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Throughput and delay analysis for hybrid radio-frequency and free-space-optical (RF/FSO) networks

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Abstract In this paper the per-node throughput and end-to-end delay of randomly deployed (i.e. ad-hoc) hybrid radio frequency - free space optics (RF/FSO) networks are studied. The hybrid RF/FSO network consists of an RF ad hoc network of n nodes, $f(n)$ of them, termed 'super nodes', are equipped with an additional FSO transceiver with transmission range $s(n)$. Every RF and FSO transceiver is able to transmit at a maximum data rate of W_1 and W_2 bits/sec, respectively. An upper bound on the per node throughput capacity is derived. In order to prove that this upper bound is achievable, a hybrid routing scheme is designed whereby the data traffic is divided into two classes and assigned different forwarding strategies. The capacity improvement with the support of FSO nodes is evaluated and compared against the corresponding results for pure RF wireless networks. Under optimal throughput scaling, the scaling of average end-to-end delay is derived. A significant gain in throughput capacity and a notable reduction in delay will be achieved if $f(n) = \Omega\left(\frac{1}{s(n)}\sqrt{\frac{n}{\log n}} \cdot \frac{W_1}{W_2}\right)$. Furthermore, it is found that for fixed W_1, $f(n)$ and n where $f(n) < n$, there is no capacity incentive to increase the FSO data rate beyond a critical value. In addition, both throughput and delay can achieve linear scaling by properly adjusting the FSO transmission range and the number of FSO nodes.

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1 Introduction

The capacity of Radio Frequency (RF) wireless networks is constrained by provable limits and does not scale well with the increase in number of nodes in the network due to [1] interference between concurrent transmissions from neighboring nodes [2]. On the other hand, free space optics (FSO) technology can provide high data rates and highly directional transmissions using free-space laser beams, whose beam divergence angle is on the order of *milli*-radians.This perfect directionality in transmission makes the FSO communications nearly immune from interference. However, this interference-free benefit comes with a key limitation; unlike RF links, FSO links need to maintain line-of-sight (LOS), and FSO link availability can be further limited by adverse weather conditions like fog and heavy snowfall.

This complementary interference properties of RF and FSO motivates the design of hybrid RF/FSO ad-hoc networks in which the weaknesses of individual link types could be mitigated and the benefits leveraged in a network setting. A natural objective is to view this RF/FSO combination as a way to solve the capacity scarcity problem in pure RF wireless networks, or at least reduce its severity. Several practical designs of such hybrid networks have been proposed and/or implemented e.g. [3–7], which may improve the network performance by providing higher throughput and/or better reliability. However, there is still no theoretical work that gives fundamental results on how much the performance can be improved and how. This work aims to fill this void by deriving the fundamental limit on the capacity of hybrid ah-hoc RF/FSO networks and show exactly how much this may deviate from the well known capacity results of pure RF networks [2].

In this paper we consider a random network scenario, where n nodes are independently and uniformly distributed on a unit area, each capable of transmitting at W_1 bits/sec within a fixed communication radius $r(n)$ via a shared RF channel. Only $f(n)$ of these nodes, which are called *super nodes*, are equipped with an additional FSO transceiver, each capable of transmitting along the line-of-sight at W_2 bits/sec within a fixed communication distance $s(n)$ and tuning its transmit angle freely. For each node, its destination is randomly chosen, and let $\lambda(n)$ (in bits/sec) denote the maximum data rate at which each source-destination pair is required to transmit and receive. We define this λ as the *throughput capacity* of this hybrid RF/FSO network, which is similar to the definition introduced in [2]. Here we let this random network satisfy two asymptotic connectivity assumptions: (a) all the n nodes are asymptotically connected by RF links; (b) all the $f(n)$ super nodes are asymptotically connected by FSO links. This allows us to have two stand-alone networks consisting of n nodes connected only by RF links and $f(n)$ nodes connected only by FSO links, respectively.

Note that unless all the nodes are super nodes, the throughput capacity of a hybrid RF/FSO network is not simply the sum of the capacities of the two individual pure RF and pure FSO networks. This is because when $f(n) < n$, there are at least $(n - f(n))$ source-destination pairs whose communication demands cannot be purely met by the FSO network.

The derivation of the throughput capacity can be divided into two parts. First an upper bound on the throughput capacity λ over all routing and transmission strategies is derived. Then, a hybrid routing scheme is constructed in which the data traffic is divided into two classes and use different routing strategies: a portion of data is forwarded with the (partial) support of super nodes in a hierarchical routing fashion, and the rest is purely routed through RF links in a multi-hop fashion. By properly balancing the two classes of data traffic, we show that the upper bound scaling is achievable.

We then study the end-to-end delay scaling of hybrid RF/FSO networks. It is shown that, among all schemes that achieve optimal throughput scaling, our proposed hybrid routing scheme achieves the minimum delay scaling.

The throughput and delay results obtained are then compared against the results of pure RF wireless networks evaluated in [2] in order to characterize the throughput improvement and delay reduction contributed by those super nodes. It is observed that, when the number of super nodes $f(n)$ is high enough, a noticeable throughput improvement and delay reduction will be achieved; and it is even possible to make both the throughput and delay scale linearly.

The main contributions of this paper are summarized as follows: (a) derived the scaling laws for the throughput capacity and average end-to-end delay of hybrid RF/FSO networks as a function of the total number of nodes n of which $f(n) \le n$ are super nodes and the RF/FSO transmission rates; (b) presented a routing and transmission scheme that achieves the derived throughput and delay, which may guide the practical routing protocol design for the hybrid RF/FSO networks; (c) compared with the results against pure RF wireless networks, analyzed the capacity improvement and delay reduction of hybrid networks, and characterized the number of super nodes necessary in order to cause a non-trivial increase in the per node throughput and decrease in the end-to-end delay.

The rest of this paper is organized as follows. A brief overview of related work is presented in Sect. 2. Section 3 outlines the system model that is considered throughout the paper. Section 4 presents an upper bound on throughput capacity of RF/FSO networks with minimum transmit power consumption. A constructive lower bound on throughput capacity of RF/FSO networks is presented in Sect. 5. Section 6 discusses the capacity results obtained, and provides some practical implications. Section 7 derives and discusses the delay scaling of RF/FSO networks under optimal throughput scaling. Section 8 concludes the paper and outlines future directions.

2 Related work

To the best of our knowledge, there is no prior work in the literature on the capacity analysis of ad-hoc RF/FSO multi-hop networks. However, several attempts have been made to provide capacity improvement by utilizing directional antennas or introducing infrastructure support. We review this related work here.

The authors in [8] analyzed the capacity improvement of RF wireless networks using directional antennas in three cases. The capacity gain is shown to be (a) $\sqrt{\frac{2\pi}{\alpha}}$ when using directional transmission with divergence angle α and omni-reception, (b) $\sqrt{\frac{2\pi}{\beta}}$ when using omni-transmission and directional reception with divergence angle β, and (c) $\frac{2\pi}{\sqrt{\alpha\beta}}$ when both transmission and reception are directional. These capacity gains can only be achievable when the divergence angle is not too small. Thus the results derived in [8] are not applicable to the FSO model since FSO links have an infinitesimally small divergence angle while the results in [8] hold only for directional antennas with divergence angles above a certain finite threshold.

Prior works [9–13] try to improve the capacity by the introduction of infrastructure support (access points or base stations), and we summarize each individual work as follows.

Authors in [9] depict the infrastructure network as a cellular network, where the base stations are wired by a broadband network and placed at the center of hexagonal cells. They investigate how the number of base stations should scale with the number of ad hoc nodes to achieve significant capacity improvement over the pure RF ad hoc wireless networks. They apply different routing strategies in which the ad hoc nodes are divided into two groups depending on whether they use the cellular network to reach the destination or not. The decision criteria in forming the groups may not be the optimal routing strategies. They show that the number of base stations should at least scale with $\sqrt{n}$ to achieve a noticeable gain.

The infrastructure network used in [10] consists of randomly located access points which are pre-wired and allocated infinite capacity. The authors in [10] model the node distribution and traffic pattern in the same manner as the random network model used in [2]. They assume that the number of ad hoc nodes per access point is bounded above, and each wireless node is able to transmit at W bits/sec using a fixed transmission range. Under this random network scenario, they show that a per node capacity of $\Theta(\frac{W}{\log n})$ can be achieved. To do this, they specify the upper bound of throughput capacity over all routing and transmission strategies, and then design a specific routing and transmission scheme to achieve this upper bound.

The results in [11] extend the work of [10] by allowing nodes to perform power control and properly choosing the number of access points, and further show that it is possible to provide a throughput of $\Theta(1)$ to any fraction f, $0 < f < 1$, of nodes.

In [12] the network under study are wireless mesh networks (WMN) with regularly placed mesh routers and gateways, and the throughput capacity of such networks is derived as a function of numbers of clients, mesh routers and gateways. Their results indicate that in certain cases WMN can achieve the same throughput scaling as that of hybrid ad hoc networks with infinite capacity infrastructure support.

Li et al. [13] studies the capacity and delay of hybrid wireless broadband access networks, where the network under study consists n RF ad hoc nodes, plus m regularly placed base stations connected by optical backbone with infinite capacity. The throughput and delay scalings for such a network are derived and it is shown that the throughput and delay can scale linearly when $m = \Omega(n)$.

Although overlaps may exist between our work and the prior work such as [9–13], there are major differences that underline the contribution of our work: (a) We consider the situation where the super nodes are randomly deployed rather than being carefully placed according to a regular structure e.g [9, 12, 13]; (b) Unlike base stations or mesh routers, the super nodes are considered as users of the network and they are not only forwarding but also generating data traffic. This makes it more difficult to improve the per node throughput; (c) We do not impose strong assumptions such as bounding the number of nodes per super node (as in [10]) or specifically choosing the number (as in [11]); (d) We allow the maximum data rate of each FSO transceiver, W_2, be some finite value, as compared to the infinite capacity assumptions used in [10], [11] and [13]. We believe these assumptions are more realistic in ad-hoc scenarios.

Some recent advances [14, 15] in studying the capacity of wireless networks have reported linear capacity scaling achieved by intelligent node cooperation and distributed MIMO communication. In addition the RF transceivers are assumed to be able to smoothly adjust the power and the data rate. These functionalities may require very complicated hardware and software design on each node. Node cooperation is not studied in this paper. However, our results show that a simple network design can also bring a scalable network performance.

3 System model

3.1 Network architecture

We study a random hybrid network, where n nodes each equipped with an RF transceiver are randomly located, i.e., independently and uniformly distributed on a surface S^2 of a three-dimensional sphere of area $1m^2$, and only $f(n)$ of them, which are called super nodes, are each equipped with an additional FSO transceiver. Our purpose in studying S^2 is to separate edge effects from other phenomena. Here the number of super nodes $f(n)$[1] is a function of n with

$$\lim_{n\to\infty} f(n) = \infty \tag{1}$$

Each node has a randomly chosen destination to which it wishes to send packets at $t(n)$ bits/sec, and this data rate $t(n)$ is called *per-node throughput* of the hybrid network. The destination for each node is independently chosen as the node nearest to a randomly located point on S^2. Since the destination nodes are randomly chosen, then the mean distance between any source-destination (S-D) pair scales as $\Theta(1)$. Note that all the distances are measured on the surface S^2 of the sphere by segments of great circles connecting two points. The reason for using S^2 and measuring the distance this way (rather than using the Euclidean distance) is that we want to study two-dimensional

[1] For the sake of simplicity, in this paper we sometimes just write f instead of $f(n)$. The same treatment is also applied to other network parameters e.g. $r(n)$, $s(n)$, $\lambda(n)$ and so on.

networks and at the same time avoid dealing with any edge effects so as to ensure mathematical tractability.

Now we present definitions of several terms for the hybrid network:

Throughput Capacity. A per-node throughput $t(n)$ is said to be *feasible* if there exists a spatial and temporal scheduling scheme such that every node can send packets at $t(n)$ bits/sec to its chosen destination. The *throughput capacity* of the hybrid network, denoted by $\lambda(n)$, is the maximum feasible throughput.

Average Per Bit Delay. The per bit delay in the hybrid network is the time it takes a bit to reach the destination after it leaves the source. The average per bit delay, denoted by $\overline{D}(n)$, is obtained by averaging over all bits and all source-destination pairs of the network.

3.2 RF communication model

Each RF transceiver is equipped with an omni-directional antenna and is able to transmit at W_1 bits/sec. Time is assumed to be slotted such that over each time slot an RF transceiver is able to transmit or receive B bits, where B does not depend on n. Therefore the time slot length for the RF channel is $\tau_1 = \frac{B}{W_1}$ seconds.

We use the Protocol Model introduced in [2] as the RF interference model here. All nodes employ a common range $r(n)$ for all their transmissions. Let X_i denote the location of a node. When node i transmits to a node j over the RF channel, the data will be successfully received by j if

1. The distance between i and j is no more than r, i.e.,

$$|X_i - X_j| \leq r \tag{2}$$

2. For every other node k simultaneously transmitting over the RF channel

$$|X_k - X_j| \geq (1+\Delta)r \tag{3}$$

The quantity $\Delta > 0$ models situations where a guard zone is specified by the protocol to prevent a neighboring node from transmitting on the RF channel at the same time. It also allows for imprecision in the achieved range of transmissions.

3.3 FSO communication model

Every super node is equipped with an FSO transceiver. Each FSO transceiver can transmit along the line-of-sight at W_2 bits/sec within a common distance $s(n)$ and can receive from others omni-directionally. Every FSO transceiver can transmit and receive at the same time. Again we assume time is slotted for FSO transmission, with each time slot length being $\tau_2 = \frac{B}{W_2}$ seconds.

We assume that the orientation of each FSO transmitter can be steered to any possible direction within S^2. Then every super node can transmit data to any other super node within the distance s. In practice, the beam steering functionality has been realized in several ways. Milner et al. [3] implemented the beam steering using mechanical devices, while Khan et al. [16] designed a 3-dimensional wide-angle no-moving-parts laser beam steering method. Many other design choices are also available and can be found from [17–20]. The omni-directionality assumption for the FSO receiver is also reasonable, as omni-directional FSO receivers are also implemented in [5].

4 An upper bound on throughput capacity of hybrid RF/FSO networks

In this section, we analyze the throughput capacity of hybrid RF/FSO networks, which is provided in Theorem 1 and its proof.

Theorem 1 *The throughput capacity of hybrid RF/FSO networks* $\lambda(n)$ *is bounded from above by*

1. *Case 1*:

$$\lambda(n) \leq c_1 W_1 \sqrt{\frac{1}{n \log n}} + c_2 W_2 \frac{f(n)s(n)}{n} \tag{4}$$

when f, n, s, W_1 *and* W_2 *satisfy*

$$\frac{(n-f(n))s(n)}{n}\sqrt{\frac{f(n)\log n}{n}} \cdot \frac{W_2}{W_1} = o(1) \tag{5}$$

2. *Case 2*:

$$\lambda(n) \leq \frac{c_3 W_1}{n-f(n)}\sqrt{\frac{nf(n)}{\log n}} \tag{6}$$

when f, n, s, W_1 *and* W_2 *satisfy*

$$\frac{(n-f(n))s(n)}{n}\sqrt{\frac{f(n)\log n}{n}} \cdot \frac{W_2}{W_1} = \Omega(1) \tag{7}$$

where c_1, c_2 and c_3 *are constant.*

Proof Each bit in this hybrid network may traverse a number of RF hops and a number of FSO hops. Let d_{1k} (d_{2k}) denote the number of RF (FSO) hops traversed by a randomly chosen single packet bit k, and $\overline{d}_1$ ($\overline{d}_2$) is the sample average of d_1 (d_2) averaged over all bits in the network.

From Lemma 5.4 in [2], the number of simultaneous transmissions on the RF channel is no more than $\frac{c_4}{r^2}$. (All the c_i's used above and throughout are constants.) Since each source generates λ bits/sec, there are n sources, and each bit needs to be relayed on the average by $\overline{d}_1$ RF hops, it follows that the total number of bits per second served by the RF part of the entire network is $n\lambda\overline{d}_1$. To ensure that all the required RF traffic is carried, we therefore need

$$n\lambda\overline{d}_1 \leq \frac{c_4 W_1}{r^2} \tag{8}$$

On the other hand, since FSO can operate with negligible interference, the number of simultaneous transmissions on FSO channel is no more than f. To ensure that all the required FSO traffic is carried, we therefore need

$$n\lambda\overline{d}_2 \leq W_2 f \tag{9}$$

Now let L_{ij} denote the distance of a randomly chosen S-D pair (i, j), and $\overline{L}$ is the sample average of the length L_{ij} of averaged over all the n S-D pairs on S^2. Then by the law of large numbers we have

$$E[L_{ij}] - \epsilon_1 < \overline{L} < E[L_{ij}] + \epsilon_1 \tag{10}$$

true for any $\epsilon_1 > 0$ and sufficiently large n. Since it is obvious that $E[L_{ij}] = \Theta(1)$, then we have

$$\overline{L} = \Theta(1) \tag{11}$$

true with high probability. From now on we will use the phrase 'with high probability', abbreviated as *whp* to stand for 'with probability approaching to 1 as $n \to \infty$'.

Since for each bit k traveling from source i to destination j, we can write

$$d_{1k} r + d_{2k} s \geq L_{ij} \tag{12}$$

Then by averaging over all bits transported between any S-D pair in the network, we have:

$$\overline{d}_1 r + \overline{d}_2 s \geq \overline{L} \tag{13}$$

Recall that we make the assumption that all the nodes are asymptotically connected by RF links, and all the super nodes are asymptotically connected by FSO links. To satisfy these two asymptotic connectivity assumptions [21], shows that the transmission ranges, r and s, should satisfy

$$r \geq c_5 \sqrt{\frac{\log n}{n}} \tag{14}$$

$$s \geq c_5 \sqrt{\frac{\log f}{f}} \tag{15}$$

From Eqs. 8, 9, 13 and 14, we have

$$\begin{aligned} n\lambda\overline{L} &\leq n\lambda\overline{d}_1 r + n\lambda\overline{d}_2 s \\ &\leq \frac{c_4 W_1}{r^2} \cdot r + W_2 f s \\ &= \frac{c_4 W_1}{r} + W_2 f s \\ &\leq \frac{c_4}{c_5} \cdot W_1 \sqrt{\frac{n}{\log n}} + W_2 f s \end{aligned}$$

Then

$$\lambda \leq \frac{1}{\overline{L}} \left(\frac{c_4}{c_5} \cdot W_1 \sqrt{\frac{1}{n \log n}} + W_2 \frac{fs}{n} \right) \tag{16}$$

Here Eq. 16 gives an upper bound on λ, and it holds with equality if and only if Eqs. 8, 9, 13 and 14 all hold with equality. It follows that the upper bound suggested by Eq. 16 may be achievable if

$$\overline{d}_1 = \frac{\frac{c_4 W_1}{c_5^2 \log n} \overline{L}}{\frac{c_4}{c_5} \cdot W_1 \sqrt{\frac{1}{n \log n}} + W_2 \frac{fs}{n}} \tag{17}$$

$$\overline{d}_2 = \frac{\frac{f W_2}{n} \overline{L}}{\frac{c_4}{c_5} \cdot W_1 \sqrt{\frac{1}{n \log n}} + W_2 \frac{fs}{n}} \tag{18}$$

Here Eq. 17 implies that the number of RF hops traversed by a single bit averaged over all bits, $\overline{d}_1$, may become arbitrarily small with the increase of $\frac{W_2}{W_1}$. However, this cannot be true when $f < n$, as there are $(n - f)$ regular nodes, and each bit generated by these nodes must be forwarded via RF transmission for one hop or more in the beginning of its traversal. Thus for large $\frac{W_2}{W_1}$, the upper bound on λ suggested by Eq. 16 needs to be tightened by exploiting lower bounds on $\overline{d}_1$.

For a regular node i, let $g(i)$ denote the nearest super node to i. For every bit generated by i, in the beginning of its traversal it needs to travel via RF multi-hop transmission for a distance of at least $|X_i - X_{g(i)}|$. Define the random variable $T_i = |X_i - X_{g(i)}|$. Let T refer to any of the T_i as they are i.i.d. Let $\overline{T}$ denote the sample average of the distance T_i averaged over all the regular nodes. Since $\overline{d}_1$ is the number of RF hops traversed by a single bit averaged over all bits in the network, and there are $(n - f)$ regular nodes, then we have the following inequality holds:

$$n\overline{d}_1 r \geq (n - f)\overline{T} \tag{19}$$

If assuming $n - f = \omega(1)$, then by the law of large numbers, the sample average $\overline{T}$ must converge in probability towards the expected value $E[T]$. In other words, when $(n - f)$ is sufficiently large, we have $E[T] - \epsilon_2 \leq \overline{T} \leq E[T] + \epsilon_2$ true for any $\epsilon_2 > 0$. This and Eq. 19 suggest that for any $\epsilon_2 > 0$,

$$n\overline{d}_1 r \geq (n - f)(E[T] - \epsilon_2) \tag{20}$$

is true *whp*.

In the following we will derive $E[T]$ by finding the probability distribution of T, and then can get a lower bound on $\overline{d}_1$ from Eq. 20.

For an arbitrary regular node i on S^2, let D_z denote the disk centered at X_i of area $\frac{z^2 \log f}{f}$, $z \in \mathbb{R}^+$, and let u_z denote the radius of the disk D_z. Then for large f and $0 \leq z \ll \sqrt{f / \log f}$, we have

$$u_1 = \sqrt{\frac{\log f}{\pi f}} \quad (21)$$

$$u_z = z\sqrt{\frac{\log f}{\pi f}} = z u_1 \quad (22)$$

and

$$\begin{aligned} Pr\{T \geq z u_1\} &= Pr\{T \geq u_z\} \\ &= Pr\{\text{none of the f super nodes is in } D_z\} \\ &= \lim_{f \to \infty} \left(1 - \frac{z^2 \log f}{f}\right)^f \\ &= \lim_{f \to \infty} \left(\left(1 - \frac{z^2 \log f}{f}\right)^{\frac{f}{z^2 \log f}}\right)^{z^2 \log f} \\ &= e^{-z^2 \log f} \\ &= f^{-z^2} \end{aligned} \quad (23)$$

Define the random variable Z by $Z = T/u_1$, then the cumulative distribution function (CDF) and the probability density function (PDF) of Z, denoted by $P_Z(z)$ and $p_Z(z)$, respectively, are given by

$$P_Z(z) = 1 - Pr\{T \geq z u_1\} = 1 - f^{-z^2} \quad (24)$$

and

$$p_Z(z) = P'_Z(z) = (2 \log f) \cdot z f^{-z^2} \quad (25)$$

for $0 \leq z << \sqrt{f/\log f}$. Then

$$\begin{aligned} E[Z] &= \int_0^\infty z p_Z(z) dz \\ &\geq \int_0^1 (2 \log f) \cdot z^2 f^{-z^2} \cdot dz \\ &\geq \left(\frac{\sqrt{\pi}}{2} erf(1) - \frac{\sqrt{\log f}}{f}\right) \frac{1}{\sqrt{\log f}} \end{aligned} \quad (26)$$

For large f, it is implied by Eqs. 26 and 21 that

$$\begin{aligned} E[T] &= E[Z] u_1 \\ &\geq \left(\frac{\sqrt{\pi}}{2} erf(1) - \frac{\sqrt{\log f}}{f}\right) \frac{1}{\sqrt{\log f}} \cdot \sqrt{\frac{\log f}{\pi f}} \\ &\geq \frac{c_6}{\sqrt{f}} \end{aligned} \quad (27)$$

Then for the case $n - f = \omega(1)$, according to Eqs. 8, 14, 20 and Eq. 27 and letting $\epsilon_2 = \frac{c_6}{2\sqrt{f}}$ we have the following true *whp*:

$$\begin{aligned} \lambda &\leq \frac{c_4 W_1}{n \bar{d}_1 r^2} \\ &\leq \frac{c_4 W_1}{(n-f)(E[T] - \epsilon_2) r} \\ &\leq \frac{2c_4}{c_5 c_6} \cdot \frac{W_1}{n-f} \sqrt{\frac{fn}{\log n}} \end{aligned} \quad (28)$$

Thus by Eqs. 16 and 28 we obtain the following *whp* for $n - f = \omega(1)$:

$$\begin{aligned} \lambda &\leq \min\left\{\frac{1}{\bar{L}}\left(\frac{c_4}{c_5} \cdot W_1 \sqrt{\frac{1}{n \log n}} + W_2 \frac{fs}{n}\right), \frac{c_4}{c_5 c_6} \cdot \frac{W_1}{n-f} \sqrt{\frac{fn}{\log n}}\right\} \\ &= \begin{cases} \frac{1}{\bar{L}}\left(\frac{c_4}{c_5} \cdot W_1 \sqrt{\frac{1}{n \log n}} + W_2 fs\right), \\ \quad \text{if } \frac{(n-f)s}{n} \sqrt{\frac{f \log n}{n}} \cdot \frac{W_2}{W_1} < \frac{c_4}{c_5}\left(\frac{2\bar{L}}{c_6} - \frac{n-f}{n\sqrt{f}}\right); \\ \frac{c_4}{c_5 c_6} \cdot \frac{W_1}{n-f} \sqrt{\frac{fn}{\log n}}, \quad \text{if otherwise.} \end{cases} \end{aligned} \quad (29)$$

Note the fact that the first of the two inequalities of Eq. 29 is true regardless of the scaling of $(n - f)$ (see Eq. 16); and the condition for the second inequality to hold, i.e.

$$\frac{(n-f)s}{n} \sqrt{\frac{f \log n}{n}} \cdot \frac{W_2}{W_1} \geq \frac{c_4}{c_5}\left(\frac{2\bar{L}}{c_6} - \frac{n-f}{n\sqrt{f}}\right)$$

actually implies that $n - f = \omega(1)$. Therefore (29) holds even if the constraint $n - f = \omega(1)$ is relaxed. Hence we have proved the theorem. □

5 A constructive lower bound on throughput capacity of hybrid RF/FSO networks

In this section we derive a lower bound for $\lambda(n)$ by constructing a specific routing scheme which we call the *hybrid routing scheme*. The lower bound results and the design of the routing scheme are provided in Theorem 2 and its proof.

In order to establish Theorem 2, first we introduce a lemma where we only study a pure FSO network of $f(n)$ nodes with only FSO transceivers. This lemma is a step towards analyzing the hybrid RF/FSO networks later in this section.

Lemma 1 *Consider a random (i.e. randomly deployed) FSO network with $f(n)$ nodes randomly and identically distributed on S^2. Every node is equipped with an FSO transceiver. The FSO communication range $s(n)$ is chosen such that the entire network is asymptotically connected. $n(\geq f(n))$ traffic sessions with identical data rates are assigned to the network, with each node being selected as* *source for at least one session, and the destination node for each session is chosen as the node nearest to a randomly located point on S^2. Then the maximum feasible per session throughput $\lambda'(n)$ of this random FSO network scales as*

$$\lambda'(n) = \Theta\left(\frac{W_2 f(n) s(n)}{n}\right) \tag{30}$$

Proof of Lemma 1 Please see Appendix 1. □

The lower bound on $\lambda(n)$ is given by the following theorem.

Theorem 2 *The throughput capacity of hybrid RF/FSO networks $\lambda(n)$ is bounded from below by*

1. *Case 1*:

$$\lambda(n) \geq c_7 W_1 \sqrt{\frac{1}{n \log n}} + c_8 W_2 \frac{f(n)s(n)}{n} \tag{31}$$

when f, n, s, W_1 and W_2 satisfy

$$\frac{(n - f(n))s(n)}{n} \sqrt{\frac{f(n) \log n}{n}} \cdot \frac{W_2}{W_1} = o(1) \tag{32}$$

2. *Case 2*:

$$\lambda(n) \geq \frac{c_9 W_1}{n - f(n)} \sqrt{\frac{n f(n)}{\log n}} \tag{33}$$

when f, n, s, W_1 and W_2 satisfy

$$\frac{(n - f(n))s(n)}{n} \sqrt{\frac{f(n) \log n}{n}} \cdot \frac{W_2}{W_1} = \Omega(1) \tag{34}$$

Proof In order to show that the lower bounds on λ presented above are achievable, we design a hybrid routing scheme in which the data traffic is divided into two classes and use different routing strategies: a portion of data will be forwarded with the (partial) support of super nodes in a hierarchical routing fashion, and the rest will be purely routed through RF links in a multi-hop fashion. The lower bounds on λ can thus be derived by properly balancing the load between these two classes of traffic.

The proof is organized as follows: The hybrid routing scheme is proposed in Sect. 5.1, in which we design an RF multi-hop routing strategy (Sect. 5.1.1) and a hierarchical routing strategy (Sect. 5.1.2) for the two classes of traffic. In Sect. 5.2, the maximum throughput achieved by the hybrid routing scheme is derived. Seven lemmas (Lemma 1–7) are used in the entire proof, as is illustrated in Fig. 1.

5.1 Hybrid routing scheme

We employ a hybrid routing scheme by dividing the per node throughput λ into two classes. Each source-destination pair transmits packets with data rate λ_1 by purely using RF links, and transmits packets with data rate λ_2 by (fully or partly) using FSO links. Hence we have

$$\lambda = \lambda_1 + \lambda_2 \tag{35}$$

The detailed routing strategies for these two classes of traffic are as follows.

5.1.1 An RF multi-hop routing strategy for λ_1-traffic

The routing strategy for λ_1-traffic is similar to the one in [2]. Let

$$\rho(n) := \text{radius of a disk of area } 100 \log n / n \tag{36}$$

We can construct a Voronoi tessellation $\mathcal{V}_n$ such that

(V1) Every Voronoi cell contains a disk of radius $\rho(n)$;
(V2) Every Voronoi cell is contained in a disk of radius 2 $\rho(n)$;
(V3) We choose the range r of each transmission such that

$$r = 8\rho(n) \tag{37}$$

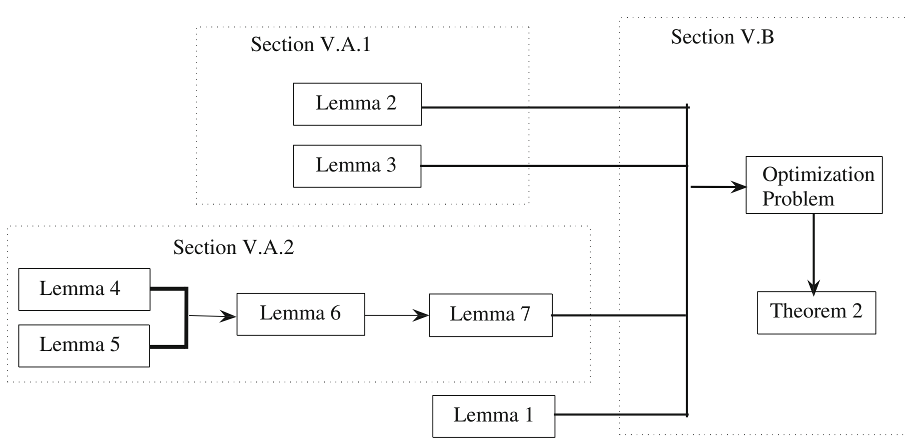

Fig. 1 The logical structure of the proof for Theorem 2

This range allows direct communication within a cell and between adjacent cells. According to [2], we have the following lemmas.

Lemma 2 *Every cell in $\mathcal{V}_n$ has no more than c_{10} interfering neighbors. Furthermore, there is a schedule for transmitting packets such that in every $c_{11}(=(1+c_{10}))$ slots, each cell in the $\mathcal{V}_n$ gets one slot in which to transmit, and such that all transmissions are successfully received within the transmission and reception coverage.*

Lemma 3 *There is a $\delta_1(n) \to 0$ such that*

$$Pr\left\{\sup_{V\in\mathcal{V}_n}(\text{RF } \lambda_1 - \text{traffic needing to be carried by cell } V) \leq c_{12}\lambda_1\sqrt{n\log n}\right\} \geq 1-\delta_1(n) \tag{38}$$

5.1.2 A hierarchical routing strategy for λ_2-traffic

The routing strategy for λ_2-traffic is described by the following steps:

- Step 1: The packets originated by a source node are routed through RF links to the super node nearest to the source node, if needed;
- Step 2: The packets are then routed through FSO links to the super node nearest to the destination node;
- Step 3: The packets are then routed through RF links to the destination node, if needed.

Here the RF routing scheme used in steps 1 and 3 is the same as the one used in [2]. The FSO routing scheme used in Step 2 is described in the proof of Lemma 1. Note that the routing scheme used here for λ_2-traffic is similar to hierarchical state routing e.g. [6].

The rest of this subsection Eq. 39 presents how we derive an upper bound on the RF λ_2-traffic carried by each cell $V \in \mathcal{V}_n$.

It can be noted that the λ_2-traffic carried by RF links consists of a number of RF traffic segments, which may be of two types: (a) a *type-1 RF λ_2-traffic route* is one that is started by a regular node and then reaches the super node nearest to it, and (b) a *type-2 RF λ_2-traffic route* is one that terminates at a regular node, and its starting point is the nearest super node to that regular node.

Let random variable $\mathcal{N}$ denote the total number of RF λ_2-traffic segments. The following lemma bounds $\mathcal{N}$ (probabilistically).

Lemma 4 *There is $\delta_2(n) \to 0$ such that*

$$Pr\{\mathcal{N} \leq (2+c_{13})(n-f)\} \geq 1-\delta_2(n) \tag{39}$$

Proof of Lemma 4 Please see Appendix 2. □

By the property (V2) on the tessellation $\mathcal{V}_n$, each cell $V \in \mathcal{V}_n$ is contained in a disk of radius 2ρ. This allows us to bound the probability that an RF λ_2-traffic route intersects a given cell $V \in \mathcal{V}_n$.

Lemma 5 *Randomly pick up an RF λ_2-traffic route with endpoints i and $g(i)$, where node i is a regular node, and node $g(i)$ is the super node closest to i. Let L_i denote the segment joining node i and node $g(i)$. Then for any cell $V \in \mathcal{V}_n$,*

$$p_V := Pr\{L_i \text{ intersects } V\} \leq c_{14}\sqrt{\frac{\log n}{fn}} \tag{40}$$

Proof of Lemma 5 Please see Appendix 3. □

Based on Lemma 4 and Lemma 5, we can then bound the number of RF λ_2-traffic segments served by every cell.

Lemma 6 *Let random variable $\mathcal{M}$ denote the number of RF λ_2-traffic segments that intersect cell V. Then there is a $\delta_3(n) \to 0$ such that*

$$Pr\left\{\mathcal{M} \leq c_{15}(n-f)\sqrt{\frac{\log n}{fn}}\right\} \geq 1-\delta_3(n) \tag{41}$$

Proof of Lemma 6 Please see Appendix 4. □

Note that the λ_2-traffic handled by each Voronoi cell is proportional to the number of RF λ_2-traffic segments passing through it. Since each RF λ_2-traffic route carries traffic of rate λ_2 bits/sec, then we can readily obtain the following bound.

Lemma 7 *There is a $\delta_3(n) \to 0$ such that*

$$Pr\left\{\sup_{V\in\mathcal{V}_n}(\text{RF } \lambda_2 - \text{traffic needing to be carried by cell } V) \leq c_{15}\lambda_2(n-f)\sqrt{\frac{\log n}{fn}}\right\} \geq 1-\delta_3(n) \tag{42}$$

5.2 Maximum throughput achieved by the hybrid routing scheme

From Lemma 2, Lemma 3 and Lemma 7, we can conclude that the data transmission utilizing RF links can be accommodated by all cells if

$$c_{12}\lambda_1\sqrt{n\log n} + c_{15}\lambda_2(n-f)\sqrt{\frac{\log n}{fn}} \leq \frac{W_1}{c_{11}} \tag{43}$$

According to Lemma 1, the data transmission (partially) utilizing FSO links can be accommodated by the FSO network if

$$\lambda_2 \le c_{16} W_2 \frac{fs}{n} \tag{44}$$

Then we can derive the maximum throughput achieved by the proposed routing scheme by solving the following optimization problem:

maximize & λsubject to &

$$\begin{cases} \lambda_1 + \lambda_2 = \lambda \\ c_{12}\lambda_1\sqrt{n\log n} + c_{15}\lambda_2(n-f)\sqrt{\frac{\log n}{fn}} \le \frac{W_1}{c_{11}} \\ \lambda_2 \le c_{16} W_2 \frac{fs}{n} \end{cases}$$

The optimal solution falls into the following two mutually exclusive cases:

Case 1: When f, n, s, W_1 and W_2 satisfy

$$\frac{W_1}{W_2} > c_{11}c_{15}c_{16}\frac{(n-f)s}{n}\sqrt{\frac{f\log n}{n}} \tag{45}$$

the maximum throughput λ and the data rates of the two classes of traffic, λ_1 and λ_2, are given by

$$\begin{cases} \lambda = \frac{W_1}{c_{11}c_{12}\sqrt{n\log n}} + c_{16}\left(1 - \frac{c_{15}}{c_{12}}\frac{n-f}{n\sqrt{f}}\right) W_2 \frac{fs}{n} \\ \lambda_1 = \frac{W_1}{c_{11}c_{12}\sqrt{n\log n}} - \frac{c_{15}c_{16}W_2(n-f)\sqrt{f}s}{c_{12}n^2} \\ \lambda_2 = c_{16} W_2 \frac{fs}{n} \end{cases} \tag{46}$$

Case 2: When m, n, W_1 and W_2 satisfy

$$\frac{W_1}{W_2} \le c_{11}c_{15}c_{16}\frac{(n-f)s}{n}\sqrt{\frac{f\log n}{n}} \tag{47}$$

the maximum throughput λ and the data rates of the two classes of traffic, λ_1 and λ_2, are given by

$$\begin{cases} \lambda = \frac{W_1}{c_{11}c_{15}}\frac{1}{n-f}\sqrt{\frac{fn}{\log n}} \\ \lambda_1 = 0 \\ \lambda_2 = \frac{W_1}{c_{11}c_{15}}\frac{1}{n-f}\sqrt{\frac{fn}{\log n}} \end{cases} \tag{48}$$

Hence we have proved Theorem 2. □

6 Practical interpretations on capacity results

Theorem 1 and Theorem 2 provide tight bounds on the capacity of hybrid RF/FSO networks, as we can generalize the results as follows:

$$\lambda = \begin{cases} \Theta\left(\frac{W_1}{\sqrt{n\log n}}\right) + \Theta\left(W_2\frac{f(n)s(n)}{n}\right), \\ \quad \text{if } \frac{(n-f(n))s(n)}{n}\sqrt{\frac{f(n)\log n}{n}} \cdot \frac{W_2}{W_1} = o(1); \\ \Theta\left(\frac{W_1}{n-f(n)}\sqrt{\frac{nf(n)}{\log n}}\right), \\ \quad \text{if } \frac{(n-f(n))s(n)}{n}\sqrt{\frac{f(n)\log n}{n}} \cdot \frac{W_2}{W_1} = \Omega(1). \end{cases} \tag{49}$$

This result allows us to observe several practical implications which designers may want to consider.

6.1 Capacity improvement

The capacity of pure RF wireless networks is $\Theta\left(\frac{W_1}{\sqrt{n\log n}}\right)$, which is given by [2]. In the hybrid RF/FSO network scenario, the capacity is improved by equipping $f(n)$ nodes with an additional FSO transceiver. Here we characterize this capacity improvement quantitatively.

We define the *capacity gain ratio* $G(n)$, as the capacity ratio between hybrid RF/FSO and pure RF ad hoc networks.

First we take Case 1 into consideration. By comparing with the capacity results in [2], we can show that G may only scales as $\Theta(1)$ when

$$f(n) = o\left(\frac{1}{s(n)}\sqrt{\frac{n}{\log n}} \cdot \frac{W_1}{W_2}\right) \tag{50}$$

On the other hand, we can achieve a significant capacity improvement (i.e. $G(n) = \omega(1)$) by adding FSO transceivers to $f(n)$ nodes if and only if

$$f(n) = \Omega\left(\frac{1}{s(n)}\sqrt{\frac{n}{\log n}} \cdot \frac{W_1}{W_2}\right) \tag{51}$$

For Case 2, Eq. 48 implies that the capacity gain ratio $G(n)$ will always tend to infinity as n grows. Thus a significant capacity improvement can be achieved with high probability under Case 2.

Thus we can generalize the results for Cases 1 and 2 by making the following statement:

As compared to pure RF wireless networks, a hybrid RF/FSO network can achieve a significant throughput capacity improvement if

$$f(n) = \Omega\left(\frac{1}{s(n)}\sqrt{\frac{n}{\log n}} \cdot \frac{W_1}{W_2}\right) \tag{52}$$

This result characterizes the number of super nodes needed in order to guarantee a significant capacity improvement as compared to the case of pure RF wireless networks.

Moreover, given $n > f(n)$, it can be noted that when

$$W_2 \ge \frac{n}{c_0(n-f(n))s(n)}\sqrt{\frac{n}{f(n)\log n}} \cdot W_1 =: W_2^{(critical)} \tag{53}$$

λ will seize to increase as W_2 increases. Therefore, we can make the following statement:

For fixedW_1, $s(n)$, $f(n)$ *and* n *where* $f(n) < n$, *there is no incentive (in the sense of capacity improvement) to increase the FSO data rate beyond some critical value* $W_2^{(critical)}$.

6.2 Conditions for achieving linear capacity scaling

In the previous subsection we have determined the case under which there is a significant capacity improvement with the introduction of FSO transceivers. Now we ask a more ambitious question: can the per-node throughput of hybrid RF/FSO networks scale linearly with the number of nodes n? The answer to this question is yes and results are generalized in the following statement.

The per node throughput capacity of a hybrid RF/FSO network $\lambda(n)$ scales as $\Theta(1)$ if and only if the FSO transmission range $s(n)$ scales as $\Theta(1)$, and the number of regular nodes, $n - f(n)$, scales as

$$n - f(n) = O\left(\frac{n}{\sqrt{\log n}} \cdot \frac{W_1}{W_2}\right) \tag{54}$$

When the number of super nodes $f(n) = kn$ with $0 < k < 1$ some constant, The per node throughput capacity $\lambda(n)$ scales as $\Theta(1/\sqrt{\log n})$, i.e., the The per node throughput capacity scales close to linearity.

This statement can be easily validated from Eq. 49. This result implies that, in order to have the per node throughput grows close to linearity with n, we should have the number of super nodes grows as fast as $\Theta(n)$. And in order to achieve strict linear throughput scaling, we have to limit the number of regular nodes by making sure that it grows no faster than $O\left(\frac{n}{\sqrt{\log n}} \cdot \frac{W_1}{W_2}\right)$.

7 Delay analysis in hybrid RF/FSO networks

In previous sections we developed scaling results for the throughput capacity of hybrid RF/FSO networks. In this section we study the average packet delay $\overline{D}(n)$ in RF/FSO networks that achieve optimal throughput scaling. In Sect. 7.1 we derive the scaling results for $\overline{D}(n)$ (see Theorem 3 and Theorem 4). Then we present some practical interpretations of these delay results in Sect. 7.2.

7.1 Delay scaling results

Similar to our throughput scaling derivations in previous sections, we study the delay by first deriving a lower bound on $\overline{D}$ over all schemes that achieve optimal throughput scaling (Theorem 3), then we show that under the hybrid routing scheme proposed in Sect. 5 this lower bound scaling can be achieved (Theorem 4).

Theorem 3 *When the per-node throughput $t(n)$ of a hybrid RF/FSO network achieves optimal scaling, i.e. $t(n) = \Theta(\lambda(n))$, the average (end-to-end) delay $\overline{D}(n)$ scales by*

$$\overline{D}(n) = \begin{cases} \Omega\left(\frac{1}{W_1}\sqrt{\frac{n}{\log n}}\right), \\ \quad \text{if } \lambda(n) = \Theta\left(\frac{W_1}{\sqrt{n\log n}}\right); \\ \Omega\left(\frac{n-f}{W_1\sqrt{nf\log n}}\right) + \Omega\left(\frac{1}{W_2 s}\right), \\ \quad \text{if } \lambda(n) = \omega\left(\frac{W_1}{\sqrt{n\log n}}\right). \end{cases} \tag{55}$$

Proof We study this problem under the following two cases: Case 1: $\lambda(n) = \Theta\left(\frac{W_1}{\sqrt{n\log n}}\right) + \Theta\left(W_2\frac{\sqrt{f(n)s(n)}}{n}\right)$.

In this case, we can rewrite Eq. 16 as

$$t\overline{L} \le t\overline{d}_1 r + t\overline{d}_2 s \le \frac{c_4}{c_5} W_1 \frac{1}{\sqrt{n\log n}} + W_2\frac{fs}{n} \tag{56}$$

Since $t(n) = \Theta(\lambda(n))$, therefore

$$t\overline{d}_1 r + t\overline{d}_2 s = \Theta(\lambda(n)) \tag{57}$$

That is,

$$\overline{d}_1 r + \overline{d}_2 s = \Theta(1) \tag{58}$$

Then we should either have

$$\begin{cases} \overline{d}_1 r = \Theta(1) \\ \overline{d}_2 s = O(1) \end{cases} \tag{59}$$

or

$$\begin{cases} \overline{d}_1 r = o(1) \\ \overline{d}_2 s = \Theta(1) \end{cases} \tag{60}$$

When Eq. 59 is true, it is implied by Eqs. 8, 9 and 14 that

$$\Theta(\lambda(n)) = t\overline{d}_1 r \le \frac{c_4 W_1}{nr} \le \frac{c_4}{c_5} W_1 \frac{1}{\sqrt{n\log n}} \tag{61}$$

Therefore

$$\lambda(n) = \Theta\left(\frac{W_1}{\sqrt{n\log n}}\right) \tag{62}$$

and we must have $r = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$. Thus

$$\overline{d}_1 = \Theta\left(\sqrt{\frac{n}{\log n}}\right) \tag{63}$$

Since the time slot length for RF transmission is $\tau_1 = B/W_1$, then the average delay $\overline{D}$ is lower bounded by

$$\begin{aligned} \overline{D} &\ge \overline{d}_1 \tau_1 \\ &= \Theta\left(\frac{1}{W_1}\sqrt{\frac{n}{\log n}}\right) \end{aligned} \tag{64}$$

Now suppose Eq. 61 is true. Then it is readily known that $\lambda(n) = \omega\left(\frac{W_1}{\sqrt{n\log n}}\right)$, and $\overline{d}_2 = \Theta\left(\frac{1}{s}\right)$. It is implied by Eqs. 20 and 27 that

$$\overline{d}_1 = \Omega\left(\frac{n-f}{\sqrt{nf}\log n}\right) \quad (65)$$

Since the time slot length for FSO transmission is $\tau_2 = B/W_2$, then

$$\begin{aligned}\overline{D} &\geq \overline{d}_1\tau_1 + \overline{d}_2\tau_2 \\ &= \Omega\left(\frac{n-f}{W_1\sqrt{nf}\log n}\right) + \Theta\left(\frac{1}{W_2 s}\right)\end{aligned} \quad (66)$$

Combining Eqs. 64 and 66 we can write

$$\overline{D}(n) = \begin{cases} \Omega\left(\frac{1}{W_1}\sqrt{\frac{n}{\log n}}\right), \\ \quad \text{if } \lambda(n) = \Theta\left(\frac{W_1}{\sqrt{n\log n}}\right); \\ \Omega\left(\frac{n-f}{W_1\sqrt{nf}\log n}\right) + \Omega\left(\frac{1}{W_2 s}\right), \\ \quad \text{if } \lambda(n) = \omega\left(\frac{W_1}{\sqrt{n\log n}}\right). \end{cases} \quad (67)$$

Case 2: $\lambda(n) = \Theta\left(\frac{W_1}{n-f(n)}\sqrt{\frac{nf(n)}{\log n}}\right)$.

In this case, it is implied by Eqs. 20, 27 and 28 that

$$\begin{cases} n\overline{d}_1 r = \Theta((n-f)\overline{T}) \\ \overline{T} = \Theta\left(\frac{1}{\sqrt{f}}\right) \\ r = \sqrt{\frac{\log n}{n}} \end{cases} \quad (68)$$

Therefore

$$\overline{d}_1 = \Theta\left(\frac{n-f}{\sqrt{nf}\log n}\right) \quad (69)$$

As a result we have $\overline{d}_1 r = \Theta\left(\frac{n-f}{n\sqrt{f}}\right) = o(1)$. Then it is implied by Eq. 13 that

$$\overline{d}_2 s = \Omega(1) \quad (70)$$

thus $\overline{d}_2 = \Theta(\frac{1}{s})$. Then $\overline{D}$ is lower bounded by

$$\begin{aligned}\overline{D}(n) &\geq \overline{d}_1\tau_1 + \overline{d}_2\tau_2 \\ &= \Theta\left(\frac{n-f}{W_1\sqrt{nf}\log n}\right) + \Omega\left(\frac{1}{W_2 s(n)}\right)\end{aligned} \quad (71)$$

Combining the results for Case 1 and Case 2 we get Eq. 55. Hence we have proved the theorem. □

Theorem 4 *When the per node throughput of an RF/FSO network achieves optimal scaling $\Theta(\lambda(n))$ under the hybrid routing scheme, the average per bit delay $\overline{D}(n)$ scales as*

$$\overline{D}(n) = \begin{cases} \Theta\left(\frac{1}{W_1}\sqrt{\frac{n}{\log n}}\right), \\ \quad \text{if } \lambda(n) = \Theta\left(\frac{W_1}{\sqrt{n\log n}}\right); \\ \Theta\left(\frac{n-f}{W_1\sqrt{nf}\log n}\right) + \Theta\left(\frac{1}{W_2 s}\right), \\ \quad \text{if } \lambda(n) = \omega\left(\frac{W_1}{\sqrt{n\log n}}\right). \end{cases} \quad (72)$$

Proof Assume that the data traffic is packetized in such a way that the packet size scales as $\Theta(\lambda_1(n))$ for the λ_1-traffic, and as $\Theta(\lambda_2(n))$ for the λ_2-traffic. It is shown in [22, 23] that, under such assumption the per bit delay scaling and the per packet delay scaling are the same. Then we only need to determine the per packet delay scaling for each class of traffic.

For the λ_1-traffic, the average number of RF hops traveled by a packet scales as $\Theta\left(\sqrt{\frac{n}{\log n}}\right)$, then the average packet delay for λ_1-traffic, denoted by $\overline{D}_1(n)$, scales as

$$\overline{D}_1(n) = \Theta\left(\frac{1}{W_1}\sqrt{\frac{n}{\log n}}\right) \quad (73)$$

For the λ_2-traffic, the average number of RF hops traveled by a packet scales as $\Theta\left(\frac{n-f}{\sqrt{nf(n)}\log n}\right)$, and the average number of FSO hops traveled by a packet scales as $\Theta\left(\frac{1}{s(n)}\right)$, then the average packet delay for λ_2-traffic, denoted by $\overline{D}_2(n)$, scales as

$$\overline{D}_2(n) = \Theta\left(\frac{n-f}{W_1\sqrt{nf(n)}\log n}\right) + \Theta\left(\frac{1}{W_2 s(n)}\right) \quad (74)$$

Since the average per bit delay $\overline{D}(n)$ scales as

$$\overline{D}(n) = \Theta\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\overline{D}_1(n) + \frac{\lambda_2}{\lambda_1+\lambda_2}\overline{D}_2(n)\right) \quad (75)$$

Then by substituting Eqs. 46 and 48 into Eq. 75 we get Eq. 72. Hence we have proved the theorem. □

Theorem 3 and Theorem 4 imply that, among all schemes that achieve optimal throughput scaling $\Theta(\lambda(n))$, the hybrid routing scheme produces the minimum delay scaling. Some practical interpretations of these delay results will be presented in the following subsection.

7.2 Practical interpretations of delay results

Theorem 3 and Theorem 4 have shown that under optimal throughput scaling, the minimum average delay scales as

$$\overline{D}(n) = \begin{cases} \Theta\left(\frac{1}{W_1}\sqrt{\frac{n}{\log n}}\right), \\ \quad \text{if } \lambda(n) = \Theta\left(\frac{W_1}{\sqrt{n\log n}}\right); \\ \Theta\left(\frac{n-f}{W_1\sqrt{nf\log n}}\right) + \Theta\left(\frac{1}{W_2 s}\right), \\ \quad \text{if } \lambda(n) = \omega\left(\frac{W_1}{\sqrt{n\log n}}\right). \end{cases} \tag{76}$$

Then we observe serveral practical implications based on this result.

7.2.1 Delay reduction

It is already known that the delay scaling of pure RF wireless networks is $\Theta\left(\frac{1}{W_1}\sqrt{\frac{n}{\log n}}\right)$ [22]. In hybrid RF/FSO network scenario, the delay is reduced by equipping $f(n)$ nodes with an additional FSO transceiver. To characterize this reduction quantitatively, let $R(n)$ denote the delay ratio between hybrid RF/FSO and pure RF ad hoc networks. Then clearly there is a significant delay reduction (i.e. $R(n) \to 0$) when there is a significant throughput improvement, i.e. $\lambda(n) = \omega\left(\frac{W_1}{\sqrt{n\log n}}\right)$. Therefore the condition of achieving a significant delay reduction is the same as the one of getting throughput improvement, i.e., the number of super nodes should scale as

$$f(n) = \Omega\left(\frac{1}{s(n)}\sqrt{\frac{n}{\log n}}\cdot\frac{W_1}{W_2}\right) \tag{77}$$

7.2.2 Conditions for achieving linear delay scaling

It is implied by Eq. 76 that, in order to achieve linear delay scaling, the FSO transmission range $s(n)$ should scale as $\Theta(1)$, and the number of super nodes $f(n)$ should scale as $\Omega\left(\sqrt{\frac{n}{\log n}}\right)$. This is a weaker condition as compared to that of achieving linear throughput scaling. Therefore, in order to have the delay and per node throughput both scaling linearly, in addition to making $s(n)$ scale as $\Theta(1)$, one still should have the number of regular nodes $(n - f(n))$ scale as $O\left(\frac{n}{\sqrt{\log n}}\cdot\frac{W_1}{W_2}\right)$.

8 Concluding remarks and future work

In this paper, we have studied the per node throughput capacity and end-to-end delay of hybrid RF/FSO networks. A hybrid RF/FSO network consists of an RF wireless network with random deployment and connected by RF links, and only a portion of nodes (so called super nodes) are equipped with an additional FSO transceiver. All the super node are connected by the FSO links and thus may form a stand-alone FSO network. The objective of this paper is to derive the asymptotic capacity and delay of such hybrid networks, and evaluate the benefit of using this hybrid RF/FSO network architecture over the pure RF wireless networks.

We consider a hybrid RF/FSO network with a total number of n nodes, and $f(n)$ of them are super nodes. Every RF and FSO transceiver is able to transmit at a maximum data rate of W_1 and W_2 bits/sec, respectively. We show that the throughput capacity of hybrid RF/FSO networks with the constraint of minimum power consumption, λ, is bounded by $\lambda \le c_1 W_1\sqrt{\frac{1}{n\log n}} + c_2 W_2\frac{f(n)s(n)}{n}$, when $\frac{(n-f(n))s(n)}{n}\sqrt{\frac{f(n)\log n}{n}}\cdot\frac{W_2}{W_1} = o(1)$, and by $\lambda \le \frac{c_3 W_1}{n-f(n)}\sqrt{\frac{nf(n)}{\log n}}$ when $\frac{(n-f(n))s(n)}{n}\sqrt{\frac{f(n)\log n}{n}}\cdot\frac{W_2}{W_1} = \Omega(1)$, where $s(n)$ is the FSO transmission range, and c_0, c_1, c_2 and c_3 are positive constants. In order to evaluate the tightness of this upper bound, we design a hybrid routing scheme as we divide the data traffic into two classes and use different routing strategies: a portion of data will be forwarded with the (partial) support of super nodes in a hierarchical routing fashion, and the rest will be purely routed through RF links in a multi-hop fashion. By properly balancing the loads between these two classes of traffic, we have proved that the upper bound is a tight one as a per node throughput of same order can be achieved.

Further, we studied the scaling properties of average end-to-end delay in RF/FSO networks under optimal throughput scaling. The results show that our proposed hybrid routing scheme achieves the minimum delay scaling among all possible schemes that achieve the optimal throughput scaling.

We have evaluated the throughput capacity improvement and delay reduction with the support of FSO nodes, as compared with the results for RF wireless networks in [2]. We have shown that a significant capacity gain as well as delay reduction will be achieved if $f(n) = \Omega\left(\frac{1}{s(n)}\sqrt{\frac{n}{\log n}}\cdot\frac{W_1}{W_2}\right)$. We further conclude that for fixed W_1, $f(n)$, n and $s(n)$ where $f(n) < n$, there is no incentive (in the sense of capacity improvement) to increase the FSO data rate any further as long as it exceeds a critical value $W_2^{(critical)} = \frac{n}{c_0(n-f(n))}\sqrt{\frac{n}{\log f(n)\log n}}\cdot W_1$. In addition, we also show that RF/FSO networks can achieve linear throughput scaling when the number of super nodes $f(n)$ scales as $\Omega\left(\sqrt{\frac{n}{\log n}}\right)$, and achieve linear delay scaling when the number of regular nodes $n - f(n)$ scales $O\left(\frac{n}{\sqrt{\log n}}\cdot\frac{W_1}{W_2}\right)$.

The entire analysis of this paper is based on the assumption that nodes are distributed uniformly and randomly over the network area of interest, which is a reasonable model for the ad hoc nature of many wireless computer networks such as several military networks and wireless sensor networks. We require the entire network to be asymptotically connected, which is a necessary condition of enabling any-to-any communications. With these assumptions we are able to characterize the performance of these ad-hoc networks.

The analysis and results will probably change if networks with non-uniform nodal distributions are considered. The network may achieve much better or much worse performance results depending on specific distributions. For example, if all the S-D pairs happen to be very close to each other to allow direct (rather than multi-hop) communications, then each sender can simply have its communication radius to be the minimum to reach the receiver. In this case the entire network may be far from being connected. In this case the network will not be able to perform any-to-any communications and therefore may be of very limited use. On the other hand, if a network is deployed with outliers or has bottlenecks, the performance can be far below what we would expect. These circumstances, however, are shown to be unlikely to happen under the assumptions used in this paper.

Possible avenues for future work include extending the results to the case when RF and FSO transceivers may become unreliable (e.g., under adverse weather conditions).

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Appendix 1

Proof of Lemma 1 To prove this lemma we need to show there exists constants c_4 and c_5 such that λ' is upper bounded by

$$\lambda' \leq \frac{c_{16} W_2 f(n) s(n)}{n} \tag{78}$$

and there exists a spatial and temporal scheduling scheme such that a per session throughput

$$\lambda' = \frac{c_{17} W_2 f(n) s(n)}{n} \tag{79}$$

is achievable.

To derive the upper bound for λ', we notice that the number of simultaneous FSO transmissions is no more than m, thus the total data rate served by the entire FSO network is no more than $W_2 f(n)$. Now let $\overline{L}$ denote the mean length of a line connecting two independently and uniformly distributed points on S^2. Then the mean length of the path of packets is at least $\overline{L} - o(1)$. Thus the mean number of hops taken by a packet is at least $\frac{\overline{L}-o(1)}{s(n)}$. Then the total number of bits per second served by the entire network needs to be at least $\frac{(\overline{L}-o(1))n\lambda'}{s(n)}$. Then we have the following inequality:

$$\frac{(\overline{L} - o(1))n\lambda'}{s} \leq W_2 f(n) \tag{80}$$

Therefore we have the following upper bound on λ':

$$\lambda' \leq \frac{c_{16} W_2 f(n) s(n)}{n} \tag{81}$$

To show that this upper bound is order tight, we design a routing scheme using similar techniques as in [2]. Let

$$\rho'(n) := \text{radius of a disk of area } \frac{\pi s^2(n)}{64} \tag{82}$$

Then it is readily known that

$$s(n) \geq 8\rho'(n) \tag{83}$$

According to Lemma 4.1 in [2], we can construct a Voronoi tessellation $\mathcal{V}'_n$ such that

(A1) Every Voronoi cell contains a disk of radius $\rho'(n)$;
(A2) Every Voronoi cell is contained in a disk of radius 2 $\rho'(n)$;

This setting allows direct communication within a cell and between adjacent cells, i.e., every node in a cell is within this distance $s(n)$ from every other node in its own cell or adjacent cell. The routing strategy forwards the packets along the sequence of hops that approximate the straight line that connects the source and destination, i.e. packets are passing the cells that the straight-line intersects.

Now we show that the number of nodes in each cell is bounded from below with high probability. Let $\mathcal{F}$ denote the class of disks of area $\frac{\pi s^2(n)}{64}$. Then it is implied by Vapnik-Chervonenkis Theorem (see [2] and references therein) that

$$Pr\left\{\sup_{D\in\mathcal{F}}\left|\frac{\text{number of nodes in D}}{f(n)} - \frac{\pi s^2(n)}{64}\right| \leq \epsilon(n)\right\} > 1 - \delta(n) \tag{84}$$

whenever

$$n > \max\left\{\frac{24}{\epsilon(n)}\log\frac{16e}{\epsilon(n)}, \frac{4}{\epsilon(n)}\log\frac{2}{\delta(n)}\right\} \tag{85}$$

Note that $s(n)$ is chosen such that the FSO network is asymptotically connected, and $s(n)$ must be less than the maximum distance between any two points in S^2 which is $\sqrt{\pi}/2$, then

$$c_5\sqrt{\frac{\log f(n)}{f(n)}} \le s(n) \le \frac{\sqrt{\pi}}{2} \tag{86}$$

then Eq. 85 is satisfied by choosing $\epsilon(n) = s^2(n)/64$ and $\delta(n) = 1/n$ when n is large enough.

Since each cell V in $\mathcal{V}'_n$ contains a disk of area $\frac{\pi s^2(n)}{64}$, then for large n we have

$$Pr\left\{\text{number of nodes in } V \ge \frac{f(n)s^2(n)}{32}\right\} \ge 1-\delta(n) \tag{87}$$

Next we show that the traffic to be served by each cell V is bounded from above with high probability. Towards this objective we let i and j denote two randomly located points on S^2, and L_{ij} denote the line segment connecting these two points. We claim that for any cell $V \in \mathcal{V}'_n$,

$$Pr(L_{ij} \text{ intersects } V) \le c_{24}s(n) \tag{88}$$

This is proved as follows[2]. From property (A2), every cell $V \in \mathcal{V}'_n$ is contained in a disk of radius $s(n)/4$. If i lies at a distance x from the disk, then the angle θ subtended at i by the disk is no more than $c_{18}s(n)/x$. The area of the sector so formed is no more than $\frac{c_{19}\theta}{2\pi}$. If j does not lie in this sector, Then the line L_{ij} cannot intersect the disk containing the cell V. Hence for a point i at a distance x from the disk of radius $s(n)/4$ containing the cell V, the probability that the line L_{ij} intersects the disk is no more than $c_{20}s(n)/x$. Since i is uniformly distributed on S^2, the probability density that it is at a distance x from the disk is bounded above by $2c_{21}\pi(x + s(n)/4)$. Integrating, we obtain

$$\begin{aligned} Pr(L_{ij} \text{ intersects } V) \le & \int_{s(n)/4}^{\sqrt{\pi}/2}\left(\frac{c_{20}s(n)}{x}\right) \\ & \cdot 2c_{21}\pi\left(x+\frac{s(n)}{4}\right)dx \\ & \le c_{24}s(n) \end{aligned} \tag{89}$$

By a similar reasoning through Lemma 4.10–4.14 in [2] we have

$$\begin{aligned} & Pr\{\sup_{V\in\mathcal{V}'_m} (\text{traffic needing to be carried by cell V}) \\ & \le c_{24}\lambda' ns(n)\} \ge 1-\delta_1(m) \end{aligned} \tag{90}$$

It can be implied by Eq. 87 that each cell $V \in \mathcal{V}'_n$ is able to transmit at least $\frac{f(n)s^2(n)}{32}W_2$ bits/sec. Then with high probability, the rate $c_{24}\ \lambda'\ ns(n)$ can be accommodated by all cell if

$$c_5\lambda' ns(n) \le \frac{f(n)s^2(n)}{32}W_2 \tag{91}$$

Then the per session achievable throughput is given by

$$\lambda' = \frac{W_2 f(n)s(n)}{32c_{24}n} \tag{92}$$

which proves Lemma 1. □

Appendix 2

Proof of Lemma 4 Clearly the number of type-1 RF λ_2-traffic segments is $(n-f)$. Let random variable $\mathcal{B}$ denote the number of type-2 RF λ_2-traffic segments. Then

$$\mathcal{N} = \mathcal{B} + n - f \tag{93}$$

For each source-destination pair, the destination node is a regular node with probability $\frac{n-f}{n}$. Then $\mathcal{B}$ follows binomial distribution $B(n, \frac{n-f}{n})$. It follows from the law of large numbers that given any $\epsilon > 0$,

$$\lim_{n\to\infty} Pr\left\{\left|\frac{\mathcal{B}}{n} - \frac{n-f}{n}\right| < \epsilon\right\} = 1 \tag{94}$$

Let $\epsilon = c_{22}(n-f)/n$, then for large n and $f < n$, there is $\delta_2(n) \to 0$ such that

$$Pr\{\mathcal{B} \le (1+c_{22})(n-f)\} \ge 1-\delta_2(n) \tag{95}$$

Then Eq. 39 follows directly from Eqs. 93 and 95. □

Appendix 3

Proof of Lemma 5 This proof uses a similar technique introduced in the proof of Lemma 4.9 in [2].

According to the property (V2), there exists a disk of radius 2ρ that contains the Voronoi cell V. Let D denote such a disk. If i lies at a distance x from the disk D, then the angle $\alpha(\le\pi)$ of the sector subtended at node i by the disk is no more than $\frac{c_{23}}{x}\sqrt{\frac{\log n}{n}}$. If the segment L_i joining node i and node $g(i)$ intersects the disk D, then node $g(i)$ should lie inside the sector and $|X_i - X_{g(i)}| \ge x$. Then the probability that L_i intersects the disk D is no more than $\frac{\alpha}{2\pi}Pr\{|X_i - X_{g(i)}| \ge x\}$. Note that $|X_i - X_{g(i)}|$ follows the probability distribution of random variable T defined in Section IV, which is given by Eq. 23. Since node i is uniformly distributed on S^2, the probability density that it is at a distance x from the disk is bounded above by $c_{24}(x + 2\rho)$. Let $x = z\ u_1$, then

[2] The proof technique used here is similar to Lemma 4.9 in [2].

$$\begin{aligned} p_V &= Pr\{L_i \text{ intersects} V\} \\ &\leq \int_0^{\infty} \frac{\alpha}{2\pi} Pr\{|X_i - X_{g(i)}| \geq x\} c_{24}(x+2\rho)dx \\ &\leq \int_0^{\infty} \min\left\{\pi, \frac{c_{22}}{x}\sqrt{\frac{\log n}{n}}\right\} \frac{c_{23}}{2\pi} \cdot f^{-z^2}(x+2\rho)dx \\ &= \int_0^{\frac{c_{22}}{\pi}\sqrt{\frac{\log n}{n}}} \pi \cdot \frac{c_{23}}{2\pi} \cdot f^{-z^2}(x+2\rho)dx \\ &\quad + \int_{\frac{c_{22}}{\pi}\sqrt{\frac{\log n}{n}}}^{\infty} \frac{c_{22}}{x}\sqrt{\frac{\log n}{n}} \cdot \frac{c_{23}}{2\pi} \cdot f^{-z^2}(x+2\rho)dx \\ &\leq c_{14}\sqrt{\frac{\log n}{fn}} \end{aligned} \tag{96}$$

Hence we have proved the lemma. □

Appendix 4

Proof of Lemma 6 Given $\mathcal{N} = N$, the number of RF λ_2-traffic segments $\mathcal{M}$ follows the binomial distribution $B(N, p_V)$.

If $N = \omega(1)$, i.e. N is large, then it follows from the law of large numbers that given any $\epsilon > 0$,

$$\lim_{n\to\infty} Pr\left\{\left|\frac{\mathcal{M}}{N} - p_V\right| < \epsilon\right\} = 1 \tag{97}$$

Let $\epsilon = p_V$, then according to Lemma 4, there is a $\delta_4(n) \to 0$ such that

$$Pr\left\{\mathcal{M} \leq 2c_{14}N\sqrt{\frac{\log n}{fn}}\right\} \geq 1 - \delta_4(n) \tag{98}$$

If $N = O(1)$, then for large n,

$$E\{\mathcal{M}\} \leq c_{14}N\sqrt{\frac{\log n}{fn}} << 1 \tag{99}$$

Thus for large n,

$$\begin{aligned} Pr\left\{\mathcal{M} \leq 2c_{14}N\sqrt{\frac{\log n}{fn}}\right\} &\geq Pr\{\mathcal{M} \leq E\{\mathcal{M}\}\} \\ &= Pr\{\mathcal{M} = 0\} \\ &\geq \left(1 - c_{14}\sqrt{\frac{\log n}{fn}}\right)^N \\ &\to 1 \end{aligned} \tag{100}$$

Note that Eqs. 98 and 23 indicate that for any $N \in \mathbb{N}$, there is a $\delta_5(n) \to 0$ such that

$$Pr\left\{\mathcal{M} \leq 2c_{14}\mathcal{N}\sqrt{\frac{\log n}{fn}}\middle|\mathcal{N} = N\right\} \geq 1 - \delta_5(n) \tag{101}$$

According to Lemma 3, we have

$$\begin{aligned} &Pr\left\{\mathcal{M} \leq (2+c_{13})(n-f) \cdot 2c_{14}\sqrt{\frac{\log n}{fn}}\right\} \\ &\geq Pr\left\{\mathcal{M} \leq 2c_{14}\mathcal{N}\sqrt{\frac{\log n}{fn}}, \mathcal{N} \leq (2+c_{13})(n-f)\right\} \\ &= Pr\{\mathcal{N} \leq (2+c_{13})(n-f)\} \\ &\cdot Pr\left\{\mathcal{M} \leq 2c_{14}\mathcal{N}\sqrt{\frac{\log n}{fn}}\middle|\mathcal{N} \leq (2+c_{13})(n-f)\right\} \\ &\geq (1-\delta_2(n))(1-\delta_5(n)) \end{aligned} \tag{102}$$

Hence we have proved the lemma. □

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