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Coordinated Control of Wind Turbine Generator and Energy Storage System for Frequency Regulation under Temporal Logic Specifications

Zhe Xu, Agung Julius and Joe H. Chow

Abstract— **In this paper, we present a coordinated control method for wind turbine generator and energy storage system for frequency regulation with provable probabilistic guarantees in the stochastic environment of wind power generation. The regulation requirement is specified in the form of metric temporal logic (MTL). We present the stochastic control bisimulation function, which bounds the divergence of the trajectories of the stochastic control system and the diffusionless deterministic control system in a probabilistic fashion. We first design a feedforward controller by solving an optimization problem for the nominal trajectory of the deterministic control system with robustness against initial state variations and stochastic uncertainties. Then we generate a feedback control law from the data of the simulated trajectories. We implement our control method on a four-bus system and test the effectiveness of the method with a generation loss disturbance. We also test the advantage of the feedback controller over the feedforward controller when unexpected disturbance occurs.**

I. Introduction

With the increasing penetration of renewable energy into the power grid, the stochastic nature of the renewable energy such as wind energy makes the planning and operation of power grid more challenging. Energy storage systems such as battery energy storage systems can be utilized to compensate the volatility brought by the renewable energy and regulate the grid frequency within the allowable range [1], [2], [3], [4]. On the other hand, the renewable energy can also be utilized for the frequency regulation when disturbances such as generation loss or line failure occurs. For example, it has been shown that wind turbine generators can adjust its power output for restoring the grid frequency after a disturbance [5].

As different generators and energy storage systems have different response time, the regulated frequency could have different temporal properties. Therefore, temporal logics can be utilized to provide time-related specifications such as "after a disturbance, the grid frequency should be restored to 60Hz$\pm$0.5Hz within 2 seconds and to 60Hz$\pm$0.3Hz within 20 seconds". There is much research on the control of wind turbine generators or energy storage systems for economic or stability purposes, while incorporating temporal logic constraints into the controller synthesis problem is still a novel approach.

There have been various methods on how to design controllers to meet temporal logic specifications in stochastic environment [6], [7]. For discrete-time temporal logic specifications such as linear temporal logic (LTL) specifications, the usual approach is to abstract the system as a Markov Decision Process (MDP), then the control design is transformed to a problem of finding the control strategy that maximizes the probability of producing a sequence of states in the MDP satisfying the LTL specification [8]. For dense-time temporal logic specifications, the system can be abstracted as a timed automaton [9], [10] and the design process reduces to reachability analysis after the timed automaton is constructed.

In this paper, we present a controller synthesis approach for metric temporal logic specifications with stochastic environment by designing the controller for the trajectory of the diffusionless version of the process with robustness against initial state variations and stochastic uncertainties. Our work is motivated by the works of probabilistic testing for stochastic systems in [11], [12]. We present the stochastic control bisimulation function, which bounds the divergence of the trajectories of the stochastic control system and the diffusionless deterministic control system (nominal system) in a probabilistic fashion. Thus all the controller synthesis methods for the deterministic system can be used for designing the optimal input signals, and the same input signals can be applied to the stochastic system with a probabilistic guarantee. To account for unexpected disturbances, we generate a feedback control law from the data of the simulated trajectories to form a feedback controller. We apply the controller synthesis method in regulating the grid frequency of a four-bus system by the coordinated control of wind turbine generators and energy storage systems. **We test the effectiveness of the controllers with the stochastic wind generation after a large generation loss disturbance, and also the case when unexpected disturbances are added to the situation**. Simulations have shown that the trajectories with the feedforward and the feedback controllers can satisfy the MTL specification with a probabilistic guarantee. Besides, simulations show that when even unexpected disturbances occur, the feedback controller has better performance in comparison with the feedforward controller.

II. Stochastic Control Bisimulation Function

A. Stochastic Control Bisimulation Function

We consider the following general stochastic control system

$$dx = F(x,u)dt + G(x,u)dw, \tag{1}$$

Zhe Xu, Agung Julius and Joe H. Chow are with the Department of Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, USA e-mail: xuz8, juliua2, chowj@rpi.edu. This research was partially supported by the National Science Foundation through grants CNS-1218109, CNS-1550029 and CNS-1618369.

where the state $x \in \mathcal{X} \in \mathbb{R}^n$, the input $u \in \mathcal{U} \in \mathbb{R}^p$, w is an $\mathbb{R}^m$-valued standard Brownian motion.

Note that our modeling framework is essentially the same as that in [11] when the input signal $u(\cdot)$ is given and bounded, where the existence and uniqueness of the solution of (1) can be guaranteed with the conditions given in [11].

We also consider the **nominal system** of (1) as the diffusionless deterministic version:

$$dx^* = F(x^*, u)dt, \tag{2}$$

The trajectories generated by the nominal system are referred to as the **nominal trajectories**. For the nominal system (2), a control autobisimulation function can be formed [13].

Definition 1: A continuously differentiable function $\psi : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_{\geqslant 0}$ is a control autobisimulation function of the nominal system (2) if for any $x, \tilde{x} \in \mathbb{R}^n$ $(x \neq \tilde{x})$ there exists a function $u : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^p$ such that $\psi(x, \tilde{x}) > 0$, $\psi(x, x) = 0$ and $\nabla_x \psi(x, \tilde{x}) F(x, u(x, t)) + \nabla_{\tilde{x}} \psi(x, \tilde{x}) F(\tilde{x}, u(\tilde{x}, t)) \leq 0$.

In the following, we extend the concept of control autobisimulation function to the stochastic setting.

Definition 2: A twice differentiable function $\phi : \mathcal{X} \times \mathcal{X} \to \mathbb{R}_{\geqslant 0}$ is a **stochastic control bisimulation function** between (1) and its nominal system (2) if it satisfies

$$\begin{aligned} &\phi(x, \tilde{x}) > 0, \forall x, \tilde{x} \in \mathcal{X}, x \neq \tilde{x}, \\ &\phi(x, x) = 0, \ \forall x \in \mathcal{X}, \end{aligned} \tag{3}$$

and there exist $\mu, \alpha > 0$ and a function $u : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^p$ such that

$$\begin{aligned} &\frac{\partial \phi}{\partial x} F(x, u(x, t)) + \frac{\partial \phi}{\partial \tilde{x}} F(\tilde{x}, u(\tilde{x}, t)) \\ &+ \frac{1}{2} G^T(x, u(x, t)) \frac{\partial^2 \phi}{\partial x^2} G(x, u(x, t)) \leq -\mu \phi + \alpha, \end{aligned} \tag{4}$$

for any $x, \tilde{x} \in \mathcal{X}$.

The stochastic control bisimulation function establishes a bound between the trajectories of system (1) and its nominal system (2). We denote $B_\psi(x, r) \triangleq \{\tilde{x} \in \mathcal{X} | \psi(x, \tilde{x}) \leq r\}$.

B. Stochastic Control Bisimulation Function for Linear Dynamics

In this subsection, we consider the stochastic control system with linear dynamics described as below:

$$dx = (Ax + Bu)dt + \Sigma dw, \tag{5}$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times p}$, $\Sigma \in \mathbb{R}^{n \times m}$.

If the system is stable, i.e. A is Hurwitz, we can construct a stochastic control bisimulation function of the form

$$\phi(x, \tilde{x}) = (x - \tilde{x})^T M (x - \tilde{x}),$$

where M is a symmetric positive definite matrix. In order for this function to qualify as a stochastic control bisimulation function, we need to have $M \succ 0$, and

$$\begin{aligned} &\frac{\partial \phi}{\partial x}(Ax + Bu) + \frac{\partial \phi}{\partial \tilde{x}}(A\tilde{x} + Bu) + \\ &\qquad trace(\frac{1}{2}\Sigma^T(\frac{\partial^2 \phi(x, \tilde{x})}{\partial x^2})\Sigma) \\ =&2(x - \tilde{x})^T MA(x - \tilde{x}) + trace(\Sigma^T M \Sigma) \\ \leq& -\mu(x - \tilde{x})^T M(x - \tilde{x}) + \alpha. \end{aligned} \tag{6}$$

for some $\mu, \alpha > 0$. If we pick $\alpha = trace(\Sigma^T M \Sigma)$, the inequality (6) becomes a linear matrix inequality (LMI) $A^T M + MA + \mu M \preceq 0$. We denote the system trajectory starting from x_0 with the input signal $u(\cdot)$ as $\xi(\cdot; x_0, u)$. It can be seen that (6) holds for any input signal $u(\cdot)$, so $u(\cdot)$ is free to be designed. It can also be seen that the matrix M that satisfies $A^T M + MA + \mu M \preceq 0$ also satisfies $A^T M + MA \preceq 0$. Thus it can be verified that $\psi(x, \tilde{x}) = \phi(x, \tilde{x}) = (x - \tilde{x})^T M (x - \tilde{x})$ is also a control bisimulation function of the nominal system

$$dx^* = (Ax^* + Bu)dt. \tag{7}$$

We denote the nominal system trajectory starting from x_0 with the input signal $u(\cdot)$ as $\xi^*(\cdot; x_0, u)$.

Proposition 1: If ϕ is a stochastic control bisimulation function between the stochastic system (5) and its nominal system (7), then for any $T > 0$,

$$P\{\sup_{0 \leq t \leq T} \phi(\xi^*(t; x_0, u), \xi(t; x_0, u)) < \gamma\} > 1 - \frac{\alpha T}{\gamma}. \tag{8}$$

It can be seen from (8) that ϕ provides a probabilistic upper bound for the distance between the states of the stochastic system and its nominal system in a finite time horizon.

III. Stochastic Controller Synthesis

A. Feedforward Controller Synthesis

The syntax and semantics of the Metric Temporal Logic are described in [2]. We denote the set of states that satisfy the predicate p as $\mathcal{O}(p) \subset \mathcal{X}$, where $\mathcal{X}$ is the domain of the state x. In this paper, we consider a fragment of MTL formulae in the following form:

$$\varphi = \Box_{[\tau_1, T_{\text{end}}]} p_1 \wedge \Box_{[\tau_2, T_{\text{end}}]} p_2 \wedge \cdots \wedge \Box_{[\tau_q, T_{\text{end}}]} p_q \tag{9}$$

where $\tau_1 < \tau_2 < \ldots \tau_q \leq T_{\text{end}}$, T_{end} is the end of the simulation time, $\mathcal{O}(p_q) \subset \mathcal{O}(p_{q-1}) \subset \cdots \subset \mathcal{O}(p_1)$, each predicate p_k is in the following form:

$$p_k \triangleq \left(\bigwedge_{\nu=1}^{n_k} a_{k,\nu}^T x < b_{k,\nu} \right), a_{k,\nu} \in \mathbb{R}^n, b_{k,\nu} \in \mathbb{R}, \tag{10}$$

where $a_{k,\nu}$ and $b_{k,\nu}$ denote the parameters that define the predicate, n_k is the number of atomic predicates in the k-th predicate. We constraint $\|a_{k,\nu}\|_2 = 1$ to reduce redundancy.

The MTL formulae in the above-defined form is actually specifying a series of regions to be entered before certain deadlines and stayed thereafter, with larger regions corresponding to tighter deadlines. The MTL formulae in this form is especially useful in power system frequency regulations as discussed in Section IV.

The $\delta_{k,\nu}$-robust modified formula $\hat{\varphi}_\delta$ is defined as follows:

$$\hat{\varphi}_\delta \triangleq \Box_{[\tau_1, T_{\text{end}}]} \hat{p}_1 \wedge \Box_{[\tau_2, T_{\text{end}}]} \hat{p}_2 \wedge \cdots \wedge \Box_{[\tau_q, T_{\text{end}}]} \hat{p}_q \quad (11)$$

where each predicate $\hat{p}_k$ is modified from (10) as follows:

$$\hat{p}_k \triangleq \left(\bigwedge_{\nu=1}^{n_k} a_{k,\nu}^T x < b_{k,\nu} - \delta_{k,\nu} \right), a_{k,\nu} \in \mathbb{R}^n, b_{k,\nu} \in \mathbb{R}, \quad (12)$$

We use $[[\varphi]](s,t)$ to denote the robustness degree of the trajectory s with respect to the formula φ at time t.

Theorem 1: If for every $k \in \{1,\ldots,q\}$ and $\nu \in \{1,\ldots,n_k\}$, there exist $z_{k,\nu}, \epsilon > 0$ such that $z_{k,\nu}^2 a_{k,\nu} a_{k,\nu}^T \preceq M$ and $[[\varphi_{\hat{\delta}}]](\xi^*(\cdot; x_0^*, u), 0) \geq 0$, where $\varphi_{\hat{\delta}}$ is the $\hat{\delta}_{k,\nu}$-robust modified formula of φ, $\hat{\delta}_{k,\nu} = (\sqrt{r} + \sqrt{\hat{\gamma}})/z_{k,\nu}$, $\hat{\gamma} = \frac{\alpha T_{\text{end}}}{\epsilon}$, then for any $x_0 \in B_\psi(x_0^*, r)$, the trajectory $\xi(\cdot; x_0, u)$ satisfies MTL specification φ with probability at least $1-\epsilon$, i.e. $P\{[[\varphi]](\xi(\cdot; x_0, u), 0) \geq 0\} > 1 - \epsilon$.

From Theorem 1, if we can design the input signal $u(\cdot)$ such that the nominal trajectory $\xi^*(\cdot; x_0^*, u)$ of the nominal system (2) satisfies the $\hat{\delta}_{k,\nu}$-robust modified formula of φ ($\hat{\delta}_{k,\nu} \triangleq (\sqrt{r} + \sqrt{\hat{\gamma}})/z_{k,\nu}$), then all the trajectories of the stochastic system (1) starting from the initial set $B_\psi(x_0^*, r)$ are guaranteed to satisfy the MTL specification φ with probability at least $1-\epsilon$. To make the robust modification as tight as possible, for every $k \in \{1,\ldots,q\}$ and $\nu \in \{1,\ldots,n_k\}$, we compute the maximal $z_{k,\nu}$ such that $z_{k,\nu}^2 a_{k,\nu} a_{k,\nu}^T \preceq M$. We denote the maximal value of $z_{k,\nu}$ as $z_{k,\nu}^*$, $\hat{\delta}_{k,\nu}^* \triangleq (\sqrt{r} + \sqrt{\hat{\gamma}})/z_{k,\nu}^*$, and the $\hat{\delta}_{k,\nu}^*$-robust modified formula as $\varphi_{\hat{\delta}^*}$ (the predicates in $\varphi_{\hat{\delta}^*}$ are denoted as $\hat{p}_k^*$).

The optimization problem to find the optimal input signal such that the nominal trajectory satisfies the $\delta_{k,\nu}^*$-robust modified formula $\varphi_{\hat{\delta}^*}$ is formulated as follows:

$$\begin{aligned} &\arg\min_{u(\cdot)} \; J(u(\cdot)) \\ &\text{subject to } \; [[\varphi_{\hat{\delta}^*}]](\xi^*(\cdot; x_0^*, u), 0) \geq 0. \end{aligned}$$

The performance measure $J(u(\cdot))$ can be set as the control effort $\|u(\cdot)\|_2$ or $\|u(\cdot)\|_1$. For linear systems, the above optimization problem can be converted to a linear program (LP) problem [14], [15] and it can be solved efficiently by LP solvers. **As we only focus on the time horizon $[0, T_{\text{end}}]$ (e.g. the most critical time period in the transient response of the power system), we assume that some other controllers that make $\mathcal{O}(\hat{p}_q^*)$ a control invariant set for the nominal trajectories will take over after T_{end}.**

B. Feedback Controller Synthesis

In this section, we design a feedback control law to replace the optimal input signals of the feedforward controller. The advantage of a feedback controller is that it is more robust to unexpected disturbances. When the states and inputs of the trajectories are calculated using numeric simulators such as ODE or CVODE, the data are discrete and therefore in the following we use $\xi_\ell^*[j] \triangleq \xi_\ell^*(j; x_{0,\ell}^*, u_\ell)$ and $u_\ell[j]$ ($\ell = 1,2,\ldots,N$, $j = 0,1,\ldots,N_t$) to denote the flow solution and the input of the ℓth nominal trajectory of the nominal system (2) at the jth time instant, respectively. As we have assumed in Section III-A, we extend each signal $u_\ell(\cdot)$ after T_{end} and when $t[j] > T_{\text{end}}$, $u_\ell[j]$ is a control input that makes $\mathcal{O}(\hat{p}_q^*)$ a control invariant set for the ℓth nominal trajectory.

The algorithm to generate the feedback law is shown in Algorithm 1. We apply the following feedback law $\chi_u(x,t)$ which depends both on the current state x and the current time instant j:

$$\chi_u(x,t[j]) \triangleq \begin{cases} u_1[0], & \text{if } (x,t[j]) \in \mathcal{X}_{0,1}[j] \times [0,t[0]], \\ \quad\vdots & \quad\vdots \\ u_N[0], & \text{if } (x,t[j]) \in \mathcal{X}_{0,N}[j] \times [0,t[0]], \\ \quad\vdots & \quad\vdots \\ u_1[i], & \text{if } (x,t[j]) \in \mathcal{X}_{i,1}[j] \times [0,t[i]], \\ \quad\vdots & \quad\vdots \\ u_N[i], & \text{if } (x,t[j]) \in \mathcal{X}_{i,N}[j] \times [0,t[i]], \\ \quad\vdots & \quad\vdots \\ u_1[N_t], & \text{if } (x,t[j]) \in \mathcal{X}_{N_t,1}[j] \times [0,T_{\text{end}}], \\ \quad\vdots & \quad\vdots \\ u_N[N_t], & \text{if } (x,t[j]) \in \mathcal{X}_{N_t,N}[j] \times [0,T_{\text{end}}], \\ u_{\hat{\ell}}[\hat{j}], & \text{otherwise}, \end{cases} \quad (13)$$

where $t[j]$ is the time at the jth time instant, each region $\mathcal{X}_{i,\ell}[j]$ is changing with time and defined as follows:

$$\mathcal{X}_{i,\ell}[j] = \begin{cases} \hat{\mathcal{X}}_{i,\ell} \setminus \bigcup_{i'>i} \hat{\mathcal{X}}_{i',\ell}, & \text{if } \ell = \hat{\ell}[j], \\ B_\psi(\xi_\ell^*[i], r) \setminus \Big(\big(\bigcup_{i'} \hat{\mathcal{X}}_{i',\hat{\ell}}\big) \cup \big(\bigcup_{\ell'>\ell} \bigcup_{i'} (\xi_{\ell'}^*[i'], r) \big) & \\ \cup \big(\bigcup_{i'>i} B_\psi(\xi_\ell^*[i'], r) \big) \Big), & \text{if } \ell \neq \hat{\ell}[j]. \end{cases} \quad (14)$$

where $\hat{\ell}$ is initially assigned as $\min\{\ell | x_0 \in B_\psi(x_{0,\ell}^*, r)\}$ and then assigned according to lines 5-9 of Algorithm 1, $\hat{j}$ is initially assigned as 0 and then assigned according to lines 5-9 of Algorithm 1, $\hat{\mathcal{X}}_{i,\ell} = \{x | \phi(x, \tilde{x}) \leq \hat{\gamma}, \tilde{x} \in B_\psi(\xi_\ell^*[i], r)\}$ is defined as the **stochastic robust neighbourhood with probability** $(1-\epsilon)$, where $\hat{\gamma} = \frac{\alpha T_{\text{end}}}{\epsilon}$.

Algorithm 1 Feedback law generation.

1: $j \leftarrow 0$, $\hat{j} \leftarrow 0$
2: $\hat{\ell}[1] \leftarrow \min\{\ell | x_0 \in B_\psi(x_{0,\ell}^*, r)\}$
3: For every i, ℓ, obtain $\mathcal{X}_{i,\ell}[1]$ according to (14)
4: **while** $j < N_t$ **do**
5:     **if** there exists $\ell' \neq \hat{\ell}[j]$ and $i \geq j$ such that $x \in \mathcal{X}_{i,\ell'}[j]$ **then**
6:         $\hat{\ell}[j+1] \leftarrow \ell'$, $\hat{j} \leftarrow i+1$
7:     **else**
8:         $\hat{\ell}[j+1] \leftarrow \hat{\ell}[j]$, $\hat{j} \leftarrow \hat{j}+1$
9:     **end if**
10:     Obtain $\chi_u(x, t[j])$ according to (13)
11:     $j \leftarrow j+1$
12:     For every i, ℓ, obtain $\mathcal{X}_{i,\ell}[j]$ according to (14)
13: **end while**

We use $\xi(j; x_0, \chi_u)$ to denote the flow solution of the trajectory of the stochastic system starting from x_0 at the jth time instant when the feedback control law $\chi_u(x,t)$ is applied. As shown in Fig. 1, for the two possible trajectories (realizations) of the stochastic system, the trajectory (realization) in Fig. 1 (a) stays within the stochastic robust neighbourhoods around the nominal trajectory $\xi^*_\ell[\cdot]$ throughout the time while the trajectory (realization) in Fig. 1 (b) exits the stochastic robust neighbourhoods around $\xi^*_\ell[\cdot]$ and enters $\mathcal{X}_{i,\ell'}[j]$ at time instant j, thus $\hat{\ell}$, $\hat{j}$ and the following inputs are changed according to Algorithm 1 such that the trajectory stays within the stochastic robust neighbourhoods around the nominal trajectory $\xi^*_{\ell'}[\cdot]$ with probability $(1-\epsilon)$ since time instant j.

Theorem 2: If for every ℓ, $[[\varphi_{\hat{\delta}^*}]]\,(\xi^*(\cdot; x^*_{0,\ell}, u_\ell), 0) \geq 0$ ($\varphi_{\hat{\delta}^*}$ is the $\hat{\delta}^*_{k,\nu}$-robust modified formula, where $\hat{\delta}^*_{k,\nu} = (\sqrt{r} + \sqrt{\hat{\gamma}})/z^*_{k,\nu}$, $\hat{\gamma} = \frac{\alpha T_{\text{end}}}{\epsilon}$), then for any $x_0 \in \bigcup\limits_{\ell=1}^{N} B_\psi(x^*_{0,\ell}, r)$ the trajectory $x(\xi(\cdot; x_0, \chi_u))$ satisfies the MTL specification φ with probability at least $1-\epsilon$, i.e. $P\{[[\varphi]]\,(\xi(\cdot; x_0, \chi_u), 0) \geq 0\} > 1-\epsilon$.

While Theorem 2 gives provable probabilistic guarantees when there is no unexpected disturbance, the following theorem considers the situation when unexpected disturbance occurs.

Theorem 3: Assume that for every ℓ, $[[\varphi_{\hat{\delta}^*}]]$ $(\xi^*(\cdot; x^*_{0,\ell}, u_\ell), 0) \geq 0$ ($\varphi_{\hat{\delta}^*}$ is defined in the same way as in Theorem 2). For any $x_0 \in \bigcup\limits_{\ell=1}^{N} B_\psi(x^*_{0,\ell}, r)$, if at any time instant j, unexpected disturbances can perturb the state x to another state $x' \in \bigcup\limits_{\ell=1}^{N} B_\psi(\xi^*_\ell[i], r)$, where $i \geq j$, then the trajectory $x(\xi(\cdot; x_0, \chi_u))$ still satisfies the MTL specification φ with probability at least $1-\epsilon$.

IV. Wind Turbine Generator Controller Synthesis

In this section, we apply the controller synthesis method in designing a coordinated controller for regulating the grid frequency of a four-bus system with a 600 MW thermal plant made up of four identical units, a wind farm consisting of 200 identical 1.5 MW Type-C wind turbine generators (WTG) and an energy storage system (ESS), as shown in Fig. 2. The configuration parameters of each Type-C WTG can be found in Appendix B of [16]. For each Type-C WTG, the differential equations are given as follows:

$$\begin{cases} \dot{E}'_{qD} = -\frac{1}{T'_0}(E'_{qD} + (X_s - X'_s)I_{ds}) + \omega_s \frac{X_m}{X_r} V_{dr} \\ \qquad - (\omega_s - \omega_r)E'_{dD}, \\ \dot{E}'_{dD} = -\frac{1}{T'_0}(E'_{dD} - (X_s - X'_s)I_{qs}) - \omega_s \frac{X_m}{X_r} V_{qr} \\ \qquad + (\omega_s - \omega_r)E'_{qD}, \\ d\omega_r = \frac{\omega_s}{2H_D}(T_m - E'_{dD}I_{ds} - E'_{qD}I_{qs})dt + k_w dw, \\ \dot{x}_1 = K_{I1}(P_{\text{ref}} - P_{\text{gen}}), \\ \dot{x}_2 = K_{I2}(K_{P1}(P_{\text{ref}} - P_{\text{gen}}) + x_1 - I_{qr}), \\ \dot{x}_3 = K_{I3}(Q_{\text{ref}} - Q_{\text{gen}}), \\ \dot{x}_4 = K_{I4}(K_{P3}(Q_{\text{ref}} - Q_{\text{gen}}) + x_3 - I_{dr}), \end{cases}$$

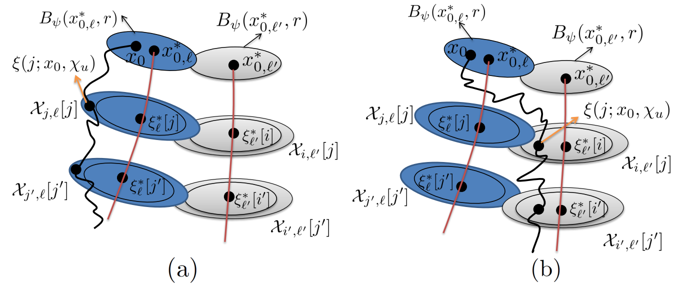


Fig. 1. Two possible trajectories (realizations) of the stochastic system (black), two nominal trajectories (brown) and the stochastic robust neighbourhoods around the nominal trajectories.

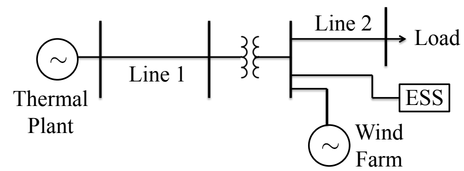


Fig. 2. The four-bus system [5] with a thermal plant, a wind farm and an energy storage system (ESS).

where E'_{dD}, E'_{qD} and ω_r are the d, q axis voltage and rotor speed of the WTG, respectively, k_w is a positive factor corresponding to the stochastic part of the wind power generation, x_1 to x_4 are proportional-integral (PI) regulator induced states, K_{I1}, K_{I2}, K_{I3}, K_{I4}, K_{P1}, K_{P2}, K_{P3}, K_{P4} are parameters of the PI regulator, T_m is the mechanical torque generated by the wind, $V_{dr}, V_{qr}, I_{dr}, I_{qr}$ are the rotor d, q axis voltage and current, respectively, I_{ds}, I_{qs} are the stator d, q axis current, respectively, P_{gen} and Q_{gen} are the WTG active and reactive power output, respectively, and

$$\begin{aligned} & P_{\text{ref}} = C_{\text{opt}}\omega_r^3 + u^w, Q_{\text{ref}} = Q_{\text{set}}, \\ & T'_0 = \frac{X_r}{\omega_s R_r},\ X'_s = X_s - \frac{X_m^2}{X_r}, \lambda = \frac{2k\omega_r R_t}{pv_{\text{wind}}}, \\ & \lambda_i = (\frac{1}{\lambda + 0.08\theta_t} - \frac{0.035}{\theta_t^3 + 1})^{-1}, \\ & C_p = 0.22(\frac{116}{\lambda_i} - 0.4\theta_t - 5)e^{-\frac{12.5}{\lambda_i}}, \\ & T_m = \frac{1}{2}\frac{\rho\pi R_t^2 \omega_b C_p v_{\text{wind}}^3}{S_b \omega_r}, \end{aligned}$$

where u^w is a control input (by adjusting the input u^w, the wind turbine generator can adjust its power output for restoring the grid frequency to allowable ranges after a disturbance), the explanations of the other parameters can be found in Section 2.1.2 of [16]. We set S_b=1MVA, $P_{\text{gen}} = 1.5$, v^3_{wind}=12m/s for the operating condition, C_{opt}.

The algebraic equations of each Type-C WTG are given as follows:

$$\begin{cases}0 = K_{P2}(K_{P1}(P_{\text{ref}} - P_{\text{gen}}) + x_1 - I_{qr}) + x_2 - V_{qr},\\ 0 = K_{P4}(K_{P3}(Q_{\text{ref}} - Q_{\text{gen}}) + x_3 - I_{dr}) + x_4 - V_{dr},\\ 0 = -P_{\text{gen}} + E'_{dD}I_{ds} + E'_{qD}I_{qs} - R_s(I_{ds}^2 + I_{qs}^2)\\ \quad - (V_{qr}I_{qr} + V_{dr}I_{dr}),\\ 0 = -Q_{\text{gen}} + E'_{qD}I_{ds} - E'_{dD}I_{qs} - X'_s(I_{ds}^2 + I_{qs}^2),\\ 0 = -I_{dr} + \dfrac{E'_{qD}}{X_m} + \dfrac{X_m}{X_r}I_{ds},\\ 0 = -I_{qr} - \dfrac{E'_{dD}}{X_m} + \dfrac{X_m}{X_r}I_{qs},\end{cases}$$

The network algebraic equations are given as follows (details of the Type-C wind turbine generator network can be found in Figure 2.3 of [16]):

$$\begin{aligned}E'_{qD} - jE'_{dD} &= (R_s + jX'_s)(I_{qs} - jI_{ds}) + V_D,\\ V_D e^{j\theta_D} &= jX_t(I_{qs} - jI_{ds} - I_{GC})e^{j\theta_D} + Ve^{j\theta},\end{aligned}$$

where V_D and θ_D are voltage magnitude and angle of the bus to which the WTG is connected, and

$$I_{GC} = \frac{V_{qr}I_{qr} + V_{dr}I_{dr}}{V_D}.$$

By linearizing the system of differential-algebraic equations at the equilibrium point (the equilibrium point can be found by calculating the root of the algebraic equations and the right-hand side of the differential equations equal to zero), we have

$$\begin{aligned}&d\begin{bmatrix}\Delta x\\ 0\end{bmatrix} = \begin{bmatrix}A_s & B_s\\ C_s & D_s\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}dt + \begin{bmatrix}M_s\\ N_s\end{bmatrix}u^w dt + \begin{bmatrix}\Sigma_{s1}\\ \Sigma_{s2}\end{bmatrix}dw,\\ &\Delta P_{\text{gen}} = \begin{bmatrix}E_s & F_s\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix},\end{aligned} \tag{15}$$

where $\Delta x = [\Delta E_{qD}, \Delta E_{dD}, \Delta\omega_r, \Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4]^T$, $\Delta y = [\Delta P_{\text{gen}}, \Delta Q_{\text{gen}}, \Delta V_{dr}, \Delta V_{qr}, \Delta I_{dr}, \Delta I_{qr}, \Delta I_{ds}, \Delta I_{qs}, \Delta V_D, \Delta\theta_D]^T$, $\triangle P_{\text{gen}}$ is the active power variation from each WTG.

Through the Kron reduction, we have

$$\begin{aligned}d\Delta x &= A_{\text{kr}}\Delta x dt + B_{\text{kr}}u^w dt + \Sigma_{\text{kr}}dw,\\ \Delta P_{\text{gen}} &= C_{\text{kr}}\Delta x + D_{\text{kr}}u^w + E_{\text{kr}}dw/dt,\end{aligned} \tag{16}$$

where

$$\begin{aligned}A_{\text{kr}} &= A_s - B_s D_s^{-1}C_s, & B_{\text{kr}} &= M_s - B_s D_s^{-1}N_s,\\ C_{\text{kr}} &= E_s - F_s D_s^{-1}C_s, & D_{\text{kr}} &= -F_s D_s^{-1}N_s,\\ \Sigma_{\text{kr}} &= \Sigma_{s1} - B_s D_s^{-1}\Sigma_{s2}, & E_{\text{kr}} &= -F_s D_s^{-1}\Sigma_{s2}.\end{aligned}$$

For the four-bus system, the system frequency response model is as follows (we choose base MVA as 1000MVA):

$$\begin{aligned}\triangle\dot{\omega} &= \frac{\omega_s}{2H}(\triangle P_m + u^s - \triangle P_d + 200\Delta P_{\text{gen}}/1000 - \frac{D}{\omega_s}\triangle\omega),\\ \triangle\dot{P}_m &= \frac{1}{\tau_{\text{ch}}}(\triangle P_v - \triangle P_m),\\ \triangle\dot{P}_v &= \frac{1}{\tau_g}(-\triangle P_v - \frac{1}{2\pi R}\triangle\omega),\end{aligned} \tag{17}$$

where $\triangle\omega$ is the grid frequency deviation, $\triangle P_m$ is the governor mechanical power variation, $\triangle P_v$ is the governor valve position variation and $\triangle P_d$ denotes a large disturbance (e.g. generation loss or abrupt load changes). $\triangle P_{\text{gen}}$ times 200 as there are 200 WTGs, and it is divided by 1000 as the base MVA for each WTG and the power system are 1MVA and 1000 MVA, respectively. We set $\omega_s = 2\pi \times 60$rad/s, D=1, H=4s, τ_{ch}=0.3s, τ_g=0.1s, R=0.05.

With (16) and (17), we have the following linear system:

$$d\hat{x} = (\hat{A}\hat{x} + \hat{B}u)dt + \hat{\Sigma}dw, \tag{18}$$

where $\hat{x} = [\triangle E_{qD}, \triangle E_{dD}, \triangle\omega_r, \triangle x_1, \triangle x_2, \triangle x_3, \triangle x_4, \triangle\omega, \triangle P_m, \triangle P_v]^T$, the input $u = [u^w, u^s]^T$. As the matrix $\hat{A}$ is computed as Hurwitz, the system is stable.

We consider a disturbance of generation loss of 150 MW (loss of one unit, $\Delta P_d = 0.15$). We use the following MTL specification for frequency regulation after the disturbance:

$$\begin{aligned}\varphi =&\Box_{[0,T_{\text{end}}]}p_1 \wedge \Box_{[2,T_{\text{end}}]}p_2,\\ p_1 =&(-0.5\text{Hz} \le \Delta f \le 0.5\text{Hz}) \wedge (-10\text{Hz} \le \Delta f_r \le 10\text{Hz}),\\ p_2 =&(-0.4\text{Hz} \le \Delta f \le 0.4\text{Hz}),\end{aligned} \tag{19}$$

where $\Delta f = \frac{\triangle\omega}{2\pi}$, $\Delta f_{\text{r}} = \frac{\triangle\omega_{\text{r}}}{2\pi}$. The specification means "After a disturbance, the grid frequency deviation should never exceed $\pm$0.5Hz, the WTG rotor speed deviation should never exceed $\pm$10Hz, after 2 seconds the grid frequency deviation should always be within $\pm$0.4Hz.

We set $k_w = 1$, $T_{\text{end}} = 5$ (s), $\epsilon = \alpha T_{\text{end}}/\hat{\gamma} = 5\%$, so $\alpha = 0.05\hat{\gamma}/T_{\text{end}} = 0.01\hat{\gamma}$. As $\alpha = trace(\hat{\Sigma}^T M\hat{\Sigma}) = k_w^2 M(3,3)$, we have $\hat{\gamma} = 100k_w^2 M(3,3) = 100M(3,3)$. We assume that the initial state variations can be covered by $B_\psi(\hat{x}_0^*, r)$, where $r = 4\hat{\gamma}$ ($4 = 2^2$ is chosen as the initial state variations due to the time needed for running the algorithm to generate the controller, which is about twice the simulation time), $\hat{x}_0^*$ is zero in every dimension. It can be seen from (19) that the allowable variation range of the grid frequency variation $\triangle\omega$ is much smaller than that of the wind turbine rotor speed variation $\triangle\omega_{\text{r}}$. Therefore, in order to decrease the conservativeness of the probabilistic bound as much as possible, we further optimize both $z_{k,i}$ and the matrix M such that the outer bounds of the stochastic robust neighbourhoods in the dimension of the grid frequency variation $\hat{\delta}_{1,1}^*$ ($\hat{\delta}_{1,1}^* = \hat{\delta}_{1,2}^* = \hat{\delta}_{2,1}^* = \hat{\delta}_{2,2}^*$) are much smaller than the outer bounds in the dimension of the wind turbine rotor speed variation $\hat{\delta}_{1,3}^*$ ($\hat{\delta}_{1,3}^* = \hat{\delta}_{1,4}^*$). As $\hat{\delta}_{k,i} = (\sqrt{r} + \sqrt{\hat{\gamma}})/z_{k,i}$ and $\hat{\gamma} = 100M(3,3)$, minimizing $\hat{\delta}_{1,1}$ can be achieved by minimizing $M(3,3)$ and maximizing $z_{1,1}$. The combined optimization to obtain both M^* and $z_{1,1}^*$ is as follows:

$$\begin{aligned}&\min. -\mathrm{z}_{1,1}^2\\ &\text{s.t. } M \succ 0, \hat{A}^T M + M\hat{A} + \mu M \preceq 0,\\ &\quad e_3^T M e_3 \le \eta, M - z_{1,1}^2 a_{1,1}a_{1,1}^T \succeq 0.\end{aligned} \tag{20}$$

where $e_3 = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0]^T$, $\mu = 10^{-10}$, η is tuned manually to be as small as possible while the optimization problem is feasible.

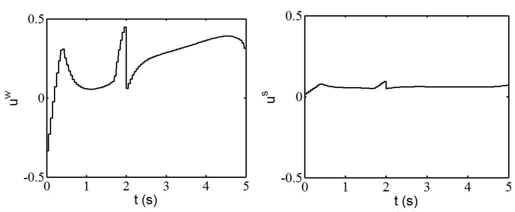
With the M^* obtained from (20), we compute the tightest outer bound in the dimension of $\triangle\omega_\mathrm{r}$ as follows:

$$\begin{aligned} &\min. -\mathrm{z}_{1,3}^2 \\ &\text{s.t. } M^* - z_{1,3}^2 a_{1,3} a_{1,3}^T \succeq 0. \end{aligned} \tag{21}$$

From (20) and (21), we obtain the $\hat{\delta}_{k,i}^*$-robust modified formula:

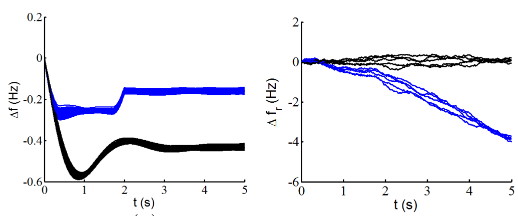
$$\begin{aligned} \varphi_{\hat{\delta}^*} = & \Box_{[0,T_\mathrm{end}]} \hat{p}_1^* \wedge \Box_{[2,T_\mathrm{end}]} \hat{p}_2^*, \\ \hat{p}_1^* = & (-0.5\mathrm{Hz} + 0.217\mathrm{Hz} \leq \Delta f \leq 0.5\mathrm{Hz} - 0.217\mathrm{Hz}) \wedge \\ & (-10\mathrm{Hz} + 6.08\mathrm{Hz} \leq \Delta f_r \leq 10\mathrm{Hz} - 6.08\mathrm{Hz}), \\ \hat{p}_2^* = & (-0.4\mathrm{Hz} + 0.217\mathrm{Hz} \leq \Delta f \leq 0.4\mathrm{Hz} - 0.217\mathrm{Hz}). \end{aligned}$$

Simulations: For the feedforward controller synthesis, we set $J(u(\cdot)) = \|u^w(\cdot)\|_2 + \lambda \|u^s(\cdot)\|_2$, where $\lambda = 100$ (larger λ encourages power input from the wind turbine generator)[1]. The obtained optimal input signals are shown in Fig. 3. As shown in Fig. 4, all of the 100 trajectories (realizations) starting from $B_\psi(\hat{x}_0^*, r)$ with the obtained optimal input signals satisfy the specification φ.



(a) (b)

Fig. 3. The obtained optimal input signals.

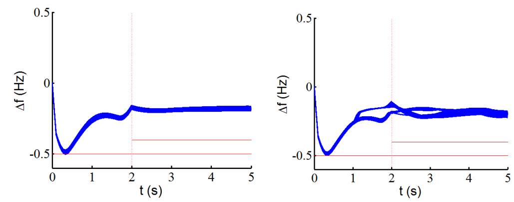


(a) (b)

Fig. 4. 100 trajectories (realizations) of Δf and Δf_r without control (black) and with the feedforward controller (blue).

We further design a feedback controller based on the obtained optimal input signals of the feedforward controller. To make a comparison between the feedback controller and the feedforward controller, we add an unexpected disturbance of per unit value 0.35 to ΔP_d during the first 0.1 second while generating 100 trajectories (realizations) of the stochastic system with both the feedforward and the feedback controller. As shown in Fig. 5, 99% of the trajectories generated with the feedforward controller still satisfy the MTL specification φ, while all the trajectories generated with the feedback controller still satisfy the MTL specification φ with the minimal robustness degree of 0.0016.

[1]The Matlab codes for the simulations can be found in https://github.com/david00710/WTGcontroller.



(a) (b)

Fig. 5. 100 trajectories (realizations) of Δf (a) with the feedforward controller and (b) with the feedback controller, with an unexpected disturbance of per unit value 0.35 to ΔP_d during the first 0.1 second.

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