ECSE 6460: Multivariable Control Systems

Take Home Midterm Exam

Due date: Tuesday, 3 November 2009, **in class.**

Attention: Please treat this as a real exam. No cooperation is allowed. If you have any question about the interpretation of the problems, send me an email.

Points: Problem 1 = 7+8 pts, Problem 2 = 10+15+5+15+15, Problem 3 = 15+10

Problem 1. Given a transfer matrix

$$G(s) = \frac{1}{s^2 + 3s + 2} \begin{bmatrix} s & -1 \\ 100 & s \end{bmatrix}.$$

(a) Plot the largest singular value of $G(j\omega)$ as function of ω in a log-log scale plot.
(b) What are the zeros of this MIMO system, and compute their (complex) directions.

Problem 2. Given a MIMO plant in state-space representation

$$\dot{x} = Ax + Bu, \tag{1}$$
$$y = Cx, \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 10 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 10 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 10 \end{bmatrix}, \tag{3}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{4}$$

(a) Determine if this is a minimal representation.
(b) Determine if the Geometric Input-Output Decoupling Problem is solvable. If so, compute a decoupling linear feedback

$$u = Fx + Gv$$

such that the TF from v to y is diagonal.
(c) Determine if the decoupled system has any unstable hidden (i.e. unobservable or uncontrollable) dynamics.
(d) Consider the following control problem: Design a feedback controller so that the three outputs track three reference signals simultaneously. We want the the rise times of all three tracking control to be less than 1 second with no more than 5% overshoot, and zero steady state tracking error for step reference signals. Design a controller to solve this control problem using the result of part (b) above. Explain your design.
(e) Solve the same control problem, assuming that only output feedback is available. You can use any design method, but explain your design.

Problem 3. For the same model above, define the matrix formed by the second and third rows of C as

$$C_{2,3} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

(a) Compute V^*, the largest controlled invariant subspace that is contained in kernel of $C_{2,3}$.
(b) Based on Problem 2 above, even before computing V^* you can already tell that its dimension is at least two. Explain the basis of this statement.