ECSE 6460: Multivariable Control Systems

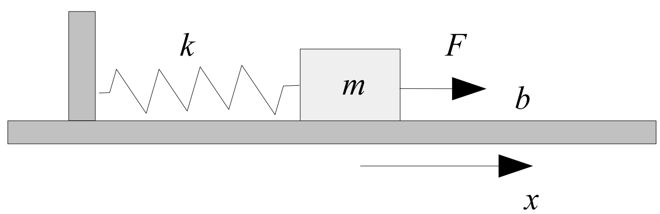
Take Home Final Exam

Due date: Tuesday, 15 December 2009 6pm, **in JEC 6044.**

Attention: Please treat this as a real exam. No cooperation is allowed. If you have any question about the interpretation of the problems, send me an email.

Points: Problem 1 = 2+5+3+10+5+5+10+15 pts, Problem 2 = 10, Problem 3 = 5+10+10+10

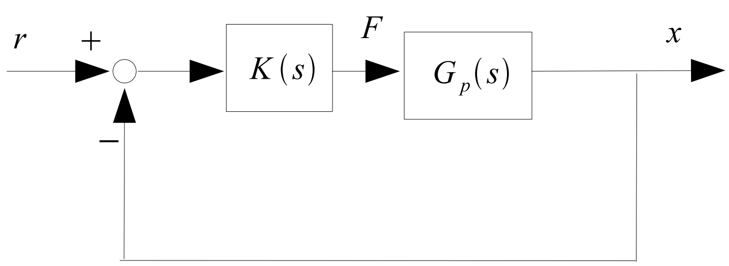
Problem 1. Consider the following problem from HW set 5. A mass-spring-friction control system:



The parameters of the system and their range of uncertainty are given as follows:

Quantity (unit)	Value
Mass m (kg)	0.1 - 0.3
Spring constant k (N/m)	10
Friction coefficient b (Ns/m)	1 - 2

We want to design a stabilizing feedback controller $K(s)$, to be implemented in the following loop, such that the following control performance criteria is achieved. Minimum gain crossover frequency (=bandwidth, see p. 39) is 5 rad/s, steady state step tracking error is practically 0, and $\|S\|_\infty \leq 1.2$.

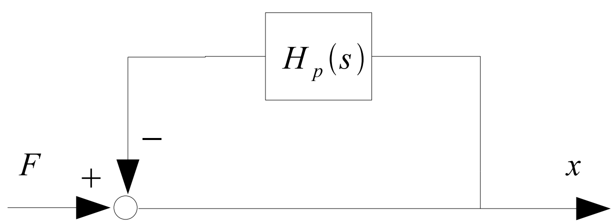


We want the controller $K(s)$ to satisfy all the requirements above **robustly**, i.e. for any value of the parameters above. However, unlike in HW5, we want to solve this problem using structured uncertainty formulation, which is less conservative than the previous approach.

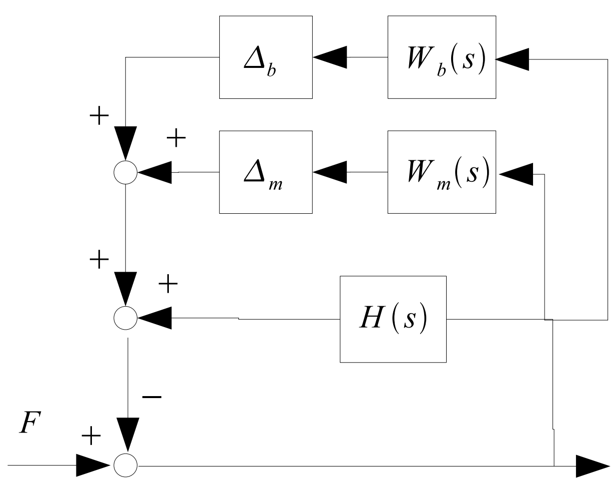
(a) We have computed that the uncertain plant model is

$$G_p(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k}.$$

Express $G_p(s)$ as the following block diagram. Compute $H_p(s)$.



(b) Assume that the nominal values for m and b are 0.2 and 1.5 respectively. Express the uncertainty in the plant model as the following block diagram, where $\Delta_m \in [-1, 1]$ and $\Delta_b \in [-1, 1]$ represent the uncertainties in m and b respectively. What are $W_m(s)$ and $W_b(s)$?



(c) Express the desired closep loop performance as an H_∞ condition.

(d) Compute the general plant model P. You need to identify what w, z, u, v, $u\Delta$, and $y\Delta$ are in this problem. Also, make sure that the uncertainties are modeled tightly as structured uncertainty.

(e) From this point on, you can assume that Δ_m and Δ_b are complex scalar. Derive the conditions for robust stability (RS) and robust performance (RP) for this uncertain system.

(f) Design a controller $K_{nom}(s)$ for your nominal plant model. You can use any design technique. Verify that the desired closep loop performance is achieved for the nominal plant.
(g) Check if the controller that you design in part (f) satisfies the RS and RP conditions.
(h) Using DK iteration, design a robust controller for your plant. You need to demonstrate the process of DK iteration (i.e. do not use automatic design procedures such as `dksyn` or `dkit`). You might need several iterations before stopping. If that's the case just show your first step and your final result. For numerical stability, you might want to use `balreal` to obtain a balanced realization of your plant and weights.

Problem 2. Consider the following transfer matrix

$$G(s) = \begin{bmatrix} \frac{s}{s+1} & 0 & 0 \\ 0 & \frac{s+2}{s(s+1)} & \frac{1}{s+1} \\ 0 & -1 & \frac{s}{s+2} \end{bmatrix}.$$

Find all the poles and zeros of this system, and compute their respective input AND output directions.

Problem 3. Consider the following uncertain plant model

$$G_p(s) = \begin{bmatrix} \frac{s}{s+p} & 1 & \frac{s+z}{(s+1)(0.1s+1)} \\ 1 & \frac{s+z}{s(s+1)} & \frac{1}{s+1} \end{bmatrix},$$

where

$$-2.5 \le p \le -3.5,$$
$$1 \le z \le 2.$$

(a) Represent the uncertainty as **unstructured** input and output side multiplicative uncertainty (i.e. one representation as input side and one representation as output side).
(b) Represent the uncertainty as **tight structured** uncertainty.
(c) Design a robustly stabilizing controller, $K(s)$, for the uncertainty model you derived in part (b).
(d) Implement the controller in a close loop with the plant, and assess the structured singular values for robust stability for all three uncertainty representations that you have computed (i.e. two from (a) and one from (b)). Compare the μ plots and comment on how they relate to the conservativeness of the uncertainty representations.