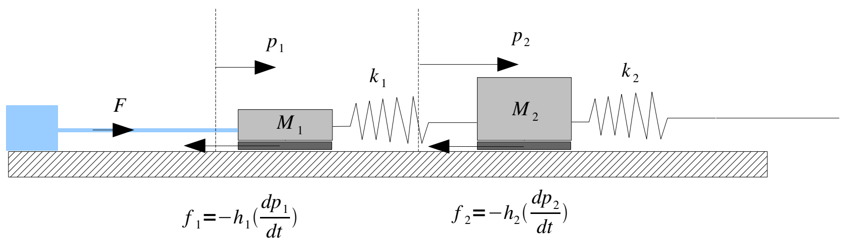
ECSE 6420: Nonlinear Control Systems

Homework set 3. Due date: 26 March 2009

Points: Problem 1 = 8+3+10+3+6 pts, Problem 2 = 25 pts, Problem 3 = 20 pts, Problem 4 = 25 pts

Problem 1. Consider the nonlinear mechanical system involving two rigid objects, two ideal springs, and nonlinear frictions as shown in figure below. The system is connected to an external component through a mechanical link, and receives force input $F(t)$.



Suppose that M_1 and M_2 are the masses of the the objects, k_1 and k_2 are the spring constants as shown in the diagram. The position of the first and second objects are denoted by $p_1(t)$ and $p_2(t)$, respectively, and the springs are in the unforced position if $p_1 = p_2$ and $p_2 = 0$, respectively. The nonlinear friction forces that act on the objects are given by:

$$f_1(t) = -h_1\left(\frac{dp_1}{dt}\right), \; f_2(t) = -h_2\left(\frac{dp_2}{dt}\right),$$

for some functions $h_1(\cdot)$ and $h_2(\cdot)$. Note that all variables are 1D vectors, with the convention that the positive direction points to the right (in the diagram).
(a) Derive a state model

$$\begin{aligned} \dot{x} &= f(x,u), \\ y &= h(x,u), \end{aligned}$$

where $u(t) := F(t)$ and $y(t) := \frac{dp_1}{dt}$. (Notice that the product $y(t)u(t)$ is the power delivered by the external component).
(b) Write the total energy in the system (potential and kinetic) as a function of the states $V(x)$.
(c) Derive a sufficient condition for $h_1(\cdot)$ and $h_2(\cdot)$, such that the system is

passive.

(d) Suppose that h_2 is known to belong to the sector $[0, \infty]$. Show that the system is output-feedback-passive.

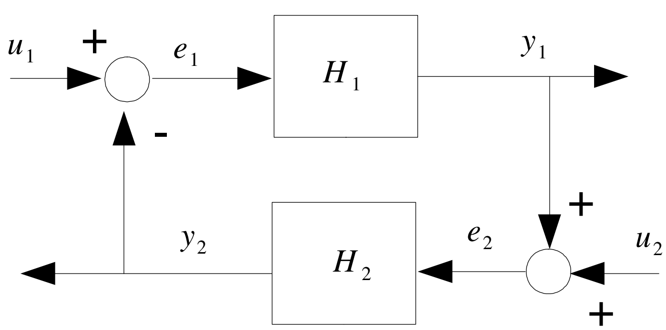
(e) Using part (d), redefine the input via output-feedback, such that the system is passive. What is the physical interpretation of this new input?

Problem 2. (Problem 5.12) Consider the system

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -y - h(y) + u, \\ y &= x_1 + x_2, \end{aligned}$$

where h is continuously differentiable, $h(0) = 0$, and $zh(z) > az^2$ for all $z \in \mathbb{R}$, for some $a > 0$. Show that the system is finite-gain $\mathcal{L}_p$ stable for each $p \in [1, \infty]$.

Problem 3. Consider the feedback loop shown below.



Suppose that the feedback loop is well-posed, and that the systems H_1 and H_2 are finite gain L_p stable for some $p \in [1, \infty]$. Show that the I/O system from (u_1, u_2) to (e_1, e_2) is finite gain L_p stable if and only if the I/O system from (u_1, u_2) to (y_1, y_2) is finite gain L_p stable.

Problem 4. Consider the following system

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -h_1(x_1) - h_2(x_2) + u, \\ y &= kx_2, \end{aligned}$$

where $k > 0$, and h_1 and h_2 belong to the sector $(0, \infty)$. Show that the system is strictly passive by constructing a suitable storage function $V(x_1, x_2)$. Note: Strict passivity means $y^T u - \dot{V} \geq \psi(x_1, x_2)$, for some positive definite function ψ. Hint: Draw inspiration from second order mechanical systems; consider x_1 as position and x_2 as velocity.