Network-Layer Scheduling and Relaying in Cooperative Spectrum Sharing Networks

Dibakar Das, Electrical, Computer and Systems Engineering
Rensselaer Polytechnic Institute
Troy, NY 12180
Email: dasd2@rpi.edu
Alhussein A. Abouzeid, Electrical, Computer and Systems Engineering
Rensselaer Polytechnic Institute
Troy, NY 12180
Email: abouzeid@ecse.rpi.edu
Marian Codreanu, University of Oulu
P.O. Box 4500 FIN-90014, Finland
Email: codreanu@ee.oulu.fi

Abstract

This work considers network-layer cooperation in spectrum sharing networks whereby the secondary users can relay primary users' packets, in return for more favorable spectrum access rules. Under this cooperative scheme, the paper investigates whether, and under what conditions, the primary and secondary networks can be stabilized without explicit knowledge of the packet arrival rates. We consider a primary packet generation process wherein constant amounts of bit arrive in every time-slot from upper-layers of the primary transmitter. These bits are arranged into a primary packet once sufficient bits have been accumulated. For this primary packet-generation model we develop a relaying and scheduling algorithm using Lyapunov drift techniques that does not require knowledge of packet arrival rates. A *guaranteed stability region* consisting of packet generation rates is also constructed, for which the network can be stabilized under this algorithm. The set of secondary packet arrival rate vectors for which the network can be stabilized do not decrease under cooperation when the primary packet generation rate is lower than what can be maximally supported without cooperation. For higher primary packet generation rates the algorithm stabilizes the network for a non-empty set of secondary packet arrival rate vectors.

I. Introduction

The increase in the number of wireless devices has resulted in increasing demand for wireless spectrum. However, at any given time, many bands in the licensed spectrum are often under-utilized. This observation has led to the widespread study of dynamic spectrum sharing radio networks. In one realization of spectrum sharing regulations, unlicensed or secondary users can opportunistically access the spectrum when it is not being used by primary or licensed users. Typically primary and secondary networks are thought to be non-cooperative, i.e., the users in the respective networks do not assist in each others transmissions. However if secondary nodes somehow assist the transmission of primary users, it may reduce the fraction of time the channel is used by primary users. This in turn may increase transmission opportunities for secondary users.

Even in conditions where cooperation is beneficial for primary users and some secondary users, a question remains on whether this would also be beneficial to *all* secondary users. Intuitively, it can be seen that in a general network some secondary users may benefit from cooperation while others may not. For example, some secondary users located close to a secondary relay node but far away from the primary transmitter may obtain fewer transmission opportunities when there is cooperation. This occurs due to increased transmission activity by the secondary relays. An important network problem is therefore how to find a balanced cooperative scheduling policy under which the primary network can improve its performance while maintaining secondary network stability.

We study cooperation in a network consisting of a single primary source-destination (s-d) pair and multiple secondary s-d pairs. Secondary users can help primary users without harming primary user traffic (otherwise cooperation would be simply disabled). Scheduling and relaying decisions are made at a centralized network controller whose goal is to keep the lengths of all queues in the network bounded. The controller has no knowledge about packet generation rates; however it can observe the instantaneous state of the queues at every node.

Our main contribution in this work is to develop a cooperative relay and scheduling algorithm with stability performance guarantees. The primary packet generation rate supported by this algorithm can be *greater* than what is supported by the primary network alone. The algorithm is constructed using Lyapunov drift techniques. The policy requires knowledge of only *instantaneous queue-lengths* at secondary nodes and *inter-arrival times* of primary packets[1]. In order to analyze the stability performance of the algorithm we construct a region, referred to as guaranteed stability region, consisting of primary and secondary arrival rates. We show that our scheduling algorithm stabilizes the network for all arrival rates in the interior of this region. For tractable analysis the definition of guaranteed stability region contains additional constraints beyond typical stability constraints. As a result this region is not the capacity region of the network. However, it includes the capacity region for the network in absence of cooperation. Hence there is no reduction in stability performance of secondary users when primary packet generation rate is lower than what can be supported without cooperation.

Related Works: Cooperation between primary and secondary networks have been widely studied from a physical layer perspective. Some of these works (e.g. [1]–[3]) study it as an information-theoretic problem. Other works such as [4]–[6] involve the primary network leasing the spectrum to secondary nodes in return for cooperation and the objective therein is to maximize utility functions corresponding to link-rates. However, interactions between primary and secondary users also affect higher-layer operations such as queuing and prioritized scheduling. The above studies do not address this issue. Prior works that did study the problem from a network-layer perspective include [7]–[12]. In [8] the authors consider two links: a primary and a secondary where a secondary relay node re-transmits packets that were not successfully received by the intended primary destination node but correctly received at the relay node. A similar model is used in [9] where the authors consider a single primary s-d pair in the presence of multiple secondary nodes that act as relay for primary network. In [10] the authors obtain stable throughput region for the primary and secondary users in a 5 node network with 2 primary transmitters and one common secondary relay. In [11] the authors consider uplink of a TDMA-based primary network in the presence of two sets of cognitive nodes: pure relay nodes that assist the primary network by re-transmitting some primary packets that were not received

[1]In this work we refer to the time difference between the arrival time of a given packet at the primary source node and that of the previous primary packet to be the inter-arrival time of the former packet.

successfully at the base-station and another set of cognitive non-relaying nodes that form an ad-hoc network and communicate using slotted Aloha protocol. In all these works the primary transmitter is assumed to be oblivious to the existence of secondary users which is consistent with *commons* model of cognitive radio [4]. In [7] the authors find optimal cooperative power allocations in a network of multiple secondary users and a single primary user. The authors assume independent and identically distributed (iid) packet arrivals and develop a cooperative scheduling scheme using the concept of renewal frames. However, their analysis is valid only for the range of primary packet arrival rates for which the primary network is stable even without any cooperation from secondary nodes. It is because they used renewal frame based analysis which requires bounded frame-lengths. This property cannot be guaranteed for higher primary packet generation rates. In [12] the authors extend [7] to include cases of higher primary packet arrival rates.

However none of these works addressed the case where cooperation can be beneficial to some secondary nodes while being *harmful to others*. Besides, the above works have also not considered more general network models where multiple secondary users can transmit simultaneously. In our work we address these issues. We observe that the work in [12] is closest to our work. However they too have not considered case of simultaneous transmissions from secondary users and that cooperation can be beneficial to some secondary nodes and harmful to others. In addition there is the following difference between their work and ours. In [12] the authors maximize a function of throughputs of secondary users by solving a convex optimization problem with the knowledge of primary packet arrival rate. On the other hand, we find a scheduling algorithm by solving a max-weight problem with knowledge of *only instantaneous queue-lengths and inter-arrival time* of the Head-of-Line packet (HOL) at primary transmitter. Thus our work is in accordance with the wide body of works on max-weight based scheduling policies for communication networks that require knowledge of only instantaneous queue-states.

We assume the following:

1) primary network is *aware of the existence* of the secondary network and can request cooperation from the latter to improve latency of its own packets, and
2) packets can be transmitted across multiple time-slots.

Both assumptions are similar to the *spectrum leasing* model of cognitive radio which has been used in [4]–[6]. In those works it is assumed that a slot used for direct transmission of data from a primary source to a primary destination can be further divided into smaller intervals. In each such interval one of the following events take place: transmission from a primary source to a relay node or transmission from relay node to primary destination or transmission of secondary network's own data. We assume a similar model here. It is to be noted that [4]–[6] study the cooperative relaying problem from a physical-layer perspective and do not investigate the network-layer aspects.

The remainder of the paper is organized as follows. Section II describes the system model. In Section III we present our scheduling policy. In Section IV we construct a guaranteed stability region and claim that the network is stable for all secondary packet arrival rate vectors in the interior of this region. The proof is shown in Section V. In Section VI we extend our analysis to a case where the amount of primary data arrival at any slot is not constant. In Section VII we present simulation results. Section VIII concludes the paper.

II. System model

We consider a single primary s-d pair in the presence of multiple secondary s-d pairs. One or more secondary node(s) that can act as relay for primary traffic. There is one primary transmitter (PT) and S secondary transmitters: ST_1, ST_2, ..., ST_S. We denote the primary receiver and the secondary receiver corresponding to ST_i (where $i = 1, 2, \ldots, S$) respectively as PR and SR_i respectively. We assume ideal sensing process, i.e., the sensing results are always accurate and take place in an infinitesimal time-duration. Details of our system model is presented next. Important notations are summarized in Table I.

A. Primary packet transmission model

PT transmits at fixed power to PR. PT can also transmit a packet with lower power to some secondary transmitters within its transmission range that can act as intermediate relays. Each secondary transmitter uses fixed power to transmit any packet. A transmission link is defined by the ordered node-pair (n_1, n_2), where $n_1 \in \{\text{PT}, \text{ST}_1, \ldots, \text{ST}_S\}$, $n_2 \in \{\text{PR}, \text{ST}_1, \ldots, \text{ST}_S, \text{SR}_1, \ldots, \text{SR}_S\}$ and n_2 is located within n_1's transmission range. Links with one of the nodes as PT or PR can be used to transmit primary packets and are referred to as *primary links*. The capacity of any link (in bits/s) is a rational number. Due to this assumption the length of a slot can be defined such that (s.t.) the time taken to transmit a primary packet through any primary

TABLE I
LIST OF NOTATIONS

S	number of secondary transmitters	R	number of relay nodes
PT (PR)	primary transmitter (receiver)	ST_j (SR_j)	j'th secondary transmitter (receiver)
L	set of all links	L_p	links used to transmit primary packets
λ_p	primary packet generation rate (no fluctuation case)	D_p	primary packet size in bits
$d_p(t)$	amount of data-arrival at PT (in bits) at slot t	λ_i	packet generation rate of ST_i
Φ	set of feasible activation vectors	$\Lambda_0(\lambda_p)$	capacity region without cooperation
$U_p(t)$	queue-length of PT at slot t	$U_i(t)$	queue-length at ST_i at slot t
Γ	set of feasible average transmission rate vectors	$\pi_{(n_1,n_2)}$	probability of link (n_1, n_2) being used
π_0	probability of no primary packet transmission	$t_{r,j}$	j'th slot in r'th frame
(n_1, n_2)	link with n_1 as transmitter and n_2 as receiver		
$K_{(n_1,n_2)}$	number of slots required to transmit primary packet via (n_1, n_2)		
$\mathbf{\Pi}(\mathbf{E})$	offered secondary transmission rate vector corresponding to activation vector $\mathbf{E}$		
$I(n_1, n_2)$	set of transmission rate vectors feasible if (n_1, n_2) is used		
I_0	set of transmission rate vectors feasible if no primary packet being transmitted		
f_i	effective primary packet transmission capacity via ST_i in packets/slot		
f_0	effective packet transmission capacity of link (PT,PR) in packets/slot		

link is a multiple of the length of a slot. We denote as $K_{(n_1,n_2)}$ the number of slots required to transmit a primary packet through link (n_1, n_2).

A secondary transmitter ST_j can act as relay for primary network if:

1) ST_j and PR are within transmission range of PT and ST_j respectively and
2) overall latency for a primary packet transmitted via ST_j is at least as good as one obtained from using direct link i.e.,

$$K_{(\text{PT},\text{ST}_j)} + K_{(\text{ST}_j,\text{PR})} \le K_{(\text{PT},\text{PR})}. \tag{1}$$

Let R (where $1 \le R \le S$) denote the number of such relay nodes. Without loss of generality (w.l.o.g.) we denote ST_1,..., ST_R to be those nodes. Let L denote the set of all links that can be used, i.e., $L = \{(\text{PT}, \text{PR}), (\text{PT}, \text{ST}_i),\ (\text{ST}_i, \text{PR}),\ (\text{ST}_j, \text{SR}_j) : 1 \le i \le R, 1 \le j \le S\}$. We denote the set of links that are used exclusively for primary packet transmission by L_p, i.e., L_p=$\{(\text{PT}, \text{PR}),\ (\text{PT}, \text{ST}_i),\ (\text{ST}_i, \text{PR}) : 1 \le i \le R\}$.

PT transmits packets whenever its buffer is non-empty because primary packets have higher

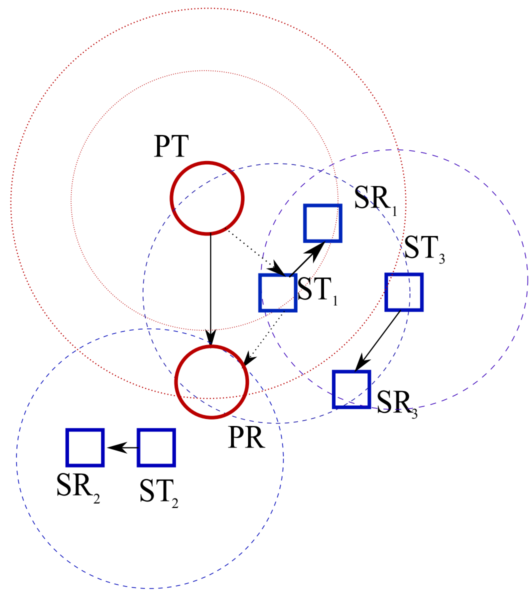


Fig. 1. A network with one primary s-d pair and three secondary s-d pairs. Each of the three blue-dashed circles correspond to transmission range of one of the ST$_i$s (where $i = 1, 2, 3$). No node within the circle drawn with ST$_i$ as center can simultaneously receive a packet from any node except ST$_i$ when ST$_i$ is transmitting. The large and small red, dotted circles drawn with PT as center correspond to PT's transmission range when it is directly transmitting and relaying respectively. No node located within the larger circle, except PR, can receive a packet when PT is directly transmitting a packet. No node located within the smaller circle, except ST$_1$, can receive a packet when PT is relaying a packet. The dotted lines from PT to ST$_1$ and from ST$_1$ to PR represent cooperative transmission with ST$_1$ as a relay node.

transmission priority in the channel. If PT begins transmitting a packet to PR directly at slot t, then clearly for slots $[t,\, t + K_{(\text{PT,PR})} - 1]$ it is busy transmitting the packet. Instead if at slot t the packet is scheduled to be relayed via ST$_j$ (where $1 \leq j \leq R$), then PT transmits the packet to ST$_j$ during slots $[t,\, t+K_{(\text{PT,ST}_j)} - 1]$. During slots $[t+K_{(\text{PT,ST}_j)},\, t+K_{(\text{PT,ST}_j)} + K_{(\text{ST}_j\text{,PR})} - 1]$, node ST$_j$ relays the packet to PR. Due to (1) cooperative relaying always reduces latency of primary packets as compared to direct transmission.

Fig. 1 shows example of a network with $R = 1$, S=3.

B. Primary packet arrival model

The packet generation process at PT results from aggregation of constant stream of data that arrives from higher layer to the transmission layer of PT. We assume the system begins at slot 0 with no data being initially present at transmission layer of PT. At slot t, $d_p(t)$ bits arrive from the upper layers of PT to its transmission layer. We assume $d_p(t)$ is constant and equal to b_p for every t. Whenever the accumulated data is greater than D_p bits, those D_p bits are aggregated as a primary packet and moved to the transmission queue of PT. We denote the number of primary packet arrivals in slot t as $A_p(t) \in \{0, 1\}$. The primary packet generation rate, denoted as λ_p, is $\lambda_p = \frac{b_p}{D_p}$. Since b_p, D_p are integers, λ_p is a rational number, i.e., $\lambda_p \in \mathbb{Q}$. Further, since there is no data present at transmission layer initially, the primary packet generation process is periodic. The period is the denominator of λ_p after it has been expressed as the ratio of two co-prime integers.

For example, consider the case when $\lambda_p = \frac{5}{13}$. The first primary packet is generated at slot 2 when the accumulated data is $\frac{15D_p}{13}$ bits. Then the sequence of $A_p(t)$ starting at $t = 0$ is: 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, $\ldots$. The process is periodic with the period consisting of 13 slots. The inter-arrival time of the first and second packet is 3 slots and that of the third one is 2 slots.

C. Secondary packet arrival and transmission model

Every slot a secondary packet arrives at the transmission layer of ST$_i$ with probability λ_i (where $i = 1, 2, \ldots, S$). The packet generation process is iid across slots. We denote as $A_i(t)$ (where $A_i(t) \in \{0, 1\}$) the number of packet arrivals at ST$_i$ at slot t. The vector of secondary arrival-rates is denoted as $\boldsymbol{\lambda}_s$. For simplicity we assume all secondary transmitter-receiver links have the same capacity of 1 secondary packet per slot.

D. Interference model

The interference model is based on the protocol model of interference whereby a node can transmit to another node within its transmission range. Such a transmission is allowed only if the receiving node is not within range of another node that is transmitting in the same slot.

We represent a set of transmissions that can occur simutaneously by an *activation vector*. Each component of the vector corresponds to a unique link in L and its length is equal to the cardinality of L i.e., $S + 2R + 1$. Since every component in an activation-vector $\boldsymbol{E}$ corresponds to a unique link, with slight abuse of notation, we denote as $E_{(n_1,n_2)}$ the component of $\boldsymbol{E}$ corresponding to link (n_1, n_2). An activation vector is binary; for every $(n_1, n_2) \in L$ the component $E_{(n_1,n_2)}$ in activation vector $\boldsymbol{E}$ is set to 1 if n_1 is transmitting to n_2, otherwise it is set to 0. An activation vector $\boldsymbol{E}$ that is feasible under protocol model of interference is constructed as follows: any component $E_{(n_1,n_2)}$ is set to 1 only if $E_{(n_3,n_4)} = 0$ for every $(n_3, n_4) \in L$ s.t. n_2 is within transmission range of n_3. The set consisting of all feasible activation vectors is denoted by Φ.

E. Scheduling and control model

Whenever PT is about to transmit a new packet, the network controller makes a decision about whether to transmit the packet directly to PR or relay it to PR by a cooperating secondary node. At every slot t the controller also offers transmission rate-vector $\boldsymbol{\mu}_s(t)$ to secondary nodes for their own transmissions where $\boldsymbol{\mu}_s(t) = (\mu_1(t), \ldots, \mu_S(t))^T$ and $\mu_i(t) \in \{0, 1\}$ denotes the transmission rate offered (in secondary packets/slot) at slot t for transmission from ST$_i$ to SR$_i$ (where $i \in 1, 2, \ldots, S$).

We introduce some new notations which will be used extensively in rest of the paper. We denote as $I(n_1, n_2)$ the set of transmission rate vectors that can be offered to secondary users for their own transmissions in any slot if n_1 is transmitting a primary packet to n_2 (where$(n_1, n_2) \in L_p$). This set can be written as, $I(n_1, n_2)$=$\{\Pi(\boldsymbol{E}) : \boldsymbol{E} \in \Phi,\ \boldsymbol{E}_{(n_1,n_2)} = 1\}$ where $\Pi(\boldsymbol{E})$ denotes the offered secondary transmission rate vector obtained from a feasible activation vector $\boldsymbol{E} \in \Phi$ [2]. For the particular case when no node in the network is transmitting a primary packet in some slot, the set of transmission rate vectors that can be offered to secondary users in that slot is denoted by I_0. This set can be written as, $I_0 = \{\Pi(\boldsymbol{E}) : \boldsymbol{E} \in \Phi, \boldsymbol{E}_{(n_1,n_2)} = 0 \quad \forall (n_1, n_2) \in L_p\}$.

F. Queuing model

Let $U_p(t)$ and $U_i(t)$ denote the queue-lengths of PT and ST$_i$ (where $i = 1, 2, \ldots, S$) at slot t respectively. Queue-length at PT evolves as

$$U_p(t + 1) = U_p(t) - C(t) + A_p(t), \tag{2}$$

where $C(t)$ is an indicator variable which is 1 if a primary packet transmission is completed at t and is 0 otherwise.

[2]The offered secondary transmission rate vector in any slot can be obtained from the binary activation vector used in that slot by eliminating the components corresponding to links used to transmit primary packets. For example, for the network in Fig. 1 the offered secondary transmission rate vector corresponding to activation vector $\boldsymbol{E}$ can be obtained by eliminating components: $E_{(\text{PT},\text{PR})}$, $E_{(\text{PT},\text{ST}_1)}$ and $E_{(\text{ST}_1,\text{PR})}$.

The queue for secondary packets evolve as:

$$U_i(t+1) = \max[U_i(t) - \mu_i(t), 0] + A_i(t). \tag{3}$$

Let $\boldsymbol{U}_s(t)$ denote the queue-length vector $(U_1(t), \ldots, U_S(t))^T$ at slot t.

III. DYNAMIC RELAYING AND SCHEDULING POLICY

In this section we present a dynamic Scheduling and Cooperative Relay Policy (SCRP). The policy is based on Lyapunov drift techniques introduced by Tassiulas and Ephremides in [13], which has been widely used to develop throughput-optimal algorithms in computer networks. At any slot, if there is no primary packet in the system, then secondary packets are scheduled according to backpressure policy. Otherwise, if the HOL packet at PT is not being served currently, then it is scheduled (either via the direct link (PT,PR) or a relay) by solving optimization problem P1. For slots in which a primary packet is being transmitted, secondary packets are scheduled according to backpressure policy (problems P2, P3 and P4) where links used for or interfered by primary packet transmission are excluded.

The intuition behind problem P1 is following: for a HOL primary packet with inter-arrival time of k slots P1 minimizes the k-slot conditional drift[3] of the Lyapunov function $\sum_{n=1}^{S} U_n^2(t)$. This minimization is performed under the assumption that no new primary packet arrives in next k slots and the primary packet is transmitted by slot $t+k$. Note, only relay nodes (and the direct transmission path) with overall transmission time not greater than k slots are considered as feasible options for transmission of this primary packet. This restriction, referred to as *deadline constraint* for primary packets, is required to analyze stability performance of the algorithm (to be discussed in detail in the next section). Depending on whether k is less than $K_{(\text{PT,PR})}$ or not, P1 has two expressions given as (8) and (6) respectively.

Before we outline the SCRP algorithm we introduce some additional notations. Let f_i (where $1 \leq i \leq R$) denote the maximum primary packet arrival rate that can be supported if every primary packet is transmitted via relay ST$_i$. Clearly, this is just reciprocal of the overall latency of a primary packet transmitted via ST$_i$, i.e., $f_i = \frac{1}{K_{(\text{PT,ST}_i)}+K_{(\text{ST}_i,\text{PR})}}$. W.l.o.g. we assume the ST$_1$,…,ST$_R$ are indexed in ascending order of their capacity to relay primary packets, i.e.,

[3]The k-slot conditional drift of a Lyapunov function of instantaneous queue-lengths, $V(\mathbf{U_s}(t))$ is $\mathbb{E}[V(t+k) - V(t)|\mathbf{U_s}(t)]$ [14].

$f_j \leq f_{j+1}$ for every $j \in \{1, \ldots, R-1\}$. Let f_0 denote the maximum primary packet arrival rate that can be supported if every primary packet is directly transmitted. Clearly, $f_0 = \frac{1}{K_{(\text{PT,PR})}}$. Due to (1), this rate is lower than transmission rate offered by other relays, i.e., $f_0 \leq f_1$.

A slot is considered *idle* if there is no primary packet at PT. A *busy period* consists of slots when the transmission of some primary packet takes place. A busy period always consists of contiguous slots because a relay node begins transmitting any primary packet in the next slot after receiving the packet. A slot when the network is in a busy period is called a *busy slot*.

SCRP algorithm makes the following scheduling and relay decisions:

1) *Cooperative Relaying Decision for Primary Packets:* If the transmission queue of PT is non-empty and the HOL packet in its queue is not served at slot t, then its service begins at t in the following manner:

(i) We find the transmission rate vector $\boldsymbol{v}^*_{(n_1,n_2)}(t)$ for each possible link $(n_1, n_2) \in L_p$ as:

$$\boldsymbol{v}^*_{(n_1,n_2)}(t) \in \underset{\boldsymbol{v} \in I(n_1,n_2)}{\text{argmax}}\, \boldsymbol{U}_s{}^T(t)\boldsymbol{v}. \tag{4}$$

We also find the transmission rate vector $\boldsymbol{v}^*_0(t)$ as solution to the following maximization problem over all transmission rate vectors in the set I_0:

$$\boldsymbol{v}^*_0(t) \in \underset{\boldsymbol{v} \in I_0}{\text{argmax}}\, \boldsymbol{U}_s{}^T(t)\boldsymbol{v}. \tag{5}$$

(ii) If the inter-arrival time of the primary packet is greater than or equal to $\frac{1}{f_0}$ slots, then we solve the following maximization problem:

$$\begin{aligned} \text{P1:}\ \underset{1 \leq n \leq R}{\text{argmax}}\, \boldsymbol{U}_s{}^T(t)\{&K_{(\text{PT,ST}_n)}\boldsymbol{v}^*_{(\text{PT,ST}_n)}(t) + K_{(\text{ST}_n\text{,PR})}\boldsymbol{v}^*_{(\text{ST}_n\text{,PR})}(t) \\ &+ (K_{(\text{PT,PR})} - K_{(\text{PT,ST}_n)} - K_{(\text{ST}_n\text{,PR})})\boldsymbol{v}^*_0(t)\} \end{aligned} \tag{6}$$

s.t.

$$\begin{aligned} \boldsymbol{U}_s{}^T(t)K_{(\text{PT,PR})}\boldsymbol{v}^*_{(\text{PT,PR})}(t) \ \leq\ \boldsymbol{U}_s{}^T(t)\{&K_{(\text{PT,ST}_n)}\boldsymbol{v}^*_{(\text{PT,ST}_n)}(t) + K_{(\text{ST}_n\text{,PR})}\boldsymbol{v}^*_{(\text{ST}_n\text{,PR})}(t) \\ &+ (K_{(\text{PT,PR})} - K_{(\text{PT,ST}_n)} - K_{(\text{ST}_n\text{,PR})})\boldsymbol{v}^*_0(t)\} \end{aligned} \tag{7}$$

Notice that, the solution of P1 maximizes, among all relays via which the packet can be transmitted, the dot-product of $\boldsymbol{U}_s{}^T(t)$ and the sum of offered secondary transmission-rate vectors for slots $[t, t + K_{(\text{PT,PR})} - 1]$ if only one primary packet is transmitted during those slots. If there is no feasible solution it means the dot-product is maximized by transmitting the primary packet via direct link (PT,PR). The term $K_{(\text{PT,PR})}\boldsymbol{v}^*_{(\text{PT,PR})}(t)$ is the sum of offered secondary transmission-rate vectors during those slots if the primary packet is directly transmitted to PR and $\boldsymbol{v}^*_{(\text{PT,PR})}(t)$ is the offered secondary transmission-rate vector in each of those slots. Similarly $\{K_{(\text{PT,ST}_\text{n})}\boldsymbol{v}^*_{(\text{PT,ST}_\text{n})}(t)+K_{(\text{ST}_\text{n},\text{PR})}\boldsymbol{v}^*_{(\text{ST}_\text{n},\text{PR})}(t)+(K_{(\text{PT,PR})}-K_{(\text{PT,ST}_\text{n})}-K_{(\text{ST}_\text{n},\text{PR})})\boldsymbol{v}^*_0(t)\}$ is the sum of offered secondary transmission rate-vectors during those slots if the primary packet is transmitted via ST$_n$ (where $1 \le n \le R$), and $\boldsymbol{v}^*_{(\text{PT,ST}_n)}(t)$, $\boldsymbol{v}^*_{(\text{ST}_n,\text{PR})}(t)$ and $\boldsymbol{v}^*_0(t)$ are the offered secondary transmission-rate vectors for slots $[t,\, t+K_{(\text{PT,ST}_n)}-1]$, $[t+K_{(\text{PT,ST}_n)},\, t+K_{(\text{PT,ST}_n)}+K_{(\text{ST}_n,\text{PR})}-1]$ and $[t+K_{(\text{PT,ST}_n)}+K_{(\text{ST}_n,\text{PR})},\, t+K_{(\text{PT,PR})}-1]$ respectively.

(iii) Otherwise, if the inter-arrival time of the primary packet is less than $\frac{1}{f_0}$ slots, then we solve the following maximization problem:

$$\text{P1: } \underset{k\le n\le R}{\operatorname{argmax}} \boldsymbol{U}_s{}^T(t)\{K_{(\text{PT,ST}_n)}\boldsymbol{v}^*_{(\text{PT,ST}_n)}(t)+K_{(\text{ST}_n,\text{PR})}\boldsymbol{v}^*_{(\text{ST}_n,\text{PR})}(t) \\ +(K_{(\text{PT,ST}_k)}+K_{(\text{ST}_k,\text{PR})}-K_{(\text{PT,ST}_n)}-K_{(\text{ST}_n,\text{PR})})\boldsymbol{v}^*_0(t)\} \quad (8)$$

where ST$_k$ ($1 \le k \le R$) is s.t. inter-arrival time of the packet is less than $\frac{1}{f_{k-1}}$ slots but greater than or equal to $\frac{1}{f_k}$ slots. Similar to (6) the solution of (8) maximizes the dot-product of $\boldsymbol{U}_s{}^T(t)$ and the sum of offered secondary transmission-rate vectors for slots $[t, t+K_{(\text{PT,ST}_k)}+K_{(\text{ST}_k,\text{PR})}-1]$ if only one primary packet is transmitted during those slots and the deadline constraint is satisfied.

(iv) Suppose, i^* is the solution of problem P1 (in case of multiple solutions pick an i^* arbitrarily). Then use ST$_{i^*}$ as relay and transmit the HOL primary packet from PT to ST$_{i^*}$ in slots $[t,\, t+K_{(\text{PT,ST}_{i^*})}-1]$ and from ST$_{i^*}$ to PR in slots $[t+K_{(\text{PT,ST}_{i^*})},\, t+K_{(\text{PT,ST}_{i^*})}+K_{(\text{ST}_{i^*},\text{PR})}-1]$. If there is no solution for (6) then directly transmit the primary packet to PR in slots $[t,\, t+K_{(\text{PT,PR})}-1]$.

2) *Secondary Scheduling Decision in Busy Slots:* Suppose the decision about transmitting the primary packet in the previous step was to relay the same via ST$_{i^*}$. Then the secondary transmission rate vector $\boldsymbol{\mu}^*_s(\tau)$ to be used in slots $\tau \in [t,\, t+K_{(\text{PT,ST}_{i^*})}+K_{(\text{ST}_{i^*},\text{PR})}-1]$ are obtained as follows:

(i) For slots $\tau \in [t,\, t+K_{(\text{PT,ST}_{i^*})}-1]$ i.e. for transmission from PT to ST$_{i^*}$, $\boldsymbol{\mu}^*_s(\tau)$ is obtained as:

$$\text{P2: } \boldsymbol{\mu}^*_s(\tau) \in \underset{\boldsymbol{v}\in I(\text{PT,ST}_{i^*})}{\operatorname{argmax}} \boldsymbol{U}_s{}^T(\tau)\boldsymbol{v}. \quad (9)$$

(ii) For slots $\tau \in [t + K_{(\text{PT},\text{ST}_{i^*})},\, t + K_{(\text{PT},\text{ST}_{i^*})} + K_{(\text{ST}_{i^*},\text{PR})} - 1]$ i.e. for transmission from ST_{i^*} to PR, $\boldsymbol{\mu}_s^*(\tau)$ is obtained as:

$$\text{P3: } \boldsymbol{\mu}_s^*(\tau) \in \underset{\boldsymbol{v} \in I(\text{ST}_{i^*},\text{PR})}{\operatorname{argmax}}\ \boldsymbol{U}_s{}^T(\tau)\boldsymbol{v}. \tag{10}$$

If the decision about transmission of primary packet was to directly transmit the same then the secondary transmission rate vector $\boldsymbol{\mu}_s^*(\tau)$ to be used in slots $\tau \in [t,\, t + K_{(\text{PT,PR})} - 1]$ are obtained as:

$$\text{P4: } \boldsymbol{\mu}_s^*(\tau) \in \underset{\boldsymbol{v} \in I(\text{PT,PR})}{\operatorname{argmax}}\ \boldsymbol{U}_s{}^T(\tau)\boldsymbol{v}. \tag{11}$$

3) *Scheduling Decision in Idle Slots:* At any idle slot τ, the network selects a secondary transmission-rate vector $\boldsymbol{\mu}_s^*(\tau)$ according to a max-weight scheduling policy:

$$\text{P5: } \boldsymbol{\mu}_s^*(\tau) \in \underset{\boldsymbol{v} \in I_0}{\operatorname{argmax}} \boldsymbol{U}_s{}^T(\tau)\boldsymbol{v}. \tag{12}$$

4) *Transmission and Queue-update:* Transmit $\min(\mu_i^*(t), U_i(t))$ secondary packets from ST_i in slot t for every $i = 1, \ldots,$S. If t is the last slot in any busy period, remove the HOL primary packet from PT's transmission queue at the end of t.

It should be noted that the SCRP algorithm makes decision based only on instantaneous queue-states and requires no knowledge of the primary packet generation rate.

Implementing the SCRP algorithm might be difficult in practice as it involves a centralized controller collecting queue-length information from all secondary nodes and then selecting a transmission rate vector by searching from a combinatorial set of transmission rate vectors. In literature, greedy maximal scheduling algorithms have been suggested as approximation to such max-weight based algorithms [15]. In addition to low complexity, such algorithms are also suited for distributed implementation [16]. These algorithms typically work by selecting the link with maximum weight at any time, followed by the link with next highest weight among the set of remaining links and so on. We note that a greedy algorithm can also be used for our network model. However it is beyond the scope of this paper to analyze such greedy approximations of the SCRP policy.

IV. Stability Analysis

In this section we present a guaranteed stability region for the network under SCRP algorithm. The region is constructed by imposing deadline constraints for transmitted primary packets (due to nature of the SCRP algorithm) in addition to regular flow-conservation constraints at the primary and secondary nodes. This formulation is similar to the capacity region description in [17]. We first discuss properties of the primary packet generation process, the motivation behind having deadline constraints and effective transmission rate for secondary users and use those concepts to define the guaranteed stability region.

A. Properties of Primary Packet Generation Process

Since λ_p is rational it can be expressed as ratio of two co-prime integers, denoted as $M(\lambda_p)$ and $N(\lambda_p)$ respectively, i.e., $\lambda_p = \frac{M(\lambda_p)}{N(\lambda_p)}$. $N(\lambda_p)$ and $M(\lambda_p)$ signify the length of period of $A_p(.)$ and the number of primary packet arrivals in that period respectively. Note, there exists a unique positive integer k_1 s.t. $\frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1}$. Then,

Property 1: the inter-arrival time of any primary packet is either $k_1 + 1$ slots or k_1 slots.

For a given λ_p we denote as $\kappa^{(1)}(\lambda_p)$ and $\kappa^{(2)}(\lambda_p)$ the number of primary packet arrivals, during any interval of length $N(\lambda_p)$ slots, with inter-arrival times of $k_1 + 1$ and k_1 slots respectively. Then,

Property 2: total number of such packet arrivals within any period is $M(\lambda_p)$, i.e.,

$$\kappa^{(1)}(\lambda_p) + \kappa^{(2)}(\lambda_p) = M(\lambda_p), \text{ and} \tag{13}$$

Property 3: sum of inter-arrival times of all primary packet arrivals within any period is equal to the length of the period, i.e.,

$$(k_1 + 1)\kappa^{(1)}(\lambda_p) + k_1\kappa^{(2)}(\lambda_p) = N(\lambda_p). \tag{14}$$

For example, Fig. 2 shows primary packet arrival process when $\lambda_p = \frac{3}{8}$. We observe, the inter-arrival time of any primary packet is either 3 or 2 slots which is consistent with $\frac{3}{8}$ being less than $\frac{1}{2}$ but greater than $\frac{1}{3}$. Total number of packet arrivals in any 8 slots is 3. In any 8 slots, there are 2 and 1 primary packet arrivals with inter-arrival time of 3 and 2 slots respectively and the sum of inter-arrival times is therefore 8.

In the rest of the section we assume $\lambda_p \in \mathbb{Q}$ and refer to $M(\lambda_p)$ and $N(\lambda_p)$ simply as M and N whenever there is no confusion.

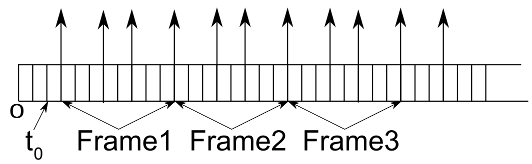


Fig. 2. Primary packet arrival process when $\lambda_p = \frac{3}{8}$. The time-line is partitioned into frames starting at slot 3 with $t_0 = 2$. Each small rectangle represents a slot. The arrival of a primary packet at the transmission queue of PT during any slot is indicated by a vertical arrow at the boundary between the slot and the one immediately after it.

B. Intuition Behind Deadline Constraints

Deadline constraints allow us to apply Lyapunov-drift techniques and analyze stability performance of SCRP. The explanation is provided below.

Lyapunov-drift techniques are helpful in constructing efficient backpressure-type scheduling policies without knowledge of packet arrival rates. However, it is difficult to develop such policies in networks where there is cooperation between primary and secondary users. This is because such policies assign higher priority of transmission to queues with high backlogs. On the other hand, in cognitive networks primary users always have highest priority of transmission regardless of queue-length. Moreover in our model the evolution of primary queue depends on the actions of secondary users. We solve this problem by using renewal frame-based techniques. Due to periodicity of the primary packet arrival process we first define the *system-state* to be the queue-length of PT at the arrival-time of every $(nM+1)$'th primary packet (where $n = 0, 1, 2, \ldots$). The system-state is *refreshed* every time its value becomes 0. A renewal frame is defined as the time-slots between successive system-state refresh events. Existence of renewal frames whose frame-length is constant facilitates construction of a Lyapunov drift based algorithm [14]. The use of primary queue-lengths as system state and then applying renewal frame based techniques is motivated by [7][4]. We observe, a renewal frame whose length is N slots can be obtained if M primary packets are transmitted every N slots. A sufficient condition to ensure this is to require that overall transmission time of every primary packet is less than its inter-arrival time.

Assume system starts at slot 0 and denote the slot in which the first primary packet arrives

[4]In [7] the authors employ a renewal frame based analysis to find a near-optimal cooperative power allocation policy for secondary nodes by defining the system state to be the queue-length at primary transmitter.

as t_0. The time-line can then be partitioned into a finite interval $[0, t_0]$ and successive non-overlapping frames of length N slots each as: $[t_0+1, t_0+N]$, $[t_0+N+1, t_0+2N]$,.... Fig. 2 shows partition of the time-line into frames for the case where $\lambda_p = \frac{3}{8}$.

C. Effective Transmission Rate for Secondary Users

Since for every slot t, $\boldsymbol{\mu}_s(t)$ depends on which primary link is being used in that slot, the effective transmission rate offered to secondary users is defined in a time average sense. Let $\pi_{(n_1,n_2)}$ denote, under some policy, the time-averaged probability of the event[5]: "n_1 is transmitting to n_2" (where $(n_1, n_2) \in L_p$). Let π_0 denote the time-averaged probability of the event: no node is transmitting a primary packet. Note, all such events are mutually exclusive. For the event: "n_1 is transmitting to n_2", the time-averaged transmission rate vector offered to secondary nodes belongs to the convex hull[6] of $I(n_1, n_2)$ since any of the transmission rate vectors in $I(n_1, n_2)$ can be used during this event. Similarly the time-averaged transmission rate vector offered to secondary nodes when no primary packet is being transmitted belongs to the convex hull of I_0. Averaging over all such events, we observe that any effective transmission rate vector offered to secondary users belongs to the set $\Gamma(\boldsymbol{\pi})$ defined as,

$$\begin{aligned}\Gamma(\boldsymbol{\pi}) &= \pi_{(\text{PT},\text{PR})}\mathbf{conv}I(\text{PT},\text{PR}) + \sum_{j=1}^{R}\left\{\pi_{(\text{PT},\text{ST}_j)}\mathbf{conv}I(\text{PT},\text{ST}_j)\right. \\ &+ \left.\pi_{(\text{ST}_j,\text{PR})}\mathbf{conv}I(\text{ST}_j,\text{PR})\right\} + \pi_0\mathbf{conv}I_0. \qquad (15)\end{aligned}$$

where $\boldsymbol{\pi}$ denotes the vector $(\pi_{(\text{PT},\text{PR})}, \pi_{(\text{PT},\text{ST}_1)}, \ldots, \pi_{(\text{PT},\text{ST}_R)}, \pi_{(\text{ST}_1,\text{PR})}, \ldots, \pi_{(\text{ST}_R,\text{PR})}, \pi_0)^T$. The "+" operator in (15) indicates Minkowski addition of sets[7].

D. Guaranteed Stability Region

The guaranteed stability region is defined as the set $\{(\lambda_p, \boldsymbol{\lambda}_s) : \lambda_p \leq f_R, \boldsymbol{\lambda}_s \in \text{Interior}(\Lambda(\lambda_p))\}$ where $\Lambda(\lambda_p)$ is defined as follows. Suppose k_1 is the unique positive integer s.t. λ_p is less than

[5]For the time being assume that such an average exists. In next section we show the existence of a stationary policy which achieves such averages.

[6]The convex hull of a set W is the set of all convex combinations of elements $w \in W$.

[7]The Minkowski addition of two sets of vectors W_1 and W_2 is the set formed by adding every vector in W_1 to every vector in W_2 i.e. the set $\{w_1 + w_2 | w_1 \in W_1, w_2 \in W_2\}$ [18].

$\frac{1}{k_1}$ but is greater than or equal to $\frac{1}{k_1+1}$. Then $\Lambda(\lambda_p)$ is the set of secondary packet arrival rate vectors $\boldsymbol{\lambda}_s$ for which there exists variables $\pi_0, \pi^{(i)}_{(n_1,n_2)}$ (where $(n_1, n_2) \in L_p$ and $i = 1, 2$) s.t.:

$$\frac{\kappa^{(i)}(\lambda_p)}{N} = \frac{\pi^{(i)}_{(\text{PT,PR})}}{K_{(\text{PT,PR})}} + \sum_{1\leq j\leq R} \frac{\pi^{(i)}_{(\text{PT,ST}_j)}}{K_{(\text{PT,ST}_j)}} \quad \forall i = 1, 2 \tag{16}$$

$$\pi_0, \pi^{(i)}_{(n_1,n_2)} \geq 0 \quad \forall (n_1, n_2) \in L_p, \quad i = 1, 2 \tag{17}$$

$$\frac{\pi^{(i)}_{(\text{PT,ST}_j)}}{K_{(\text{PT,ST}_j)}} = \frac{\pi^{(i)}_{(\text{ST}_j,\text{PR})}}{K_{(\text{ST}_j,\text{PR})}} \quad \forall i = 1, 2,\ 1 \leq j \leq R \tag{18}$$

$$\pi^{(1)}_{(\text{PT,PR})} = 0, \quad \text{if } K_{(\text{PT,PR})} > k_1 + 1 \tag{19}$$

$$\pi^{(2)}_{(\text{PT,PR})} = 0, \quad \text{if } K_{(\text{PT,PR})} > k_1 \tag{20}$$

$$\pi^{(1)}_{(\text{PT,ST}_j)} = 0 \quad \forall 1 \leq j \leq R,\ \text{if } K_{(\text{PT,ST}_j)} + K_{(\text{ST}_j,\text{PR})} > k_1 + 1 \tag{21}$$

$$\pi^{(2)}_{(\text{PT,ST}_j)} = 0 \quad \forall 1 \leq j \leq R,\ \text{if } K_{(\text{PT,ST}_j)} + K_{(\text{ST}_j,\text{PR})} > k_1 \tag{22}$$

$$\pi_{(n_1,n_2)} = \pi^{(1)}_{(n_1,n_2)} + \pi^{(2)}_{(n_1,n_2)} \quad \forall (n_1, n_2) \in L_p \tag{23}$$

$$\pi_0 = 1 - \sum_{(n_1,n_2)\in L_p} \pi_{(n_1,n_2)} \tag{24}$$

$$\lambda_i \leq X_i \quad \forall i = 1, 2, \ldots, S \tag{25}$$

for some $\boldsymbol{X}_s = (X_1, \ldots, X_S)^T \in \Gamma(\boldsymbol{\pi})$. Terms $\pi^{(1)}_{(n_1,n_2)}$ and $\pi^{(2)}_{(n_1,n_2)}$ represents the long-term average probability of the events "node n_1 is transmitting a packet with inter-arrival time of $k_1 + 1$ slots to node n_2" and "node n_1 is transmitting a packet with inter-arrival time of k_1 slots to node n_2" respectively. The equality constraint (16) is a conservation constraint which represents that arrival rate of primary packets of either type is equal to their departure rate from PT. Terms $\frac{\pi^{(1)}_{(\text{PT,ST}_j)}}{K_{(\text{PT,ST}_j)}}$ and $\frac{\pi^{(2)}_{(\text{PT,ST}_j)}}{K_{(\text{PT,ST}_j)}}$ in (16) respectively represent the average number of primary packets with inter-arrival times of $k_1 + 1$ and k_1 slots transmitted per slot via ST_j. Similarly terms $\frac{\pi^{(1)}_{(\text{PT,PR})}}{K_{(\text{PT,PR})}}$ and $\frac{\pi^{(2)}_{(\text{PT,PR})}}{K_{(\text{PT,PR})}}$ in (16) respectively represent the average number of primary packets with inter-arrival times of $k_1 + 1$ and k_1 slots transmitted per slot directly to PR. Constraint (18) represents that the average number of primary packets of either type that enter any relay node is equal to that transmitted by the relay node to PR. Deadline constraints are introduced in (19)-(22). They require that primary packets with inter-arrival times of $k_1 + 1$ and k_1 slots are not transmitted directly or via a relay if such a transmission takes more than $k_1 + 1$ and k_1 slots respectively. The relations (17), (23) and (24) represent that probability of all events are non-negative and their sum is 1. The inequality constraint (25) represents stability condition for secondary transmitters.

We show that under SCRP the network is strongly stable for packet arrival rate vectors in the guaranteed stability region.

Theorem 1: For all $\lambda_p < f_R$, under the SCRP policy, $U_p(t)$ and $U_i(t)$ (where $i = 1, 2, \ldots, S$) are strongly stable for all secondary packet arrival rate vectors in the interior of $\Lambda(\lambda_p)$.

When λ_p is 0 the algorithm reduces to traditional back-pressure theorem with capacity region $\Lambda(0)$ whose proof can be found in [13]. For the case when $\lambda_p \neq 0$ the theorem is proven in next section.

Before we prove Theorem 1 we discuss some observations that follow immediately from Theorem 1.

1) Due to presence of deadline constraints (19)-(22) the guaranteed stability region is not the capacity region of the network under cooperation.
2) Corresponding to any given λ_p not greater than what can be supported without cooperation f_0, the set of $\boldsymbol{\lambda}_s$ for which the secondary network is stable, denoted as $\Lambda_0(\lambda_p)$, does not decrease with cooperation. This is because $\Lambda_0(\lambda_p)$ can be obtained by setting the variable $\pi^{(i)}_{(\text{PT},\text{ST}_j)}$ to 0 in (16)-(25) (i.e. the relations used to obtain the guaranteed stability region) for every $j \in \{1, \ldots, \text{R}\}$ and $i = 1, 2$. Ignoring the set of secondary packet arrival rate vectors forming the boundary of $\Lambda(\lambda_p)$ we observe that cooperation does not adversely affect stability performance of secondary nodes when $\lambda_p \leq f_0$.
3) The set of $\boldsymbol{\lambda}_s$ for which the secondary network can be stabilized, for a given λ_p not greater than f_0, is expanded with cooperation. For higher λ_p the set of $\boldsymbol{\lambda}_s$ for which the secondary network can be stabilized with cooperation may not include all $\boldsymbol{\lambda}_s$ for which the network can be stabilized without cooperation. However, in this case cooperation may result in win-win scenarios for both PT and some secondary transmitter. We illustrate these observations by the following example. Consider the network in Fig. 1 with $K_{(\text{PT},\text{ST}_1)} = K_{(\text{ST}_1,\text{PR})} = 1$ and $K_{(\text{PT},\text{PR})} = 3$. The interference model is described in Table II.

   The maximum λ_p that can be supported are 0.5 and 0.33 with and without cooperation respectively. In Fig. 3 we plot the set of $\boldsymbol{\lambda}_s$, in terms of λ_2 and λ_3, that belongs to $\Lambda(\lambda_p)$ when λ_p are 0.4 and 0.167 respectively and λ_1 is 0. We also plot, in terms of λ_2 and λ_3, all $\boldsymbol{\lambda}_s$ for which the secondary network can be stabilized without cooperation

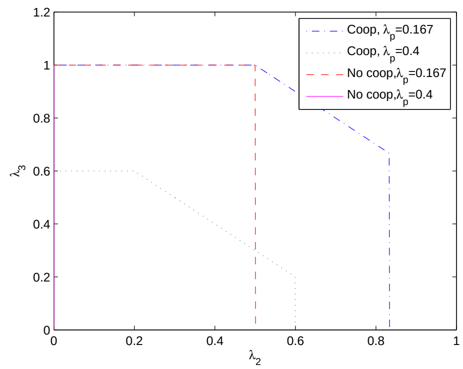


Fig. 3. Stability region without cooperation and guaranteed stability region under SCRP when λ_1 is 0 and λ_p is 0.4 and 0.167 respectively.

TABLE II

TABLE LISTING SECONDARY TRANSMITTERS THAT CAN SIMULTANEOUSLY TRANSMIT THEIR OWN PACKETS IN SOME TIME-SLOT AGAINST THE CORRESPONDING PRIMARY PACKET TRANSMISSION EVENT IN THAT TIME-SLOT, FOR THE NETWORK IN FIG. 1.

Primary transmission activity	Simultaneously transmitting STs
PT transmits to PR (direct)	ST_3
PT transmits to ST_1 (relay)	ST_2
ST_1 transmits to PR (relay)	None
None	ST_2 and either ST_1 or ST_3

when λ_p are 0.4 and 0.167 respectively and λ_1 is 0. When λ_p is 0.4 this region is just the vertical line from the point (λ_2, λ_3)=(0,0) to (0,1). The values of 0.4 and 0.167 are arbitrarily chosen as examples of λ_p for which the network is stabilizable only with and even without cooperation respectively.

All points, with λ_2 greater than 0.5, in the region below the line marked "Coop, $\lambda_p = 0.167$" correspond to an increase in set of secondary arrival-rate vectors due to cooperation when λ_1 is 0 and λ_p is 0.167.

All points, with positive λ_2, in the region below the line marked "Coop, $\lambda_p = 0.4$" correspond to secondary arrival-rate vectors that constitute win-win scenarios for PT and ST_2. This is because, without cooperation, maximum λ_p that can be supported is 0.33 and ST_2 can not transmit any packet if λ_p is greater than or equal to 0.33. Note, this region does not include the entire set of $\boldsymbol{\lambda}_s$ for which the secondary network can be stabilized without cooperation when λ_p is 0.4[8].

V. Proof of Theorem 1

In this section we prove Theorem 1. In the proof we compare SCRP against a Stationary Scheduling Policy (SSP) that stabilizes the network for all arrival rates in the guaranteed stability region without any knowledge of secondary queue-lengths.

A. Stationary Scheduling Policy

Consider the variables $X_1, \ldots, X_S$ and $\pi_0, \pi^{(i)}_{(n_1,n_2)}$ (where $i = 1, 2$ and $(n_1, n_2) \in L_p$) that satisfy (16)-(25) for any $\boldsymbol{\lambda}_s \in \Lambda(\lambda_p)$. Suppose we index vectors in the set $I(n_1, n_2)$ (where $(n_1, n_2) \in L_p$) and I_0 as $\boldsymbol{\nu}_{(n_1,n_2),1}, \ldots, \boldsymbol{\nu}_{(n_1,n_2),|I(n_1,n_2)|}$ and $\boldsymbol{\nu}_{0,1}, \ldots, \boldsymbol{\nu}_{0,|I_0|}$ respectively. Then from (15) it follows that there exists variables $p^{\text{SSP}}_{(n_1,n_2),m}$, $p^{\text{SSP}}_{0,l}$ (where $(n_1, n_2) \in L_p$, $1 \leq m \leq |I(n_1, n_2)|$ and $1 \leq l \leq |I_0|$) s.t.

$$\boldsymbol{X}_s = \pi_{(\text{PT,PR})} \sum_{m=1}^{|I(\text{PT,PR})|} p^{\text{SSP}}_{(\text{PT,PR}),m} \boldsymbol{\nu}_{(\text{PT,PR}),m} + \sum_{j=1}^{R} \{ (\pi_{(\text{PT,ST}_j)} \sum_{m=1}^{|(I(\text{PT,ST}_j)|} p^{\text{SSP}}_{(\text{PT,ST}_j),m} \boldsymbol{\nu}_{(\text{PT,ST}_j),m}) + (\pi_{(\text{ST}_j,\text{PR})} \sum_{m=1}^{|(I(\text{ST}_j,\text{PR})|} p^{\text{SSP}}_{(\text{ST}_j,\text{PR}),m} \boldsymbol{\nu}_{(\text{ST}_j,\text{PR}),m}) \} + \pi_0 \sum_{l=1}^{|I_0|} p^{\text{SSP}}_{0,l} \boldsymbol{\nu}_{0,l} \tag{26}$$

$$\sum_{m=1}^{|I(\text{PT,PR})|} p^{SSP}_{(\text{PT,PR}),m} = 1, \quad \sum_{l=1}^{|I_0|} p^{\text{SSP}}_{0,l} = 1 \tag{27}$$

$$\sum_{m=1}^{|I(\text{PT,ST}_j)|} p^{\text{SSP}}_{(\text{PT,ST}_j),m} = 1, \quad \sum_{m=1}^{|I(\text{ST}_j,\text{PR})|} p^{\text{SSP}}_{(\text{ST}_j,\text{PR}),m} = 1, \text{ where } 1 \leq j \leq R. \tag{28}$$

The policy SSP performs the following steps in each frame:

a) *Cooperative Relay Decision for Primary Packets:* Transmit all primary packets with inter-arrival time of $k_1 + 1$ and k_1 slots directly with probability $\pi^{(1)}_{(\text{PT,PR})}$ and $\pi^{(2)}_{(\text{PT,PR})}$ respectively, or transmit them via ST_j with probability $\pi^{(1)}_{(\text{PT,ST}_j)}$ and $\pi^{(2)}_{(\text{PT,ST}_j)}$ respectively.

[8] Simply disabling cooperation will allow the secondary network to be stabilized for all $\boldsymbol{\lambda}_s$ in the latter set.

b) *Secondary Scheduling Decision in Busy Slots:* At slots when a primary packet is being transmitted from n_1 to n_2 use $\boldsymbol{\nu}_{(n_1,n_2),m}$ as transmission rate vector $\boldsymbol{\mu}_s(t)$ with probability $p^{\text{SSP}}_{(n_1,n_2),m}$.

c) *Secondary Scheduling Decision in Idle Slots:* At idle slots use $\boldsymbol{\nu}_{0,l}$ as transmission rate vector $\boldsymbol{\mu}_s(t)$ with probability $p^{\text{SSP}}_{0,l}$.

d) *Queue Update:* Update the queues as in SCRP.

It can be shown that for any $\lambda_p \le f_R$, SSP stabilizes the network for all secondary arrival rate vectors in the region $\Lambda(\lambda_p)$.

Lemma 1: Under the SSP policy for any $\lambda_p \le f_R$ and $\boldsymbol{\lambda}_s \in \Lambda(\lambda_p)$ the queue-length at PT is bounded and for all $r = 1, 2, \ldots$, we have

$$\mathbb{E}[\sum_{\tau=t_0+1+(r-1)N}^{t_0+rN} \mu_i^{\text{SSP}}(\tau)] \ge \lambda_i N \quad \forall i = 1, 2, \ldots, S, \tag{29}$$

where $\mu_i^{\phi}(t)$ denotes the transmission rate offered to ST_i at slot t under policy ϕ.

Proof: Note, under SSP transmission time of every primary packet is no greater than its inter-arrival time. Therefore, M primary packets are transmitted in each frame and the queue-length at PT is bounded.

Since the selections of relays and transmission-rate vectors are done based on a solution of (16)-(25), from (25) it follows that $\mathbb{E}[\sum_{\tau=t_0+1+(r-1)N}^{t_0+rN} \mu_i^{\text{SSP}}(\tau)] = X_i N \ge \lambda_i N$ for every $i \in \{1, 2, \ldots, S\}$. ■

We next consider an Alternate Scheduling Policy (ASP) that we will use later in the proof of Theorem 1.

B. Alternate Scheduling Policy

ASP maximizes the following function in r'th frame (where $r = 1, 2, \ldots$) over the set of all scheduling policies ϕ in which transmission time of every primary packet is less than or equal to its inter-arrival time:

$$\psi^{\phi}(t_{r,1}) \triangleq \sum_{i=1}^{S} U_i(t_{r,1}) \mathbb{E}[\sum_{\tau=t_{r,1}}^{t_{r,1}+N-1} \mu_i^{\phi}(\tau) | \boldsymbol{U}_s(t_{r,1})], \tag{30}$$

where $t_{r,j}$ denotes the j'th slot (where j=1,..., N) in r'th frame.

Lemma 2: For any $\lambda_p \leq f_R$ and $r = 1, 2, \ldots$ we have

$$\psi^{\text{ASP}}(t_{r,1}) \geq \psi^{\text{SSP}}(t_{r,1}). \tag{31}$$

Proof: ASP maximizes $\psi^{\phi}(t_{r,1})$ among all policies in which transmission time of every primary packet is no greater than its inter-arrival time. Since SSP is one such policy therefore $\psi^{\text{ASP}}(t_{r,1}) \geq \psi^{\text{SSP}}(t_{r,1})$. ■

By comparing $\psi^{\text{ASP}}(.)$ with $\psi^{\text{SCRP}}(.)$ for each frame we observe,

Lemma 3: For any $\lambda_p \leq f_R$ and $r = 1, 2, \ldots$

$$\psi^{\text{SCRP}}(t_{r,1}) \geq \psi^{\text{ASP}}(t_{r,1}) - B, \tag{32}$$

where $B \geq 0$ is a finite constant.

Proof: Proof is shown in Appendix A. ■

Proof of Theorem 1: Consider $\boldsymbol{\lambda}_s \in$ Interior $(\Lambda(\lambda_p))$ where $\lambda_p < f_R$.

Note under SCRP and for any time-slot t, if $\lambda_p < f_R$, then M packets are transmitted in each frame and therefore, $U_p(t) \leq M$. Therefore $U_p(t)$ is strongly stable.

For $n = 1, 2, \ldots, S$ we consider the secondary queue-lengths at the beginning of r'th frame (where $r = 1, 2, \ldots$), $Z_n(r) \stackrel{\triangle}{=} U_n(t_{r,1})$.

We denote the vector $(Z_1(r), Z_2(r), \ldots, Z_S(r))^T$ as $\boldsymbol{Z}_s(r)$. We define a Lyapunov function $V(\boldsymbol{Z}_s(r)) \stackrel{\triangle}{=} \sum_{n=1}^{S} Z_n^2(r)$ The conditional drift $\triangle(r)$ is defined as

$$\triangle(r) \stackrel{\triangle}{=} \mathbb{E}[V(\boldsymbol{Z}_s(r+1)) - V(\boldsymbol{Z}_s(r)) | \boldsymbol{Z}_s(r)]. \tag{33}$$

Now for every $n \in \{1, 2, \ldots, S\}$,

$$U_n(t_{r,1} + N) \leq \max[U_n(t_{r,1}) - \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{SCRP}}(\tau), 0] + \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_n(\tau). \tag{34}$$

Since maximum arrival or transmission rate of secondary packets is less than or equal to 1 secondary packet per slot,

$$\triangle(r) \leq SN^2(1+1) - 2\sum_{n=1}^{S} Z_n(r)\mathbb{E}[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{SCRP}}(\tau) - A_n(\tau) | \boldsymbol{Z}_s(r)], \tag{35}$$

$$= 2SN^2 - 2\mathbb{E}[\sum_{n=1}^{S} Z_n(r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{SCRP}}(\tau) | \boldsymbol{Z}_s(r)] + 2\sum_{n=1}^{S} Z_n(r)\mathbb{E}[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_n(\tau)] \tag{36}$$

$$\leq 2SN^2 + 2B - 2\mathbb{E}[\sum_{n=1}^{S} Z_n(r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{ASP}}(\tau)|\boldsymbol{Z}_s(r)] + \sum_{n=1}^{S} Z_n(r) 2\mathbb{E}[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_n(\tau)] \tag{37}$$

$$\leq 2SN^2 + 2B - 2\mathbb{E}[\sum_{n=1}^{S} Z_n(r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{SSP}}(\tau)|\boldsymbol{Z}_s(r)] + \sum_{n=1}^{S} Z_n(r) 2\mathbb{E}[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_n(\tau)] \tag{38}$$

$$= 2SN^2 + 2B - \sum_{n=1}^{S} Z_n(r) 2\mathbb{E}[(\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_n^{\text{SSP}}(\tau) - A_n(\tau))] \tag{39}$$

$$\leq 2SN^2 + 2B - \sum_{n=1}^{S} 2N\epsilon Z_n(r) \tag{40}$$

where $\epsilon > 0$ is a constant s.t. $(\lambda_1+\epsilon, \lambda_2+\epsilon, \ldots, \lambda_S+\epsilon)^T \in (\Lambda(\lambda_p))$. Inequalities (37), (38) and (40) follow from Lemma 3, 2 and 1 respectively. Therefore by Theorem 4.1 of [14] the sampled queue-length processes $U_n(t_{r,1})$ are strongly stable. Since frame-length is finite, therefore the queue-length processes $U_n(t)$ are strongly stable. ∎

VI. PRIMARY DATA ARRIVAL PROCESS WITH FLUCTUATIONS

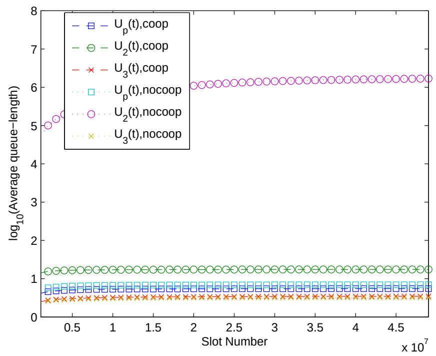
In this section we extend the stability analysis to a case where the amount of primary data that arrives in each slot is not constant but has bounded fluctuations around a fixed number. In particular, we consider a network with one relay. We consider the case where $K_{(\text{PT,PR})}$ is 3, $K_{(\text{PT},\text{ST}_1)} = K_{(\text{ST}_1,PR)}$ is 1 and the primary data arrival process $d_p(t)$ satisfies the following conditions:

a) At any slot t the upper and lower bound of $d_p(t)$ lies in the open interval $(\frac{1}{3}, \frac{1}{2})$.
b) Time-averaged value of $d_p(t)$ converges to a rational number $\lambda_p D_p$ i.e., for any t and strictly positive ϵ there exists a finite integer N_0 s.t. for all $n > N_0$ we have

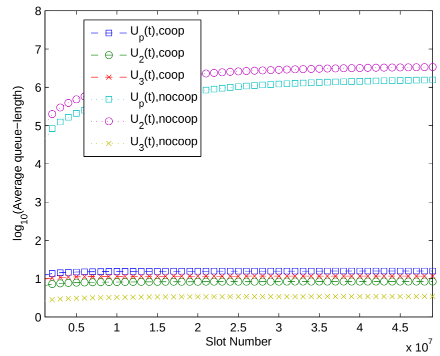
$$\left| \frac{\sum_{\tau=t}^{t+n} d_p(\tau)}{D_p n} - \lambda_p \right| < \epsilon \tag{41}$$

This property is satisfied almost surely, by strong law of large numbers, when $d_p(t)$ variables are iid with expected mean $D_p\lambda_p$.

We claim that for any λ_p between $\frac{1}{3}$ and $\frac{1}{2}$, the set of $\boldsymbol{\lambda}_s$ for which the network is guaranteed to be stable under SCRP, is same as that of a system with constant primary data arrival



(a) Primary packet generation rate=0.167



(b) Primary packet generation rate=0.4167

Fig. 4. Plot of time-averaged queue-lengths of PT, ST_2 and ST_3 versus number of time-slots under SCRP and a non-cooperative scheduling policy. For Fig. 4a λ_p is 0.167 and $\boldsymbol{\lambda}_s$ is $(0,0.6,0.2)^T$; for Fig. 4b λ_p is 0.4167 and $\boldsymbol{\lambda}_s$ is $(0,0.2,0.2)^T$.

rate of $D_p\lambda_p$ bits per slot. Recall, $\frac{1}{3}$ and $\frac{1}{2}$ are the maximum primary packet generation rate that can be supported without and with cooperation respectively.

Lemma 4: SCRP stabilizes the network for any $\boldsymbol{\lambda}_s \in$ Interior $(\Lambda(\lambda_p))$ where $\frac{1}{3} < \lambda_p < \frac{1}{2}$, $R = 1$, $K_{(\mathrm{PT,PR})} = 3$ and $K_{(\mathrm{PT,ST}_1)} = K_{(\mathrm{ST}_1\mathrm{,PR})} = 1$.

Proof: Proof is provided in Appendix B. ■

Lemma 4 can be extended to more general cases where $K_{(\mathrm{PT,PR})}$, $K_{(\mathrm{PT,ST}_1)}$ and $K_{(\mathrm{ST}_1\mathrm{,PR})}$ can take any integer value. However for simplicity of analysis we restrict ourselves to the particular case considered above.

VII. Simulations

In this section we validate the performance of the SCRP policy through simulations in C-programming language. We consider the network depicted in Fig.1. We assume, $K_{(\mathrm{PT,PR})}$ is 3 and $K_{(\mathrm{PT,ST}_1)} = K_{(\mathrm{ST}_1\mathrm{,PR})}$ is 1. Recall from our discussion in Section IV that the maximum λ_p that can be supported by the network with and without cooperation are respectively 0.5 and 0.33 respectively. For comparison we also consider a non-cooperative policy based on the work in [19] wherein a backpressure-type scheduling is performed at secondary transmitters when PT is not transmitting. Only ST_3 is allowed to transmit whenever PT is transmitting. The policy is throughput-optimal for secondary packet arrival rate vectors among the set of all non-cooperative policies.

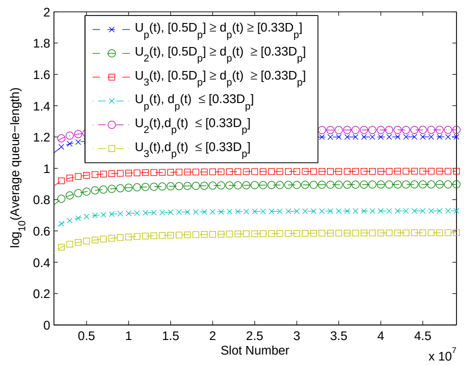


Fig. 5. Plot of time-averaged queue-lengths of PT, ST_2 and ST_3 versus number of time-slots under SCRP with random primary packet generation process. The parameters used are: $d_p(t)$ between 0 and $[\frac{D_p}{3}]$,$\boldsymbol{\lambda}_s$ =(0,0.6,0.2)^T; $d_p(t)$ between $[\frac{D_p}{3}]$ and $[\frac{D_p}{2}]$, $\boldsymbol{\lambda}_s$ =(0,0.2,0.2)^T.

All simulations are run for 50,000,000 time-slots. In every simulation we set λ_1 as 0 and plot the base-10 logarithm of time-averaged queue-lengths of PT, ST_2 and ST_3 for a single run. Plotting logarithm of queue-lengths allows us to clearly observe very high queue-lengths, that may result from non-cooperative policy, along with relatively small queue-lengths from the SCRP policy.

We consider two values of λ_p: 0.167 and 0.4167, corresponding to cases when the primary network can and cannot be stabilized without cooperation respectively. For each case we select a $\boldsymbol{\lambda}_s$ for which the secondary network is unstable without cooperation. Specifically we select $\boldsymbol{\lambda}_s$ equal to (0,0.6,0.2)^T and (0,0.2,0.2)^T when λ_p is 0.167 and 0.4167 respectively. In Fig. 4a and 4b we plot time-averaged queue-lengths under SCRP and non-cooperative policy when λ_p is 0.167, $\boldsymbol{\lambda}_s$ is (0,0.6,0.2)^T, and when λ_p is 0.4167, $\boldsymbol{\lambda}_s$ is (0,0.2,0.2)^T respectively. As expected the queue-length of ST_2 becomes unbounded with time under the non-cooperative policy in both cases; queue-length of PT becomes unbounded under the non-cooperative policy when λ_p is 0.4167. However, all queue-lengths are bounded under SCRP.

Next in Fig. 5 we observe the performance of SCRP when the primary data-arrival process d_p has fluctuations. We consider two cases of d_p. In both cases, $d_p(t)$ is an iid uniformly distributed random variable. In one case, $d_p(t)$ is any integer between 0 and $[\frac{D_p}{3}]$; in other case $d_p(t)$ is any integer between $[\frac{D_p}{3}]$ and $[\frac{D_p}{2}]$. For any real x, the function $[x]$ denotes the largest integer not greater than x. The value of D_p used is 8192 which corresponds to a packet-size 1KB. In Fig. 5 we plot time-averaged queue-lengths versus time-slot under SCRP when $d_p(t)$ lies between 0 and $[\frac{D_p}{3}]$ and $\boldsymbol{\lambda}_s$ is $(0,0.6,0.2)^T$, as well as when $d_p(t)$ lies between $[\frac{D_p}{3}]$ and $[\frac{D_p}{2}]$ and $\boldsymbol{\lambda}_s$ is $(0,0.2,0.2)^T$. We observe that queue-lengths are bounded under SCRP even with variable d_p.

VIII. Conclusion

In this work we studied opportunistic cooperation in a cognitive network where some nodes may benefit from cooperative relaying while others may suffer loss of transmission of opportunities. For a deterministic periodic primary packet arrival process we develop a scheduling algorithm using Lyapunov drift techniques that balances the trade off between cooperation and network stability. The periodic nature of the primary packet arrival process is exploited to render a renewal frame based analysis even for primary packet generation rates greater than what can be supported without relays. A guaranteed stability region for the algorithm is found and its key features are studied. Interesting research possibilities would be to extend this analysis to more general networks where the service-time of packets in different links are stochastic, cases involving multiple primary s-d pairs and also to search for efficient decentralized suboptimal algorithms.

Appendix A
Proof of Lemma 3

Proof: For any two integers h_1 and h_2 we define δ_{h_1,h_2} as $\delta_{h_1,h_2} \triangleq |h_1 - h_2|$. Since arrival rate and maximum transmission rate of secondary packets per slot are both no greater than 1, for any two slots t_1 and t_2 (where t_2 is greater than t_1 and $n = 1, 2, \ldots, S$),

$$|U_n(t_2) - U_n(t_1)| \leq \delta_{t_1,t_2} \tag{42}$$

Consider any λ_p s.t. $\frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} \leq f_R$ (where k_1 is a positive integer). We index the $\kappa^{(1)}(\lambda_p)$ and $\kappa^{(2)}(\lambda_p)$ primary packets that are generated in r'th frame (where $r = 1, 2, \ldots$) as $x_{r,1}, x_{r,2}, \ldots, x_{r,\kappa^{(1)}(\lambda_p)}$ and $\grave{x}_{r,1}, \grave{x}_{r,2}, \ldots, \grave{x}_{r,\kappa^{(2)}(\lambda_p)}$ respectively. For all i s.t. $1 \leq i \leq \kappa^{(1)}(\lambda_p)$ let $w^{\phi}_{r,i}$ denote the slot when primary packet $x_{r,i}$'s transmission begins under policy ϕ. If it is transmitted using a relay ST_{n_1} (where $1 \leq n_1 \leq R$) for which

$\frac{1}{f_{n_1}}$ is less than k_1+1 then it creates $(k_1+1)-\frac{1}{f_{n_1}}$ idle slots in the same frame denoted as $y^{n_1,\phi}_{r,i,1}, y^{n_1,\phi}_{r,i,2}, \ldots, y^{n_1,\phi}_{r,i,(k_1+1-\frac{1}{f_{n_1}})}$ respectively. Similarly for all $\grave{i}$ s.t. $1 \leq \grave{i} \leq \kappa^{(2)}(\lambda_p)$ let $\grave{w}^{\phi}_{r,\grave{i}}$ denote the slot when primary packet $\grave{x}_{r,\grave{i}}$'s transmission begins under policy ϕ. If it is transmitted using a relay ST_{n_2} (where $1 \leq n_2 \leq R$) for which $\frac{1}{f_{n_2}} < k_1$ then it creates $k_1 - \frac{1}{f_{n_2}}$ idle slots in r'th frame denoted as $\grave{y}^{n_2,\phi}_{r,\grave{i},1}, \grave{y}^{n_2,\phi}_{r,\grave{i},2}, \ldots, \grave{y}^{n_2,\phi}_{r,\grave{i},(k_1-\frac{1}{f_{n_2}})}$ respectively.
Let $U^{\text{SCRP}}_n(\tau)$ and $U^{\text{ASP}}_n(\tau)$ (where $\tau \in [t_{r,1}, t_{r,1}+(N-1)]$) denote the transmission queue-length of ST_n at time-slot τ under SCRP and ASP respectively. Similarly we denote the vector $\boldsymbol{v}^*_0(\tau)$, $\boldsymbol{v}^*_{(n_1,n_2)}(\tau)$ (where $(n_1,n_2) \in L_p$) obtained under policies SCRP and ASP as $\boldsymbol{v}^*_{0,\text{SCRP}}(\tau)$, $\boldsymbol{v}^*_{(n_1,n_2),\text{SCRP}}(\tau)$ and $\boldsymbol{v}^*_{0,\text{ASP}}(\tau)$, $\boldsymbol{v}^*_{(n_1,n_2),\text{ASP}}(\tau)$ respectively.
Next assume $x_{r,i}$ is relayed via ST_{θ_1} under ASP and by ST_{θ_2} under SCRP (where $1 \leq \theta_1, \theta_2 \leq R$ and $\frac{1}{f_{\theta_1}}, \frac{1}{f_{\theta_2}} \leq (k_1+1)$). For convenience, we abbreviate (PT, PR), $(\text{PT}, \text{ST}_{\theta_1})$, $(\text{ST}_{\theta_1}, \text{PR})$, $(\text{PT}, \text{ST}_{\theta_2})$, $(\text{ST}_{\theta_2}, \text{PR})$, SCRP, ASP as z_0, z_1, z_2, z_3, z_4, ϕ_1 and ϕ_2 respectively. Then we have

$$\mathbb{E}[\sum_{\tau=w^{\phi_1}_{r,i}}^{w^{\phi_1}_{r,i}+\frac{1}{f_{\theta_2}}-1} \sum_{n=1}^{S} U_n(t_{r,1})\mu^{\phi_1}_n(\tau) + \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} \sum_{n=1}^{S} U_n(t_{r,1})\mu^{\phi_1}_n(y^{\theta_2,\phi_1}_{r,i,\hat{n}})|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq \mathbb{E}[\sum_{\tau=w^{\phi_1}_{r,i}}^{w^{\phi_1}_{r,i}+K_{z_3}-1} \sum_{n=1}^{S}(U^{\phi_1}_n(\tau)-\delta_{\tau,t_{r,1}})\mu^{\phi_1}_n(\tau) + \sum_{\tau=w^{\phi_1}_{r,i}+K_{z_3}}^{w^{\phi_1}_{r,i}+\frac{1}{f_{\theta_2}}-1} \sum_{n=1}^{S}(U^{\phi_1}_n(\tau)-\delta_{\tau,t_{r,1}})\mu^{\phi_1}_n(\tau)$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} \sum_{n=1}^{S}(U^{\phi_1}_n(y^{\theta_2,\phi_1}_{r,i,\hat{n}}) - \delta_{y^{\theta_2,\phi_1}_{r,i,\hat{n}},t_{r,1}})\mu^{\phi_1}_n(y^{\theta_2,\phi_1}_{r,i,\hat{n}})|\boldsymbol{U}_s(t_{r,1})] \quad (43)$$

$$\geq \mathbb{E}[\sum_{\tau=w^{\phi_1}_{r,i}}^{w^{\phi_1}_{r,i}+K_{z_3}-1} \sum_{n=1}^{S} U^{\phi_1}_n(\tau)\mu^{\phi_1}_n(\tau) + \sum_{\tau=w^{\phi_1}_{r,i}+K_{z_3}}^{w^{\phi_1}_{r,i}+\frac{1}{f_{\theta_2}}-1} \sum_{n=1}^{S} U^{\phi_1}_n(\tau)\mu^{\phi_1}_n(\tau)$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} \sum_{n=1}^{S} U^{\phi_1}_n(y^{\theta_2,\phi_1}_{r,i,\hat{n}})\mu^{\phi_1}_n(y^{\theta_2,\phi_1}_{r,i,\hat{n}})|\boldsymbol{U}_s(t_{r,1})] - 3N^2S \quad (44)$$

$$\geq \mathbb{E}[\sum_{\tau=w^{\phi_1}_{r,i}}^{w^{\phi_1}_{r,i}+K_{z_3}-1} (\boldsymbol{U}^{\phi_1}_s(\tau))^T\boldsymbol{v}^*_{z_3,\phi_1}(w^{\phi_1}_{r,i}) + \sum_{\tau=w^{\phi_1}_{r,i}+K_{z_3}}^{w^{\phi_1}_{r,i}+\frac{1}{f_{\theta_2}}-1} (\boldsymbol{U}^{\phi_1}_s(\tau))^T\boldsymbol{v}^*_{z_4,\phi_1}(w^{\phi_1}_{r,i})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} (\boldsymbol{U}_s^{\phi_1}(y_{r,i,\hat{n}}^{\theta_2,\phi_1}))^T \boldsymbol{v}_{0,\phi_1}^*(w_{r,i}^{\phi_1}) | \boldsymbol{U}_s(t_{r,1})] - 3N^2S \quad (45)$$

$$\geq \mathbb{E}[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+K_{z_3}-1} (\boldsymbol{U}_s^{\phi_1}(w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{z_3,\phi_1}^*(w_{r,i}^{\phi_1}) + \sum_{\tau=w_{r,i}^{\phi_1}+K_{z_3}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{\theta_2}}-1} (\boldsymbol{U}_s{}^{\phi_1}(w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{z_4,\phi_1}^*(w_{r,i}^{\phi_1})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} (\boldsymbol{U}_s{}^{\phi_1}(w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{0,\phi_1}^*(w_{r,i}^{\phi_1}) | \boldsymbol{U}_s(t_{r,1})] - 6N^2S \quad (46)$$

$$\geq \mathbb{E}[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+K_{z_1}-1} (\boldsymbol{U}_s^{\phi_1}(w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{z_1,\phi_2}^*(t_{r,1}) + \sum_{\tau=w_{r,i}^{\phi_1}+K_{z_1}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{\theta_1}}-1} (\boldsymbol{U}_s^{\phi_1}((w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{z_1,\phi_2}^*(t_{r,1})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} (\boldsymbol{U}_s^{\phi_1}((w_{r,i}^{\phi_1}))^T \boldsymbol{v}_{0,\phi_2}^*(t_{r,1}) | \boldsymbol{U}_s(t_{r,1})] - 6N^2S \quad (47)$$

$$\geq \mathbb{E}[\sum_{\tau=w_{r,i}^{\phi_1}}^{w_{r,i}^{\phi_1}+K_{z_1}-1} (\boldsymbol{U}_s(t_{r,1}))^T \boldsymbol{v}_{z_1,\phi_2}^*(t_{r,1}) + \sum_{\tau=w_{r,i}^{\phi_1}+K_{z_1}}^{w_{r,i}^{\phi_1}+\frac{1}{f_{\theta_1}}-1} (\boldsymbol{U}_s(t_{r,1}))^T \boldsymbol{v}_{z_2,\phi_2}^*(t_{r,1})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} (\boldsymbol{U}_s(t_{r,1}))^T \boldsymbol{v}_{0,\phi_2}^*(t_{r,1}) | \boldsymbol{U}_s(t_{r,1})] - 9N^2S \quad (48)$$

$$= \mathbb{E}[\sum_{\tau=w_{r,i}^{\phi_2}}^{w_{r,i}^{\phi_2}+\frac{1}{f_{\theta_1}}-1} \sum_{n=1}^{S} U_n(t_{r,1}) \mu_n^{\phi_2}(\tau) + \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} \sum_{n=1}^{S} U_n(t_{r,1}) \mu_n^{\phi_2}(y_{r,i,\hat{n}}^{\theta_1,\phi_2}) | \boldsymbol{U}_s(t_{r,1})] - 9N^2S. (49)$$

Inequality (43) follows from (42). Inequality (44) follows because each of k_1+1, K_{z_3}, K_{z_4} and $\delta_{\tau,t_{r,1}}$ is less than N. Inequalities (45), (47) follows from the definition of SCRP: at any slot SCRP selects transmission-rate vectors based on instantenous queue-lengths. Inequalities (46) and (48) follows from (42) and because $k_1+1, K_{z_3}, K_{z_4}, K_{z_1}, K_{z_2}, \delta_{\tau,t_{r,1}}$ is less than N.

We denote idle slots created when $x_{r,i}$ is transmitted directly from PT to PR as $y_{r,i,1}^{0,\phi}$, $y_{r,i,2}^{0,\phi}, \ldots,$

$y^{0,\phi}_{r,i,(k_1+1-\frac{1}{f_0})}$ respectively. Repeating the above analysis for cases where one of the algorithms use direct transmission and while other uses a relay to transmit $x_{r,i}$ or both use direct transmission we obtain

$$\mathbb{E}[\sum_{\tau=w^{\phi_1}_{r,i}}^{w^{\phi_1}_{r,i}+\frac{1}{f_{i'}}-1}\sum_{n=1}^{S}U_n(t_{r,1})\mu^{\phi_1}_n(\tau)+\sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{i'}}}\sum_{n=1}^{S}U_n(t_{r,1})\mu^{\phi_1}_n(y^{i',\phi_1}_{r,i,\hat{n}})|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq \mathbb{E}[\sum_{\tau=w^{\phi_2}_{r,i}}^{w^{\phi_2}_{r,i}+\frac{1}{f_{i''}}-1}\sum_{n=1}^{S}U_n(t_{r,1})\mu^{\phi_2}_n(\tau)+\sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{i''}}}\sum_{n=1}^{S}U_n(t_{r,1})\mu^{\phi_2}_n(y^{i'',\phi_2}_{r,i,\hat{n}})|\boldsymbol{U}_s(t_{r,1})]-B_b, \quad (50)$$

where $B_b > 0$ is a finite constant and i', i'' are some non-negative integer less than R. Similarly repeating the above procedure for $\grave{x}_{r,\grave{i}}$s and assuming packet $\grave{x}_{r,\grave{i}}$ is relayed via ST$_{\grave{\theta}_1}$ under ASP and by ST$_{\grave{\theta}_2}$ under SCRP (where $1 \leq \grave{\theta}_1, \grave{\theta}_2 \leq R$ and $\frac{1}{f_{\grave{\theta}_1}}, \frac{1}{f_{\grave{\theta}_2}} \leq k_1$) we obtain

$$\mathbb{E}[\sum_{\tau=\grave{w}^{\phi_1}_{r,\grave{i}}}^{\grave{w}^{\phi_1}_{r,\grave{i}}+\frac{1}{f_{\grave{\theta}_2}}-1}\sum_{n=1}^{S}U_n(t_{r,1})\mu^{\phi_1}_n(\tau)+\sum_{\hat{n}=1}^{k_1-\frac{1}{f_{\grave{\theta}_2}}}\sum_{n=1}^{S}U_n(t_{r,1})\mu^{\phi_1}_n(\grave{y}^{\grave{\theta}_2,\phi_1}_{r,\grave{i},\hat{n}})|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq \mathbb{E}[\sum_{\tau=\grave{w}^{\phi_2}_{r,\grave{i}}}^{\grave{w}^{\phi_2}_{r,\grave{i}}+\frac{1}{f_{\grave{\theta}_2}}-1}\sum_{n=1}^{S}U_n(t_{r,1})\mu^{\phi_2}_n(\tau)+\sum_{\hat{n}=1}^{k_1-\frac{1}{f_{\grave{\theta}_1}}}\sum_{n=1}^{S}U_n(t_{r,1})\mu^{\phi_2}_n(\grave{y}^{\grave{\theta}_1,\phi_2}_{r,\grave{i},\hat{n}})|\boldsymbol{U}_s(t_{r,1})]-B_c, \quad (51)$$

where $B_c > 0$ is a finite constant. Using the results from the above cases and adding them up for all $x_{r,i}$ and $\grave{x}_{r,\grave{i}}$ we obtain

$$\psi^{\text{SCRP}}(t_{r,1}) \geq \psi^{\text{ASP}}(t_{r,1}) - B \quad (52)$$

where $B = M\max(B_b, B_c, 9N^2S) > 0$ is a finite constant. ■

APPENDIX B

PROOF OF LEMMA 4

The proof requires some results which are stated in Lemma 5, 6, 7 and 8 respectively. In Lemma 5 we show that for any $\boldsymbol{\lambda}_s$ that belongs to interior of $\Lambda(\lambda_p)$ there exists a rational number $\hat{\lambda}_p$ arbitrarily close to λ_p s.t. $\boldsymbol{\lambda}_s$ belongs to interior of $\Lambda(\hat{\lambda}_p)$ as well. We refer to the system with fluctuating primary data-arrival process $d_p(t)$ as System 1 and that with constant primary data arrival-rate of $\hat{\lambda}_p D_p$ bits per slot as System 2. We partition the time-line into a collection of frames where length of each frame is a multiple of the period of the primary packet arrival process in System 2. This multiple is selected, according to Lemma 6 and 7, so as to satisfy certain conditions on the number of primary packets generated in each frame under both systems. Those conditions are utilized in Lemma 8 where, analogous to Lemma 3, we compare the value of utility function ψ for System 1 using SCRP with that of System 2 using ASP respectively. Finally we use Lemma 8 to complete the proof.

Lemma 5: When $R = 1$, $K_{(\text{PT,PR})} = 3$ and $K_{(\text{PT,ST}_1)} = K_{(\text{ST}_1\text{,PR})} = 1$, if $\boldsymbol{\lambda}_s \in$ Interior $(\Lambda(\lambda_p))$ for some rational $\lambda_p \in (\frac{1}{3}, \frac{1}{2})$ then there exists rational $\hat{\lambda}_p > \lambda_p$ s.t. $\boldsymbol{\lambda}_s \in$ Interior $(\Lambda(\hat{\lambda}_p))$.

Proof: For this network L_p is $\{(\text{PT}, \text{PR}), (\text{PT}, \text{ST}_1), (\text{ST}_1, \text{PR})\}$. Since $\boldsymbol{\lambda}_s \in$ Interior(λ_p) there exists $\epsilon > 0$ and non-negative variables $\boldsymbol{X}_s$, $\pi_0, \pi_{(n_1,n_2)}$, $p_{(n_1,n_2),m}$, $p_{0,l}$ (where $(n_1, n_2) \in L_p$, $1 \leq m \leq |I(n_1, n_2)|$, $1 \leq l \leq |I_0|$) that satisfies (16)- (25) and (15). Those relations are re-written, ignoring the deadline constraints for convenience of analysis as well as due to their redundancy[9], as:

$$\lambda_p = \pi_{(\text{PT,PR})}\frac{1}{3} + \pi_{(\text{PT,ST}_1)} \tag{53}$$

$$\pi_{(\text{PT,PR})}, \pi_{(\text{PT,ST}_1)} \geq 0 \tag{54}$$

$$\pi_{(\text{PT,ST}_1)} = \pi_{(\text{ST}_1\text{,PR})} \tag{55}$$

$$\pi_0 + \pi_{(\text{PT,PR})} + \pi_{(\text{PT,ST}_1)} + \pi_{(\text{ST}_1\text{,PR})} = 1 \tag{56}$$

$$\lambda_n + \epsilon \leq X_n \quad \forall n = 1, 2, .., S. \tag{57}$$

[9]In order to see redundancy of deadline constraints first note that any $\pi_{(\text{PT,PR})}, \pi_{(\text{PT,ST}_1)}$ that satisfies (15)-(25) also satisfies (53)-(60). Conversely, whenever $\pi_{(\text{PT,PR})}$, $\pi_{(\text{PT,ST}_1)}$ satisfies (53)-(60), it also satisfies (15)-(25). In order to see this set $\pi^{(1)}_{(\text{PT,PR})} = \pi_{(\text{PT,PR})}$ and $\pi^{(2)}_{(\text{PT,PR})} = 0$. Note, (53)-(56) defines sufficient condition for stability of queue at PT. Since arrival-time of every primary packet at PT is either 3 or 2 slots, clearly for stability of this queue we must have $\pi_{(\text{PT,ST}_1)}$ atleast equal to the average number of packets with inter-arrival time of 2 slots generated in each slot i.e., $\pi_{(\text{PT,ST}_1)} > \frac{\kappa^{(2)}(\lambda_p)}{N}$ (formal proof is skipped). Set $\pi^{(2)}_{(\text{PT,ST}_1)} = \frac{\kappa^{(2)}(\lambda_p)}{N}$ and $\pi^{(1)}_{(\text{PT,ST}_1)} = \pi_{(\text{PT,ST}_1)} - \pi^{(2)}_{(\text{PT,ST}_1)}$. Therefore, given $\pi_{(\text{PT,PR})}$, $\pi_{(\text{PT,ST}_1)}$ that satisfies (53)-(56), there always exists variables $\pi^{(1)}_{(\text{PT,PR})}$, $\pi^{(1)}_{(\text{PT,ST}_1)}$, $\pi^{(2)}_{(\text{PT,PR})}$, $\pi^{(2)}_{(\text{PT,ST}_1)}$ that satisfies (16)-(24). This property along with the fact that $\Gamma()$ depends only on average number of primary packets transmitted directly or via relay, regardless of their inter-arrival time, implies any $\pi_{(\text{PT,ST}_1)}$ $\pi_{(\text{PT,PR})}$ that satisfies (53)-(60) also satisfies (15)-(25).

$$\boldsymbol{X}_s = \pi_{(\text{PT,PR})} \sum_{m=1}^{|I(\text{PT,PR})|} p_{(\text{PT,PR}),m} \boldsymbol{v}_{(\text{PT,PR}),m} + \pi_{(\text{PT,ST}_1)} \sum_{m=1}^{|(I(\text{PT,ST}_1)|} p_{(\text{PT,ST}_1),m} \boldsymbol{v}_{(\text{PT,ST}_1),m}$$

$$+\pi_{(\text{ST}_1,\text{PR})} \sum_{m=1}^{|(I(\text{ST}_1,\text{PR})|} p_{(\text{ST}_1,\text{PR}),m} \boldsymbol{v}_{(\text{ST}_1,\text{PR}),m} + \pi_0 \sum_{l=1}^{|I_0|} p_{0,l} \boldsymbol{v}_{0,l} \tag{58}$$

$$\sum_{m=1}^{|I(\text{PT,PR})|} p_{(\text{PT,PR}),m} = 1, \quad \sum_{l=1}^{|I_0|} p_{0,l} = 1 \tag{59}$$

$$\sum_{m=1}^{|I(\text{PT,ST}_1)|} p_{(\text{PT,ST}_1),m} = 1, \quad \sum_{m=1}^{|I(\text{ST}_1,\text{PR})|} p_{(\text{ST}_1,\text{PR}),m} = 1 \tag{60}$$

Assume $\pi_0 > 0$; later in this proof we will show there exists $\pi_0 > 0$, $\pi_{(\text{PT,PR})}$, $\pi_{(\text{PT,ST}_1)}$, $\pi_{(\text{ST}_1,\text{PR})}$, ϵ that is a solution for (53)-(60) if $\boldsymbol{\lambda}_s \in \text{Interior}(\lambda_p)$. Since ϵ is strictly positive and the set of real numbers is dense in the set of rational numbers, there exists arbitrary small strictly positive numbers ϵ', ϵ'' s.t.

a) $\frac{1}{3}(\pi_{(\text{PT,PR})} + \epsilon') + \pi_{(\text{PT,ST}_1)} + \epsilon''$ is a rational number in the interval $(\frac{1}{3}, \frac{1}{2})$.

b) The product of $(2\epsilon'' + \epsilon')$ and maximum component of $\sum_{l=1}^{|I_0|} p_{0,l} \boldsymbol{v}_{0,l}$ is less than ϵ i.e.,

$$(2\epsilon'' + \epsilon') \left\| \sum_{l=1}^{|I_0|} p_{0,l} \boldsymbol{v}_{0,l} \right\|_\infty < \epsilon \tag{61}$$

c) The term $(2\epsilon'' + \epsilon')$ is less than π_0 i.e.,

$$(2\epsilon'' + \epsilon') < \pi_0 \tag{62}$$

For any two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, both of length n, let $\boldsymbol{u} \leq \boldsymbol{v}$ denote the component-wise inequality i.e.,

$$\boldsymbol{u} \leq \boldsymbol{v} \iff u_i \leq v_i \quad \forall i = 1, \ldots, n \tag{63}$$

Then,

$$(\pi_{(\text{PT,PR})} + \epsilon') \sum_{m=1}^{|I(\text{PT,PR})|} p_{(\text{PT,PR}),m} \boldsymbol{v}_{(\text{PT,PR}),m} + (\pi_{(\text{PT,ST}_1)} + \epsilon'') \sum_{m=1}^{|(I(\text{PT,ST}_1)|} p_{(\text{PT,ST}_1),m} \boldsymbol{v}_{(\text{PT,ST}_1),m}$$

$$+(\pi_{(\text{ST}_1,\text{PR})} + \epsilon'') \sum_{m=1}^{|(I(\text{ST}_1,\text{PR})|} p_{(\text{ST}_1,\text{PR}),m} \boldsymbol{v}_{(\text{ST}_1,\text{PR}),m}) + (\pi_0 - 2\epsilon'' - \epsilon') \sum_{l=1}^{|I_0|} p_{0,l} \boldsymbol{v}_{0,l}$$

$$\geq \pi_{(\text{PT,PR})} \sum_{m=1}^{|I(\text{PT,PR})|} p_{(\text{PT,PR}),m} \boldsymbol{v}_{(\text{PT,PR}),m} + \pi_{(\text{PT,ST}_1)} \sum_{m=1}^{|(I(\text{PT,ST}_1)|} p_{(\text{PT,ST}_1),m} \boldsymbol{v}_{(\text{PT,ST}_1),m}$$

$$+\pi_{(\text{ST}_1,\text{PR})}\sum_{m=1}^{|(I(\text{ST}_1,\text{PR})|} p_{(\text{ST}_1,\text{PR}),m}\boldsymbol{v}_{(\text{ST}_1,\text{PR}),m}+(\pi_0-2\epsilon''-\epsilon')\sum_{l=1}^{|I_0|}p_{0,l}\boldsymbol{v}_{0,l} \tag{64}$$

$$=\boldsymbol{X}_s-(2\epsilon''+\epsilon')\sum_{l=1}^{|I_0|}p_{0,l}\boldsymbol{v}_{0,l} \tag{65}$$

$$\geq \boldsymbol{X}_s-(2\epsilon''+\epsilon')\|\sum_{l=1}^{|I_0|}p_{0,l}\boldsymbol{v}_{0,l}\|_\infty \tag{66}$$

$$>\boldsymbol{X}_s-\epsilon \tag{67}$$

$$\geq \boldsymbol{\lambda}_s \tag{68}$$

Inequalities (67) and (68) follow from (61) and (57) respectively.

Therefore, for $\hat{\lambda}_p \triangleq \lambda_p+\frac{\epsilon'}{3}+\epsilon''$ replacing $\pi_{(\text{PT},\text{PR})},\pi_{(\text{PT},\text{ST}_1)},\pi_{(\text{ST}_1,\text{PR})}$ in (53)- (60) with $\hat{\pi}_{(\text{PT},\text{PR})}\triangleq\pi_{(\text{PT},\text{PR})}+\epsilon',\hat{\pi}_{(\text{PT},\text{ST}_1)}\triangleq\pi_{(\text{PT},\text{ST}_1)}+\epsilon'',\hat{\pi}_{(\text{ST}_1,\text{PR})}\triangleq\pi_{(\text{ST}_1,\text{PR})}+\epsilon''$ respectively, we observe that $\boldsymbol{\lambda}_s\in$ Interior $(\Lambda(\hat{\lambda}_p))$.

In order to complete the proof we next show by construction that if $\boldsymbol{\lambda}_s\in$ Interior $(\Lambda(\lambda_p))$ then there exists $\epsilon>0$ and non-negative variables $\boldsymbol{X}_s$, $\pi_0,\pi_{(n_1,n_2)}$, $p_{(n_1,n_2),m}$, $p_{0,l}$ (where $(n_1,n_2)\in L_p$, $1\leq m\leq|I(n_1,n_2)|$, $1\leq l\leq|I_0|$) that satisfies (16)- (25) and (15) with π_0 being strictly greater than zero.

Since $\boldsymbol{\lambda}_s\in$ Interior $(\Lambda(\lambda_p))$ there exists $\tilde{\epsilon}>0$ and non-negative variables $\tilde{\pi}_0$, $\tilde{\boldsymbol{X}}_s$, $\tilde{\pi}_{(n_1,n_2)}$, $p_{(n_1,n_2),m}$,$p_{0,l}$ (where $(n_1,n_2)\in L_p$, $1\leq m\leq|I(n_1,n_2)|$, $1\leq l\leq|I_0|$) that satisfies (16)-(25) and (15). If $\tilde{\pi}_0>0$ we are done. Otherwise, we define $\tilde{\epsilon}'$ as $\tilde{\epsilon}'\triangleq\frac{1}{6}\min(\tilde{\pi}_{(\text{PT},\text{PR})},\tilde{\epsilon})$. Since $\lambda_p<\frac{1}{2}$ and $\tilde{\pi}_0=0$ we must have $\tilde{\pi}_{(\text{PT},\text{PR})}>0$. We define $\pi_{(\text{PT},\text{PR})}\triangleq\tilde{\pi}_{(\text{PT},\text{PR})}-3\tilde{\epsilon}',\pi_{(\text{PT},\text{ST}_1)}\triangleq\tilde{\pi}_{(\text{PT},\text{ST}_1)}+\tilde{\epsilon}',\pi_{(\text{ST}_1,\text{PR})}\triangleq\tilde{\pi}_{(\text{ST}_1,\text{PR})}+\tilde{\epsilon}'$, $\pi_0\triangleq\tilde{\epsilon}'$ respectively. Clearly, $\pi_{(\text{PT},\text{PR})}$, $\pi_{(\text{PT},\text{ST}_1)}$, $\pi_{(\text{ST}_1,\text{PR})}$ and π_0 satisfies (53)-(56). Then,

$$\pi_{(\text{PT},\text{PR})}\sum_{m=1}^{|I(\text{PT},\text{PR})|}p_{(\text{PT},\text{PR}),m}\boldsymbol{v}_{(\text{PT},\text{PR}),m}+\pi_{(\text{PT},\text{ST}_1)}\sum_{m=1}^{|(I(\text{PT},\text{ST}_1)|}p_{(\text{PT},\text{ST}_1),m}\boldsymbol{v}_{(\text{PT},\text{ST}_1),m}$$

$$+\pi_{(\text{ST}_1,\text{PR})}\sum_{m=1}^{|(I(\text{ST}_1,\text{PR})|}p_{(\text{ST}_1,\text{PR}),m}\boldsymbol{v}_{(\text{ST}_1,\text{PR}),m}+\pi_0\sum_{l=1}^{|I_0|}p_{0,l}\boldsymbol{v}_{0,l}$$

$$\geq\tilde{\pi}_{(\text{PT},\text{PR})}\sum_{m=1}^{|I(\text{PT},\text{PR})|}p_{(\text{PT},\text{PR}),m}\boldsymbol{v}_{(\text{PT},\text{PR}),m}+\tilde{\pi}_{(\text{PT},\text{ST}_1)}\sum_{m=1}^{|(I(\text{PT},\text{ST}_1)|}p_{(\text{PT},\text{ST}_1),m}\boldsymbol{v}_{(\text{PT},\text{ST}_1),m}$$

$$+\tilde{\pi}_{(\mathrm{ST}_1,\mathrm{PR})} \sum_{m=1}^{|(I(\mathrm{ST}_1,\mathrm{PR})|} p_{(\mathrm{ST}_1,\mathrm{PR}),m} \boldsymbol{\nu}_{(\mathrm{ST}_1,\mathrm{PR}),m} + \tilde{\pi}_0 \sum_{l=1}^{|I_0|} p_{0,l} \boldsymbol{\nu}_{0,l} - 3\tilde{\epsilon}' \sum_{m=1}^{|I(\mathrm{PT},\mathrm{PR})|} p_{(\mathrm{PT},\mathrm{PR}),m} \boldsymbol{\nu}_{(\mathrm{PT},\mathrm{PR}),m} \quad (69)$$

$$\geq \tilde{\boldsymbol{X}}_s - 3\tilde{\epsilon}' \quad (70)$$

$$\geq \boldsymbol{\lambda}_s \quad (71)$$

The last inequality follows as $3\tilde{\epsilon}' < \tilde{\epsilon}$. Therefore, selecting ϵ as $\epsilon = \tilde{\epsilon} - 3\tilde{\epsilon}'$ we complete the proof.

■

Lemma 6: Assume there is no data-arrival in System 1 and 2 prior to slot 0. Then for any finite positive number n_3 there exists finite positive constants Y_1, Y_2 s.t. number of primary packet arrivals with inter-arrival time of 2 slots during slots [0, $Y_2 Y_1 N(\hat{\lambda}_p)$] in System 2 is greater than that in System 1 by at least n_3.

Proof: We define ϵ_2 as $\epsilon_2 = \frac{\hat{\lambda}_p - \lambda_p}{2}$. Since time-averaged value of $d_p(t)$ converges to λ_p there exists N_0 s.t. for every $n \geq N_0$ we have

$$\hat{\lambda}_p \geq \frac{\sum_{\tau=0}^{n} d_p(\tau)}{n D_p} + \epsilon_2 \quad (72)$$

Select Y_1 s.t. $Y_1 N(\hat{\lambda}_p) > N_0$. Since $\hat{\lambda}_p > \lambda_p$ the total amount of data-arrival (in packets) during slots [0, $Y_2 Y_1 N(\hat{\lambda}_p)$] in System 2, $\hat{\lambda}_p Y_2 Y_1 N(\hat{\lambda}_p)$, is greater than that in System 1, $\frac{\sum_{\tau=0}^{Y_2 Y_1 N(\hat{\lambda}_p)} d_p(\tau)}{D_p}$, by $Y_2 Y_1 N(\hat{\lambda}_p)\epsilon_2$ i.e.,

$$Y_2 Y_1 N(\hat{\lambda}_p)\hat{\lambda}_p \geq Y_2 Y_1 N(\hat{\lambda}_p)\epsilon_2 + \frac{\sum_{\tau=0}^{Y_2 Y_1 N(\hat{\lambda}_p)} d_p(\tau)}{D_p} \quad (73)$$

Let n_1 and n_2 denote the number of primary packets with inter-arrival time of 2 slots that arrive during slots [0, $Y_2 Y_1 N(\hat{\lambda}_p)$] in System 1 and 2 respectively. Since inter-arrival time of any packet in both systems is either 3 slots or 2 slots, the maximum number of primary packets that arrive during slots [0, $Y_2 Y_1 N(\hat{\lambda}_p)$] in System 1 is $\frac{Y_2 Y_1 N(\hat{\lambda}_p) - 2n_1}{3} + n_1$. Number of primary packets that arrive during slots [0, $Y_2 Y_1 N(\hat{\lambda}_p)$] in System 2 is *always* $\frac{Y_2 Y_1 N(\hat{\lambda}_p) - 2n_2}{3} + n_2$ due to periodicity of the primary packet generation process.

Therefore, (73) can be rewritten as,

$$\frac{Y_2Y_1N(\hat{\lambda}_p)-2n_2}{3}+n_2 \geq Y_2Y_1N(\hat{\lambda}_p)\epsilon_2+\frac{Y_2Y_1N(\hat{\lambda}_p)-2n_1}{3}+n_1 \quad (74)$$

$$\text{i.e. } \frac{n_2}{3} \geq Y_2Y_1N(\hat{\lambda}_p)\epsilon_2+\frac{n_1}{3} \quad (75)$$

Lemma 6 follows by selecting Y_2 s.t. $3Y_2Y_1N(\hat{\lambda}_p)\epsilon_2$ is greater than n_3.

■

We partition the time-line into a collection of frames of length $Y_2Y_1N(\hat{\lambda}_p)$ each with the first frame beginning at slot $[\frac{1}{\hat{\lambda}_p}]$. Note, such frames are renewal frames for System 2 but not for System 1. We denote the j'th slot (where $j = 1,\ldots,Y_2Y_1N(\hat{\lambda}_p)$) in r'th frame as $t_{r,j}$. Assume SCRP and ASP are employed as scheduling policies for System 1 and 2 respectively.

Lemma 7: Consider System 1. For any Y_1, Y_2 s.t. $Y_2Y_1N(\hat{\lambda}_p)\epsilon_2$ is greater than 5, there exists at least 2 primary packets, in every frame, with inter-arrival time of 3 slots s.t. :

a) transmission of both packets begin and end in that frame and,
b) if any of those packets is relayed it creates an idle slot within that frame.

Proof: We prove by contradiction. If possible, assume that there exists only one primary packet with inter-arrival time of 3 slots, created only from data-bits that arrived during r'th frame (where $r = 1,\ldots,$) s.t. (a) its transmission begins and ends in that frame and, (b) if it is relayed it creates an idle slot within that frame. Recall, direct transmission requires 3 slots and relay transmission requires 2 slots.

For the r'th frame, note that any primary packet transmission that began prior to $t_{r,1}$ but continued in r'th frame lasts for at most 2 slots in the r'th frame. Further, one more primary packet could already be present at $t_{r,1}$ along with some non-zero amount of data at transmission-layer. Together they correspond to at most 2 packets formed, at least partially, from data that arrived prior to beginning of the r'th frame. Assume both these packets are present, they are relayed and do not lead to creation of any idle slot[10]. Therefore total number of slots in r'th frame that corresponds to transmission of primary packets formed

[10]Note, if any of these packets is directly transmitted during r'th frame, or relayed and creates an idle slot in r'th frame, then the proof is complete.

from data that arrived prior to $t_{r,1}$ is at most 6 (including any primary packet transmission that continued from the previous frame).

We consider slots in r'th frame excluding those 6 slots and the 3 slots corresponding to the transmission of the primary packet with inter-arrival time of 3 slots as mentioned earlier (i.e., the 3 slots when PT transmits to PR in case of direct transmission, or the 2 slots when PT transmits to ST_1 and ST_1 transmits to PR and the resultant idle slot created in r'th frame). This means, in the remaining $Y_2Y_1N(\hat{\lambda}_p) - 9$ slots in r'th frame there is no idle slot and one of the following events occur:

a) *Case 1:* In all these slots either PT transmits to ST_1 or ST_1 transmits to PR.

b) *Case 2:* An incomplete direct transmission from PT to PR takes place in the last (or the last 2) slots in r'th frame. In the remaining $Y_2Y_1N(\hat{\lambda}_p) - 10$ (or $Y_2Y_1N(\hat{\lambda}_p) - 11$) slots either PT transmits to ST_1 or ST_1 transmits to PR.

All such packets are created from data that arrive during r'th frame. Total number of primary packets created only from data that arrive in r'th frame, is greater than or equal to the number of primary packets transmitted completely or partially during those slots along with the primary packet with interarrival time of 3 slots that is assumed to have been transmitted during the frame. Hence, total number of primary packets created only from data that arrive in r'th frame, is greater than or equal to $\left\lceil \frac{Y_2Y_1N(\hat{\lambda}_p)-9}{2} \right\rceil + 1$ in case 1 and is greater than or equal to $\left\lceil \frac{Y_2Y_1N(\hat{\lambda}_p)-11}{2} \right\rceil + 2$ in case 2. For case 1 we have,

$$\left\lceil \frac{Y_2Y_1N(\hat{\lambda}_p) - 9}{2} \right\rceil + 1 \geq \left\lceil \frac{Y_2Y_1N(\hat{\lambda}_p)}{2} - 5 \right\rceil + 1 \tag{76}$$

$$\geq \frac{Y_2Y_1N(\hat{\lambda}_p)}{2} - 5 \tag{77}$$

For case 2 we have,

$$\left\lceil \frac{Y_2Y_1N(\hat{\lambda}_p) - 11}{2} \right\rceil + 2 \geq \left\lceil \frac{Y_2Y_1N(\hat{\lambda}_p)}{2} - 6 \right\rceil + 2 \tag{78}$$

$$\geq \frac{Y_2Y_1N(\hat{\lambda}_p)}{2} - 5 \tag{79}$$

Since $Y_2Y_1N(\hat{\lambda}_p)\epsilon_2$ is greater than 5, due to (74), total number of primary packets created during r'th frame in System 2 is at least 5 more than that created during r'th frame in System 1. Therefore, total number of primary packets created in System 2 during r'th frame is greater than $\frac{Y_2 Y_1 N(\hat{\lambda}_p)}{2} - 5 + 5$ i.e. $\frac{Y_2 Y_1 N(\hat{\lambda}_p)}{2}$. This is a contradiction, because number of packets created in $Y_2 Y_1 N(\hat{\lambda}_p)$ slots in System 2 cannot be greater than $\frac{Y_2 Y_1 N(\hat{\lambda}_p)}{2}$ since $\hat{\lambda}_p$ is less than $\frac{1}{2}$. Similar contradiction exists when less than 2 primary packets are formed from data-arrivals in earlier frames or if the primary packet transmission transmission continuing from the previous frame takes less than 2 slots. Hence there cannot be only one primary packet with inter-arrival time of 3 slots that satisfies the condition stated in Lemma 7. Similarly the case with existence of no such primary packets can also be shown to be non-feasible. This completes the proof. ■

In the rest of the paper we assume Y_1, Y_2 is s.t. $Y_2 Y_1 N(\hat{\lambda}_p)\epsilon_2$ is greater than 5. Let $\mu_n^{\text{SYS1}}(\tau)$ and $\mu_n^{\text{SYS2}}(\tau)$ denote the transmission rate offered to ST_n in System 1 under policy SCRP and in System 2 under policy ASP respectively.

Lemma 8: For every $r = 1, \ldots,$ the value of the utility function $\psi^{\text{SCRP}}(t_{r,1})$ in System 1 is greater than $\psi^{\text{ASP}}(t_{r,1})$ in System 2 minus a finite positive constant, i.e.,

$$\mathbb{E}[\sum_{\tau=t_{r,1}}^{t_{r+1,1}-1} \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})] \geq \mathbb{E}[\sum_{\tau=t_{r,1}}^{t_{r+1,1}-1} \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS2}}(\tau)|\boldsymbol{U}_s(t_{r,1})] - \hat{B} \quad (80)$$

where $\hat{B} > 0$ is a finite constant.

Proof: We partition slots in r'th frame into 4 distinct collections of slots: $\tilde{T}_{r,1}$, $\tilde{T}_{r,2}$, $\tilde{T}_{r,3}$ and $\tilde{T}_{r,4}$ respectively (exact definition of those collections will be provided later). The partition of slots is done based on the type of primary packet transmission that occurs in a slot in System 1, i.e. whether the transmission of the packet began and is completed in r'th frame, inter-arrival time of the transmitted primary packet and whether or not it is an idle slot. We show that sum of $\mathbb{E}[\sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})]$ terms over slots in each collection is greater than the sum of $\mathbb{E}[\sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS2}}(\tau)|\boldsymbol{U}_s(t_{r,1})]$ terms over another set of slots in r'th frame with equal cardinality[11], minus a finite positive constant. The proof is completed by adding the above terms over all collections.

A. $\mathbb{E}[\sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\mathit{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})]$ *over slots in* $\tilde{T}_{r,1}$*:*

In System 1 there are two possible scenarios concerning primary packet transmissions at beginning of r'th frame:

[11]This set is distinct for each collection.

a) *Case 1:* PT transmits a packet to ST_1 at $t_{r,1}-1$. This packet will be transmitted from ST_1 to PR at $t_{r,1}$.

b) *Case 2:* PT begins transmitting a packet to PR at $(t_{r,1}-2)$ or $(t_{r,1}-1)$. This transmission will continue in r'th frame at slots $t_{r,1}$, or $t_{r,1}$ and $t_{r,1}+1$ respectively.

Let $x_{r,j}^{\mathrm{SYS1}}$ denote the j'th primary packet (where $j=1,2$) in System 1 with inter-arrival time of 3 slots that is completely transmitted during r'th frame[12]. Let $w_{r,j}^{\mathrm{SYS1}}$ denote the slot when $x_{r,j}^{\mathrm{SYS1}}$'s transmission begins. If it is relayed, an idle slot $y_{r,j}^{\mathrm{SYS1}}$ is created during r'th frame.

We define $\tilde{T}_{r,1}$ as

a) $\tilde{T}_{r,1} \triangleq \{t_{r,1}, t_{r,2}\}$ if Case 2 is true and a primary packet transmission starts at $t_{r,1}-1$.

b) $\tilde{T}_{r,1} \triangleq \{t_{r,1}, w_{r,1}^{\mathrm{SYS1}}, w_{r,1}^{\mathrm{SYS1}}+1, w_{r,1}^{\mathrm{SYS1}}+2\}$ if $x_{r,1}^{\mathrm{SYSI}}$ is directly transmitted and either Case 1 is true, or Case 2 is true and a primary packet transmission starts at $t_{r,1}-2$.

c) $\tilde{T}_{r,1} \triangleq \{t_{r,1}, w_{r,1}^{\mathrm{SYS1}}, w_{r,1}^{\mathrm{SYS1}}+1, y_{r,1}^{\mathrm{SYS1}}\}$ if $x_{r,1}^{\mathrm{SYSI}}$ is relayed and either Case 1 is true, or Case 2 is true and a primary packet transmission starts at $t_{r,1}-2$.

We first consider Case 1.

Assume $x_{r,1}^{\mathrm{SYS1}}$ was directly transmitted. Then utilizing (42) and using same techniques as in proof of Lemma 3 we show,

$$\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\mathrm{ST}_1,\mathrm{PR})}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] + \sum_{n=1}^{S}\sum_{\tau=w_{r,1}^{\mathrm{SYS1}}}^{w_{r,1}^{\mathrm{SYS1}}+2}\mathbb{E}[U_n(t_{r,1})\mu_n^{\mathrm{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\mathrm{ST}_1,\mathrm{PR})}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] + \sum_{n=1}^{S}\sum_{\tau=w_{r,1}^{\mathrm{SYS1}}}^{w_{r,1}^{\mathrm{SYS1}}+2}\mathbb{E}[U_n(\tau)\mu_n^{\mathrm{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})] - 3SY_2Y_1N(\hat{\lambda}_p) \tag{81}$$

$$\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\mathrm{ST}_1,\mathrm{PR})}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] + 3\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\mathrm{PT},\mathrm{PR})}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] - 6SY_2Y_1N(\hat{\lambda}_p) \tag{82}$$

$$\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\mathrm{ST}_1,\mathrm{PR})}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] + \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\{\boldsymbol{v}^*_{(\mathrm{PT},\mathrm{ST}_1)}(t_{r,1}) + v^*_{(\mathrm{ST}_1,\mathrm{PR})}(t_{r,1}) + \boldsymbol{v}^*_0(t_{r,1})\}|\boldsymbol{U}_s(t_{r,1})] - 6SY_2Y_1N(\hat{\lambda}_p) \tag{83}$$

$$\geq 2\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\{\boldsymbol{v}^*_{(\mathrm{PT},\mathrm{ST}_1)}(t_{r,1}) + \boldsymbol{v}^*_{(\mathrm{ST}_1,\mathrm{PR})}(t_{r,1})\}|\boldsymbol{U}_s(t_{r,1})] - 6SY_2Y_1N(\hat{\lambda}_p) \tag{84}$$

[12]Recall, due to Lemma 7, there exists at least 2 such packets.

In (81), (82) we used (42). Inequality (83) follows from the definition of SCRP. Inequality (84) follows because $\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}_0^*(t_{r,1})$ is greater than or equal to $\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}$ for any other $v \in I_0$. Similarly when $x_{r,1}^{\text{SYS1}}$ is relayed via ST_1 we can show

$$\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}) + \sum_{n=1}^{S}\sum_{\tau=w_{r,1}^{\text{SYS1}}}^{w_{r,1}^{\text{SYS1}}+1} U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau) + \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS1}}(y_{r,1}^{\text{SYS1}})|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq 2\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1})\}|\boldsymbol{U}_s(t_{r,1})] - 6SY_2Y_1N(\hat{\lambda}_p) \quad (85)$$

Now assume Case 2 is true and transmission of a primary packet from PT to PR began at $(t_{r,1}-2)$. Then,

$$\begin{aligned}
\mathbb{E}[\sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS1}}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] &= \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\text{PT},\text{PR})}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] && (86)\\
&\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\text{PT},\text{PR})}(t_{r,1}-2)|\boldsymbol{U}_s(t_{r,1})] && (87)\\
&\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1}-2)\boldsymbol{v}^*_{(\text{PT},\text{PR})}(t_{r,1}-2)|\boldsymbol{U}_s(t_{r,1})] - 2S && (88)\\
&\geq \frac{1}{3}\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1}-2)\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-2) + \\
&\quad \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}-2) + \boldsymbol{v}_0^*(t_{r,1}-2)\}|\boldsymbol{U}_s(t_{r,1})] - 2S && (89)\\
&\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1}-2)\min\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-2), \\
&\quad \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}-2)\}|\boldsymbol{U}_s(t_{r,1})] - 2S && (90)
\end{aligned}$$

Inequality (87), (89) follows from the definition of SCRP. In (88) we used (42). Assume, $\min\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-2), \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}-2)\} = \boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-2)$. Then

$$\begin{aligned}
\mathbb{E}[\sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS1}}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] &\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1}-2)\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-2)|\boldsymbol{U}_s(t_{r,1})] - 2S && (91)\\
&\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1}-2)\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] - 2S && (92)\\
&\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1})|\boldsymbol{U}_s(t_{r,1})] - 4S && (93)
\end{aligned}$$

Proceeding similarly as in Case 1 by considering primary packet $x_{r,1}^{\text{SYSI}}$ we show

$$\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\text{PT},\text{PR})}(t_{r,1}) + \sum_{n=1}^{S}\sum_{\tau=w_{r,1}^{\text{SYS1}}}^{w_{r,1}^{\text{SYS1}}+2} U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq 2\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1})\}|\boldsymbol{U}_s(t_{r,1})] - 6SY_2Y_1N(\hat{\lambda}_p) - 4S, \quad (94)$$

if $x_{r,1}^{\text{SYSI}}$ is directly transmitted. If $x_{r,1}^{\text{SYSI}}$ is relayed we can show,

$$\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\text{PT},\text{PR})}(t_{r,1}) + \sum_{n=1}^{S}\sum_{\tau=w_{r,1}^{\text{SYS1}}}^{w_{r,1}^{\text{SYS1}}+1} U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau) + \sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\text{SYS1}}(y_{r,1}^{\text{SYS1}})|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq 2\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1})\}|\boldsymbol{U}_s(t_{r,1})] - 6SY_2Y_1N(\hat{\lambda}_p) - 4S \quad (95)$$

Inequality (94) and (95) can be shown to be true also
when $\min\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-2), \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}-2)\} = \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}-2)$.
Assume Case 2 is true and direct transmission of a primary packet began at slot $t_{r,1}-1$. Then we have

$$\mathbb{E}[\sum_{n=1}^{S} U_n(t_{r,1})\{\mu_n^{\text{SYS1}}(t_{r,1}) + \mu_n^{\text{SYS1}}(t_{r,2})\}|\boldsymbol{U}_s(t_{r,1})]$$

$$\geq \mathbb{E}[\sum_{n=1}^{S}\{U_n(t_{r,1})\mu_n^{\text{SYS1}}(t_{r,1}) + U_n(t_{r,2})\mu_n^{\text{SYS1}}(t_{r,2})\}|\boldsymbol{U}_s(t_{r,1})] - S \quad (96)$$

$$\geq 2\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\boldsymbol{v}^*_{(\text{PT},\text{PR})}(t_{r,1}-1)|\boldsymbol{U}_s(t_{r,1})] - 2S \quad (97)$$

$$\geq 2\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1}-1)\boldsymbol{v}^*_{(\text{PT},\text{PR})}(t_{r,1}-1)|\boldsymbol{U}_s(t_{r,1})] - 4S \quad (98)$$

$$\geq \frac{2}{3}\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1}-1)\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-1) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}-1)$$
$$+\boldsymbol{v}^*_0(t_{r,1}-1)\}|\boldsymbol{U}_s(t_{r,1})] - 4S \quad (99)$$

$$\geq \frac{2}{3}\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1}-1)\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-1) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}-1)$$
$$+\frac{1}{2}\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-1) + \frac{1}{2}\boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}-1)\}|\boldsymbol{U}_s(t_{r,1})] - 4S \quad (100)$$

$$\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1}-1)\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}-1) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1}-1)\}|\boldsymbol{U}_s(t_{r,1})] - 4S \quad (101)$$

$$\geq \mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1})\}|\boldsymbol{U}_s(t_{r,1})] - 6S \quad (102)$$

In (96), (97), (98), (102) we have used (42). Inequality (99) follows from definiton of SCRP.

By combining results from (102), (94), (95), (85) and (84) we obtain

$$\sum_{\tau\in\tilde{T}_{r,1}}\sum_{n=1}^{S}\mathbb{E}[U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})] \geq \frac{|\tilde{T}_{r,1}|}{2}\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1})\}|\boldsymbol{U}_s(t_{r,1})] - \hat{B}_1 \quad (103)$$

where $\hat{B}_1 > 0$ is a finite constant.

B. $\mathbb{E}[\sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\textit{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})]$ *over slots in* $\tilde{T}_{r,2}$ *:*

There are 2 possible scenarios in which a primary packet transmission, in System 1, begins but is not completed in r'th frame:

a) *Case 3:* PT transmits a primary packet to ST_1 at $t_{r+1,1} - 1$. This packet will be transmitted from ST_1 to PR at $t_{r+1,1}$.

b) *Case 4:* PT begins transmitting a packet to PR at $(t_{r+1,1} - 2)$, or $(t_{r+1,1} - 1)$ and will continue in $(r + 1)$'th frame at slots $t_{r+1,1}$, or $t_{r+1,1}$ and $t_{r+1,2}$ respectively.

Accordingly we define $\tilde{T}_{r,2}$ as

a) $\tilde{T}_{r,2} \stackrel{\triangle}{=} \{t_{r+1,1} - 2, t_{r+1,1} - 1\}$ if Case 4 is true and a primary packet transmission starts at $t_{r+1,1} - 2$.

b) $\tilde{T}_{r,2} \stackrel{\triangle}{=} \{t_{r+1,1} - 1, w_{r,2}^{\text{SYS1}}, w_{r,2}^{\text{SYS1}} + 1, w_{r,2}^{\text{SYS1}} + 2\}$ if $x_{r,2}^{\text{SYS1}}$ is directly transmitted and either Case 3 is true, or Case 4 is true and a primary packet transmission starts at $t_{r+1,1} - 1$.

c) $\tilde{T}_{r,2} \stackrel{\triangle}{=} \{t_{r+1,1} - 1, w_{r,2}^{\text{SYS1}}, w_{r,2}^{\text{SYS1}} + 1, y_{r,2}^{\text{SYS1}}\}$ if $x_{r,2}^{\text{SYS1}}$ is relayed and either Case 3 is true, or Case 4 is true and a primary packet transmission starts at $t_{r+1,1} - 1$.

Proceeding similarly as in the case of slots in $\tilde{T}_{r,1}$ we can show,

$$\sum_{\tau\in\tilde{T}_{r,2}}\sum_{n=1}^{S}\mathbb{E}[U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})] \geq \frac{|\tilde{T}_{r,2}|}{2}\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1})\}|\boldsymbol{U}_s(t_{r,1})] - \hat{B}_2 \quad (104)$$

where $\hat{B}_2 > 0$ is a finite constant.

C. $\mathbb{E}[\sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{\textit{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})]$ *over slots in* $\tilde{T}_{r,3}$ *:*

Consider primary packets in System 1 with inter-arrival time of 2 slots whose transmission begins and is completed within r'th frame. The set $\tilde{T}_{r,3}$ denote the collection of slots in which such transmissions take place. Then using same techniques as used in proof of Lemma 3 we can show

$$\sum_{\tau \in \tilde{T}_{r,3}} \sum_{n=1}^{S} \mathbb{E}[U_n(t_{r,1})\mu_n^{\text{SYSI}}(\tau)|\boldsymbol{U}_s(t_{r,1})] \geq \frac{|\tilde{T}_{r,3}|}{2}\mathbb{E}[\boldsymbol{U}_s^T(t_{r,1})\{\boldsymbol{v}^*_{(\text{PT},\text{ST}_1)}(t_{r,1}) + \boldsymbol{v}^*_{(\text{ST}_1,\text{PR})}(t_{r,1})\}|\boldsymbol{U}_s(t_{r,1})] - \hat{B}_3 \quad (105)$$

where $\hat{B}_3 > 0$ is a finite constant.

Next we show that there exist $\frac{\sum_{i=1}^{3}|\tilde{T}_{r,i}|}{2}$ primary packets with inter-arrival times of 2 slots that are transmitted in r'th frame in System 2.

Let n_1 and n_2 denote the number of primary packets with inter-arrival time of 2 slots that arrive during slots $[0, Y_2Y_1N(\hat{\lambda}_p)]$ in System 1 and 2 respectively. Therefore, in System 1 number of primary packets with inter-arrival time of 2 slots whose transmissions begin and is completed during r'th frame is less than or equal to $2+n_1$[13]. Number of primary packets with inter-arrival time of 2 slots transmitted in r'th frame in System 2 is n_2 due to periodicity of the primary packet generation process. Since $Y_2Y_1N(\hat{\lambda}_p)\epsilon_2$ is greater than 5, from (75), we have $n_2 > n_1 + 15$. Note, $|\tilde{T}_{r,1}| \leq 4$, $|\tilde{T}_{r,2}| \leq 4$ and $|\tilde{T}_{r,3}| \leq 2(n_1+2)$. Therefore, $|\tilde{T}_{r,1}|$+$|\tilde{T}_{r,2}|$+$|\tilde{T}_{r,3}| \leq 2(n_1+6)$. Since $n_2 > n_1+6$, there exists $\frac{\sum_{i=1}^{3}|\tilde{T}_{r,i}|}{2}$ primary packets with inter-arrival times of 2 slots that are transmitted in r'th frame in System 2. We denote as $\tilde{T'}_{r,1}$ the set of slots in which those packets are transmitted. From (103), (104), (105) and noting that in ASP scheduling decisions for secondary users are made based on secondary queue-length at beginning of every frame, we obtain

$$\sum_{\tau \in \{\tilde{T}_{r,1}\cup\tilde{T}_{r,2}\cup\tilde{T}_{r,3}\}} \sum_{n=1}^{S} \mathbb{E}[U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})] \geq \sum_{\tau\in\tilde{T}'_{r,1}} \sum_{n=1}^{S} \mathbb{E}[U_n(t_{r,1})\mu_n^{\text{SYS2}}(\tau)|\boldsymbol{U}_s(t_{r,1})] - \hat{B}_4 \quad (106)$$

where $\hat{B}_4 > 0$ is a finite constant.

[13]In System 1, total number of primary packets with inter-arrival time of 2 slots whose transmissions begin and is completed during r'th frame can only consist of:

a) packets which may be present at transmission queue of PT at $t_{r,1}$, and
b) packets created from left-over data at transmission-layer of PT at $t_{r,1}$ along with data received during r'th frame.

Therefore, total number of such packets can at most be 2 higher than n_1.

D. $\mathbb{E}[\sum_{n=1}^{S} U_n(t_{r,1})\mu_n^{SYS1}(\tau)|\boldsymbol{U}_s(t_{r,1})]$ *over slots in* $\tilde{T}_{r,4}$ *:*

The set $\tilde{T}_{r,4}$ is defined as the set of slots in r'th frame that do not belong to $\tilde{T}_{r,1}$, $\tilde{T}_{r,2}$ or $\tilde{T}_{r,3}$ i.e., $\tilde{T}_{r,4} \triangleq \{\tau : \tau \in r\text{'th frame}, \tau \notin \tilde{T}_{r,1} \cup \tilde{T}_{r,2} \cup \tilde{T}_{r,3}\}$. The collection of slots $\tilde{T'}_{r,2}$ is defined as, $\tilde{T'}_{r,2} \triangleq \{\tau : \tau \in r\text{'th frame} , \tau \notin \tilde{T'}_{r,1}\}$.

Now since $Y_2 Y_1 N(\hat{\lambda}_p)\epsilon_2$ is greater than 5, total number of primary packets transmitted during r'th frame in System 1, completely or partially, is at least 5 minus 3 i.e., 2 lower than that transmitted in System 2[14]. However, the number of primary packets transmitted during slots $\tilde{T'}_{r,1}$ and $\{\tilde{T}_{r,1} \cup \tilde{T}_{r,2} \cup \tilde{T}_{r,3}\}$ in System 2 and System 1 respectively are equal. Therefore, number of primary packets transmitted in System 2 during $\tilde{T}'_{r,2}$ is higher than that transmitted during $\tilde{T}_{r,4}$ in System 1. Also, all slots in $\tilde{T}_{r,4}$ are used in System 1 either to transmit primary packets with inter-arrival time of 3 slots or are idle slots. As a result, using same techniques as used in proof of Lemma 3 and noting that in ASP scheduling decisions for secondary users are made based on secondary queue-length at beginning of every frame, we can show,

$$\sum_{\tau \in \tilde{T}_{r,4}} \sum_{n=1}^{S} \mathbb{E}[U_n(t_{r,1})\mu_n^{\text{SYS1}}(\tau)|\boldsymbol{U}_s(t_{r,1})] \geq \sum_{\tau \in \tilde{T}'_{r,2}} \sum_{n=1}^{S} \mathbb{E}[U_n(t_{r,1})\mu_n^{\text{SYS2}}(\tau)|\boldsymbol{U}_s(t_{r,1})] - \hat{B}_5 \quad (107)$$

where $\hat{B}_5 > 0$ is a finite constant.

Adding the results obtained from (106) and (107) we prove Lemma 8. ■

Proof of Lemma 4: Since every primary packet is transmitted in at most as many slots as its inter-arrival time, queue at PT is strongly stable. For $n = 1, 2, ..., S$ we consider the secondary queue-lengths at the beginning of r'th frame (where $r = 1, 2, \ldots,)$, $Z_n(r) \triangleq U_n(t_{r,1})$. We denote the vector $(Z_1(r), Z_2(r) \ , \ldots, Z_S(r))^T$ as $\boldsymbol{Z}_s(r)$. We define a Lyapunov function $V(\boldsymbol{Z}_s(r)) \triangleq \sum_{n=1}^{S} Z_n^2(r)$. The conditional drift $\triangle(r)$, defined as in (33), satisfies,

$$\begin{aligned} \triangle(r) &\leq S(Y_2 Y_1 N(\hat{\lambda}_p))^2(1+1) - 2\sum_{n=1}^{S} Z_n(r)\mathbb{E}[\sum_{\tau=t_{r,1}}^{\tau=t_{r+1,1}-1} (\mu_n^{\text{SYS1}}(\tau) - A_n(\tau))|\boldsymbol{Z}_s(r)] \\ &\leq 2S(Y_2 Y_1 N(\hat{\lambda}_p))^2 + 2\hat{B} - 2\mathbb{E}[\sum_{n=1}^{S} Z_n(r) \sum_{\tau=t_{r,1}}^{\tau=t_{r+1,1}-1} \mu_n^{\text{SYS2}}(\tau)|\boldsymbol{Z}_s(r)] \end{aligned} \quad (108)$$

[14]Note, there are at most 3 packets transmitted in r'th frame created from data that arrived in an earlier frame.

$$+2\sum_{n=1}^{S} Z_n(r)\mathbb{E}[\sum_{\tau=t_{r,1}}^{\tau=t_{r+1,1}-1} A_n(\tau)] \tag{109}$$

$$\leq 2S(Y_2Y_1N(\hat{\lambda}_p))^2 + 2\hat{B} - 2\epsilon_3\sum_{n=1}^{S} Z_n(r) \tag{110}$$

where $\epsilon_3 > 0$ is a finite constant. Inequality (109) follows from Lemma 8; (110) follows from Lemma 5, 1 and 2 respectively. Therefore by Theorem 4.1 of [14] and due to boundedness of the secondary packet arrival and departure processes, the secondary queue-length processes are strongly stable. ■

REFERENCES

[1] I. Maric, R. Yates, and G. Kramer, "Capacity of interference channels with partial transmitter cooperation," *Information Theory, IEEE Transactions on*, vol. 53, no. 10, pp. 3536–3548, 2007.

[2] I. Marić, A. Goldsmith, G. Kramer, *et al.*, "On the capacity of interference channels with one cooperating transmitter," *European Transactions on Telecommunications*, vol. 19, no. 4, pp. 405–420, 2008.

[3] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *Information Theory, IEEE Transactions on*, vol. 51, no. 9, pp. 3037–3063, 2005.

[4] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz, "Spectrum leasing to cooperating secondary ad hoc networks," *Selected Areas in Communications, IEEE Journal on*, vol. 26, no. 1, pp. 203–213, 2008.

[5] I. Stanojev and A. Yener, "Facilitating flexible multihop communication via spectrum leasing," in *Personal Indoor and Mobile Radio Communications (PIMRC), 2012 IEEE 23rd International Symposium on*, 2012, pp. 2160–2165.

[6] J. Zhang and Q. Zhang, "Stackelberg game for utility-based cooperative cognitive radio networks," in *Proceedings of the tenth ACM international symposium on Mobile ad hoc networking and computing*, ser. MobiHoc '09. New York, NY, USA: ACM, 2009, pp. 23–32.

[7] R. Urgaonkar and M. Neely, "Opportunistic cooperation in cognitive femtocell networks," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 3, pp. 607–616, 2012.

[8] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Stable throughput of cognitive radios with and without relaying capability," *Communications, IEEE Transactions on*, vol. 55, no. 12, pp. 2351–2360, 2007.

[9] A. Fanous and A. Ephremides, "Stable throughput in a cognitive wireless network," *Selected Areas in Communications, IEEE Journal on*, vol. 31, no. 3, pp. 523–533, 2013.

[10] I. Krikidis, N. Devroye, and J. Thompson, "Stability analysis for cognitive radio with multi-access primary transmission," *Wireless Communications, IEEE Transactions on*, vol. 9, no. 1, pp. 72–77, 2010.

[11] A. El-Sherif, A. Sadek, and K. Liu, "Opportunistic multiple access for cognitive radio networks," *Selected Areas in Communications, IEEE Journal on*, vol. 29, no. 4, pp. 704–715, 2011.

[12] E. Matskani, N. Chatzidiamantis, L. Georgiadis, I. Koutsopoulos, and L. Tassiulas, "Optimal primary-secondary user cooperation policies in cognitive radio networks," *arXiv preprint arXiv:1307.5613*, 2013.

[13] L. Tassiulas and A. Ephremides, “Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks,” *Automatic Control, IEEE Transactions on*, vol. 37, no. 12, pp. 1936–1948, 1992.

[14] M. J. Neely, “Stochastic network optimization with application to communication and queueing systems,” *Synthesis Lectures on Communication Networks*, vol. 3, no. 1, pp. 1–211, 2010.

[15] C. Joo, X. Lin, and N. B. Shroff, “Understanding the capacity region of the greedy maximal scheduling algorithm in multihop wireless networks,” *IEEE/ACM Transactions on Networking (TON)*, vol. 17, no. 4, pp. 1132–1145, 2009.

[16] C. Joo and N. B. Shroff, “Local greedy approximation for scheduling in multihop wireless networks,” *Mobile Computing, IEEE Transactions on*, vol. 11, no. 3, pp. 414–426, 2012.

[17] M. Neely, E. Modiano, and C. Rohrs, “Dynamic power allocation and routing for time varying wireless networks,” in *INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications. IEEE Societies*, vol. 1, March 2003, pp. 745–755 vol.1.

[18] E. Oks and M. Sharir, “Minkowski sums of monotone and general simple polygons,” *Discrete & Computational Geometry*, vol. 35, no. 2, pp. 223–240, 2006.

[19] R. Urgaonkar and M. J. Neely, “Opportunistic scheduling with reliability guarantees in cognitive radio networks,” *Mobile Computing, IEEE Transactions on*, vol. 8, no. 6, pp. 766–777, 2009.