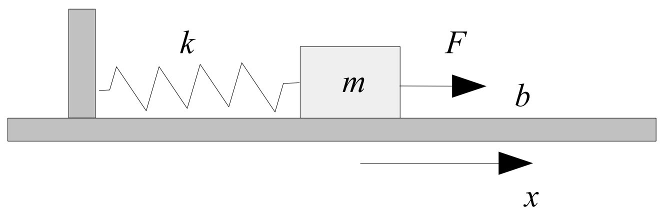
ECSE 6460: Multivariable Control Systems

Homework set 5. Due date: 8 December 2009

Points: Problem 1: 5+15+10+10+10+20 pts, Problem 2: 30 pts

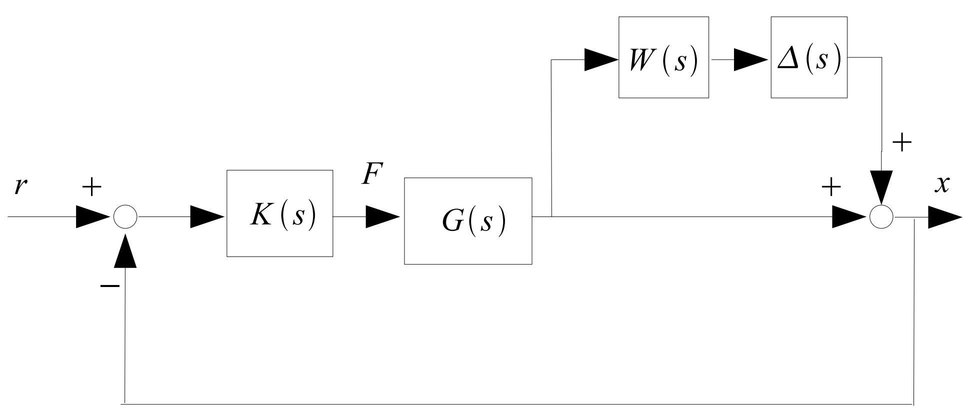
Problem 1. Consider the usual mass-spring-friction control system:



The parameters of the system and their range of uncertainty are given as follows:

Quantity (unit)	Value
Mass m (kg)	0.1 - 0.3
Spring constant k (N/m)	10
Friction coefficient b (Ns/m)	1 - 2

We want to design a stabilizing feedback controller $K(s)$, to be implemented in the following loop, such that the following control performance criteria is achieved. Rise time < 0.2 sec, Overshoot $< 5\%$, steady state tracking error for step reference signal is practically 0.



We want the controller $K(s)$ to satisfy all the requirements above **robustly**, i.e. for any value of the parameters above.
(a) Derive the transfer function $G(s) = \frac{X(s)}{F(s)}$ for the nominal values of the mass ($m_{\text{nom}} = 0.2$) and the friction coefficient ($b_{\text{nom}} = 1.5$).
(b) Represent the uncertainty in the plant as multiplicative uncertainty, with the nominal plant model given above. Compute the weight function $W(s)$.You can sample the parameter sets. Hint: Example 7.6 in the textbook.
(c) Using H_∞ synthesis, design a controller that satisfy the design requirement for the nominal plant (i.e. not necessarily robust). Explain/motivate your design, and simulate the performance of the system.
(d) Derive an H_∞ norm condition for robust stability (RS) for controller design. Hint: Section 7.5 in the text book.
(e) Derive an H_∞ norm condition for robust performance (RP) for controller design. Hint: Section 7.6 in the text book.
(f) Design a controller that is both RS and RP. Explain/motivate your design, and simulate the performance of the system for a set of perturbed plant models. You can sample the parameter sets. Use at least 10 samples.

Problem 2. For each of the six uncertainty configurations shown in Figure 8.5 (page 293) in the textbook, derive the transfer matrix M. Hint: Recall the $M\Delta$ loop, where M is the transfer function from the output of the Δ block to its input. You can use page 303 to check if your derivation is correct.