d(y)$*), if congestion cost (delay) is any convex function with* $d(y) \leq ay^n$*, then with* $\beta > n+1$*, the PoA is bounded by* $\frac{\beta}{2}$.

Characteristics of the PoA bounds: The PoA bounds for different values of n with $d(y) = ay^n$ are shown for Zero and Proportional Pricing in Figure 2. We can see that, although the equations of the bounds were quite complicated, both the bounds are quite well-behaved. With Zero Pricing (which is a special case of Proportional Pricing with $\beta = 1$), if the delay cost is a linear function ($n = 1$), then the PoA is 1.33, which means irrespective of the shape of the $\lambda(y)$ curve (i.e., the values of the external routing costs of ISPs and the IXP), the worst equilibrium will only cost 33% more than the optimum cost. The results also show that the social cost benefits of proportional pricing (for a well chosen β value) over zero pricing can be quite significant.

For Proportional Pricing, the case is a bit more complicated with two variables n and β, still the bounds exhibit a simple linear-like behavior. If the value of n is increased from 1, then with $\beta > n+1$, the PoA starts to decrease and goes to 1 when $\beta = n+1$; after that with $\beta < n+1$ the PoA starts to increase. The value of PoA becoming 1 at $\beta = n+1$ coincides with the cost function $c(y)$ becoming equal to $E'(y)$. Overall, the PoA for social cost remains very small for reasonable values of n and β, showing that it is possible for IXPs to make a nice profit while still attaining a traffic exchange solution between ISPs that is close to optimal in terms of social cost.

2) PoA for Social Cost under Queuing Delay Functions: Next we consider the queuing delay function[6] (modeling the public switch at the IXP as a single server) expressed as:

$$d(y) = \frac{1}{\mu - y}, \tag{6}$$

where μ is the processing rate of traffic at the public switch. Note that $d(y)$ can become unbounded as the aggregate traffic load on the switch approaches capacity μ. These type of congestion cost (delay) functions are not analyzable in the same framework as polynomial delay functions considered earlier; they need to be treated separately, as we do here. Normally, switches are operated under 60-70% of the full capacity, as otherwise congestion delays would become too high to exchange traffic smoothly. This fact will be utilized in deriving the PoA for social cost.

Definition III.3. *Utilization factor,* U_f*, satisfying* $0 < U_f < 1$*, is the ratio of traffic load on a network to the total capacity of the network. If the total traffic at equilibrium is* y_e*, then we define it to be* $U_f = y_e/\mu$.

The following results provide the PoA bounds for social cost under Proportional Pricing and Zero Pricing with queuing delay functions. The result for Zero Pricing again follows as a special case ($\beta = 1$) of the Proportional Pricing result.

Theorem III.3. *For Proportional Pricing (i.e.,* $c(y) = \beta d(y)$*) and congestion cost (delay) function* $d(y) = \frac{1}{\mu - y}$*, the PoA is bounded by*

i) $\frac{U_f\sqrt{\frac{1-U_f}{\beta}}}{(1-U_f)[2-\sqrt{1-U_f}(\frac{1+\beta}{\sqrt{\beta}})]}$, *when* $U_f \geq 1 - \frac{1}{\beta}$;

ii) $\frac{\left(\sqrt{\beta}-\sqrt{U_f(\beta-1)}\right)^2}{1-U_f}$, *when* $U_f < 1 - \frac{1}{\beta}$.

Corollary III.3.1. *For Zero Pricing (i.e.,* $c(y) = d(y)$*) and congestion cost (delay) function* $d(y) = \frac{1}{\mu - y}$*, the PoA is bounded by* $\frac{U_f\sqrt{1-U_f}}{2(1-U_f)(1-\sqrt{1-U_f})}$.

Characteristics of the PoA bounds: Figure 3 shows the PoA upper bound with the increase of utilization factor, U_f of the switch. We see that the upper bound on PoA becomes 1 when $U_f = 1 - 1/\beta$. Also, PoA upper bound increases at an exponential rate on both sides of $U_f = 1 - 1/\beta$, but the rate of increment is higher on the side where $U_f > 1 - 1/\beta$. Typically, routers and switches maintain a utilization factor in the range of 40% to 70%, and for that operational range we see that PoA is below 1.2 for $\beta = 2$ and 3. This means that the worst equilibrium has social cost only 20% higher than the optimum under normal operating loads at the IXP.

B. PoA under Asymmetric External Routing Costs

All the results given until now had the assumption of $\lambda_{ij} = \lambda_{ji}$. However, as mentioned earlier, it is possible that the two ISPs i and j may encounter different per-unit costs for routing traffic externally between themselves, due to differences in transit pricing, or cabling/leasing cost associated with private peering. In this subsection, we will discuss the effect of $\lambda_{ij} \neq \lambda_{ji}$ on the PoA bounds calculated. The consideration of asymmetry between the external routing costs makes the proofs substantially longer and more difficult.

The consideration of asymmetric external routing costs requires us to revisit the definition of equilibrium. From the discussion of Section II-A, we know that when an ISP i has

[6]Note that the queuing delay function we consider represents that of the basic M/M/1 queue. The PoA results easily generalize to functions up to a multiplicative constant, i.e., delay functions of the form $a/(\mu - y)$; extension of the results to other more general forms of delay functions remains open for future investigation.

some traffic to exchange with ISP j, it compares the value λ_{ij} of that traffic with the cost $c(y)$ of exchanging that traffic via the IXP. As for the current case $\lambda_{ij} \neq \lambda_{ji}$, suppose that $\lambda_{ij} > c(y) > \lambda_{ji}$. Hence, ISP i will want to exchange traffic (since, then, its cost will decrease by λ_{ij} and increase by $c(y)$), whereas ISP j will not want to (since, then, its cost will decrease by λ_{ji} and increase by $c(y)$). Since both participants are needed to form a peering connection at an IXP, the traffic exchange will not happen in this case. Thus, in an equilibrium state, exchange of traffic between two ISPs is only possible when $\min(\lambda_{ij}, \lambda_{ji}) > c(y)$.

Definition III.4. *A traffic flow $\vec{y}_n$ with $y_n = |\vec{y}_n|$ is said to be an equilibrium flow for the case of $\lambda_{ij} \neq \lambda_{ji}$, when all the traffic with $\min(\lambda_{ij}, \lambda_{ji}) > c(y_n)$ is sent and the traffic with $\min(\lambda_{ij}, \lambda_{ji},) < c(y_n)$ is not sent.*

We will now jump directly to the PoA results for the three cases: socially optimal pricing (Theorem III.4), and Proportional Pricing (Theorem III.5 and Theorem III.6, which discusses the cases of polynomial and queuing delay functions, respectively). Recall that $c(y) = p(y) + d(y)$, and note that PoA results for Zero Pricing follows from Theorems III.5 and III.6 by setting $\beta = 1$.

Theorem III.4. *If $c(y) = E'(y)$, i.e., $p(y) = d'(y)y$, when $\lambda_{ij} \neq \lambda_{ji}$ with $\alpha = \max\{\frac{\lambda_{ij}}{\lambda_{ji}}\}$, PoA is bounded by $\frac{1+\alpha}{2}$.*

Theorem III.5. *If $c(y) = \beta d(y)$ (i.e., Proportional Pricing) and $d(y) = ay^n$, when $\lambda_{ij} \neq \lambda_{ji}$ with $\alpha = \max\{\frac{\lambda_{ij}}{\lambda_{ji}}\}$, PoA is bounded by*
I) $(\frac{1+\alpha}{2})(\frac{\beta}{n+1}) \times$(PoA bounds for $\lambda_{ij} = \lambda_{ji}$); when $\beta > n+1$;
II) $(\frac{1+\alpha}{2}) \times$ (PoA bounds for $\lambda_{ij} = \lambda_{ji}$); when $\beta \leq n+1$.

Theorem III.6. *If $c(y) = \beta d(y)$ (i.e., Proportional Pricing) and $d(y) = \frac{1}{\mu - y}$, when $\lambda_{ij} \neq \lambda_{ji}$ with $\alpha = \max\{\frac{\lambda_{ij}}{\lambda_{ji}}\}$, PoA is bounded by*
I) $\frac{1+\alpha}{2} \times \beta(1-U_f) \times$(the PoA bound when $\lambda_{ij} = \lambda_{ji}$) ; when $U_f < (1 - \frac{1}{\beta})$;
II) $\frac{1+\alpha}{2} \times$ (the PoA bound when $\lambda_{ij} = \lambda_{ji}$); when $U_f \geq (1 - \frac{1}{\beta})$.

Roughly speaking, Theorems III.4-III.6 state that when the notion of equilibrium is defined as in Definition III.4, the PoA bounds for the asymmetric external routing cost scenario differs from the symmetric case by about $\frac{1+\alpha}{2}$, where α represents the degree of asymmetry between the costs. Note that in the above theorems, the degree of asymmetry α is calculated only over ISP pairs i, j at the IXP that are interested in sending traffic to one another, i.e., $B_{ij} > 0$.

Paid Peering. Note that the PoA bounds in Theorems III.4-III.6 are large when the degree of asymmetry in the external routing costs is high. However, our simulation results in Section IV show that, in practice, the efficiency at equilibrium compared with the optimum is typically much better than these worst case bounds. Further, when one ISP encounters a much higher cost than the other in enabling traffic exchange between the two, *paid peering* would make sense. Indeed, paid peering has been suggested by some as a solution to growing asymmetry in costs experienced (or benefits realized) between two peering ISPs [14]. The following result states that in the case of asymmetric costs, if every pair of ISPs share the external routing costs via *Nash Bargaining* [34], it results in the same PoA as in the symmetric case. This is because if i pays λ_{ij} and j pays λ_{ji} (where $\lambda_{ji} > \lambda_{ij}$, say) in external routing costs, it is not difficult to see that the Nash Bargaining solution would result in a payment ("settlement") of $(\lambda_{ji} - \lambda_{ij})/2$ from i to j. Thus, when using a Nash Bargaining protocol, one ISP would pay the other exactly enough so that the effective external costs for not using the IXP would become equal, thus resulting in the same bounds as if λ_{ij} were always equal to λ_{ji}.

Theorem III.7. *If ISPs are allowed to pay each other (paid peering), and use Nash Bargaining to determine payments, then the PoA bounds are the same as in the symmetric case (i.e., as in Section III-A).*

TABLE II
SUMMARY ON IXPS AND ISPS IN THE USA

Total IXPs (PoP locations)	140
Total Facilities	1078
Facilities supporting public peering only	293
Facilities supporting both type of peering	830
Total ISPs	2821
ISPs doing public peering only	723
ISPs doing private peering only	728
ISPs doing both types of peering	1370

IV. SIMULATION RESULTS

A. Data Collection

To achieve realistic traffic demand values B_{ij} and external routing costs λ_{ij}, data from PeeringDB and CAIDA databases were collected and analyzed. PeeringDB was utilized for obtaining information about the locations of the IXPs, the ISPs peering in that location (also called Point-of-Presence (PoP)) and the port capacity each ISP has purchased. A short summary of the current statistics of ISPs and IXPs in the USA is given in Table II; note that an IXP can constitute of multiple facilities, which are typically located close to one another. Moreover, a map of the USA showing the location of the IXPs and the number of ISPs in those IXPs are shown in Figure 5. On the other hand, we utilized CAIDA to obtain the number of active routers and their approximate location (at a city level) for each ISP, to approximate the amount of traffic that may be generated for that ISP at that location.

B. Simulation Setup

1) Generating External Routing Cost (λ) values: To generate the $\lambda(y)$ curves we need two sets of values: i) the traffic demand between ISPs (B_{ij}), and ii) the per-unit external routing costs (λ_{ij}) for that traffic. While the exact values for these are very difficult to estimate closely, we make several reasonable approximations based on the PoP locations (obtained from PeeringDB), router densities (obtained from CAIDA) and previously published models on traffic demand and pricing. The traffic demand between two ISPs serving at two different PoP locations is determined using the gravity model [35]. If ISP i has R_A number of routers serving at location A and ISP j has R_B number of routers serving at location B, then the traffic demand between these two ISPs for these two locations is thus approximated as $Y_{AB} = \frac{R_A \times R_B}{d_{AB}^2}$. Then the summation of all these values over all the possible pairs of PoP locations gives us the total traffic demand between these two ISPs, hence $B_{ij} = \sum_{A,B} Y_{AB}$. To calculate the external routing cost for B_{ij} we follow [31], which models transit costs as being linearly or logarithmically proportional to the distance that traffic has to travel. Since traffic between different locations of the same ISP pair (say Y_{AB}) is going to travel different distances (d_{AB}), we use the weighted average of these distances: for some ISP pair (i, j), we set $d_{ij} = \frac{\sum_{A,B} Y_{AB} d_{AB}}{\sum_{A,B} Y_{AB}}$. Thus, we have the per-unit external routing cost as, $\lambda_{ij} = a \times d_{ij}$ or $\lambda_{ij} = a \times log(d_{ij})$, for an appropriately chosen constant a.

Note that the total traffic B_{ij} of ISP pair (i, j) may be split across the different PoP locations that the two ISPs have in common. To find how much of of this traffic will flow through each of these common points, we used three different approaches. The first approach sends all the traffic through the IXP that minimizes the total end-to-end geographical distance; the second approach divides the traffic equally among all common IXP locations; and third approach splits the traffic as inversely proportional to total end-to-end geographical distance of each path. Although these three methods yielded different traffic values at the IXPs, the nature of the external routing cost curves ($\lambda(y)$ curves) were quite similar. Since our performance results mainly depend on the shape of these $\lambda(y)$ curves and they ended up being similar across the three approaches; therefore, in the following we only present the results for the third approach. Figure 4 shows the $\lambda(y)$ curves for the largest (in terms of number of participating ISPs) 28 IXPs in USA, as generated by this approach.

2) Simulations: Simulations were done for the largest 28 IXPs among the 140 IXPs present in USA. Most of the remaining (smaller) IXPs have a very small number of participating ISPs, resulting in a few discrete $\lambda(y)$ values and making the study of the equilibrium uninteresting. Also, from the PeeringDB port capacity data, it was found that more than 95% of the total port capacities (which can be seen as an indicator of the traffic flowing through these IXPs) are accounted for by considering the largest 28 IXPs. To find the PoA values for proportional pricing with polynomial delay functions, we used different values of a, where $d(y) = ay^n$; the PoA value was calculated considering the delay function (or, equivalently, the value of a) that resulted in the worst PoA. Since the PoA value also critically depends on the $\lambda(y)$ curves which will differ across IXPs, both worst and average case PoA values were calculated by taking the the worst value and average values over all the $\lambda(y)$ curves, respectively. Due to space limitations,

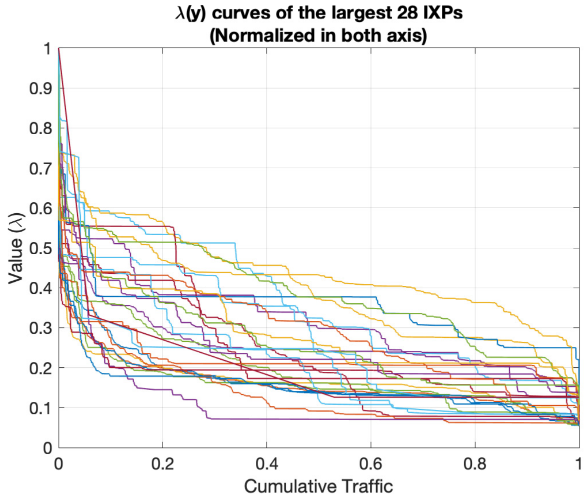

Fig. 4. Generated external routing cost ($\lambda(y)$) curves.

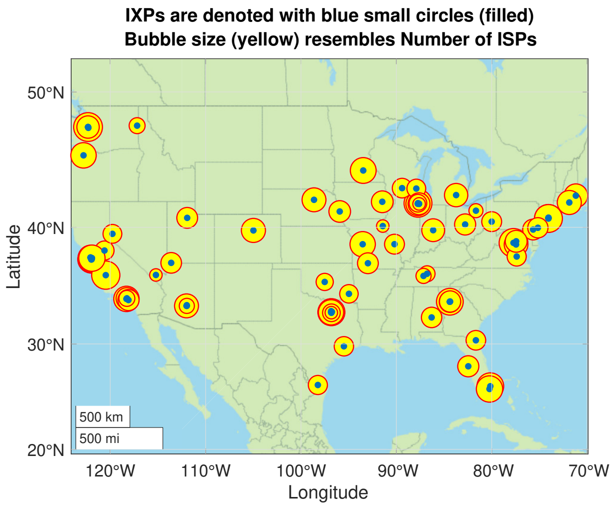

Fig. 5. USA map with IXP locations.

we only show the average PoA; the worst-case PoA results were quite close of the theoretical bounds derived in Section III showing that the bounds are fairly tight. For the case of queuing delay functions, the $\lambda(y)$ curves were normalized with respect to the value of μ (recall that $d(y) = \frac{a}{\mu - y}$), from which results for different utilization factors (U_f) were generated.

C. Results and Discussion

The average value of PoA (which we denote as $Avg(PoA)$) for Proportional pricing obtained from simulation, along with their corresponding theoretical bounds are plotted in Figures 6 to 9. To find $Avg(PoA)$, the value of a (recall, $d(y) = ay^n$) that resulted in the worst PoA value for each $\lambda(y)$ curve was considered; the corresponding PoA values were then averaged over all 28 $\lambda(y)$ curves. In other words, $Avg(PoA) = \frac{\sum_{i=1}^{N} \max_a PoA_i}{N}$, where PoA_i is the PoA values found using the i^{th} $\lambda(y)$ curve and N is the total number of $\lambda(y)$ curves, once for each of the $N = 28$ IXPs under consideration. In addition to the PoA for social cost (SC), we also plot the PoA for the revenue (Rev) earned by the IXP. The definition of PoA of revenue is similar to the definition of PoA of social cost: we define it as the ratio of maximum achievable revenue for some $\lambda(y)$ curve to the revenue attained at equilibrium, where revenue of the IXP is, $Rev(\vec{y}, p(y)) = p(y)\sum_i \sum_{(ij)\ni i} y_{ij} = 2p(y)y$. In Figures (6 to 9), the curves marked $Theo$ represents the corresponding theoretical bounds; whereas the curves marked Sim are the PoA values found through simulation.

We note from Figures 6 and 8, that the $Avg(PoA)$ values obtained through simulation are well below their corresponding theoretical bounds. For the case of $Max(PoA)$ values, the same trend was observed, but unlike the $Avg(PoA)$ values, the $Max(PoA)$ values followed the theoretical bounds very closely. From Figures 6 and 7 we can observe that the $Avg(PoA(SC))$ and $Avg(PoA(Rev))$ for proportional pricing with polynomial delay are quite small for a wide range of β (from 1 to 8) and n (from 1 to 4). The same can be said about $Avg(PoA(SC))$ and $Avg(PoA(Rev))$ for proportional pricing with queuing delay functions as well, where instead of n, the value of U_f is varied (Figures 8 and 9). Looking closely at $Avg(PoA(SC))$ for both type of delay functions we see that if β is chosen to be within a value of 2 to 4, then the PoA values are less than 1.5. On the other hand, the $Avg(PoA(Rev))$ for both type of delay functions is quite small (less than 1.5) for $\beta = 3$ to 5. Since IXPs may want to make good revenue while also keeping a low social cost, we observe from our results that $\beta = 3$ or 4 can be a good choice for any IXP to get a good balance between Revenue and Social Cost. From Figures 6 and 8, we also observe that proportional pricing with an appropriate value of β (say 2 to 4) attains better social cost compare to zero pricing (the $\beta = 1$ case), while providing the IXP with a positive profit. Note that zero pricing only lets the IXP recover its costs (i.e., the IXP makes no profit), and is therefore suited for non-profit IXPs. Our results show that if for-profit IXPs employ proportional pricing (with a carefully chosen proportionality constant) they can not only attain near-optimal profit, but can also induce traffic exchanges that are near-optimal in terms of social cost.

V. Conclusion

We considered the question of pricing of ISP traffic at IXPs, with the goal of attaining an equilibrium solution that ensures efficient social cost (smaller value of PoA). Through theoretical analysis we have bounded the $PoA(SC)$ values for proportional pricing, which maintained a small value for a wide range of model parameters. Simulations generated from real data showed that a good tradeoff can be achieved where both the SC and IXP Revenue are close to the maximum achievable values. A practical benefit of proportional pricing is that a good choice of the price (proportionality constant) does not require the IXP to know the external routing costs of the participating ISPs. This corresponds to the price proportionality factor $(\beta - 1)$ being about 2 to 3 times the congestion cost, for which the PoA for both social cost and revenue end up being quite small (i.e, less than 2 in general, and often quite close to 1), for both polynomial and queuing congestion cost (delay) functions. Our results also show the performance benefits of proportional pricing (with the proportionality constant chosen appropriately) over a zero pricing scheme where the ISPs experience congestion but no additional price is charged by the IXP.

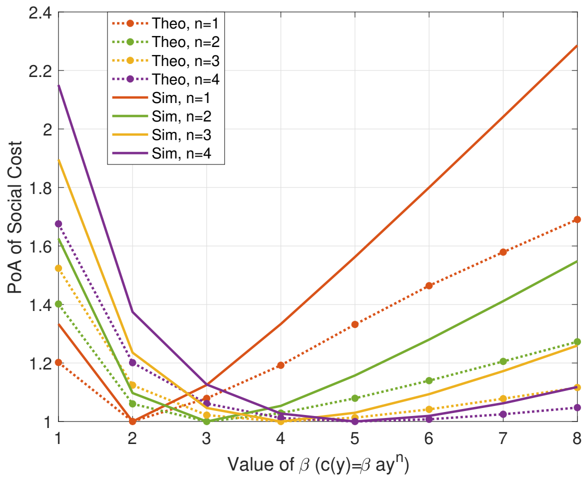

Fig. 6. Avg PoA of SC (Sim) with Theoretical bounds (Proportional pricing with polynomial delay function).

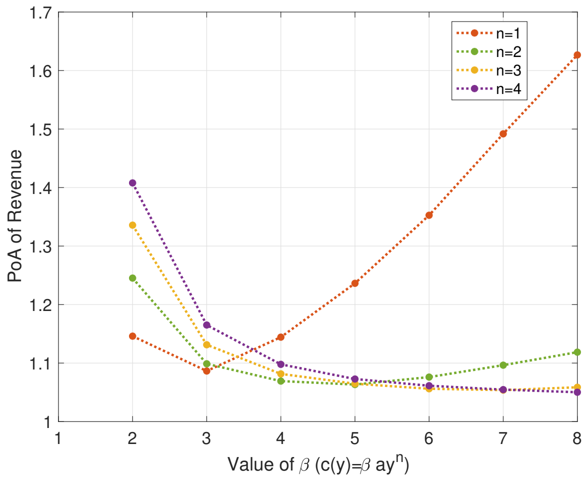

Fig. 7. Avg PoA of Rev - Simulated (Proportional pricing with polynomial delay function).

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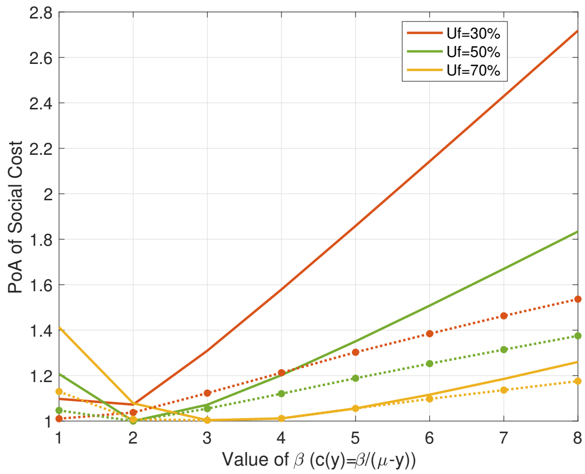

Fig. 8. Avg PoA of SC (Sim) with Theoretical bounds (Proportional pricing with queuing delay function).

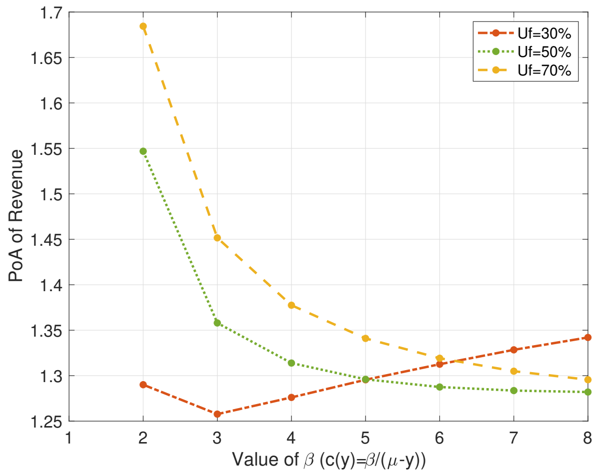

Fig. 9. Avg PoA of Rev - Simulated (Proportional pricing with queuing delay function).

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