

Parallel-Channel Flow Instabilities and Active Control Schemes in Two-Phase Microchannel Heat Exchanger Systems

TieJun Zhang, John T. Wen, Agung Julius, He Bai, Yoav Peles, Michael K. Jensen
Rensselaer Polytechnic Institute, Troy, NY 12180

Abstract—Parallel-channel flow mal-distribution and pressure-drop flow oscillations are two of the most severe dynamic instabilities for boiling flow especially in microchannel systems. This paper presents a framework for the transient analysis and active control of microchannel flow instabilities at a system-level view. A lumped two-phase flow system model is derived from the momentum balance equation to capture the characteristics of the microchannel heat exchangers. Bifurcations of flow distribution and inlet pressure can arise in parallel-channel two-phase flow systems. This paper investigates the control-theoretic properties with different control devices, including inlet valves and supply pump. Individual control valves at the inlet of each channel can be used to suppress both flow mal-distribution and flow oscillations effectively, although this scheme is subject to higher pressure loss and potential higher supply pumping power. Using the pump alone can only suppress pressure-drop flow oscillations, but not for flow mal-distribution in two identical parallel channels. However, we make an interesting observation that with different channel properties, we regain controllability from the pump and observability from a single channel flow rate measurement.

I. INTRODUCTION

Thermal challenges in next-generation electronic systems are attracting more attention due to the rapidly increasing demands of high-power density electronics [1]. The heat dissipation rate of defense radars, directed-energy laser, and electromagnetic weapons will exceed 1000 W/cm² in the near future [2], while the surface temperatures of chips and devices need to be maintained below 85 °C in naval all-electric surface ships [3]. In recent years, microchannel boiling has become a popular scheme in these high heat flux electronics cooling challenges [2], [4]–[6] since boiling utilizes the latent heat of vaporization with a lower mass flow rate. However, two-phase microchannel heat sinks have a critical operation problem: cooling systems with microchannel heat sinks are prone to various boiling flow instabilities (e.g., pressure-drop and thermal oscillations, parallel-channel instabilities [5], [6]).

Therefore, understanding of flow instabilities is of particular importance for the design, control, and performance prediction of any two-phase system, especially the design of mini/microchannel evaporators [4]–[6], [8]. If there is no compressible volume upstream of a boiling system, the

The authors are with the Center for Automation Technologies and Systems. Wen and Julius are also with Electrical, Computer, & Systems Engineering Dept., Zhang, Peles, and Jensen are also with Mechanical, Aerospace & Nuclear Engineering Dept. Bai is currently with Mechanical Engineering Dept., Northwestern University, USA. {zhangt6, wenj, juliua2, pelesy, jensem}@rpi.edu

equilibrium in the two-phase negative-slope region is unstable [7], [9], [10], [12]. This static instability is called the Ledinegg instability. When an upstream compressible volume is present and the flow operates in the two-phase negative-slope region, a dynamic instability (oscillation) would occur [8]. This is called the pressure-drop instability, one of the well-known dynamic flow instabilities in two-phase flows. A compressible volume of gas may exist in long boiling channels (length/diameter, L/D , $\geq \sim 150$) or can be artificially introduced by placing a surge tank upstream of the heated section [7], [9]. Such L/D ratios are typically encountered in microchannels, and indeed, corresponding oscillations have been observed by many researchers [5], [6], [12]–[14]. Although there has been considerable research efforts focused on dynamic flow instabilities in microchannels, most of the existing work has been experimental demonstrations and visualization. The control of the dynamic flow boiling instabilities in microchannels is still a problem. Passive control methods have been proposed in the past, with inlet restrictors [14] and reentrant cavities [13]. The inlet restrictor instability suppression methods are limited by the pumping power capability of micro or mini-pumps since a larger pressure drop could arise. The focus in this paper is on active two-phase flow control as an alternative or complementary option for instability suppression.

II. PRELIMINARIES

A. Momentum Balance

The two-phase flow in a horizontal microchannel may be modeled by the one-dimension momentum balance equation:

$$\frac{\partial \dot{m}}{\partial t} + \frac{\partial}{\partial z} \left(\frac{\dot{m}^2}{\rho A} \right) + \frac{\partial PA}{\partial z} + F_{visc} = 0 \quad (1)$$

where \dot{m} is the mass flow rate, and F_{visc} is the frictional shear force due to fluid viscosity. Integrating through the channel length, one obtains the lumped momentum balance equation:

$$I \cdot \frac{d\dot{m}}{dt} = \Delta P_S(\dot{m}) - \Delta P_D(\dot{m}) \quad (2)$$
$$I = \frac{L}{A}, \quad \Delta, \quad \Delta P_D = \Delta P_a + \Delta P_f$$

where ΔP_S is the supply pressure drop; the demand pressure drop ΔP_D includes both the acceleration and frictional pressure drops, ΔP_a and ΔP_f , respectively.

B. Flow Instabilities

A typical shape of the demand curve is shown in Fig. 1 (no valve case). From Eq. (2), it is clear that under the constant supply pressure (dotted line), the equilibrium in the two-phase region is unstable. More generally, the stability condition at the equilibrium is [7], [10]

$$\frac{\partial(\Delta P_D)}{\partial \dot{m}} > \frac{\partial(\Delta P_S)}{\partial \dot{m}} \quad (3)$$

where $\Delta P_S(\dot{m})$ and $\Delta P_D(\dot{m})$ intersect. For a constant supply pressure, the equilibrium in the positive-slope region of the demand curve corresponds to stable operating conditions (in subcooled and superheated regions), and the equilibrium in the two-phase region is unstable. The unstable equilibria means that the system will shift to the superheated or subcooled operating points, thereby causing the heat dissipation performance to deteriorate, especially in microchannels [10].

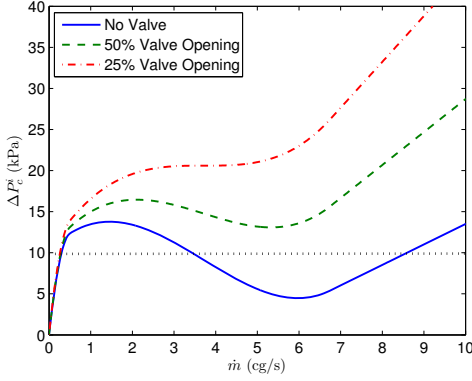


Fig. 1. Two-phase flow characteristics of individual microchannel heat exchanger (solid: without inlet valve; dashed: with inlet valve of 50% opening; dash-dotted: with inlet valve of 25% opening; dotted: constant supply pressure drop)

III. MAIN RESULTS

A. Ledinegg Instability in Parallel-Channel Flows

Suppose a heat sink has N identical parallel channels, then the overall mass flow rate through the boiling channels is $\dot{m} = \sum_{i=1}^N \dot{m}_i$, where \dot{m}_i is the mass flow rate across the i -th channel. By applying the momentum balance (2) to each boiling channel, one has

$$I_c \frac{d\dot{m}_i}{dt} = \Delta P_c^S - \Delta P_c^i = P_{in}(\dot{m}) - P_e - \Delta P_c^i(\dot{m}_i) \quad (4)$$

where I_c is the lumped channel inertia, P_{in} the inlet pressure, P_e is the exit pressure, and ΔP_c^i the demand pressure drop in the i -th channel with a shape similar to that in Fig. 1.

We shall assume that the channel exit pressure P_e is fixed. The inlet pressure P_{in} is dependent on the overall flow \dot{m} driven by the supply pump. The system (4) linearized about the equilibrium flows $\{\dot{m}_i^*\}$ is

$$I_c \frac{d(\delta \dot{m}_i)}{dt} = \frac{\partial P_{in}}{\partial \dot{m}} \cdot (\delta \dot{m}) - \frac{\partial(\Delta P_c^i)}{\partial \dot{m}_i} \cdot (\delta \dot{m}_i) \quad (5)$$

Defining

$$s_i = \frac{1}{I_c} \frac{\partial(\Delta P_c^i)}{\partial \dot{m}_i}, \quad s_0 = \frac{1}{I_c} \frac{\partial(P_{in})}{\partial \dot{m}}$$

with $\delta \dot{m} = \sum_{i=1}^N \delta \dot{m}_i$, the linearized system (5) may be expressed as

$$\frac{d(\delta \dot{m}_i)}{dt} = (s_0 - s_i) \cdot (\delta \dot{m}_i) + \sum_{j=1, j \neq i}^N s_0 \cdot (\delta \dot{m}_j) \quad (6)$$

The stability of the linearized system is determined by the eigenvalues of

$$\mathcal{A} = \begin{bmatrix} s_0 - s_1 & s_0 & \cdots & s_0 & s_0 \\ s_0 & s_0 - s_2 & \ddots & s_0 & s_0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ s_0 & s_0 & \ddots & s_0 - s_{N-1} & s_0 \\ s_0 & s_0 & \cdots & s_0 & s_0 - s_N \end{bmatrix}$$

Stability conditions for several special cases are considered below:

- $N = 1$: The system is stable if and only if $s_1 > s_0$. This is just the single-channel Ledinegg stability case in Eq. (3).
- $N = 2$: The characteristic equation is $\lambda^2 + (s_1 + s_2 - 2s_0)\lambda + s_1s_2 - (s_1 + s_2)s_0 = 0$. The system is stable if and only if $s_1 + s_2 > s_0$ and $s_1s_2 - (s_1 + s_2)s_0 > 0$. For a constant supply pressure, $s_0 = 0$, which implies flow stability if and only if both s_1 and s_2 are positive [15]. For the negative infinity supply pressure slope, $s_0 \rightarrow -\infty$, from positive displacement pump, the flow stability condition is, $s_1 + s_2 > 0$.
- $N = 3$: The corresponding stability conditions may be obtained using the Routh-Hurwitz criterion:

$$\begin{cases} t_1 = (s_1 + s_2 + s_3) - 3s_0 > 0 \\ t_2 = s_1s_2 + s_1s_3 + s_2s_3 - 2s_0(s_1 + s_2 + s_3) > 0 \\ t_3 = s_1s_2s_3 - s_0(s_1s_2 + s_1s_3 + s_2s_3) > 0 \\ t_4 = t_1t_2 - t_3 > 0. \end{cases} \quad (7)$$

Consider a two parallel-channel system as a special case, with the demand channel pressure drop curve as in Fig. 1 (no valve case). There may be up to three equilibria depending on the flow rates. For a constant supply pressure, the equilibria and their stability (bifurcation) are shown in the top graph of Fig. 2. Note that in the two-phase region, the uniform flow distribution, $\dot{m} = 2\dot{m}_1$, is the desired operating condition since it corresponds to enhanced heat transfer performance for both channels. Unfortunately, this operating state is also unstable.

The stability condition may be improved by adding in a control valve before each channel. In this case, the momentum balance of each parallel flow line (including both the control valve and heat exchanger) becomes

$$I_c \frac{d\dot{m}_i}{dt} = P_{in}(\dot{m}) - P_e - \Delta P_c^i(\dot{m}_i) - \Delta P_v^i \quad (8)$$

where ΔP_v^i is the control valve pressure drop:

$$\Delta P_v^i = \frac{\kappa_v \cdot \dot{m}_i}{A_v^i} = \kappa_v \cdot w_i \cdot \dot{m}_i \quad (9)$$

where $\kappa_v > 0$ is the valve characteristic coefficient, A_v^i the valve opening position within the range of $(0, 1]$, and $w_i = 1/A_v^i$. The stability for the constant supply pressure case then becomes:

$$s_i > -\kappa_v w_i / I_c \quad (10)$$

which improves the range of stability. As shown in Fig. 1, the negative-sloped region is reduced with greater valve pressure drop (i.e., smaller A_v^i). The bifurcation range for the two-phase flow is also reduced as in the bottom graph of Fig. 2.

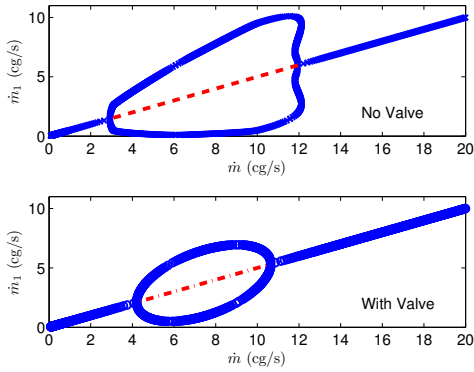


Fig. 2. Bifurcation curves of two-phase flow distribution \dot{m}_1/\dot{m} versus total flow rate \dot{m} in two parallel channels with/without inlet valves (blue marker: stable; red dashed line: unstable)

The slope of the combined valve/channel system will always be positive if the control valve resistance becomes dominant in each flow line. From our previous research [11], for a given heat load, the pressure drop of a boiling channel can be represented by a cubic function within the local two-phase region,

$$\Delta P_c^i = k_1 \dot{m}_i^3 + k_2 \dot{m}_i^2 + k_3 \dot{m}_i + k_4 \quad (11)$$

The stability condition is $\partial(\Delta P_c^i)/\partial \dot{m}_i + \partial(\Delta P_v^i)/\partial \dot{m}_i > 0$, which becomes

$$3k_1 \dot{m}_i^2 + 2k_2 \dot{m}_i + k_3 + \kappa_v w_i > 0 \quad (12)$$

A sufficient condition is

$$w_i = \frac{1}{A_v^i} > \frac{k_2^2}{3k_1 \kappa_v} - \frac{k_3}{\kappa_v} \quad (13)$$

When A_v is sufficiently small, e.g. $A_v \approx 25\%$ as in Fig. 1, the flow characteristic curve becomes monotonic, and the parallel-channel Ledinegg instability will no longer appear.

B. Parallel-Channel Flow Oscillations

The flow loop considered here may be represented by the schematic diagram Fig. 3, where the working fluid, subcooled water, is supplied to both the compressible surge tank and the parallel microscale heat exchangers. The microchannel flow meter is placed before the boiling system

to record the transient flow rate changes. Due to the laminar flow condition in microchannel applications, the differential pressure drop to flow rate calibration curve is an affine function: $\Delta P_{FM} = \alpha_1 \cdot \dot{m} + \alpha_2$ [11]. We now consider the stability of two parallel channels, each with flow characteristics as in Fig. 1, with an upstream surge tank. We shall regard the upstream flow rate, \dot{m}_0 (adjustable using the upstream pump) and the each individual channel valve opening, A_v^i , as the potential control variables.

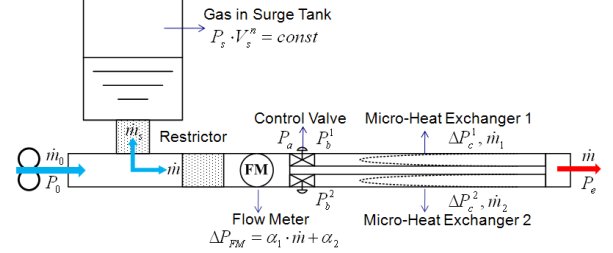


Fig. 3. Schematic of parallel heat exchangers with upstream surge tank

From mass balance, we have $\dot{m}_0 = \dot{m} + \dot{m}_s = \dot{m}_1 + \dot{m}_2 + \dot{m}_s$, where \dot{m} is the total boiling mass flow rate measured by a flow meter,

$$\Delta P_{FM} = P_0 - P_a = \alpha_1 \cdot \dot{m} + \alpha_2 \quad (14)$$

\dot{m}_i is the flow rate of channel i , and \dot{m}_s is the flow rate into the surge tank.

The momentum balance of the surge tank gives

$$I_s \frac{d\dot{m}_s}{dt} = P_0 - P_s - \Delta P_d \quad (15)$$

Since laminar subcooled liquid flows into the surge tank, its demand pressure drop from the flow restrictor becomes

$$\Delta P_d = \gamma \cdot \dot{m}_s, \quad \gamma > 0$$

where γ is a lumped coefficient for the pressure loss in the line connecting the surge tank to the main flow line.

In the tank, the gas is assumed to be inert. Therefore, the pressure P_s and the gas volume V_s are related by

$$P_s \cdot V_s^n = constant \quad (16)$$

where n is a fixed polytropic index of expansion. Differentiating (16) yields

$$\frac{dP_s}{dt} \cdot V_s^n + P_s \cdot n V_s^{n-1} \frac{dV_s}{dt} = 0$$

It follows

$$\frac{dP_s}{dt} = -\frac{n P_s}{V_s} \frac{dV_s}{dt} \quad (17)$$

The compressible gas volume change in the surge tank is proportional to the upstream liquid inflow with a density ρ_f ,

$$\dot{m}_s = -\rho_f \frac{dV_s}{dt}$$

Substituting dV_s/dt into (17), one obtains

$$\frac{dP_s}{dt} = C_s \cdot \dot{m}_s, \quad C_s = \frac{n P_s}{\rho_f V_s} \quad (18)$$

When there exists a control valve before each channel as in Fig. 3, with the valve characteristics in (9), the momentum balance of the channel (including both the control valve and heat exchanger) becomes

$$I_c \frac{d\dot{m}_i}{dt} = P_a(\dot{m}, \dot{m}_s) - \Delta P_c^i(\dot{m}_i) - \Delta P_v^i(\dot{m}_i) \quad (19)$$

where ΔP_c^i is the channel i pressure drop, and ΔP_v^i the channel i valve pressure drop.

This is a three-state system with state variables $(\dot{m}_s, \dot{m}_1, \dot{m}_2)$. To explicitly show the influence of the control variable \dot{m}_0 , we transform the system to one with state variables $(\dot{m}, \dot{m}, \dot{m}_1)$, where \dot{m} is the total flow rate. From mass balance, we have

$$\dot{m}_s = \dot{m}_0 - \dot{m}, \quad \dot{m}_2 = \dot{m} - \dot{m}_1$$

Summing the boiling flow dynamics (19) for two parallel channels yields

$$I_c \frac{d(\dot{m}_1 + \dot{m}_2)}{dt} = 2P_a - 2P_e - \Delta P_b^1 - \Delta P_b^2$$

$$\frac{I_c}{2} \frac{d\dot{m}}{dt} = P_a - P_e - \frac{\Delta P_b^1 + \Delta P_b^2}{2} \quad (20)$$

where $\Delta P_b^i = \Delta P_c^i + \Delta P_v^i$ is the total channel pressure drop. Then by subtracting (20) from the first channel dynamics (19), one can easily derive

$$\frac{d\dot{m}_1}{dt} = \frac{1}{2} \frac{d\dot{m}}{dt} + \frac{\Delta P_b^2 - \Delta P_b^1}{2 \cdot I_c} \quad (21)$$

To eliminate \dot{m}_s , we differentiate Eq. (15) to obtain

$$I_s \frac{d^2 \dot{m}}{dt^2} + \gamma \frac{d\dot{m}}{dt} + C_s \dot{m} + \frac{dP_0}{dt}$$

$$= C_s \dot{m}_0 + \gamma \frac{d\dot{m}_0}{dt} + I_s \frac{d^2 \dot{m}_0}{dt^2} \quad (22)$$

From the flow meter equation (14), we have

$$\frac{d(\Delta P_{FM})}{d\dot{m}} \frac{d\dot{m}}{dt} = \frac{dP_0}{dt} - \frac{dP_a}{dt} = \alpha_1 \frac{d\dot{m}}{dt} \quad (23)$$

which may be used to eliminate dP_0/dt in Eq. (22).

To further eliminate dP_a/dt , we differentiate (20) to obtain

$$\frac{I_c}{2} \frac{d^2 \dot{m}}{dt^2} = \frac{dP_a}{dt} - \frac{1}{2} \frac{d(\Delta P_b^1)}{d\dot{m}_1} \frac{d\dot{m}_1}{dt} - \frac{1}{2} \frac{d(\Delta P_b^2)}{d\dot{m}_2} \frac{d\dot{m}_2}{dt}$$

$$- \frac{\kappa_v}{2} \left(\dot{m}_1 \frac{dw_1}{dt} + \dot{m}_2 \frac{dw_2}{dt} \right) \quad (24)$$

Substituting (21) into (24), and combining with (22) and (23), we can summarize the parallel-channel flow oscillation model as (25)-(26) (at the top of the next page) where the slope of the individual channel pressure drop $\Delta P_b^i = \Delta P_c^i + \Delta P_v^i$ includes two parts:

- Pressure-drop slope of the two-phase flow in each channel (from Eq. (11)):

$$\frac{d(\Delta P_c^i)}{d\dot{m}_i} = \delta_i (\dot{m}_i - \dot{m}_a^i) (\dot{m}_i - \dot{m}_b^i), \quad \delta_i > 0 \quad (27)$$

\dot{m}_a^i, \dot{m}_b^i corresponds to the mass flow of saturated vapor and saturated liquid, respectively, and $\dot{m}_a^i < \dot{m}_i < \dot{m}_b^i$.

- Pressure-drop slope of subcooled liquid flow through each control valve

$$\frac{d(\Delta P_v^i)}{d\dot{m}} = \kappa_v / A_v^i = \kappa_v \cdot w_i \quad (28)$$

C. Active Flow Instability Control

As shown in the flow loop schematic (Fig. 3), the system inlet flow rate \dot{m}_0 is easily manipulated by a positive displacement pump and is linearly dependent on the pump voltage in practice. As a manipulated variable, \dot{m}_0 is no longer fixed, and its change rate will definitely affect the flow dynamics of both the surge tank and the boiling channel. In addition, the control valve opening positions A_v^i (or equivalently w_i) are other manipulated variables, which can be used to maintain a balanced flow through parallel heat exchangers as mentioned in Section III-A.

When the inlet flow resistance is large enough, or equivalently, the control valve position is small enough, the pressure drop of the combined valve and heat exchanger flow path will not decrease as the flow rate increases. This also removes one of the triggering conditions for pressure-drop flow oscillations, which was mentioned in the introduction section. More specifically, for a two-phase heat exchanger with flow characteristics like Eq. (11), the stabilizing condition of control valve for both the parallel-channel and pressure-drop flow instabilities can be summarized as follows

$$A_v^i \leq \frac{3k_1 \kappa_v}{k_2^2 - 3k_1 k_3} \quad (29)$$

Once the condition is satisfied, flow stability is expected to be achieved (given in Fig. 5). Obviously, smaller valve opening position means much higher flow resistance and larger pressure loss, so it is desirable to investigate other active flow control strategies for the suppression of two-phase flow instabilities.

Consider another control variable – inlet flow rate – and fix both control valves. Assuming that $\dot{m}_0 = Z_0 + u$, where u is a variation to a reference flow rate Z_0 , one has

$$\frac{d\dot{m}_0}{dt} = \frac{du}{dt}, \quad \frac{d^2 \dot{m}_0}{dt^2} = \frac{d^2 u}{dt^2}$$

Also, let $Z = \dot{m} - Z_0$, $Y = \dot{m}_1 - Z_0/2$, accordingly

$$\Delta P_b^1 = g_1(Y), \quad \Delta P_b^2 = g_2(Z - Y),$$

$$\frac{d(\Delta P_b^1)}{d\dot{m}_1} = g_1'(Y), \quad \frac{d(\Delta P_b^2)}{d\dot{m}_2} = g_2'(Z - Y)$$

then one can get the following control-oriented parallel flow oscillation model,

$$\frac{d^2 Z}{dt^2} + \left[\frac{\gamma + \alpha_1}{I} + \frac{g_1'(Y) + g_2'(Z - Y)}{4 \cdot I} \right] \frac{dZ}{dt} + \frac{C_s}{I} \cdot Z$$

$$= \frac{C_s}{I} \cdot u + \frac{\gamma}{I} \frac{du}{dt} + \frac{I_s}{I} \frac{d^2 u}{dt^2}$$

$$+ \frac{g_2(Z - Y) - g_1(Y)}{4 \cdot I_c \cdot I} [g_2'(Z - Y) - g_1'(Y)]$$

$$:= \frac{C_s}{I} \cdot U + h(Z, Y), \quad I = \frac{I_c}{2} + I_s \quad (30)$$

$$\frac{dY}{dt} = \frac{1}{2} \frac{dZ}{dt} + \frac{g_2(Z - Y) - g_1(Y)}{2 \cdot I_c} \quad (31)$$

$$\begin{aligned} & \left(\frac{I_c}{2} + I_s \right) \frac{d^2 \dot{m}}{dt^2} + \left(\gamma + \alpha_1 + \frac{1}{4} \left[\frac{d(\Delta P_b^1)}{d\dot{m}_1} + \frac{d(\Delta P_b^2)}{d\dot{m}_2} \right] \right) \frac{d\dot{m}}{dt} + C_s \dot{m} \\ & = C_s \dot{m}_0 + \gamma \frac{d\dot{m}_0}{dt} + I_s \frac{d^2 \dot{m}_0}{dt^2} + \frac{\Delta P_b^2 - \Delta P_b^1}{4 \cdot I_c} \left[\frac{d(\Delta P_b^2)}{d\dot{m}_2} - \frac{d(\Delta P_b^1)}{d\dot{m}_1} \right] + \frac{\kappa_v \dot{m}_1}{2} \left(\frac{dw_2}{dt} - \frac{dw_1}{dt} \right) - \frac{\kappa_v \dot{m}}{2} \frac{dw_2}{dt} \end{aligned} \quad (25)$$

$$\frac{d\dot{m}_1}{dt} = \frac{1}{2} \frac{d\dot{m}}{dt} + \frac{\Delta P_b^2 - \Delta P_b^1}{2 \cdot I_c} \quad (26)$$

Notice that U in (30) is defined as

$$U = u + \frac{\gamma}{C_s} \frac{du}{dt} + \frac{I_s}{C_s} \frac{d^2 u}{dt^2}$$

and $h(Z, Y)$ represents the flow excursion and flow interaction among parallel channels,

$$\begin{aligned} h(Z, Y) &= \frac{\Delta P_c^2 - \Delta P_c^1}{4 \cdot I_c \cdot I} \left[\frac{d(\Delta P_c^2)}{d\dot{m}_2} - \frac{d(\Delta P_c^1)}{d\dot{m}_1} \right] \\ &= \frac{g_2(Z - Y) - g_1(Y)}{4 \cdot I_c \cdot I} [g_2'(Z - Y) - g_1'(Y)] \end{aligned}$$

which can be regarded as the disturbance of the overall flow system. Moreover, defining the state variables $x_1 = Z$, $x_2 = dZ/dt$, $x_3 = Y$, and letting the coefficients

$$c_1 = \frac{\gamma + \alpha_1}{I}, \quad c_2 = \frac{1}{4I}, \quad c_3 = \frac{C_s}{I}, \quad c_4 = \frac{1}{4I_c I}, \quad c_5 = \frac{1}{2I_c}$$

we obtain the following state space representation of the normalized flow system (30)-(31)

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -c_3 x_1 - x_2 \cdot f(x_1, x_3) + c_3 U + h(x_1, x_3) \\ \dot{x}_3 = 0.5 x_2 + c_5 \cdot [g_2(x_1 - x_3) - g_1(x_3)] \end{cases} \quad (32)$$

where $f(x_1, x_3) = c_1 + c_2 \cdot [g_1'(x_3) + g_2'(x_1 - x_3)]$ and $h(x_1, x_3) = c_4 \cdot [g_2(x_1 - x_3) - g_1(x_3)][g_2'(x_1 - x_3) - g_1'(x_3)]$.

The equilibrium $x^* = [x_1^*, x_2^*, x_3^*]$ is given by the solution of $x_2^* = 0$, $c_3 x_1^* = h(x_1^*, x_3^*)$, $g_2(x_1^* - x_3^*) = g_1(x_3^*)$. If the channels are identical, i.e., $g_1 = g_2$, the equally distributed flow condition, $x_1^* = 2x_3^*$, is always an equilibrium.

Around the equilibrium point x^* , the linearized system is $f(x_1, x_3)$, $h(x_1, x_3)$ and get the following linearization of the nonlinear flow system (32)

$$\dot{\mathbf{X}} = \mathbf{A} \cdot \mathbf{X} + \mathbf{B} \cdot U, \quad (33)$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ a_{21} & a_{22} & a_{23} \\ a_{31} & 0.5 & a_{33} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ c_3 \\ 0 \end{bmatrix},$$

$$\begin{aligned} a_{21} &= -c_3 - c_2 x_2^* g_2''(x_3^*) + c_4 g_2'(x_3^*) [g_2'(x_3^*) - g_1'(x_3^*)], \\ a_{22} &= -c_1 - c_2 [g_1'(x_3^*) + g_2'(x_3^*)], \\ a_{23} &= -c_2 x_2^* [g_1''(x_3^*) - g_2''(x_3^*)] - c_4 [g_2''(x_3^*) - g_1''(x_3^*)] \\ a_{31} &= c_5 \cdot g_2'(x_3^*), \quad a_{33} = -c_5 \cdot g_1'(x_3^*) - c_5 \cdot g_2'(x_3^*). \end{aligned}$$

The controllability matrix may be computed by

$$\begin{aligned} \mathbf{Q}_c &= [\mathbf{B} \quad \mathbf{A}\mathbf{B} \quad \mathbf{A}^2\mathbf{B}] \\ &= \begin{bmatrix} 0 & c_3 & c_3 a_{22} \\ c_3 & c_3 a_{22} & c_3 a_{21} + c_3 a_{22}^2 + 0.5 c_3 a_{23} \\ 0 & 0.5 c_3 & c_3 a_{31} + 0.5 c_3 a_{22} + 0.5 c_3 a_{33} \end{bmatrix} \end{aligned} \quad (34)$$

For two identical parallel channels, $a_{23} = 0$, $a_{31} + 0.5 a_{33} = 0$, the first and third rows of (34) are linearly dependent, thus \mathbf{Q}_c is not of full rank. This means that the total mass flow \dot{m} and the individual mass flow \dot{m}_1 in a uniform parallel-channel system are not fully controllable around the desired equilibrium by using the upstream pump alone. However, it is interesting to note that for the non-identical-channel flow case (when the individual flow characteristics, g_i , are not the same), the overall flow system is controllable by just using the pump without the individual valve control.

There is also the dual result on observability based on the measurement of the total mass flow rate \dot{m} , i.e., x_1 . This means $\mathbf{C} = [1 \ 0 \ 0]$, and the observability matrix can be calculated as

$$\begin{aligned} \mathbf{Q}_o^1 &= [\mathbf{C}^T (\mathbf{C}\mathbf{A})^T (\mathbf{C}\mathbf{A}^2)^T]^T \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \end{aligned} \quad (35)$$

For two uniform parallel-channel flow system, $a_{23} = 0$; hence, the overall flow system (32) is unobservable, and the state x_3 cannot be estimated based on x_1 . This implies that individual channel flow dynamics cannot be predicted from the total flow rate measurement. However, we can again easily show that for the non-identical channel case, the full state is observable from the total mass flow rate.

We have only included a simple simulation example for the identical channel case. Consider the general model structure (30) of the flow oscillation system. It is known from the analysis in [11] that the oscillation amplitude is dependent on the nonlinear damping function associated with dZ/dt . Therefore, one can design the feedback control law to force the nonlinear damping function to be always positive. To demonstrate the effectiveness, the feedback control law,

$$\dot{m}_0^U := U + Z_0 = Z_0 - K_0 x_2 / c_3 = Z_0 - K_0 \dot{Z} \cdot I / C_s \quad (36)$$

is applied since the $t = 200$ th second. The boundary control gain, $K_0 = 14.93$, is chosen based on the identified oscillation model parameters in [11]. Active flow oscillation control responses are shown in Fig. 4. Stable and uniform flow distribution are observed, but when a pulse disturbance is imposed on one of the channels, the flow mal-distribution occurs with one channel dried out and the other flooded.

This simulation shows that for identical-parallel-channel flow system, the inlet mass flow rate alone is not sufficient to control both pressure-drop and parallel-channel flow instabilities. With additional control device such as the inlet

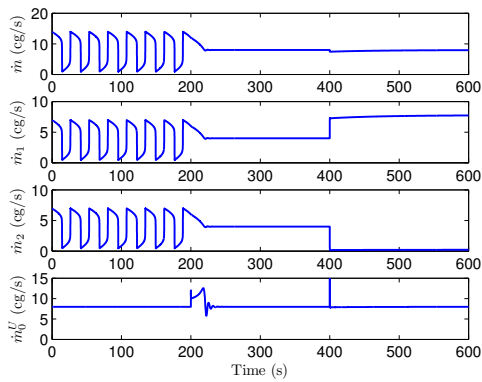


Fig. 4. Active control of pressure-drop flow oscillations coupled with parallel-channel flow mal-distribution (inlet flow control \dot{m}_0^U applied since the 200-th second and pulse disturbance imposed at the 400-th second)

control valves as in Fig. 3, the operating state of identically-distributed flows becomes stable as shown in Fig. 5.

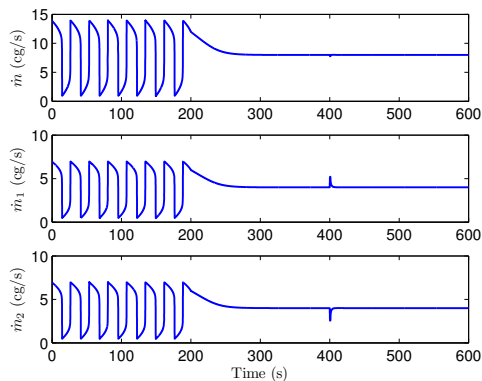


Fig. 5. Active control of pressure-drop flow oscillations coupled with parallel-channel flow mal-distribution (individual valve control A_i^i applied at the $t = 200\text{sec}$ and pulse disturbance imposed at the $t = 400\text{sec}$)

IV. CONCLUSIONS AND FUTURE WORKS

This paper analyzes the stability and control of parallel-channel flow systems. By using an inlet control valve for each individual channel, the parallel-channel instability can be suppressed and the uniform two-phase flow distribution is achieved. Although the valve-based control scheme is effective, the drawback is that the flow system has to sustain much higher pressure loss and the instrumentation is more complex. When the upstream pump is used as the control variable for an identical-channel flow system, it is only possible for the control of pressure-drop flow oscillations but not for the control of flow in the individual channels. However, we demonstrate an interesting result that if the channels are not identical, the system is full state controllable. There is also the dual result, that the channel flow rates are not observable from the total flow rate under the identical-channel case, but are observable when the channels characteristics are different. This study is being extended to

more general N -channel cases. Most of microchannel heat sinks are fabricated to have uniform multiple parallel channels. This paper shows that this is actually undesirable from the control perspective. We are now actively pursuing this research direction to exploit non-identical channel designs.

V. ACKNOWLEDGMENTS

This work is supported in part by the Office of Naval Research (ONR) under the Multidisciplinary University Research Initiative (MURI) Award GG10919 entitled “System-Level Approach for Multi-Phase, Nanotechnology-Enhanced Cooling of High-Power Microelectronic Systems,” in part by the National Science Foundation (NSF) Smart Lighting Engineering Research Center (EEC-0812056), and in part by the Center for Automation Technologies and Systems (CATS) under a block grant from the New York State Foundation for Science, Technology and Innovation (NYSTAR). The authors would like to thank Tao Tong, Je-Young Chang, Ravi Prasher in Intel Corporation for providing experimental flow oscillation data.

REFERENCES

- [1] S. V. Garimella, A. S. Fleischer, J. Y. Murthy, et al., “Thermal challenges in next-generation electronic systems”, *IEEE Trans. on Components and Packaging Technologies*, vol.31, pp.801-815, 2008.
- [2] J. Lee and I. Mudawar, “Low-temperature two-phase microchannel cooling for high-heat-flux thermal management of defense electronics”, *IEEE Trans. on Components and Packaging Technologies*, in press, vol.32, pp.453-465, 2009.
- [3] T. W. Webb, T. M. Kiehne, and S. T. Haag, “System-level thermal management of pulsed loads on an all-electric ship”, *IEEE Transactions on Magnetics*, vol.43, pp.469-473, 2007.
- [4] S. G. Kandlikar, S. Garimella, D. Li, et al., *Heat Transfer and Fluid Flow in Minichannels and Microchannels*, Elsevier, 2006.
- [5] W. Qu and I. Mudawar, “Measurement and prediction of pressure drop in two-phase micro-channel heat sinks”, *International Journal of Heat and Mass Transfer*, vol.46, pp.2737-2753, 2003.
- [6] J. R. Thome, “State-of-the-art overview of boiling and two-phase flows in microchannels”, *Heat Transfer Engineering*, vol.27, pp.4-19, 2006.
- [7] S. Kakac and B. Bon, “A review of two-phase flow dynamic instabilities in tube boiling system”, *International Journal of Heat and Mass Transfer*, vol.51, pp.399-433, 2008.
- [8] A. E. Bergles, J. H. Lienhard, G. E. Kendall, P. Griffith, “Boiling and evaporation in small diameter channels”, *Heat Transfer Engineering*, vol.24, pp.1840, 2003.
- [9] J. Yin, “Modeling and Analysis of Multiphase Flow Instabilities”, PhD Thesis, Rensselaer Polytechnic Institute, 2004.
- [10] T. J. Zhang, T. Tong, Y. Peles, R. Prasher, et al., “Ledinegg instability in microchannels”, *International Journal of Heat and Mass Transfer*, vol.52, pp.5661-5674, December, 2009.
- [11] T. J. Zhang, Y. Peles, J. T. Wen, T. Tong, R. Prasher, et al., “Analysis and active control of pressure-drop flow instabilities in boiling microchannel systems”, *International Journal of Heat and Mass Transfer*, vol.53, pp.2347-2360, May, 2010.
- [12] J. Xu, J. Zhou, and Y. Gan, “Static and dynamic flow instability of a parallel microchannel heat sink at high heat fluxes”, *Energy Conversion and Management*, vol.46, pp.313-334, 2005.
- [13] C.-J. Kuo and Y. Peles, “Pressure effects on flow boiling instabilities in parallel microchannels”, *International Journal of Heat and Mass Transfer*, vol.52, pp.271-280, 2009.
- [14] A. Kosar, C.-J. Kuo, and Y. Peles, “Suppression of boiling flow oscillations in parallel microchannels by inlet restrictors”, *ASME Journal of Heat Transfer*, vol.128, pp.251-260, 2006.
- [15] M. Ozawa, K. Akagawa, and T. Sakaguchi, “Flow instabilities in parallel-channel flow systems of gas-liquid two-phase flow mixtures”, *International Journal of Multiphase Flow*, vol.15, pp.639-657, 1989.