

# ADAPTIVE: A Dynamic Index Auction for Spectrum Sharing with Time-Evolving Values

Mehrdad Khaledi and Alhussein A. Abouzeid

Department of Electrical, Computer and Systems Engineering  
Rensselaer Polytechnic Institute  
Troy, NY 12180-3590, USA  
Email: khalem@rpi.edu, abouzeid@ecse.rpi.edu

**Abstract**—Spectrum auction is considered a suitable approach to efficiently allocate spectrum among unlicensed users. In a typical spectrum auction, Secondary Users (SUs) bid to buy spectrum bands from a Primary Owner (PO) who acts as the auctioneer. Existing spectrum auctions assume that SUs have static and known values for the channels. However, in many real world settings, SUs do not know the exact value of channel access at first, but they learn it over time. In this paper, we study spectrum auctions in a dynamic setting where SUs can change their valuations based on their experiences with the channel. We propose ADAPTIVE, a dynAmic inDex Auction for sPectrum sharing with TIme-evolving ValuEs that maximizes the social welfare of the SUs. ADAPTIVE is based on multi-armed bandit models where for each user an allocation index is independently calculated in polynomial time. ADAPTIVE has some desired economic properties that are formally proven in the analysis. Also, we provide a numerical performance comparison between ADAPTIVE and the well known Vickrey second price auction as a representative of static auctions.

**Keywords**—Cognitive Radio Networks, Spectrum Sharing, Game Theory, Auction, Multi-armed Bandit.

## I. INTRODUCTION

Spectrum scarcity has become a major challenge as a result of rapid growth in wireless communications. The Federal Communications Commission (FCC) indicates that the problem is not just the scarcity of spectrum but it is also the inefficient use of the available wireless spectrum. Measurements reported by the FCC's Spectrum Policy Task Force show that many of the allocated bands are idle or barely used in some areas [1]. To achieve better spectrum utilization, studying efficient spectrum allocation mechanisms seems imperative.

Cognitive radio network is considered as a novel communication paradigm that improves spectrum utilization by allowing dynamic spectrum sharing [2]. Dynamic spectrum sharing enables unlicensed or Secondary Users (SUs) to access idle spectrum bands that are owned by a Primary Owner (PO). For this purpose, it is necessary to design mechanisms that provide incentives for both PO and SUs to participate in spectrum sharing.

Auction-based mechanisms are very well-suited to the spectrum sharing problem. In an auction, the seller is not necessarily required to have prior knowledge about the value of items to the potential buyers. This is an advantage of auction mechanisms compared to the traditional pricing mechanisms.

by designing a mechanism that allocates to the bidders who value the items the most. Yet another advantage of auctions is that they induce less communication overhead compared to the other market mechanisms which consequently makes implementation easier and more practical.

In a simple spectrum auction, SUs bid to buy spectrum bands from a PO that sells its idle bands for a profit. An underlying assumption in existing spectrum auctions is that SUs know the exact value of channel access, and they bid accordingly. However, in real world scenarios, the value of obtaining channel access is not exactly known to the SUs a priori, but they learn it over time. In fact, SUs revise their estimates of values of channel access based upon what they experience.

In this paper, we study spectrum auctions with dynamically evolving values. We consider a cognitive radio network with one PO (a base station or an access point) who is willing to auction its idle channel to the SUs. The setting allows SUs to learn their valuations based on their experiences. In this context, SU's experience is estimated as a function of the channel quality or Signal to Noise Ratio (SNR) of the channel.

In this setting, we propose ADAPTIVE, a dynAmic inDex Auction for sPectrum sharing with TIme-evolving ValuEs. To the best of our knowledge, ADAPTIVE is the first spectrum auction that considers dynamically evolving values. ADAPTIVE is technically a repeated auction of a channel in which SUs learn their values over time. The proposed auction results in efficient allocation that maximizes the expected discounted social welfare. Also, it has some desired economic properties.

Every auction is determined by a pair of functions (or rules); the allocation function and the payment function. The allocation part of ADAPTIVE is based on the infinite horizon multi-armed bandit models [3]. The main idea is to allocate the channel based on dynamic allocation indices of SUs that are computed independent of each other. This rule provides us with the efficient allocation. To obtain the payment function, we take into account the externality that the winning SU imposes on the other SUs, which is the surplus that other SUs could have achieved in the absence of the winning SU. ADAPTIVE runs in polynomial time and has desired economic properties of Periodic Ex Post Incentive Compatibility, Periodic Ex Post Individual Rationality and No Positive Transfers.

The main contributions of this paper can be summarized

as follows. We consider a dynamic spectrum auction setting  
Also, with auctions efficient allocation can be easily obtained  
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where SUs can use their experiences with the channel to revise their valuations. This model allows learning of valuations over time and it is more realistic compared to prior work. We propose ADAPTIVE, a dynamic inDex Auction for sPectrum sharing with Time-evolving ValuEs that results in efficient allocation that maximizes the expected discounted social welfare. To the best of our knowledge, ADAPTIVE is the first spectrum auction that enables dynamically evolving valuations. We formally prove the economic properties of ADAPTIVE in the analysis, namely Periodic Ex Post Incentive Compatibility, Periodic Ex Post Individual Rationality and No Positive Transfers. Furthermore, we provide a numerical performance comparison between ADAPTIVE and the well known second price auction [4] with respect to revenue of the PO, social welfare, average payments and average utilities of SUs.

The remainder of the paper is organized as follows. In Section II, we provide a brief review and discussion of related work. Section III describes the system model that is the basis of our proposed mechanism. In Section IV, we propose the ADAPTIVE auction and prove its economic properties. Numerical results are reported and discussed in Section V. Finally, Section VI concludes this paper and provides some guidelines for future work.

## II. RELATED WORK

Several auction mechanisms have been proposed recently for wireless spectrum management in different settings [5]–[18]. In this section, we provide a brief overview of the most relevant studies.

In [5], a contract based spectrum management mechanism has been presented. The cognitive radio network consists of a PO and several primary and secondary users. In their model, the PO offers channels of different qualities. With the goal of revenue maximization, the PO acts like a monopolist and determines the qualities and prices for the available channels. This approach, however, does not let SUs to express their values (or submit bids). Thus, the PO requires some prior information about SUs' values for the channels. In [17], the authors proposed an auction based spectrum sharing mechanism with heterogeneous channels for a similar network topology. The work has been extended to a reserve price auction in [18] where the PO can impose reservation prices on the channels.

In [6], the authors present a spectrum auction with multiple POs. In their model, each SU selects one PO for bidding and POs gradually raise their trading prices until the mechanism converges to an equilibrium point where no SU and PO is interested to deviate. Similarly, the authors in [7] study the optimal pricing problem for two wireless service providers, and the optimal service provider selection problem for SUs. The authors show that the equilibrium price and its uniqueness depend on the spectrum propagation characteristics and SUs' geographical density. In [8], Niyato et al. study the dynamics of spectrum pricing in a competitive environment with multiple POs. They use noncooperative game theory to model the competition among POs and evolutionary game theory to model the behavior of SUs.

In [10], Zhou et al. proposed a general framework, called TRUST, for truthful double spectrum auctions that provide

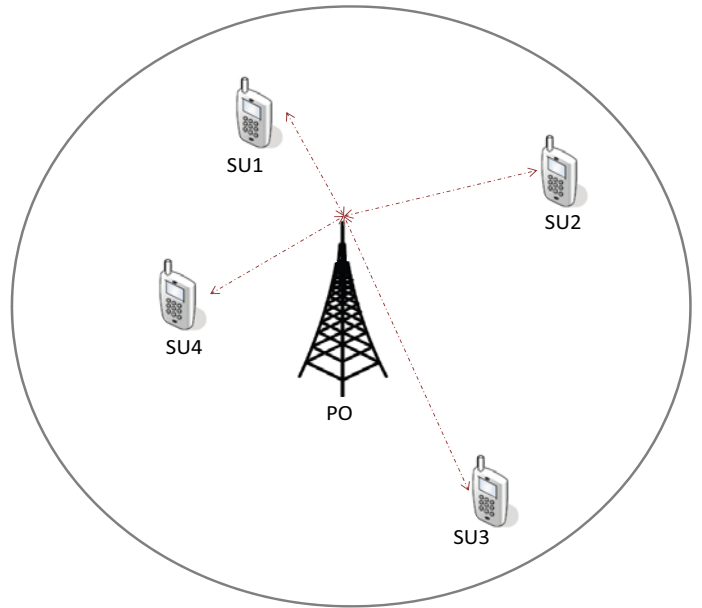


Fig. 1. A cognitive radio network with one primary owner and four secondary users.

spectrum reuse. This framework takes any reusability-driven spectrum allocation mechanism as input, and applies its own winner determination and payment rule. TAHES [9] is another truthful double auction mechanism, but works for heterogeneous spectrums. In both models, there should be an external third party who has complete information and holds the auction.

The authors in [11] consider a model in which the spectrum access opportunity is divided by frequency and time. Thus, SUs can bid for a combination of frequencies at different times. The problem then becomes a combinatorial auction and finding an efficient allocation becomes NP-Complete. The authors present approximate solutions to the general problem. In [12], a core-selecting auction has been proposed in a setting that SUs can bid for a combination of channels. The auction yields at least the revenue of the VCG mechanism [4] and it is not vulnerable to shill bidding.

Recently, a group of researchers considered online spectrum auctions where SUs can enter the auction and leave at different times [13], [14]. However, an underlying assumption is that SUs know the exact value of channel access every time they participate in the auction.

Despite all the prior work, the problem of designing a spectrum auction with dynamically evolving values for SUs has not been addressed. In this paper, we tackle this problem.

## III. SYSTEM MODEL

In this paper, we study the problem of auctioning a channel where SUs' valuations dynamically evolve over time based on their experiences. We consider a cognitive radio network with one PO (a base station or an access point) who is willing to auction its idle channel to the SUs. An example of cognitive radio network is depicted in Fig. 1.

The spectrum sharing process is modeled by an auction

in which PO acts as the auctioneer, and SUs are the bidders. The objective is to maximize social welfare while satisfying incentive compatibility and individual rationality. There are  $k$  SUs competing with each other at each time step for an infinite horizon. The type of SU  $i$  is denoted by  $\theta_i \in [0, \Theta_i]$  which is a real number reflecting how much SU  $i$  values channel access. It also captures the urgency for channel access, the more urgent the channel access to SU  $i$ , the higher the monetary value  $\theta_i$ .

SUs gain experience over time dealing with the channel. We denote SU  $i$ 's experience at time  $t$  by  $e_{i,t} \in \xi_i$  where  $\xi_i$  can be a potentially arbitrary set. It should be noted that SU's experience evolves only when he gets the channel, otherwise its experience does not change. An SU's experience at the instants that it gets the channel evolves in a Markovian model (i.e. the process is semi-Markov). That is, the probability that the next experience is  $e_{i,t+1}$  is  $P(e_{i,t+1}|e_{i,t})$ , only depends on the current experience. In our model, we consider SU's experience as the channel quality or SNR of the channel.

SU's valuation for the channel is a stationary function of its type and experience. Without loss of generality, we assume that all SUs use the same valuation function. It is worth noting that even though the function is shared, the values of the parameters of the function are private.<sup>1</sup> This actually makes sense, since SUs value a channel based on its capacity. So, the valuation functions can be similar but with different parameters, as they have different experiences and monetary preferences. SU  $i$ 's valuation for the channel at time  $t$  is defined as:

$$v(\theta_i, e_{i,t}) = \theta_i B \log(1 + e_{i,t}) \quad (1)$$

Where  $B$  is the channel bandwidth. The function  $v$  takes into account both the channel quality experienced by SUs and SU's monetary value that reflects urgency for channel access. The expected (future) value of SU  $i$  for the channel at time  $t$  equals to  $\delta^{t-1}v(\theta_i, e_{i,t})$  where  $0 < \delta < 1$  is the common discount factor.

We assume that bidders are rational which is an inherent assumption in designing truthful auction mechanisms. That implies, bidders act solely with the purpose of maximizing their own utilities. If an SU gets the channel, its utility will be the difference between its valuation for the channel and the price he has to pay.

Also, we focus on direct mechanisms by using the Revelation Principle [4]. The principle states that an outcome of any indirect mechanism can be obtained by a (truthful) direct mechanism.

#### IV. THE ADAPTIVE MECHANISM

In this section, we propose ADAPTIVE, a dynAmic inDex Auction for sPectrum sharing with TIme-evolving ValUes. ADAPTIVE is a repeated auction for an infinite horizon that allows SUs to dynamically learn and revise their values. The proposed auction has some guaranteed economic properties that will be proven in this section.

<sup>1</sup>This scheme should not be confused with common value auctions where in the latter case, the item is of the same value for all bidders but they do not know what this value is [4].

At each time step, SUs report  $(\theta_i, e_{i,t})$  to the PO who takes the role of the auctioneer and runs ADAPTIVE. The auction should determine two output functions; the allocation and the payment. Channel allocation is denoted by  $Q \in \mathbb{Q}$ , where  $\mathbb{Q}$  is the set of all possible allocation rules.  $Q$  contains  $q_{i,t} \in \{0, 1\}$  that determine which SU gets the channel at every time step. That is,  $q_{i,t} = 1$  indicates that the SU  $i$  has obtained the right to access the channel at time  $t$  and  $q_{i,t} = 0$  otherwise. Similarly,  $p_{i,t}$  represents the payment of SU  $i$  at time  $t$ .

After the channel is allocated at each round, the winner gets the chance to update its experience with the channel for the next round. It should be noted that the PO knows the Markov probability model for the evolution of experiences. Therefore, with SUs' reports, the PO can compute expected future values and make decisions accordingly.

The objective is to find an efficient allocation scheme that satisfies desired economic properties. An efficient allocation rule maximizes bidders' valuations. We can formally define the expected future social welfare at time  $t$  as:

$$S(\theta, e_t) = \max_{Q \in \mathbb{Q}} \mathbb{E} \left[ \sum_{t'=t}^{\infty} \sum_i \delta^{t'-t} q_{i,t'} v(\theta_i, e_{i,t'}) \mid \theta, e_t \right] \quad (2)$$

Where  $0 < \delta < 1$  is the common discount factor,  $\theta$  and  $e_t$  are vectors of SUs' types and experiences at time  $t$ , respectively. To achieve efficiency, we need to find an allocation scheme  $Q \in \mathbb{Q}$  that maximizes the equation (2). In the next subsection, we present a method to achieve the efficient allocation.

##### A. Efficient Allocation Policy of ADAPTIVE

The ADAPTIVE mechanism utilizes concepts from multi-armed bandit models to find the efficient channel allocation. Multi-armed bandit problems refer to a class of sequential resource allocation problems that are concerned with the dilemma of making decisions that bring high current gains or making decisions that sacrifice immediate payoffs with the hope of better future rewards [3].

In a multi-armed bandit problem, there is an operator with a collection of independent single-armed bandits. At each time step, the operator chooses to operate exactly one of the machines. The chosen machine generates a reward and updates its state. All other machines retain their current states till the next time step. The objective of the operator is to maximize the sum of rewards.

In [19], Gittins and Jones presented an index policy for the operator to obtain an optimal solution (that yields maximum sum of rewards). They introduced a *dynamic allocation index* which can be computed independently for each bandit. At each time step, the operator needs to choose the highest index bandit to achieve the optimal solution. Dynamic allocation index (also called *Gittins index*) reduces the complexity of the problem exponentially, since instead of finding the solution of a multi-armed bandit problem, the operator is required to determine Gittins indices for some single-armed bandit problems.

Now, our channel allocation problem in ADAPTIVE can be transformed into a classical multi-armed bandit problem.

Each arm in the bandit model can be thought of as an SU and rewards generated by pulling arms resemble SUs' valuations. The operator chooses a machine in the multi-armed bandit model just like the PO chooses an SU to allocate the channel to it. State change in the bandit model is similar to experience update of the winning SU in the ADAPTIVE mechanism. With the transformation, we see that the optimal solution to the multi-armed bandit problem is equivalent to the efficient channel allocation in ADAPTIVE.

Therefore, we can use the Gittins index policy to solve the efficient allocation problem. According to this policy, the PO gives the channel to the SU with the highest index. The Gittins index of SU  $i$  at time  $t$  (conditioned on its current experience and type) is defined as:

$$G_i(\theta_i, e_{i,t}) = \max_{\tau_i} \mathbb{E} \left[ \frac{\sum_{t'=t}^{\tau_i} \delta^{t'-t} v(\theta_i, e_{i,t'})}{\sum_{t'=t}^{\tau_i} \delta^{t'-t}} \middle| \theta_i, e_{i,t} \right] \quad (3)$$

An important feature of the Gittins index policy is that the index of SU  $i$  can be computed independently and without any information about other SUs. Also, it is worth noting that, the index of SU  $i$  will not change if he does not get the channel. There are several polynomial time algorithms to find the indices. For instance, Sonin in [20] proposes an algorithm to solve equation (3) in  $n^3 + O(n^2)$  operations.

In addition to the allocation policy, we need to specify the payment function of the mechanism, i.e. the price the winning SU has to pay. In the next subsection, we propose the payment rule of the ADAPTIVE.

### B. The Payment Rule of ADAPTIVE

We propose the payment rule of the ADAPTIVE mechanism in this subsection and we discuss and prove its economic properties in the next subsection. We specify payments such that under the efficient allocation policy, each SU's utility coincides with its marginal contribution to the social welfare [21].

Let  $m_{i,t}$  denote SU  $i$ 's marginal contribution to the social welfare at time  $t$ :

$$m_{i,t} = S(\theta, e_t) - S_{-i}(\theta, e_t) - \delta \mathbb{E} \left[ S(\theta, e_{t+1}) - S_{-i}(\theta, e_{t+1}) \right] \quad (4)$$

Where  $S_{-i}(\theta, e_t)$  is the expected future social welfare without SU  $i$ . Since SU's experience will not change without getting the channel,  $S_{-i}(\theta, e_t) = S_{-i}(\theta, e_{t+1})$ . Therefore, equation (4) becomes:

$$m_{i,t} = v(\theta_i, e_{i,t}) - (1 - \delta) S_{-i}(\theta, e_t) \quad (5)$$

Now, we want that SU's immediate utility coincide with its marginal contribution. That is:

$$m_{i,t} = v(\theta_i, e_{i,t}) - p_{i,t} \quad (6)$$

As a result of combining equations (5) and (6), the winning SU  $i$  at time  $t$  pays

$$p_{i,t} = (1 - \delta) S_{-i}(\theta, e_t) \quad (7)$$

If SU  $i$  does not get the channel at time  $t$ , then  $p_{i,t} = 0$ . Also, It should be noted that SU  $i$  has no control over its payment and its valuation is excluded in equation (7). This property disables SUs to manipulate their payments to gain some profit. We will discuss the prove economic properties of ADAPTIVE in the next subsection.

### C. Economic Properties

An auction is required to satisfy certain economic properties such as incentive compatibility and individual rationality. In this subsection, we define these properties and prove that the ADAPTIVE mechanism satisfies them.

An auction is called *ex post incentive compatible* if truth-telling is always the best strategy for bidders, regardless of the history and current state (i.e. type and experience) of other bidders [21]. We should note here that, SUs observe their history which includes their past states, reports and allocations. Also, it is worth noting that in dynamic settings, we have the notion of *periodic ex post incentive compatibility*. That is, the mechanism is ex post incentive compatible with respect to the information received in time  $t$ , but it is not ex post with respect to the information arriving after time  $t$ . In other words, a bidder may get some information in the future that she would regret its report at time  $t$ .

Before we formally define the properties, we need some definitions. A reporting strategy for bidder  $i$ , denoted by  $R_i$ , provides a mapping from its state (type and experience) to a report. We denote the (joint) truth-telling strategy by  $T$  in which all the bidders report truthfully. The expected future utility of bidder  $i$  under (joint) reporting strategy  $R$  is defined as:

$$U_{i,t}^R = \mathbb{E} \left[ \sum_{t'=t}^{\infty} \delta^{t'-t} (q_{i,t'}^R v(\theta_i, e_{i,t'}) - p_{i,t'}) \right] \quad (8)$$

Where  $q_{i,t}^R$  is the allocation induced by  $R$ . Now, we can define the economic properties.

- **Periodic Ex Post Incentive Compatibility;** An auction is periodic ex post incentive compatible if for every bidder  $i$  and at any time  $t$ , truth-telling is the best response to the truthfulness of the other bidders. That is  $U_{i,t}^T \geq U_{i,t}^{R_i, T_{-i}}$ , where  $U_{i,t}^{R_i, T_{-i}}$  is the utility of bidder  $i$  when  $i$  uses an arbitrary report strategy  $R_i$ , while all the other bidders use the truth-telling strategy  $T$ .
- **Periodic Ex Post Individual Rationality;** An auction is periodic ex post individually rational if for every bidder  $i$  and at any time  $t$ , we have  $U_{i,t}^T \geq 0$ . That means, bidders do not suffer as a result of participating in the auction.

*Theorem 1:* The ADAPTIVE mechanism is periodic ex post incentive compatible and periodic ex post individually rational.

*Proof:* Let  $S^{R_i}(\theta, e_t)$  be the expected future social welfare at time  $t$  when the bidder  $i$  uses the reporting strategy  $R_i$  and all other bidders report truthfully, defined as:

$$S^{R_i}(\theta, e_t) = \max_{Q \in \mathbb{Q}} \mathbb{E} \left[ \sum_{t'=t}^{\infty} \sum_j \delta^{t'-t} q_{j,t'}^{R_i} v(\theta_j, e_{j,t'}) \middle| \theta, e_t \right]$$

Where  $\mathbb{Q}$  is the set of all possible allocation rules and  $q_{j,t'}^{R_i}$  is the allocation induced by  $(R_i, T_{-i})$  (when  $i$  uses the reporting strategy  $R_i$  and others report truthfully). The expected future social welfare at time  $t$  and without bidder  $i$ , can be defined similarly:

$$S_{-i}(\theta, e_t) = \max_{Q \in \mathbb{Q}_{-i}} \mathbb{E} \left[ \sum_{t'=t}^{\infty} \sum_{j \neq i} \delta^{t'-t} q_{j,t'} v(\theta_j, e_{j,t'}) \middle| \theta, e_t \right]$$

Where  $\mathbb{Q}_{-i}$  is the set of allocation rules that disregard bidder  $i$ . Now, the marginal contribution of bidder  $i$  to the social welfare, at time  $t$ , will be:

$$m_{i,t} = S^{R_i}(\theta, e_t) - S_{-i}(\theta, e_t) - \delta \mathbb{E} \left[ S^{R_i}(\theta, e_{t+1}) - S_{-i}(\theta, e_{t+1}) \right] \quad (9)$$

If bidder  $i$  does not get the channel at time  $t$ ,  $m_{i,t} = p_{i,t} = 0$ . However, if bidder  $i$  gets the channel at time  $t$ , then:

$$S^{R_i}(\theta, e_t) = v(\theta_i, e_{i,t}) + \delta \mathbb{E} \left[ S^{R_i}(\theta, e_{t+1}) \right]$$

Also, it should be noted that if bidder  $i$  gets the channel at time  $t$ , other bidders' state will not change (their experiences with the channel remain the same). That is:

$$S_{-i}(\theta, e_t) = S_{-i}(\theta, e_{t+1})$$

Therefore, the marginal contribution of bidder  $i$  (who gets the channel at time  $t$ ) can be rewritten from equation (9) as

$$\begin{aligned} m_{i,t} &= v(\theta_i, e_{i,t}) - (1 - \delta) S_{-i}(\theta, e_t) \\ &= v(\theta_i, e_{i,t}) - p_{i,t} \end{aligned} \quad (10)$$

Where the second equality uses the payment rule, equation (7). The expected future utility of bidder  $i$  at time  $t$  when  $i$  uses reporting strategy  $R_i$  and others report truthfully using  $T_{-i}$  is defined as (for simplicity of notation we omit  $T_{-i}$ ):

$$\begin{aligned} U_{i,t}^{R_i} &= \mathbb{E} \left[ \sum_{t'=t}^{\infty} \delta^{t'-t} (q_{i,t'}^{R_i} v(\theta_i, e_{i,t'}) - p_{i,t'}) \right] \\ &= \mathbb{E} \left[ \sum_{t'=t}^{\infty} \delta^{t'-t} m_{i,t'} \right] \\ &= S^{R_i}(\theta, e_t) - S_{-i}(\theta, e_t) \end{aligned}$$

Where the second equality follows from equation (10) and the third equality holds from definition of marginal contribution, equation (9), and noting that all the terms except for time  $t$  will cancel out. Clearly,  $S_{-i}$  is independent of bidder  $i$ 's reports. Also, since the social welfare is defined with respect to the true states,  $S^{R_i}$  is maximized if bidder  $i$  reports truthfully. Therefore, the auction is periodic ex post incentive compatible. We can also see that  $U_{i,t}^T \geq 0$  that is because  $S^T \geq S_{-i}$ . As a result, the ADAPTIVE mechanism is also periodic ex post individually rational. ■

It is worth noting that the ADAPTIVE mechanism has no positive transfers (another economic property), i.e. for all times  $t$  and every bidder  $i$  we have  $p_{i,t} \geq 0$ . This can be easily seen from the payment function, equation (7) which is sum of non-negative numbers multiplied by a factor  $(1 - \delta)$  in  $[0, 1]$ .

## V. NUMERICAL RESULTS

In this section, we provide a performance evaluation of the ADAPTIVE mechanism. We compare the performance of ADAPTIVE which is a dynamic valuation auction with the well-known Vickrey auction (also called second price auction) as the representative of static auctions. Thus, in the following diagrams, static refers to the second price auction and dynamic refers to our ADAPTIVE mechanism.

We set the common discount factor,  $\delta$ , to 0.7 and change the number of SUs from 3 to 21. Each setting is run 500 times in MATLAB to eliminate the effect of random initialization. Social welfare (sum of winning SUs' valuations), discounted social welfare, average payment of SUs, average utility of SUs, and revenue of the PO (sum of SUs' payments) are considered as performance metrics. In order to study the impact of the discount factor, we fix the number of SUs and compute the revenue of PO when  $\delta$  changes from 0 to 1.

The bandwidth  $B$  is set to 1 and SUs' initial experiences and types are randomly drawn from discrete uniform distributions. So, SUs' initial valuations can be computed. The ADAPTIVE mechanism proceeds with the Gittins index policy for allocation and equation (7) for determining payments. We use Sonin's algorithm [20] to compute Gittins indices in polynomial time. The winner updates its experience and the mechanism repeats. On the other hand, in the second price auction, the SU with the highest valuation gets the channel and pays the second highest value. The second price auction continues with another random instance of experiences. In fact, at every time step, each SU gets a random experience and there will be no evolution.

In ADAPTIVE, we consider Signal to Noise Ratio (SNR) of the channel experienced by the SUs as their experiences. The winner updates its experience according to an Auto-Regressive (AR) model. If SU  $i$  is the winner, we have  $e_{i,t+1} = e_{i,t} + z$ , where  $z$  is a discrete random variable. We consider limited discrete values for SNR (ranging from -30db to 30db with increments of 1db) which provides us a finite small sized state space. Also, we use a Binomial distribution with probability of 0.5 for  $z$  that gives us a good approximation of a Gaussian distributed random variable centered at 0. With this evolution model, we can easily build a Markov probability model for transitions between experiences,  $P(e_{i,t+1}|e_{i,t})$ .

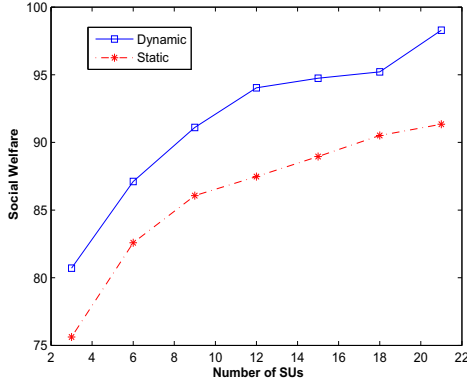


Fig. 2. Social welfare versus the number of SUs.

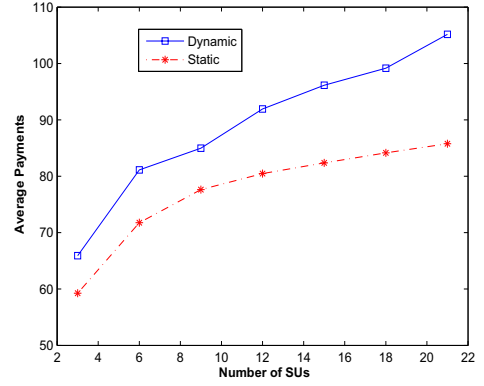


Fig. 4. Average payments versus the number of SUs.

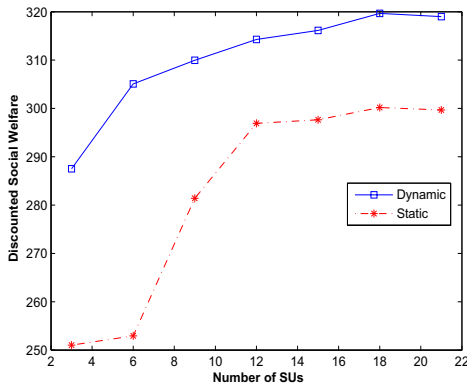


Fig. 3. Discounted social welfare versus the number of SUs.

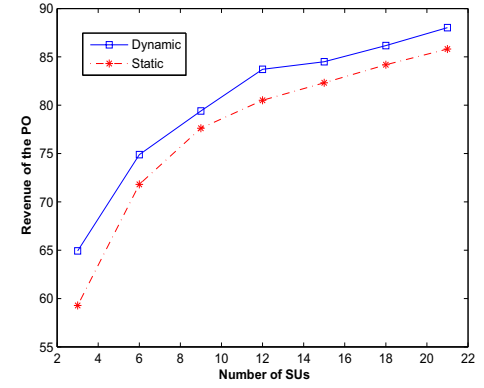


Fig. 5. Revenue of the PO versus the number of SUs.

In Fig. 2 the social welfare is shown versus number of SUs. As can be seen, in both dynamic and static auctions the social welfare increases with number of SUs. This is because when more SUs participate in the auction, there will be a wider range of valuations. Since SUs with high valuations are favored, it is more probable that the winner has a higher valuation compared to the case of having less SUs that consequently results in a higher social welfare. Fig. 2 also indicates that the dynamic ADAPTIVE mechanism results in a better social welfare than the static second price auction. This happens due to the fact that in the second price auction, SUs get random experiences at each time step and there is no evolution of experiences, which limits the chance of SUs for having higher values due to good experiences with the channel. Also, we should note that in ADAPTIVE, we have a probabilistic model of the future, because of the experience evolutions model, and the auctioneer takes into account expected future values. We observe a similar behavior in Fig. 3 that shows the discounted social welfare of the two auctions versus number of SUs.

The average payments of SUs is depicted in Fig. 4. We observe that in both auctions as the number of SUs increases, payments increase. This is because with more SUs, channel access becomes more competitive. Thus, the winner causes more externality to the other SUs, and consequently he has to pay more. This figure also shows that the ADAPTIVE mechanism induces higher payments than the second price auction. In

the payment function of the ADAPTIVE mechanism (equation (7)), the PO takes into account the future expected values and in the eye of ADAPTIVE, the winning SU causes more externality than that of the second price auction (i.e. second highest valuation). Therefore, the payments in ADAPTIVE are higher than payments in second price auction.

High payments are favorable to the PO, as it leads to more revenue. In Fig. 5 the revenue of the PO versus number of SUs is shown. Since higher payments imply higher revenue, we observe a similar behavior in this figure as in Fig. 4. In both auctions the revenue of the PO increases with number of SUs. In addition, the proposed dynamic auction yields more revenue than the static second price.

From the SUs' point of view, however, increase in the number of competitors (another SUs) is not favorable. Fig. 6 shows the average utility of SUs versus the number of SUs. We see that in both mechanisms, average utilities decreases with the number of SUs. This is due to the increase in SU payments that consequently lowers utilities (since utility is the difference between valuation and payment).

In all the diagrams so far, we had a fixed discount factor,  $\delta = 0.7$ . Now, we fix the number of SUs at 12 and change the discount factor from 0 to 1. Fig. 7 shows the revenue of the PO versus the discount factor. clearly, the revenue from second price remains constant, since it does not use the discount

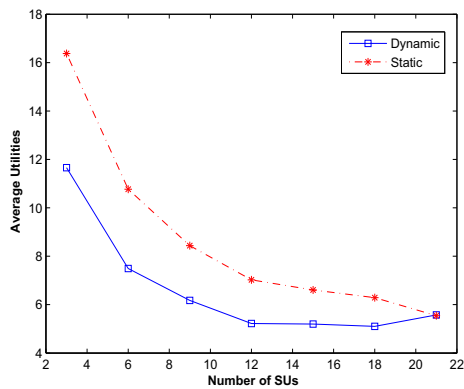


Fig. 6. Average utilities versus the number of SUs.

factor. However, we observe that the revenue from ADAPTIVE slightly increases with  $\delta$ . As  $\delta$  increases, the summation in the payment formula ( $S_{-i}(\theta, e_t)$  in equation (7)) increases as well. This increase in summation leads to higher payments and higher revenue, even though the factor  $(1 - \delta)$  in the payment formula decreases. The revenue drops to zero with  $\delta = 1$  which clearly happens because the factor  $(1 - \delta)$  becomes zero. Actually,  $\delta < 1$  but we wanted to show the extremes. Fig. 7 implies that the more we weigh the future (as opposed to the current gains), the higher revenue we get.

## VI. CONCLUSION

In this paper, we studied spectrum auctions in a realistic setting where SUs are allowed to revise their estimates of values of channel access, based upon what they experience over time. In this setting, we proposed ADAPTIVE, a dynAmic inDex Auction for sPectrum sharing with TIme-evolving ValUes. To the best of our knowledge, ADAPTIVE is the first spectrum auction that considers dynamically evolving values. ADAPTIVE runs in polynomial time and results in efficient allocation that maximizes the expected discounted social welfare. In the analysis, we formally prove the economic properties of ADAPTIVE, namely Periodic Ex Post Incentive Compatibility, Periodic Ex Post Individual Rationality and No Positive Transfers. Furthermore, we provide a numerical performance comparison between ADAPTIVE and the well known Vickrey auction (also called the second price auction) as a representative of static auctions. In our model, we assumed that SUs' population is static, that is, SUs cannot leave or enter the auction at arbitrary rounds. A possible direction for future work is to extend ADAPTIVE to a dynamic population model that will be a dynamic population and dynamic valuation mechanism.

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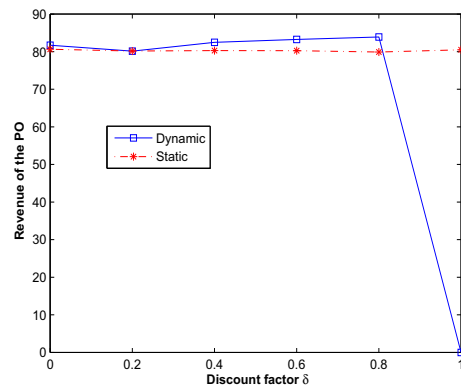


Fig. 7. Revenue of the PO versus  $\delta$ , with 12 SUs.

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