ECSE 6460: Multivariable Control Systems

Homework set 3. Due date: 20 October 2009

Points: Problem 1 = 20+25 pts, Problem 2 = 10+10+15+20 pts

Problem 1. Given a state-space representation of a system with disturbance

$$\begin{aligned} \dot{x} &= Ax + Bu + Gd, \\ y &= Cx, \end{aligned}$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}.$$

(a) Compute the largest controlled invariant subspace of $\ker C$.
(b) Can the Disturbance Decoupling Problem (DDP) be solved for this system? If so, compute the state feedback that solves it.

Problem 2. Consider the following input-state system:

$$\dot{x} = Ax + Bu,$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}; B = \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix}.$$

Assume that all the coefficients above are nonzero, unless explicitly stated.
(a) Determine if (A, B) is controllable.
(b) Show that a_{33} is one of the open loop poles.
(c) Prove that for any linear state feedback $u = -Kx$, a_{33} is one of the close loop poles.
(d) Based on (a) - (c), if given (A, B), $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ that is not controllable, describe a procedure to find all the open loop poles that cannot be moved by linear state feedback.