Opportunistic Scheduling and Relaying in a Cooperative Cognitive Network

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Abstract

This paper considers network-layer cooperation in cognitive radio networks whereby secondary users may be allowed to relay primary user's packets. Under this cooperative scheme, the paper investigates whether, and under what conditions, the primary and secondary networks can be stabilized without explicit knowledge of the arrival-rates. We consider a deterministic and periodic primary packet arrival process and develop a relaying and scheduling algorithm using Lyapunov drift techniques that does not require explicit knowledge of primary and secondary packet arrival rates. The algorithm is then shown to stabilize the transmission queues in the network for all secondary packet arrival rates that lie in the interior of a certain region. The region includes all secondary arrival-rate vectors that can be supported when the secondary nodes do not cooperate. Furthermore, when the primary data arrival-rate is greater than what could have been supported without relays but less than what can be maximally supported with relays, the algorithm stabilizes the network for a non-empty set of secondary arrival-rate vectors. The significance of these results is that they show that properly designed cooperation may result in a win-win scenario for both primary and secondary users (and not just for the primary user). Finally we extend our analysis to the case of a deterministic but aperiodic primary packet arrival process.

I. INTRODUCTION

Cooperation between primary and secondary networks have been widely studied from a physical-layer perspective. Some of these works (eg- [1], [2], [3]) study it as an information-theoretic problem. Other works such as [4], [5],

[6] involve the primary network leasing the spectrum to secondary nodes in return for cooperation and the objective therein is to maximize utility functions corresponding to link-rates. However these works do not address the higher layer features such as queuing and prioritized scheduling. One significant aspect of primary-secondary cooperation, which has received little attention, is that the actions by a cooperating secondary node can now influence the primary user-channel occupancy process [7].

In this work we address the question: whether and under what conditions a general network consisting of a single primary link and multiple secondary users, few of which can act as relay for the primary user, can be stabilized without explicit knowledge of the arrival-rates. We develop an algorithm that achieves this goal for networks with a *deterministic, periodic primary packet generation process*. The primary packets always enjoy high priority of transmission in the network even at secondary relay nodes. The primary packet generation rate in our model can be *greater* than what is supported by the primary network alone. Deterministic packet generation process has been used previously in [8] in the context of max-weight based throughput-optimal scheduling policies. We exploit the periodic aspect of the primary packet generation process to construct frames (to be defined in later sections) of fixed length that leads to tractable analysis.

As a representative network, we consider a single primary source-destination (s-d) pair in the presence of multiple secondary s-d pairs with one or more secondary node(s) that can act as relay for primary traffic. The primary s-d pair can always benefit due to the presence of the relay. Some secondary users can also benefit since the primary transmissions now occupy the channel for a smaller fraction of time. However, some other secondary users, due to their geographical locations, may obtain fewer transmission opportunities when there is cooperation due to increased transmission activity by the secondary relay. Thus in our network model we account for the *trade-off between extent of cooperation and throughput* of individual secondary users.

We assume that packets can be transmitted across multiple time-slots where the length of a time-slot is defined appropriately. This assumption can be shown to be equivalent to that used in works on spectrum leasing for cognitive radio networks such as [4], [5], [6]. In these works it is assumed that a time-slot used for direct transmission of data from a primary user to a primary destination can be further divided into smaller intervals in which transmissions from a primary user to a relay node, from relay node to primary destination and possible transmission of secondary network's own data takes place. For example, consider a simple network based on the system model in [4] with one secondary relay node, a primary user and a primary destination. The network parameters, power allocation etc. are such that the capacities of the primary user-relay link and the relay-primary destination link are 3 primary packet/time-slot each where the length of a slot is the time taken to transmit a primary packet using the primary user-primary destination direct link (note: the primary data unit in [4] is a primary code-word). Then if there is one primary packet at the transmission queue of primary user, $\frac{1}{3}$-rd of a time-slot can be used to transmit the primary packet from primary user to relay, the next $\frac{1}{3}$-rd of a slot can be used to transmit the primary packet from relay to primary destination and the remaining $\frac{1}{3}$-rd of the time-slot can be leased to the secondary network for its own transmissions. We notice that such a system model, after re-defining the length of a time-slot to be the time taken by primary user to transmit a single primary packet through the primary user-relay link, is equivalent to a model where primary user can directly transmit a primary packet to primary destination in 3 time-slots or transmit the same in 2 slots by using an intermediate relay.

The remaining of the paper is organized as follows. Section II describes our system model. In Section III we define a region consisting of secondary packet arrival-rates, as a function of primary packet arrival-rate. In Section III we also outline our objective which is to find a relay and scheduling policy that stabilizes the network for all secondary arrival-rate vectors in the interior of this region. In Section IV we develop an algorithm that makes scheduling decisions every time-slot based only on the knowledge of the instantaneous queue lengths and inter-arrival time of primary packets in the network. In Section V we show the stability of the network under this algorithm for all secondary arrival-rates that lie in the interior of the region described in Section III. The proof uses the Lyapunov-drift technique introduced by Tassiulas and Ephremides in [9], which has been widely used to develop throughput-optimal algorithms in computer networks. For the periodic primary packet arrival process in Section II the primary packet arrival rate is a rational number. In Section VI we extend the analysis to the case where the primary packet arrival rate can be an irrational number and the resultant primary packet arrival process is deterministic but aperiodic. Section VII concludes the paper.

Related Works: We mention some relevant papers that study upper-layer primary-secondary cooperation in cognitive networks. In [10] the authors consider two links: a primary and a secondary where a secondary relay node re-transmits packets that were not successfully received by the intended primary destination node but correctly received at the relay node. A similar model is used in [11] where the authors consider a single primary s-d pair in the presence of multiple secondary nodes that can act as relay. In addition, the queued primary packets at the relay nodes are required to be transmitted with a higher priority in every idle slot. In [12] the authors obtain stable throughput region for the primary and secondary users in a 5 node network with 2 primary transmitters and one common secondary relay. In [13] the authors consider uplink of a TDMA-based primary network in the presence of two sets of cognitive nodes: pure relay nodes that assist the primary network by re-transmitting some primary packets that were not received successfully at the base-station and another set of cognitive non-relaying nodes that form an ad-hoc network and communicate using slotted Aloha protocol.

In all the above works the primary transmitter is assumed to be oblivious to the existence of the secondary users which is consistent with *commons* model of cognitive radio [4]. On the other hand, in our work we assume that primary users are aware of the existence of the secondary network and can therefore request cooperation from the latter to improve latency of transmitted primary packets all the while preserving high tansmission priority for primary packets. Such assumption is related to the *spectrum leasing* model of cognitive radio which has been used in works such as [4], [5] and [6]. However, [4], [5] and [6] study the cooperative relaying problem from a physical-layer perspective and do not investigate the network-layer aspects such as queuing and scheduling. While [11] and [13] consider network with multiple secondary nodes, they do not address the scenario where the effect of cooperation may be beneficial or hurtful to certain nodes. Also in [11] and [13] at most one secondary node can transmit in any time-slot using slotted Aloha whereas in our work we consider a more general model whereby some secondary nodes are far apart from each other such that in the absence of a primary transmission, they may transmit simultaneously.

In [7] the authors find optimal cooperative power allocations in a network of multiple secondary users and a single primary user. The authors assume i.i.d packet arrivals and using the concept of renewal frames develop a cooperative scheduling scheme that requires knowledge of only queue-lengths in the network. However, a key assumption in their analysis is valid only for the range of primary packet arrival rates for which the primary network is stable even without any assistance from the secondary nodes. It is not obvious how to extend it to cases where the secondary network allows the primary user to increase its throughput beyond what is achievable without cooperation. The periodic primary packet arrival process used in our model allows us to develop scheduling policy for primary arrival rates greater than what can be supported by the network without any cooperative relay. Unlike our work, in [7] at most one secondary node can transmit at any time and the authors do not address the case where the effect of cooperative relaying can vary from node to node in the secondary network.

II. SYSTEM MODEL

We consider a network with one primary transmitter (PT) and S secondary transmitters- ST_1, $ST_2, ..., ST_S$. PR and SR_i denote the primary receiver and the secondary receiver corresponding to ST_i respectively (where $i = 1, 2, ..S$).

A. Primary packet transmission model

1) Identifying the set of relay nodes: We assume that PT always transmits at fixed power Υ_{dir} to PR. Alternatively PT can transmit a packet to some secondary transmitters at fixed power Υ_{rel}, where $\Upsilon_{rel} < \Upsilon_{dir}$. The transmission range corresponding to transmission powers Υ_{dir} and Υ_{rel} are denoted by $d_{\Upsilon_{dir}}$ and $d_{\Upsilon_{rel}}$ respectively. Without loss of generality we assume that ST_1, ST_2,…, $ST_{S_{rel}}$ (where $1 \leq S_{rel} \leq S$) are the secondary transmitters that are within a distance $d_{\Upsilon_{rel}}$ from PT and can therefore receive packets from PT. We assume that a secondary node ST_i (where $1 \leq i \leq S$) uses transmission power $\Upsilon_{s,i}$ to transmit any packet and denote the corresponding transmission range by $d_{s,i}$. We assume that PR is located within distance $d_{s,i}$ from ST_1 ,.., $ST_{S_{rel}}$ and therefore ST_1,…,ST_{rel} can act as relay for primary traffic.

2) Definition of a link and time-slot: A link is defined by the ordered pair (l_1, l_2) such that, by using the above transmission powers, packets can be transmitted from node l_1 to node l_2. Let L denote the set of all feasible links i.e. $L = \{(PT, PR), (PT, ST_i), (ST_i, PR), (ST_j, SR_j) : 1 \leq i \leq S_{rel}, 1 \leq j \leq S\}$. We denote the set of links $\{(PT, PR), (PT, ST_i), (ST_i, PR) : 1 \leq i \leq S_{rel}\}$ by L_p. All links in L_p are used exclusively for primary packet transmission.

Let $C(l_1, l_2)$ denote the capacity of the link (l_1, l_2) where $(l_1, l_2) \in L$. Let C_{max} denote the maximum capacity among all links in L. We assume the following:

1) There exists at least one $ST_{\tilde{j}}$ (where $1 \leq \tilde{j} \leq S_{rel}$) such that $C(PT, ST_{\tilde{j}})$ or $C(ST_{\tilde{j}}, PR) = C_{max}$. Without loss of generality we assume that the link with maximum capacity is the link $(ST_{\tilde{j}}, PR)$.

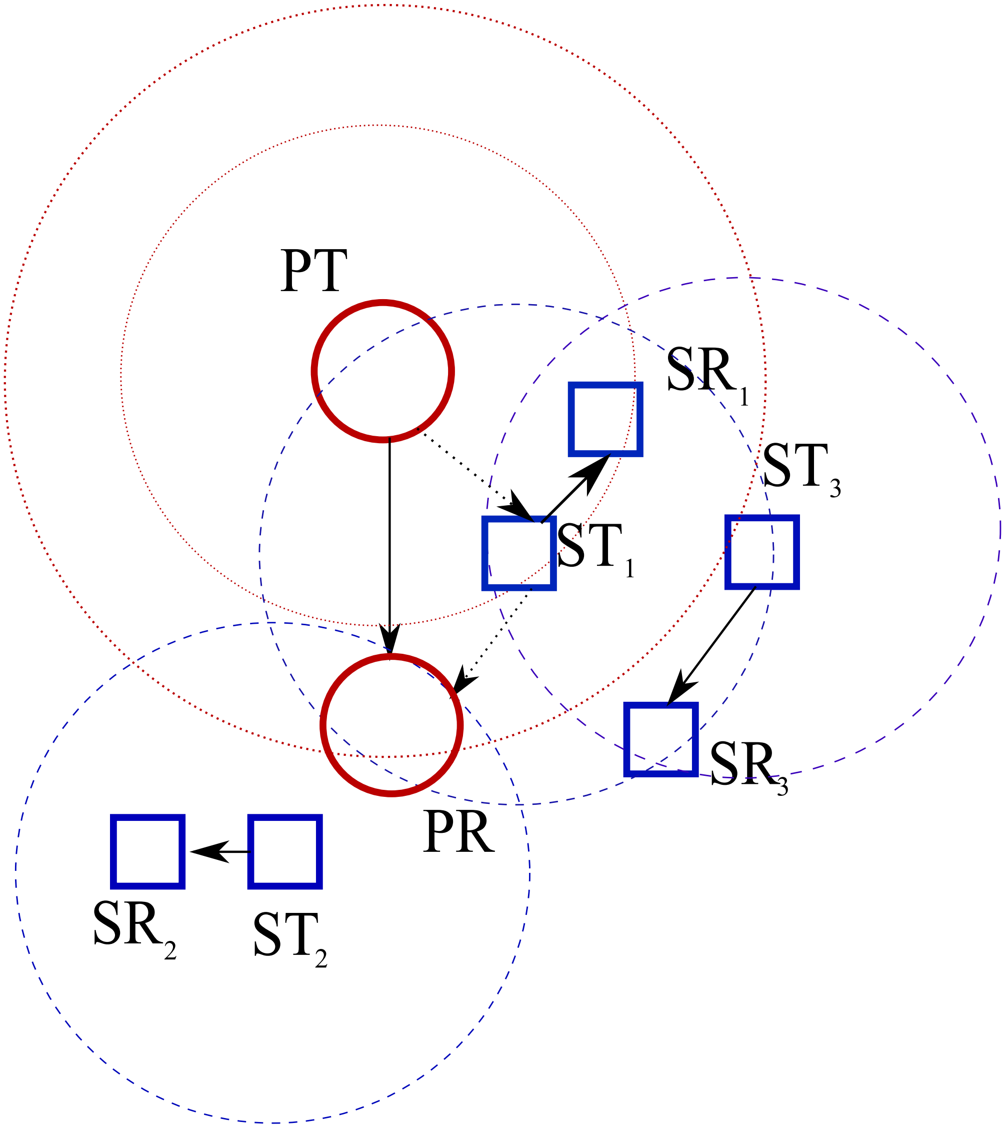

Fig. 1. A network with one primary s-d pair and three secondary s-d pairs. Each of the three blue-dashed circles with one of the ST_is (where $i = 1, 2, 3$) as center has radius $d_{\Upsilon_{rel}}$. No point within the circle drawn with ST_i (where $i = 1, 2, 3$) as center contains a node that can simultaneously receive a packet from any node except ST_i when ST_i is transmitting. The larger and smaller red, dotted circles drawn with PT as centre has radius $d_{\Upsilon_{dir}}$ and $d_{\Upsilon_{rel}}$ respectively. No node located within the larger circle, except PR, can receive a packet when PT is directly transmitting a packet. No node located within the smaller circle, except ST_1, can receive a packet when PT is directly transmitting a packet. The dotted lines from PT to ST_1 and from ST_1 to PR represent cooperative transmission with ST_1 as a relay node.

2) The ratio of capacities between any two links is a rational number i.e. $\frac{C(l_1,l_2)}{C(l_1',l_2')} = \frac{g_1((l_1,l_2),(l_1',l_2'))}{g_2((l_1,l_2),(l_1',l_2'))}$ where $g_1(.)$ and $g_2(.)$ are integers that are coprime to each other for all $(l_1, l_2), (l_1', l_2') \in L$.

We denote the length of a primary packet to be L_p bits. Let $\tilde{g_1}$ be defined as $\tilde{g_1} \overset{\triangle}{=}$ least common multiple (L.C.M) of $(g_1((PT, PR), (ST_{\tilde{j}}, PR)), g_1((PT, ST_1), (ST_{\tilde{j}}, PR))$,$\ldots, g_1((PT, ST_{S_{rel}}), (ST_{\tilde{j}}, PR)), g_1((ST_1, PR), (ST_{\tilde{j}}, PR))$, $\ldots$, $g_1((ST_{S_{rel}}, PR), (ST_{\tilde{j}}, PR)))$. The length of a time-slot is then defined to be $\frac{1}{\tilde{g_1}}$ times the time taken to transmit a single primary packet through link $(ST_{\tilde{j}}, PR)$.The transmission time of a primary packet through the (l_1, l_2) link, denoted as $K_{(l_1,l_2)}$, (where $(l_1, l_2) \in L_p$), is then obtained as

$$K_{(l_1,l_2)} = \frac{C_{max}}{C(l_1, l_2)} \tilde{g_1} \tag{1}$$

We assume that the number of slots taken to transmit every primary packet from PT to PR via *any* relay is no greater than the time taken to transmit the same directly i.e.

$$K_{(PT,ST_i)} + K_{(ST_i,PR)} \leq K_{(PT,PR)} \quad \forall 1 \leq i \leq S_{rel} \tag{2}$$

3) Constraint on primary packet scheduling: PT transmits packets whenever its buffer is non-empty. If PT begins transmitting a packet to PR directly at slot t, then for time-slots t,$t+1$,..,$t+K_{(PT,PR)}-1$ it is busy transmitting the same. Instead if at slot t the packet is scheduled to be relayed via ST_j then PT transmits the packet to ST_j during time-slots t-$(t+K_{(PT,ST_j)}-1)$ and during time-slots $t+K_{(PT,ST_j)}$,$t+K_{(PT,ST_j)}+1$,..,$t+K_{(PT,ST_j)}+K_{(ST_j,PR)}-1$ node ST_j relays the packet to PR. Due to (2) cooperative relaying always reduces latency of primary packets as compared to direct transmission.

Figure 1 shows the example of a network with $S_{rel} = 1$, S=3.

B. Primary packet arrival model

We assume within every time slot, $\lambda_p L_p$ bits, where $\lambda_p \in Q$ and Q denotes the set of rational numbers, arrive at constant rate from the upper layers of PT to the transmission layer. Whenever the accumulated data is greater than L_p bits, those L_p bits are aggregated as a primary packet and moved to the transmission queue of PT. Let $A_p(t) \in \{0, 1\}$ denote the number of primary packet arrivals in slot t. For example- when $\lambda_p = \frac{5}{13}$ and there are 0 bits at PT initially, then $A_p(t)$ starting from $t = 1$, is $\{0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, ...\}$. The process is periodic with a period of 13 slots.

C. Secondary packet arrival and transmission model

We assume that every time-slot with probability $\lambda_{s,i}$ $(i = 1, 2, .., S)$ a secondary packet arrives at the link layer of ST_i $(i = 1, 2, .., S)$ from the node's upper layers. For simplicity we assume all secondary transmitter-receiver links have same capacity and the capacity of any such link is 1 packet per slot.

D. Interference model

Our interference model is based on the protocol model of interference whereby a node can transmit to another node within its transmission range and the transmission is successful only if the latter is not within range of another node (including itself) that is transmitting in the same slot. In any slot a link $(l_1, l_2) \in L$ is said to be *active* if node l_1 is succesfully transmitting (i.e. without facing any interference from other nodes) to node l_2; otherwise it is said to be *inactive*. Due to the interference constraints not all links in the network can be simultaneously active. We represent a set of links which can be active simultenously by a *feasible activation vector* that is binary. The length of an activation vector is equal to the total number of possible links i.e. $S + 2S_{rel} + 1$. Without loss of generality the activation vectors are ordered such that the first $2S_{rel} + 1$ components correspond to links that are used to transmit primary packets, while the remaining components correspond to links used to transmit secondary packets. To be precise, in any activation vector E the first component corresponds to the link (PT, PR); the j-th and $(j + S_{rel})$-th component (where $1 \leq j \leq S_{rel}$) of E corresponds to links (PT, ST_j) and (ST_j, PR) respectively; the $(i + 2S_{rel} + 1)$-th component (where $1 \leq i \leq S$) corresponds to link (ST_i, SR_i) respectively. Any component in the activation vector is set to 1 if the corresponding link is active, otherwise it is set to 0. Due to the protocol model of interference a feasible activation vector E is obtained by setting any of its component E_e, corresponding to link $(l_{1e}, l_{2e}) \in L$, to 1 only if $E_{e'} = 0$ for every e' such that $(l_{1e'}, l_{2e'}) \in L$ and l_{2e} is within transmission range of $l_{2e'}$.

The set consisting of all feasible link activation vectors is denoted by χ. We denote the set of all feasible activation vectors, in which the component corresponding to a given link $(l_1, l_2) \in L$ is active, by $I(l_1, l_2)$. Therefore $\forall (l_1, l_2) \in L$, $I(l_1, l_2)$=$\{E \in \chi : E_e = 1\ ,\ 1 \leq e \leq 2S_{rel} + S + 1,\ E_e$ corresponds to link $(l_1, l_2)\}$.

E. Queuing model

Let $U_p(t)$, $U_{s,i}(t)$ denote the queue-length of PT and ST_i $(i = 1, 2, .., S\ )$ at slot t. $U_p(t)$ evolves as

$$U_p(t+1) = U_p(t) - C(t) + A_p(t), \tag{3}$$

where $C(t)$ is an indicator variable which is 1 if a primary packet transmission is completed at t and is 0 otherwise. The queues for ST_i $(i = 1, 2, ..., S)$ evolve as:

$$U_{s,i}(t+1) = \max[U_{s,i}(t) - \mu_{s,i}(t), 0] + A_{s,i}(t), \tag{4}$$

where $\mu_{s,i}(t) \in \{0, 1\}$ is the transmission rate offered (in secondary packets/slot) to ST_i at t for a secondary packet transmission to SR_i,$A_{s,i}(t)$ indicates the number of secondary packet arrivals to ST_i at t .

The offered secondary transmission rate to a secondary transmitter in any time-slot is a binary variable. Therefore the offered secondary transmission rate vector in any time-slot can be obtained from the activation vector in that slot by eliminating from the latter the components corresponding to links used to transmit primary packets. The secondary transmission rate vector obtained from a feasible activation vector $E \in \chi$, by eliminating its first $2S_{rel}+1$ components, is denoted by $\Pi(\boldsymbol{E})$.

If any link is used to transmit primary packets in any time-slot it constrains the transmission rate vectors that can be offered to secondary users in that slot for their own transmissions. Let $I'(l_1, l_2)$ denote the set of all transmission-rate vectors that can be offered to $ST_1, ..., ST_S$ at slot t if the link (l_1, l_2), where $(l_1, l_2) \in L_p$, is active i.e. $I'(l_1,l_2)$=$\{\Pi(\boldsymbol{E}) : E \in I(l_1,l_2)\}$.

For the particular case when no node in the network is transmitting a primary packet in some time-slot, the set of transmission-rate vectors that can be offered to secondary users in that slot is denoted by I'_0. This set can be written as,

$$I'_0 = \{\Pi(\boldsymbol{E}) : E \in \chi, \quad E_e = 0 \quad \forall 1 \leq e \leq 2S_{rel}+1\} \tag{5}$$

F. Scheduling and control model

Whenever PT is about to transmit a new packet, a decision needs to be made about whether the packet is transmitted directly to PR or it will be relayed to PR by a cooperating secondary node. Depending on that decision the scheduling for the next few slots is performed accordingly, subject to the interference constraints mentioned in Section II-D. For example, if the decision is to relay the primary packet via ST_i then for next $K_{(PT,ST_i)}$ slots the secondary transmission-rate vectors used belong to the set $I'(PT,ST_i)$ and for the subsequent $K_{(ST_i,PR)}$ slots the secondary transmission-rate vectors used belong to the set $I'(ST_i,PR)$.

We note for the example in Figure 1 with $K_{(PT,ST_1)} = K_{(ST_1,PR)} = 1$ and $K_{(PT,PR)} = 3$, ST_2 always benefits from cooperation while ST_3 always suffers due to cooperative relay.

III. STABILITY OBJECTIVE

In this section, we observe some properties of the primary packet arrival process and use them to describe a region consisting of secondary arrival-rate vectors. In next section we find an algorithm that stabilizes the network for all secondary arrival-rate vectors in the interior of this region.

A discrete-time queue-length process $\hat{U}(t)$ is said to be strongly stable if $\limsup\limits_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} E\{|\hat{U}(\tau)|\} < \infty$. In this work, by stability we imply strong stability of queue-length processes. A queuing network is said to be stable if the queue-length processes of every queue in the network is stable.

Let f_i $(1 \leq i \leq S_{rel})$ denote the maximum primary arrival-rate λ_p that can be supported if every primary packet is transmitted via relay ST_i i.e. $f_i = \frac{1}{K_{(PT,ST_i)}+K_{(ST_i,PR)}}$. Without loss of generality we assume the ST_1,...,$ST_{S_{rel}}$ are indexed such that $f_j \leq f_{j+1}$ $\forall 1 \leq j \leq S_{rel}-1$. Let f_0 denote the maximum primary arrival-rate λ_p that can be supported if every primary packet is directly transmitted i.e. $f_0 = \frac{1}{K_{(PT,PR)}}$. By our assumption in Section II-A $f_0 \leq f_1$.

Some of the f_i variables maybe equal. We form a vector $\tilde{F}$ consisting of all distinct f_i variables (where $1 \leq i \leq S_{rel}+1$) in which the components are ordered in ascending order of magnitude with the first component being f_0 and the last being $f_{S_{rel}}$.

Next we observe the following characteristics of the packet-arrival process at PT.

Lemma 1: For any $\lambda_p \in Q$, if $\frac{1}{k_1+1} \leq \lambda_p \leq \frac{1}{k_1}$ where $k_1 \in Z^+$, the set of all strictly positive integers, the inter-arrival time between any two primary packets is no greater than k_1+1 slots and no lesser than k_1 slots.

Proof: Let $\zeta(i)$ denote the time-slot when i-th primary packet is created at PT (where $i = 1, 2, ...$). Let ρ be the left-over bits at transmission-layer of PT at $\zeta(i)$ after creation of the i-th primary packet. Then $\rho < \lambda_p L_p$.

Accumulated bits at slot $\zeta(i)+(k_1-1)$ is $\rho+(k_1-1)\lambda_pL_p < k_1\lambda_pL_p \leq L_p$, since $\lambda_p \leq \frac{1}{k_1}$. Also, since $\rho \geq 0$ and $\frac{1}{k_1+1} \leq \lambda_p$, accumulated bits at slot $\zeta(i)+k_1+1$ is $\rho+(k_1+1)\lambda_pL_p \geq L_p$. Therefore, the inter-arrival time between any two primary packets is not greater than k_1+1 slots and no lesser than k_1 slots. ■

Since $\lambda_p \in Q$, $A_p(t)$ is periodic. Let N denote the length of shortest period of $A_p(t)$, let M denote the number of primary packet arrivals in that period. Then λ_p can be expressed as $\lambda_p = \frac{M}{N}$ and M, N are prime to each other. For any $\lambda_p \in Q$, $\frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1}$ (where $k_1 \in Z^+$) we denote by $\kappa^{(1)}(\lambda_p)$ and $\kappa^{(2)}(\lambda_p)$ the number of primary packet arrivals within any interval of length N slots with inter-arrival time of k_1+1 and k_1 slots respectively. Then according to Lemma 1 we have

$$\kappa^{(1)}(\lambda_p)+\kappa^{(2)}(\lambda_p) = M \quad (6)$$

$$(k_1+1)\kappa^{(1)}(\lambda_p)+(k_1)\kappa^{(2)}(\lambda_p) = N \quad (7)$$

For a given primary data arrival-rate $\lambda_p \in Q$ and $\frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} \leq f_{S_{rel}}$ (where $k_1 \in Z^+$) define a region[1] $\Lambda(\lambda_p)$ as the set of secondary arrival rate vectors $(\lambda_{s,1},\lambda_{s,2},...,\lambda_{s,S})^T$ for which there exists variables $R_{s,1},....,R_{s,S}$ and $\pi_0,\pi^{(i)}_{(l_1,l_2)}$ where $(l_1,l_2)\in L_p$, $i=1,2$ such that:

$$\frac{\kappa^{(i)}(\lambda_p)}{N} = \frac{\pi^{(i)}_{(PT,PR)}}{K_{(PT,PR)}} + \sum_{1\leq j\leq S_{rel}} \frac{\pi^{(i)}_{(PT,ST_j)}}{K_{(PT,ST_j)}} \quad i=1,2 \quad (8)$$

$$\pi_0,\pi^{(i)}_{(l_1,l_2)} \geq 0 \quad (l_1,l_2)\in L_p, \quad i=1,2 \quad (9)$$

$$\frac{\pi^{(i)}_{(PT,ST_j)}}{K_{(PT,ST_j)}} = \frac{\pi^{(i)}_{(ST_j,PR)}}{K_{(ST_j,PR)}} \quad i=1,2, \quad 1\leq j\leq S_{rel} \quad (10)$$

$$\pi^{(1)}_{(PT,PR)} = 0 \quad \text{if } K_{(PT,PR)} > k_1+1 \quad (11)$$

$$\pi^{(2)}_{(PT,PR)} = 0 \quad \text{if } K_{(PT,PR)} > k_1 \quad (12)$$

$$\pi^{(1)}_{(PT,ST_j)} = \pi^{(1)}_{(ST_j,PR)} = 0$$

$$\text{if } K_{(PT,ST_j)}+K_{(PT,ST_j)} > k_1+1, 1\leq j\leq S_{rel} \quad (13)$$

$$\pi^{(2)}_{(PT,ST_j)} = \pi^{(2)}_{(ST_j,PR)} = 0$$

$$\text{if } K_{(PT,ST_j)}+K_{(PT,ST_j)} > k_1, 1\leq j\leq S_{rel} \quad (14)$$

$$\pi_{(l_1,l_2)} = \pi^{(1)}_{(l_1,l_2)}+\pi^{(2)}_{(l_1,l_2)} \quad (l_1,l_2)\in L_p \quad (15)$$

$$\pi_0 + \sum_{(l_1,l_2)\in L_p} \pi_{(l_1,l_2)} = 1 \quad (16)$$

$$\lambda_{s,i} \leq R_{s,i} \quad i=1,2,...S \text{ for some } (R_{s,1},...,R_{s,S})^T \in \Gamma \quad (17)$$

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[1] This formulation is similar to the capacity region description in [14].

$$\begin{aligned}\text{where } \Gamma &= \pi_{(PT,PR)}\ \mathit{conv}(I'(PT,PR)) \\ &+ \sum_{j=1}^{S_{rel}}(\pi_{(PT,ST_j)}\ \mathit{conv}(I'(PT,ST_j)) \\ &+ \pi_{(ST_j,PR)}\ \mathit{conv}(I'(ST_j,PR))) \\ &+ \pi_0\ \mathit{conv}(I'_0) \end{aligned} \tag{18}$$

$\pi^{(1)}_{(PT,PR)}$ represents the long-term average probabilities of the event - "PT is directly transmitting a packet with inter-arrival time of k_1+1 slots to PR". $\pi^{(1)}_{(PT,ST_j)}$ and $\pi^{(1)}_{(ST_j,PR)}$ represents the long-term average probabilities of the events- "PT is transmitting a packet with inter-arrival time of k_1+1 slots to ST_j" and "ST_j is transmitting a packet with inter-arrival time of k_1+1 slots to PR" respectively. $\pi^{(2)}_{(PT,PR)}$, $\pi^{(2)}_{(PT,ST_j)}$ and $\pi^{(2)}_{(ST_j,PR)}$ represents long-term average probabilities for similar events for primary packets with inter-arrival time of k_1 slots. The equality (8) is a conservation constraint which indicates that average arrival rate of primary packets of either type is equal to their departure rates. The constraint (10) represents that number of primary packets of either type that enter any relay node is equal to that transmitted by the relay node to PR. Additional constraints are introduced in (11)- (14) which requires that primary packets with inter-arrival times of k_1+1 and k_1 slots are not transmitted directly or via any relay if such a transmission takes more than k_1+1 and k_1 slots respectively. This property is required to use renewal-frame based techniques (to be introduced later) which in turn leads to tractable analysis. $\pi_{(PT,ST_j)}$, $\pi_{(ST_j,PR)}$, π_0 represents the long-term average probabilities of the events - "PT is directly transmitting a packet to PR", "PT is transmitting a packet to ST_j", "ST_j is transmitting a packet to PR", "no primary packet is being transmitted by any node" respectively.The constraint (17) represents the stability condition for secondary transmitters. The "+" operator in (18) indicates Minkowski addition of sets[2]. The " *conv*" of a set of vectors is the set of all possible convex combinations of its elements. The set Γ in equation (18) characterizes the set of feasible secondary transmission-rate vectors subject to the scheduling and interference constraints mentioned in Section II.

When $\lambda_p \in Q$ and $\lambda_p < f_{S_{rel}}$ let $\Lambda_0(\lambda_p)$ denote the set of secondary arrival-rate vectors for which the network is stable without any cooperative relay. This set can be obtained by setting $\pi^{(i)}_{PT,ST_j}, \pi^{(i)}_{ST_j,PR} = 0$ for every $1 \leq j \leq S_{rel}$, $i = 1,2$ in (8)- (18). The set is empty for $\lambda_p > f_0$.

Our main contribution in this work is finding a scheduling algorithm, subject to the scheduling and interference constraints mentioned in Section II such that for a given primary packet arrival-rate $\lambda_p < f_{S_{rel}}$, $\lambda_p \in Q$ and for all secondary arrival rates in the interior of $\Lambda(\lambda_p)$, the network is stable. Since interior of $\Lambda(\lambda_p)$ contains the interior of $\Lambda_0(\lambda_p)$, the algorithm stabilizes the network for all secondary-arrival rate vectors for which the network can be stabilized even without any cooperative relay (ignoring the secondary arrival-rate vectors that form the boundary of $\Lambda_0(\lambda_p)$ for any $\lambda_p \leq f_0$ and $\lambda_p \in Q$). When $f_0 < \lambda_p < f_{S_{rel}}$ and $\lambda_p \in Q$ the interior of $\Lambda(\lambda_p)$ can include non-empty set of secondary arrival-rate vectors for which $\lambda_{s,j} > 0$ for some $1 \leq j \leq S$ such that arrival-rate $\lambda_{s,j}$ could not be supported by the network in absence of cooperation if λ_p were f_0. The proposed algorithm therefore

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[2] The Minkowski addition of two sets of vectors A and B is the set formed by adding every vector in A to every vector in B i.e. the set $\{a+b|a \in A, b \in B\}$ [15].

results in a win-win scenario for such an ST_j and PT. For the example in Figure 1, if $K_{(PT,ST_1)} = K_{(ST_1,PR)} = 1$ and $K_{(PT,PR)} = 3$ cooperation can result in win-win scenario for ST_2 and PT. For every $\lambda_p \in Q$, ignoring the set of arrival-rate vectors that are at the boundary of $\Lambda_0(\lambda_p)$ whenever it is non-empty, the set of secondary arrival-rate vectors that can be stabilized is therefore expanded under this cooperative scheduling algorithm.

IV. DYNAMIC RELAYING AND SCHEDULING POLICY

In this section we develop a dynamic Scheduling and Cooperative Relay Policy (SCRP) that, for all $\lambda_p \in Q$, $\lambda_p < f_{S_{rel}}$, satisfies the stability objective mentioned in the previous section. If in any slot the transmission queue at PT is non-empty and no primary packet is being transmitted by any node in the network, the network controller schedules transmission of a primary packet from PT either directly or via some secondary relay node by solving a max-weight problem. The algorithm uses information about instantaneous queue-lengths at secondary transmitters and the inter-arrival time of the Head-of-Line (H.O.L) primary packets at PT. The H.O.L packet is transmitted via a secondary relay or directly such that the overall transmission time is less than or equal to interarrival time of the packet. The instantaneous secondary transmission rate vectors are then obtained by solving a related max-weight problem.

If at the current slot, there is no primary packet at PT, it is considered an *idle slot*. All the slots when the transmission of j-th primary packet ($j = 1, 2, ...$) takes place is said to constitute the j-th *busy period*. Such a busy period always consists of contiguous time-slots because according to our primary transmission model, every time a relaying secondary node receives a primary packet it begins transmitting the same in the very next slot. Any time-slot when the network is in a busy period is called a *busy slot*.

Every time-slot the network controller observes the queue-length of PT, ST_i (where $i = 1, 2,.., S$) and the inter-arrival time of primary packets present at PT. Let $(U_{s,i}(t))_{i=1}^{S}$ and $(\mu_{s,i}(t))_{i=1}^{S}$ denote the queue-length vector $(U_{s,1}(t), ..., U_{s,S}(t))^T$ and secondary transmission rate-vector $(\mu_{s,1}(t), ..., \mu_{s,S}(t))^T$ at slot t. Based on the above knowledge, the algorithm makes the following scheduling and relay decisions:

1) *Scheduling decision in idle slots:* At any idle slot t, the network selects a secondary transmission-rate vector $(\mu_{s,i}(t))_{i=1}^{S}$ according to a max-weight scheduling policy:

$$(\mu_{s,i}(t))_{i=1}^{S} \in \underset{v \in I'_0}{\operatorname{argmax}} ((U_{s,i}(t))_{i=1}^{S})^T v \tag{19}$$

2) *Cooperative relaying decisions in busy slots:* If the transmission queue of PT is non-empty and the H.O.L packet in its queue is not being served currently at slot t, then its service begins at t in the following manner: (i) For each possible link $(l_1, l_2) \in L_p$ that can be used to send a primary packet find the scheduling vector that maximizes the following:

$$v^*_{(l_1,l_2)}(t) = \underset{v \in I'(l_1,l_2)}{\operatorname{argmax}} \, ((U_{s,i}(t))_{i=1}^{S})^T v \tag{20}$$

We also find the transmission-rate vector that maximizes the following max-weight expression over all transmission-rate vectors in set I_0',

$$v_0^*(t) = \underset{v \in I_0'}{\text{argmax}}((U_{s,i}(t))_{i=1}^S)^T v \tag{21}$$

(ii) If the inter-arrival time of the H.O.L primary packet is less than or equal to $\frac{1}{f_0}$ slots, then solve the following max-weight problem:

$$\begin{aligned}
&\max(K_{(PT,PR)}((U_{s,i}(t))_{i=1}^S)^T v^*_{(PT,PR)}(t), ((U_{s,i}(t))_{i=1}^S)^T \\
&\{K_{(PT,ST_1)} v^*_{(PT,ST_1)}(t) + K_{(ST_1,PR)} v^*_{(ST_1,PR)}(t) + \\
&(K_{(PT,PR)} - K_{(PT,ST_1)} - K_{(ST_1,PR)}) v_0^*(t)\}, ..., \\
&((U_{s,i}(t))_{i=1}^S)^T \{K_{(PT,ST_{S_{rel}})} v^*_{(PT,ST_{S_{rel}})}(t) \\
&+K_{(ST_{S_{rel}},PR)} v^*_{(ST_{S_{rel}},PR)}(t) + (K_{(PT,PR)} \\
&-K_{(PT,ST_{Srel})} - K_{(ST_{Srel},PR)}) v_0^*(t)\})
\end{aligned} \tag{22}$$

(iii) Otherwise if the inter-arrival time of the H.O.L primary packet is less than or equal to $\frac{1}{f_{S_{rel}}}$ slots but greater than $\frac{1}{f_0}$ slots, then solve the following max-weight problem:

$$\begin{aligned}
&\max(((U_{s,i}(t))_{i=1}^S)^T \{K_{(PT,ST_k)} v^*_{(PT,ST_k)}(t) \\
&+K_{(ST_k,PR)} v^*_{(ST_k,PR)}(t)\}, ((U_{s,i}(t))_{i=1}^S)^T \{ \\
&K_{(PT,ST_{k+1})} v^*_{(PT,ST_{k+1})}(t) + K_{(ST_{k+1},PR)} v^*_{(ST_{k+1},PR)}(t) \\
&+(K_{(PT,ST_k)} + K_{(ST_k,PR)} - K_{(PT,ST_{k+1})} - K_{(ST_{k+1},PR)}) \\
&v_0^*(t)\}, .., ((U_{s,i}(t))_{i=1}^S)^T \{K_{(PT,ST_{S_{rel}})} v^*_{(PT,ST_{S_{rel}})}(t) \\
&+K_{(ST_{S_{rel}},PR)} v^*_{(ST_{S_{rel}},PR)}(t) + (K_{(PT,ST_k)} + K_{(ST_k,PR)} \\
&-K_{(PT,ST_{S_{rel}})} - K_{(ST_{S_{rel}},PR)}) v_0^*(t)\})
\end{aligned} \tag{23}$$

where ST_k $(1 \leq k \leq S_{rel})$ is such that $f_{k-1} < \lambda_p \leq f_k$.

(iv) If there is some ST_{i^*} that maximizes (22) or (23) (depending on the inter-arrival time of the H.O.L primary packet) then use that particular ST_{i^*} as relay (in case of multiple solutions pick an ST_{i^*} arbitrarily). Transmit the H.O.L primary packet from PT to ST_{i^*} in slots t, $t+1$,..,$t + K_{(PT,ST_{i^*})} - 1$ and from ST_{i^*} to PR in slots $t + K_{(PT,ST_{i^*})}$,$t + K_{(PT,ST_{i^*})} + 1$,..,$t + K_{(PT,ST_{i^*})} + K_{(ST_{i^*},PR)} - 1$. If no such ST_{i^*} is the solution of (22) then directly transmit the primary packet to PR in slots t, $t+1$,..,$t + K_{(PT,PR)} - 1$.

3) *Secondary scheduling decisions in busy slots:* Suppose the decision about transmitting the primary packet in the previous step was to relay the same via ST_{i^*}. Then the secondary transmission rate-vector to be used in slots t,$t+1$,..,$t + K_{(PT,ST_{i^*})} + K_{(ST_{i^*},PR)} - 1$ are obtained as follows:

(i) For slots $\tau \in [t,t+K_{(PT,ST_{i^*})}-1]$ use transmission rate vector $(\mu^*_{s,1}(\tau),...,\mu^*_{s,S}(\tau))^T$ which is obtained as

$$(\mu^*_{s,i}(\tau))_{i=1}^S \in \underset{v\in I'(PT,ST_{i^*})}{\text{argmax}} \; ((U_{s,i}(\tau))_{i=1}^S)^T v \tag{24}$$

(ii) For slots $\tau \in [t+K_{(PT,ST_{i^*})},t+K_{(PT,ST_{i^*})}+K_{(ST_{i^*},PR)}-1]$ use transmission rate vector $(\mu^*_{s,1}(\tau),...,\mu^*_{s,S}(\tau))^T$ which is obtained as

$$(\mu^*_{s,i}(\tau))_{i=1}^S \in \underset{v\in I'(ST_{i^*},PR)}{\text{argmax}} \; ((U_{s,i}(\tau))_{i=1}^S)^T v \tag{25}$$

If the decision about transmission of primary packet was to directly transmit the same then the secondary transmission rate-vector $(\mu^*_{s,1}(\tau),...,\mu^*_{s,S}(\tau))^T$to be used in slots $\tau \in [t,t+K_{(PT,PR)}-1]$ are obtained as:

$$(\mu^*_{s,i}(\tau))_{i=1}^S \in \underset{v\in I'(PT,PR)}{\text{argmax}} \; ((U_{s,i}(\tau))_{i=1}^S)^T v \tag{26}$$

4) *Transmission and queue-update:* For $i = 1,2,...,S$ transmit $\min(\mu^*_{s,i}(t),U_{s,i}(t))$ secondary packets from ST_i in slot t. If t is the last slot in any busy period, remove the H.O.L primary packet from PT's transmission queue at the end of t.

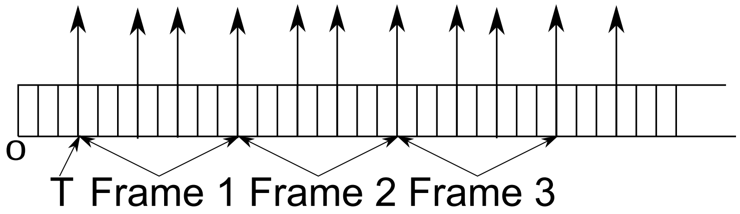
We show that under SCRP, for secondary packet arrival rates in the interior of the region described in Section III the queue-length processes in the network are strongly stable.

Theorem 1: For all $\lambda_p \in Q$, $\lambda_p < f_{S_{rel}}$, under the SCRP policy $U_p(t)$ and $U_{s,i}(t)$ $(i = 1,2,...S)$ are strongly stable for all secondary arrival-rates in the interior of $\Lambda(\lambda_p)$.

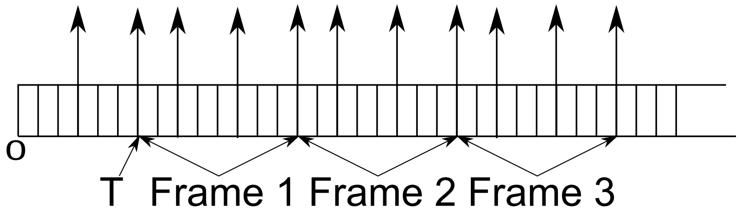
When $\lambda_p = 0$ the algorithm reduces to traditional Back-pressure theorem with capacity region $\Lambda(0)$ whose proof can be found in [9]. For the case when $\lambda_p \neq 0$ the theorem is proven in next section.

V. STABILITY ANALYSIS

In this section we prove Theorem 1. The proof uses the concept of renewal frames and relies on comparing SCRP against some other policies that are developed using the renewal frame structure. In this section we present the construction of renewal frames, description of those aforementioned policies, some useful lemmas and finally the proof of Theorem 1. We first describe the construction of renewal frames for our system model through appropriate partitioning of the time-line. Renewal frame based techniques typically use policies for which the system state is refreshed at the beginning of every frame (to be described later). We identify a class of such policies in the context of our problem and present a Stationary Scheduling Policy (SSP) and three alternate policies ALT1, ALT2 and ALT3 that belongs to this class. SSP is defined for $\lambda_p \leq f_{S_{rel}}$ and performs scheduling independent of the length of queues corresponding to secondary packets. ALT1 and ALT2 are defined for $\lambda_p \leq f_0$ and $\lambda_p \in Q$. Throughout every frame ALT1 makes scheduling decisions based on secondary queue-lengths at the beginning of the frame. ALT2 performs scheduling in idle slots like ALT1 while in other slots it performs scheduling like SCRP. We also present Lemmas 2-6. SSP, ALT1, ALT2 along with Lemmas 2-5 will be used to prove stability of SCRP when $\lambda_p \leq f_0$ and $\lambda_p \in Q$. ALT3 is defined for $f_0 < \lambda_p < f_{S_{rel}}$, $\lambda_p \in Q$ and performs scheduling throughout every frame based on secondary queue-lengths at the beginning of the frame. SSP and ALT3 along with Lemmas 2, 3



(a) $T = \zeta(1) = 3$



(b) $T = \zeta(2) = 6$

Fig. 2. Partition of time-line into frames when $\lambda_p = \frac{3}{8}$. Each small rectangle represents a time-slot. The arrival of a primary packet at the transmission queue of PT during any time-slot is indicated by a vertical arrow at the boundary between the time-slot and the one immediately after it.

and 6 will be used to prove stability of SCRP when $f_0 < \lambda_p < f_{S_{rel}}$ and $\lambda_p \in Q$. Finally the proof of Theorem 1 is provided at the end of the section.

A. Partitioning time-line into frames

For every $\lambda_p \in Q$ assuming the network was initialized at $t = 0$ when all the queues in the network were empty, the time-line can be partitioned into a finite interval $[0, T]$ and successive non-overlapping frames of length N slots each as: $[T+1, T+N]$, $[T+N+1, T+2N]$,..... By setting T to different $\zeta(j)$ where $j \in \{1, 2, ...N\}$, we obtain different partitions of the time-line. Figure 2 shows two partitions of time-lines when $\lambda_p = \frac{3}{8}$ by choosing T to be $\zeta(1)$ and $\zeta(2)$ respectively. Given a partition of time-lines, we denote the k-th indexed slot (where $k = 1, 2, .., N$) in r-th frame by $t_{r,k}$ i.e. $t_{r,k} \overset{\triangle}{=} T + k + (r-1)N$ for $r = 1, 2, ...$.

B. Relevant classes of high priority scheduling policies for primary packets

In our analysis we use renewal frame based optimization techniques as described in (Chapter 7, [16]). The frame sizes are constant in our analysis and therefore our problem is a special case of the variable frame-based optimization problems described there. If we think of the $S_{rel}+1$-dimensional vector consisting of primary packet queue-lengths at PT and ST_i ($1 \le i \le S_{rel}$) as "state" of the network, then in order to apply renewal frame based techniques we need to make sure that the system state is refreshed at the beginning of every frame. In this work we use a class of policies referred to as "non-idling and clearing for primary ($n.i.c.p$)", described below, which satisfies that requirement.

We call a scheduling and relaying policy to be "non-idling for primary ($n.i.p$)" if the transmission process of some primary packet is on-going at every time-slot t when $U_p(t) > 0$. $n.i.p$ policies thus ensure high priority for primary packet transmissions in the network. For a given partition of time-lines into frames, we call a policy to be $n.i.c.p$ if it is n.i.p and M primary packets are transmitted every frame. Since when $\lambda_p \leq f_0$ either directly transmitting or relaying any primary packet result in transmission of M primary packets in every frame, every n.i.p policy is n.i.c.p when $\lambda_p \leq f_0$. This is not true when $f_0 < \lambda_p < f_{S_{rel}}$.

C. *Stationary randomized policy*

For any $(\lambda_{s,1}, \lambda_{s,2}, .., \lambda_{s,S})^T \in$ Interior $(\Lambda(\lambda_p)$ (where $\lambda_p \in Q$) there exists $\epsilon > 0$ such that the arrival-rate vector $(\lambda_{s,1}+\epsilon, \lambda_{s,2}+\epsilon, ..., \lambda_{s,S}+\epsilon)^T \in$ Interior $(\Lambda(\lambda_p))$ $(i = 1, 2, ..., S)$. Using a similar approach as in [14], we can show the following:

Lemma 2: If $\lambda_p \in Q$ and $\frac{1}{k_1+1} < \lambda_p \leq \frac{1}{k_1} \leq f_0$ for some $k_1 \in Z^+$, partition the time-line by setting T, as mentioned in Section V-A, to be $\zeta(1)$. Otherwise if $\lambda_p \in Q$ and $f_0 \leq \frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} \leq f_{S_{rel}}$ for some $k_1 \in Z^+$, set T to be $\zeta(e-1)$ where e is the smallest non-negative integer such that inter-arrival time of e-th primary packet is k_1 slots and that of $(e-1)$-th primary packet is k_1+1 slots. Such a variable e exists because $\zeta(1)$ is always k_1+1 slots due to Lemma 1. (For example, for the arrival process in Fig. 2, if $f_0 = \frac{1}{3}$ and $f_1 = f_2 = ... = f_{S_{rel}} = \frac{1}{2}$, e is 3)

Then for all arrival-rate vector $(\lambda_{s,1}+\epsilon, \lambda_{s,2}+\epsilon, ..., \lambda_{s,S}+\epsilon)^T \in$ Interior $(\Lambda(\lambda_p))$, where $\lambda_p < f_{S_{rel}}$ and $\lambda_p \in Q$, there exists an n.i.c.p stationary scheduling policy SSP that makes scheduling and relaying decisions based on knowledge of the primary and secondary arrival rates but independent of the queue-lengths of secondary transmitters and under which for all $r = 1, 2, ...,$

$$E[\sum_{\tau=T+1+(r-1)N}^{\tau=T+rN} \mu_{s,i}^{SSP}(\tau)] \geq (\lambda_{s,i}+\epsilon)N \quad \forall i = 1, 2, .., S \tag{27}$$

Proof: Let the vectors in the set $I'(l_1, l_2)$ where $(l_1, l_2) \in L_p$ be indexed as $v_{(l_1,l_2),1}, ..., v_{(l_1,l_2),|I'(l_1,l_2)|}$; the vectors in set I'_0 be indexed as $v_{0,1}, ..., v_{0,|I'_0|}$.

Since $(\lambda_{s,1}+\epsilon, \lambda_{s,2}+\epsilon, ..., \lambda_{s,S}+\epsilon)^T \in$ Interior $(\Lambda(\lambda_p)$ there exists π_0, $\pi_{(l_1,l_2)}$, $p^{SSP}_{(l_1,l_2),n_1}$ and p^{SSP}_{0,n_2} (where $(l_1, l_2) \in L_p$, $1 \leq n_1 \leq |I'(l_1, l_2)|$, $1 \leq n_2 \leq |I'_0|$) s.t.

$$\frac{\kappa^{(i)}(\lambda_p)}{N} = \frac{\pi^{(i)}_{(PT,PR)}}{K_{(PT,PR)}} + \sum_{1\leq j\leq S_{rel}} \frac{\pi^{(i)}_{(PT,ST_j)}}{K_{(PT,ST_j)}} \quad i = 1, 2 \tag{28}$$

$$\pi^{(i)}_{(l_1,l_2)} \geq 0 \quad (l_1, l_2) \in L_p, \quad i = 1, 2 \tag{29}$$

$$\frac{\pi^{(i)}_{(PT,ST_j)}}{K_{(PT,ST_j)}} = \frac{\pi^{(i)}_{(ST_j,PR)}}{K_{(ST_j,PR)}} \quad i = 1, 2, \quad 1 \leq j \leq S_{rel} \tag{30}$$

$$\pi^{(1)}_{(PT,PR)} = 0 \quad \text{if } K_{(PT,PR)} > \frac{1}{k_1+1} \tag{31}$$

$$\pi^{(2)}_{(PT,PR)} = 0 \quad \text{if } K_{(PT,PR)} > \frac{1}{k_1} \tag{32}$$

$$\pi^{(1)}_{(PT,ST_j)} = \pi^{(1)}_{(ST_j,PR)} \quad = \quad 0 \quad \text{if } K_{(PT,ST_j)} + K_{(PT,ST_j)} > \frac{1}{k_1+1}, 1 \leq j \leq S_{rel} \tag{33}$$

$$\pi^{(2)}_{(PT,ST_j)} = \pi^{(2)}_{(ST_j,PR)} \quad = \quad 0 \quad \text{if } K_{(PT,ST_j)} + K_{(PT,ST_j)} > \frac{1}{k_1}, 1 \leq j \leq S_{rel} \tag{34}$$

$$\pi_{(l_1,l_2)} \quad = \quad \pi^{(1)}_{(l_1,l_2)} + \pi^{(2)}_{(l_1,l_2)} \quad (l_1,l_2) \in L_p \tag{35}$$

$$\pi_0 + \sum_{(l_1,l_2)\in L_p} \pi_{(l_1,l_2)} \quad = \quad 1 \tag{36}$$

$$\lambda_{s,n} + \epsilon \quad \leq \quad R_{s,n} \quad n = 1,2,...S \tag{37}$$

$$R = \pi_{(PT,PR)} \sum_{i=1}^{|I'(PT,PR)|} p^{SSP}_{(PT,PR),i} v_{(PT,PR),i} + \sum_{j=1}^{S_{rel}} \{(\pi_{(PT,ST_j)} \sum_{i=1}^{|(I'(PT,ST_j)|} p^{SSP}_{(PT,ST_j),i} v_{(PT,ST_j),i})$$

$$+(\pi_{(ST_j,PR)} \sum_{i=1}^{|(I'(ST_j,PR)|} p^{SSP}_{(ST_j,PR),i} v_{(ST_j,PR),i})\} + \pi_0 \sum_{i=1}^{|I'_0|} p^{SSP}_{0,i} v_{0,i}) \tag{38}$$

$$\sum_{i=1}^{|I'(PT,PR)|} p^{SSP}_{(PT,PR),i} = 1, \quad \sum_{i=1}^{|I'|} p^{SSP}_{0,i} = 1 \tag{39}$$

$$\sum_{i=1}^{|I'(PT,ST_j)|} p^{SSP}_{(PT,ST_j),i} = 1, \quad \sum_{i=1}^{|I'(ST_j,PR)|} p^{SSP}_{(ST_j,PR),i} = 1 \text{ where } 1 \leq j \leq S_{rel} \tag{40}$$

The policy SSP is then:

(i) Relay all primary packets with inter-arrival time of k_1+1 slots (respectively k_1 slots) via ST_j where $1 \leq j \leq S_{rel}$ with probability $\frac{\pi^{(1)}_{(PT,ST_j)}N}{K_{(PT,ST_j)}\kappa^{(1)}(\lambda_p)}$ (resp. $\frac{\pi^{(2)}_{(PT,ST_j)}N}{K_{(PT,ST_j)}\kappa^{(2)}(\lambda_p)}$) or directly transmit with probability $\frac{\pi^{(1)}_{(PT,PR)}N}{K_{(PT,PR)}\kappa^{(1)}(\lambda_p)}$ (resp. $\frac{\pi^{(2)}_{(PT,PR)}N}{K_{(PT,PR)}\kappa^{(2)}(\lambda_p)}$).

(ii) At time-slots when a primary packet is being directly transmitted use $v_{(PT,PR),i}$ (where $1 \leq i \leq |I'(PT,PR)|$) as secondary rate-transmission vector with probability $p^{SSP}_{(PT,PR),i}$. Otherwise, when a primary packet is being transmitted from PT to ST_j (where $1 \leq j \leq S_{rel}$) use $v_{(PT,ST_j),i}$ as secondary rate-transmission vector with probability $p^{SSP}_{(PT,ST_j),i}$ (where $1 \leq i \leq |I'(PT,ST_j)|$); when a primary packet is being relayed from ST_j use $v_{(ST_j,PR),i}$ as secondary rate-transmission vector with probability $p^{SSP}_{ST_j,PR,i}$ (where $1 \leq i \leq |I'(ST_j,PR)|$). For the other time-slots use $v_{(0,i)}$ with probability $p^{SSP}_{0,i}$ where $1 \leq i \leq |I'_0|$.

Due to choice of T and the scheduling scheme, number of packets transmitted in r-th frame ($r = 1,2,...$) is M and the policy is n.i.c.p. Therefore from (38) and (37) we obtain (27). ■

D. Alternate policy 1 (ALT1)

The algorithm ALT1 is defined for $\lambda_p \leq f_0$ and $\lambda_p \in Q$ and consists of the following steps:

1) *Identify frames:* Set T to be $\zeta(1)$.
2) *Primary packet transmission policy:* At the beginning of r-th frame ($r = 1,2,...$) the information about the queue-length vector $(U_{s,i}(t_{r,1}))_{i=1}^{S}$ is used to determine transmission scheme for $(1+(r-1)N)$-th to (rN)-th primary packets in the following way:

(i) Obtain $v^*_{(l_1,l_2)}(t_{r,1})$ and $v^*_0(t_{r,1})$ $\forall (l_1,l_2) \in L_p$ in the same way as in SCRP.

(ii) Solve the following max-weight problem:

$$\max(((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,PR)}(v^*_{(PT,PR)}(t_{r,1})-v^*_0(t_{r,1}))\},((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,ST_1)}(v^*_{(PT,ST_1)}(t_{r,1}) -v^*_0(t_{r,1}))+K_{(ST_1,PR)}(v^*_{(ST_1,PR)}(t_{r,1})-v^*_0(t_{r,1}))\},...,((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,ST_{S_{rel}})}(v^*_{(PT,ST_{S_{rel}})}(t_{r,1}) -v^*_0(t_{r,1}))+K_{(ST_{S_{rel}},PR)}(v^*_{(ST_{S_{rel}},PR)}(t_{r,1})-v^*_0(t_{r,1}))\}) \tag{41}$$

(iii) If at some time-slot t (where $t_{r,1} \leq t \leq t_{r,1}+(N-1)$) $U_p(t) > 0$ and no primary packet is being transmitted anywhere in the network then begin transmission of the H.O.L packet at PT at t. If there is some ST_{i^*} ($1 \leq i^* \leq S_{rel}$) that maximizes (41) then use that particular ST_{i^*} as relay (in case of multiple solutions pick an ST_{i^*} arbitrarily); transmit the H.O.L primary packet from PT to ST_{i^*} in slots t, $t+1$,..,$t+K_{(PT,ST_{i^*})}-1$ and from ST_{i^*} to PR in slots $t+K_{(PT,ST_{i^*})}$,$t+K_{(PT,ST_{i^*})}+1$,..,$t+K_{(PT,ST_{i^*})}+K_{(ST_{i^*},PR)}-1$. If in (41) no such ST_{i^*} exist then directly transmit the primary packet to PR in slots t, $t+1$,..,$t+K_{PT,PR}-1$.

3) *Scheduling for secondary nodes:* For the busy-slots created in r-th frame use $v^*_{(PT,ST_{i^*})}(t_{r,1})$ when PT is transmitting to ST_{i^*} and $v^*_{(ST_{i^*},PR)}(t_{r,1})$ when ST_{i^*} is transmitting to PR. For the idle slots in r-th frame use $v^*_0(t_{r,1})$ as secondary rate-transmission vector. For the slots prior to T arbitrarily use any $v \in I'_0$ as secondary rate-transmission vector.

E. Alternate policy 2 (ALT2)

The alternate algorithm ALT2 is defined for $\lambda_p \leq f_0$ and $\lambda_p \in Q$ and consists of the following steps:

1) *Identify frames:* Set T to be $\zeta(1)$.
2) *Primary packet transmission policy:* Make decisions about relaying a primary packet and secondary transmission-rate vector assignments in the busy-slots according to SCRP.
3) *Scheduling in idle slots:* For every idle slot in r-th frame $(r = 1, 2, ...)$ make secondary transmission rate-vector assignments in the same way as in ALT1 i.e. compute $v^*_0(t_{r,1})$ using (21) and then use it as secondary rate transmission vector for all idle slots in r-th frame. For slots prior to T use any $v \in I'_0$ arbitrarily.

F. Alternate policy 3 (ALT3)

Algorithm ALT3 is defined for $f_{S_{rel}} \geq \frac{1}{k_1} > \lambda_p \geq \frac{1}{k_1+1} \geq f_0$, $\lambda_p \in Q$, $k_1 \in Z^+$ and consists of the following steps:

1) *Identify frames:* Set T to be $\zeta(e-1)$ where e is the smallest non-negative integer such that inter-arrival time of e-th primary packet is k_1 slots and that of $e-1$-th primary packet is $k+1$ slots.
2) *Primary packet transmission policy:* In the r-th frame $(r = 1, 2, ...)$ the information of queue-length vector $(U_{s,i}(t_{r,1}))_{i=1}^{S}$ is used to determine the transmission scheme for $e-1+(r-1)N$ to $e-2+rN$-th primary packets in the following way:

   (i) Obtain $v^*_{(l_1,l_2)}(t_{r,1})$ and $v^*_0(t_{r,1})$ $\forall (l_1,l_2) \in L_p$ in the same way as in ALT1.

(ii) For a primary packet with inter-arrival time of (k_1+1) slots solve the following max-weight problem if $\frac{1}{k_1+1}=f_0$:

$$\max(((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,PR)}(v^*_{(PT,PR)}(t_{r,1})-v^*_0(t_{r,1}))\},((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,ST_1)}(v^*_{(PT,ST_1)}(t_{r,1}) -v^*_0(t_{r,1}))+K_{(ST_1,PR)}(v^*_{(ST_1,PR)}(t_{r,1})-v^*_0(t_{r,1}))\},...,((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,ST_{S_{rel}})} (v^*_{(PT,ST_{S_{rel}})}(t_{r,1})-v^*_0(t_{r,1}))+K_{(ST_{S_{rel}},PR)}(v^*_{(ST_{S_{rel}},PR)}(t_{r,1})-v^*_0(t_{r,1}))\}) \tag{42}$$

Otherwise for a primary packet with inter-arrival time of (k_1+1) slots solve the following max-weight problem if $\frac{1}{k_1+1}\neq f_0$:

$$\max(((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,ST_k)}v^*_{(PT,ST_k)}(t_{r,1})+K_{(ST_k,PR)}v^*_{(ST_k,PR)}(t_{r,1})\},((U_{s,i}(t_{r,1}))_{i=1}^{S})^T \{K_{(PT,ST_{k+1})}v^*_{(PT,ST_{k+1})}(t_{r,1})+K_{(ST_{k+1},PR)}v^*_{(ST_{k+1},PR)}(t_{r,1})+(K_{(PT,ST_k)}+K_{(ST_k,PR)}-K_{(PT,ST_{k+1})} -K_{(ST_{k+1},PR)})v^*_0(t_{r,1})\},..,((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,ST_{S_{rel}})}v^*_{(PT,ST_{S_{rel}})}(t_{r,1})+K_{(ST_{S_{rel}},PR)}v^*_{(ST_{S_{rel}},PR)}(t_{r,1}) +(K_{(PT,ST_k)}+K_{(ST_k,PR)}-K_{(PT,ST_{S_{rel}})}-K_{(ST_{S_{rel}},PR)})v^*_0(t_{r,1})\}) \tag{43}$$

where ST_k is such that $f_{k-1}<\frac{1}{k_1+1}\leq f_k$ for some k where $1\leq k\leq S_{rel}$. Similarly for a primary packet with inter-arrival time of k_1 slots solve the following problem:

$$\max(((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,ST_k)}v^*_{(PT,ST_k)}(t_{r,1})+K_{(ST_k,PR)}v^*_{(ST_k,PR)}(t_{r,1})\},((U_{s,i}(t_{r,1}))_{i=1}^{S})^T \{K_{(PT,ST_{k+1})}v^*_{(PT,ST_{k+1})}(t_{r,1})+K_{(ST_{k+1},PR)}v^*_{(ST_{k+1},PR)}(t_{r,1})+(K_{(PT,ST_k)}+K_{(ST_k,PR)}-K_{(PT,ST_{k+1})} -K_{(ST_{k+1},PR)})v^*_0(t_{r,1})\},..,((U_{s,i}(t_{r,1}))_{i=1}^{S})^T\{K_{(PT,ST_{S_{rel}})}v^*_{(PT,ST_{S_{rel}})}(t_{r,1})+K_{(ST_{S_{rel}},PR)}v^*_{(ST_{S_{rel}},PR)}(t_{r,1}) +(K_{(PT,ST_k)}+K_{(ST_k,PR)}-K_{(PT,ST_{S_{rel}})}-K_{(ST_{S_{rel}},PR)})v^*_0(t_{r,1})\}) \tag{44}$$

where ST_k is such that $f_{k-1<\frac{1}{k_1}\leq f_k}$ for some k where $1\leq k\leq S_{rel}$.

(iii) As in ALT1, use the solution of (42), (43) and (44) to select relays for primary packets numbered $e-1+(r-1)N$ to $e-2+rN$. Prior to T transmit any primary packet of inter-arrival time k_1 slots (resp. k_1+1 slots) using any relay for which the overall transmission consists of less than or equal to k_1 slots (resp. k_1+1 slots).

3) *Scheduling in idle slots:* Suppose ST_{i^*} (where $1\leq i^*\leq S_{rel}$) is the solution of (42) or (43) (whichever is applicable for given λ_p). Then in r-th frame use $v^*_{(PT,ST_{i^*})}(t_{r,1})$ as secondary rate-transmission vector in the slots when PT is transmitting a primary packet with inter-arrival time of $k+1$ slots to ST_{i^*} and $v^*_{(ST_{i^*},PR)}(t_{r,1})$ when ST_{i^*} is transmitting to PR. Similarly if ST_{i^*} (where $1\leq i^*\leq S_{rel}$) is the solution of (44) then in r-th frame use $v^*_{(PT,ST_{i^*})}(t_{r,1})$ as secondary rate-transmission vector in the slots when PT is transmitting a primary packet with inter-arrival time of k slots to ST_{i^*} and $v^*_{(ST_{i^*},PR)}(t_{r,1})$ when ST_{i^*} is transmitting to PR. For the idle slots in r-th frame use $v^*_0(t_{r,1})$ as secondary rate-transmission vector. For the slots prior to T arbitrarily use any $v\in I'_0$ as secondary rate-transmission vector.

For any n.i.c.p policy ϕ and $\lambda_p \in Q$, we define the function $\psi^{\phi}(t_{r,1})$ $(r = 1, 2, ...)$ as

$$\psi^{\phi}(t_{r,1}) \triangleq \sum_{i=1}^{S} U_{s,i}(t_{r,1}) E[\sum_{t=t_{r,1}}^{t_{r,1}+N-1} \mu^{\phi}_{s,i}(t) | (U_{s,i}(t_{r,1}))_{i=1}^{S}], \tag{45}$$

Lemma 3: For any $\lambda_p \in Q$ and given secondary arrival-rate vector $(\lambda_{s,i})_{i=1}^{S} \in$ Interior$(\Lambda(\lambda_p))$, define a policy SSP, using the procedure in proof of Lemma 2, for arrival-rate vector $(\lambda_{s,i} + \epsilon)_{i=1}^{S} \in$ Interior$(\Lambda(\lambda_p))$ where $\epsilon > 0$.

1) If $\lambda_p \leq f_0$, partition the time-line similarly as in ALT1. Then for every $r = 1, 2, ...$

$$\psi^{ALT1}(t_{r,1}) \geq \psi^{SSP}(t_{r,1}). \tag{46}$$

2) If $f_{S_{rel}} > \lambda_p > f_0$, partition the time-line similarly as in ALT3. Then for every $r = 1, 2, ...$

$$\psi^{ALT3}(t_{r,1}) \geq \psi^{SSP}(t_{r,1}). \tag{47}$$

Proof: It can be easily shown that for $r = 1, 2, ...$ and $\lambda_p \leq f_0$, $\lambda_p \in Q$ and for any n.i.c.p policy X, $\psi^{ALT1}(t_{r,1}) \geq \psi^{X}(t_{r,1})$. Similarly it can also be shown that for $r = 1, 2, ...$ and $f_0 < \lambda_p < f_{S_{rel}}$, $\lambda_p \in Q$, $\psi^{ALT3}(t_{r,1}) \geq \psi^{X}(t_{r,1})$ for all n.i.c.p policies X under which every primary packet with a certain inter-arrival time is always transmitted directly or via relay in such a way that overall transmission time is less than the inter-arrival time. Since SSP is an n.i.c.p policy satisfying the respective requirements mentioned above, (46) and (47) are true. ■

Lemma 4: For every $\lambda_p \leq f_0$, $\lambda_p \in Q$, partition the time-line similarly as in ALT1. Then for every $r = 1, 2, ...$

$$\psi^{ALT2}(t_{r,1}) \geq \psi^{ALT1}(t_{r,1}) - B_1, \tag{48}$$

where $B_1 > 0$ is a finite constant.

Proof: Proof is provided in Appendix A ■

Lemma 5: For every $\lambda_p \leq f_0$, $\lambda_p \in Q$, partition the time-line similarly as in ALT2. Then for every $r = 1, 2, ...$

$$\psi^{SCRP}(t_{r,1}) \geq \psi^{ALT2}(t_{r,1}) - B_2, \tag{49}$$

where $B_2 \geq 0$ is a finite constant.

Proof: Proof provided in Appendix B. ■

Lemma 6: If $f_0 < \lambda_p < f_{s_{rel}}$, $\lambda_p \in Q$, $r = 1, 2, ..$, and the partition of the time-line is done according to Step 1 of ALT3,

$$\psi^{SCRP}(t_{r,1}) \geq \psi^{ALT3}(t_{r,1}) - B_3, \tag{50}$$

where $B_3 > 0$ is a finite constant.

Proof: Proof provided in Appendix C. ■

Proof of Theorem 1: Consider a secondary arrival-rate vector $(\lambda_{s,1}, \lambda_{s,2}, ..., \lambda_{s,S})^T \in$ Interior $(\Lambda(\lambda_p))$ where $\lambda_p < f_{S_{rel}}$ and $\lambda_p \in Q$.

Note under SCRP and for any time-slot t, $\lambda_p < f_{S_{rel}}$ and $\lambda_p \in Q$, $U_p(t) \leq M$. Therefore $U_p(t)$ is strongly stable. For $n = 1, 2, ..., S$ we consider the secondary queue-lengths at the beginning of r-th frame $(r = 1, 2, ...)$,

$Z_{s,n}(t_r) \overset{\triangle}{=} U_{s,n}(t_{r,1})$ where $t_{r,1} = T + 1 + (r-1)N$ with $T = \zeta(1)$ if $\lambda_p \leq f_0$, $\lambda_p \in Q$, while if $f_0 < \lambda_p < f_{s_{rel}}$ T is defined as in Step1 of ALT3.

We denote the vector $(Z_{s,1}(t_r), Z_{s,2}(t_r), .., Z_{s,S}(t_r))^T$ by $(Z_{s,n}(t_r))_{n=1}^S$. We define a Lyapunov function $L((Z_{s,n}(t_r))_{n=1}^S)$ $\overset{\triangle}{=} \sum_{n=1}^S Z_{s,n}^2(t_r)$ The conditional drift $\triangle(t_r)$ is defined as

$$\triangle(t_r) \overset{\triangle}{=} E[L((Z_{s,n}(t_{r+1}))) - L((Z_{s,n}(t_r)))|(Z_{s,n}(t_r))_{n=1}^S] \tag{51}$$

Now for $n = 1, 2, ..., S$,

$$U_{s,n}(t_{r,1}+N) \leq \max[U_{s,n}(t_{r,1}) - \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_{s,n}^{SCRP}(\tau), 0] + \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_{s,n}(\tau) \tag{52}$$

Since maximum arrival or transmission rate of secondary packets is less than or equal to 1 secondary packet per slot (as maximum transmission rate of secondary nodes is 1 secondary packet per slot, if the system is stable the arrival rate of secondary packets cannot be greater than 1 secondary packet per slot) when $\lambda_p \leq f_0$,

$$\triangle(t_r)$$

$$\leq S.N^2.(1+1) - 2\sum_{n=1}^S Z_{s,n}(t_r)E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_{s,n}^{SCRP}(\tau) - A_{s,n}(\tau)|(Z_{s,n}(t_r))_{n=1}^S] \tag{53}$$

$$= 2SN^2 - 2E[\sum_{n=1}^S Z_{s,n}(t_r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_{s,n}^{SCRP}(\tau)|(Z_{s,n}(t_r))_{n=1}^S] + 2\sum_{n=1}^S Z_{s,n}(t_r)E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_{s,n}(\tau)] \tag{54}$$

$$\leq 2SN^2 + 2B_2 - 2E[\sum_{n=1}^S Z_{s,n}(t_r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_{s,n}^{ALT2}(\tau)|(Z_{s,n}(t_r))_{n=1}^S] + \sum_{n=1}^S Z_{s,n}(t_r)2E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_{s,n}(\tau)] \tag{55}$$

$$\leq 2SN^2 + 2B_2 + 2B_1 - 2E[\sum_{n=1}^S Z_{s,n}(t_r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_{s,n}^{ALT1}(\tau)|(Z_{s,n}(t_r))_{n=1}^S] + \sum_{n=1}^S Z_{s,n}(t_r)2E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_{s,n}(\tau)] \tag{56}$$

$$\leq 2SN^2 + 2B_2 + 2B_1 - 2E[\sum_{n=1}^S Z_{s,n}(t_r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_{s,n}^{SSP}(\tau)|(Z_{s,n}(t_r))_{n=1}^S] + \sum_{n=1}^S Z_{s,n}(t_r)2E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} A_{s,n}(\tau)] \tag{57}$$

$$= 2SN^2 + 2B_2 + 2B_1 - \sum_{n=1}^S Z_{s,n}(t_r)2E[(\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+N-1} \mu_{s,n}^{SSP}(\tau) - A_{s,n}(\tau))] \tag{58}$$

$$\leq 2SN^2 + 2B_2 + 2B_1 - \sum_{n=1}^S 2N\epsilon Z_{s,n}(t_r) \tag{59}$$

where $\epsilon > 0$ is a constant such that $(\lambda_{s,1}+\epsilon, \lambda_{s,2}+\epsilon, .., \lambda_{s,S}+\epsilon)^T \in$ Interior$(\Lambda(\lambda_p))$. (55), (56), (57) and (59) follows from Lemma 5, 4, 3 and 2 respectively. Therefore by Theorem 4.1 in [16], the sampled queue-length processes- $U_{s,1}(t_{r,1}), U_{s,2}(t_{r,1}), ..., U_{s,S}(t_{r,1})$, where $r = 1, 2, ..$, are strongly stable. Since $U_{s,n}(t) \leq U_{s,n}(t_{r,1}) + N$ for all $n = 1, 2, .., S$, $r = 1, 2, ..$, $t_{r,1} \leq t \leq t_{r,1} + N - 1$, and $U_{s,n}(t_{1,1}) < \infty$ therefore the queue-length processes - $U_{s,n}(t)$ are strongly stable when $\lambda_p \leq f_0$.

Using Lemma 2, 3, 6 and following similar procedures as above we can show secondary network is strongly stable for $f_0 < \lambda_p < f_{S_{rel}}$. ■

VI. GENERAL PRIMARY DATA ARRIVAL RATES

In this section we extend the analysis in previous sections to the case when λ_p is not restricted to the set of rational numbers but can be any real number less than or equal to f_0. We consider the case where $K_{(PT,PR)} = 3$, $S_{rel} = 1$, $K_{(PT,ST_1)} = K_{(ST_1,PR)} = 1$ and show the following:

Lemma 7: Let the region $\Lambda(\lambda_p)$ be defined for any $\lambda_p \in R$ s.t. $\frac{1}{k_1+1} \leq \lambda_p \leq \frac{1}{k_1} \leq f_0$ (for some $k_1 \in Z^+$), by using this value of λ_p in (9)- (18) and the following equation:

$$\lambda_p = \frac{\pi_{(PT,PR)}}{K_{(PT,PR)}} + \sum_{1 \leq j \leq S_{rel}} \frac{\pi_{(PT,ST_j)}}{K_{(PT,ST_j)}}. \tag{60}$$

When $K_{(PT,PR)} = 3$, $S_{rel} = 1$ and $K_{(PT,ST_1)} = K_{(ST_1,PR)} = 1$, SCRP stabilizes the network for any $(\lambda_{s,1}, \lambda_{s,2}, .., \lambda_{s,S})^T \in$ Interior($\Lambda(\lambda_p)$) where $0 \leq \lambda_p \leq \frac{1}{3}$.

Lemma 7 can be extended to the network with general values of S_{rel} and $K_{(l_1,l_2)}$ (where $(l_1, l_2) \in L_p$). However for simplicity of analysis we restrict ourselves to the particular case considered above.

In the rest of this section we provide proof of Lemma 7. We prove Lemma 7 by comparing the system with $\lambda_p L_p$ primary data-arrival rate (in bits per slot), where λ_p is an irrational number, to a stable system with $\hat{\lambda}_p L_p$ primary data-arrival rate (in bits per slot), where $\hat{\lambda}_p$ is a rational number that is arbitrarily close to λ_p. We first show that such $\hat{\lambda}_p$ can be found when $\lambda_p < f_0$, and $\lambda_p \notin Q$. Then we construct periodic superframes corresponding to the system with primary data arrival-rate of $\hat{\lambda}_p$ bits per slot. We compare the value of the utility function defined in (45), obtained when the primary data arrival-rate is $\hat{\lambda}_p$ bits per slot, to the system where the primary data arrival-rate is λ_p bits per slot. Finally we use the result of this comparison in a similar way as in the proof of Theorem 1 and prove Lemma 6.

When $\lambda_p \leq f_0, \lambda_p \in R, \lambda_p \notin Q, K_{(PT,PR)} = 3, S_{rel} = 1, K_{(PT,ST_1)} = K_{(ST_1,PR)} = 1$ for any $(\lambda_{s,1}, \lambda_{s,2}, .., \lambda_{s,S})^T \in$ Interior($\Lambda(\lambda_p)$), there exists $\epsilon > 0$ s.t.

$$\lambda_p = \pi_{(PT,PR)}\frac{1}{3} + \pi_{(PT,ST_1)} \tag{61}$$

$$\pi_{(PT,PR)}, \pi_{(PT,ST_1)} \geq 0 \tag{62}$$

$$\pi_{(PT,ST_1)} = \pi_{(ST_1,PR)} \tag{63}$$

$$\pi_0 + \pi_{(PT,PR)} + \pi_{(PT,ST_1)} + \pi_{(ST_1,PR)} = 1 \tag{64}$$

$$\lambda_{s,n} + \epsilon \leq R_{s,n} \quad \forall n = 1, 2, .., S \text{ where} \tag{65}$$

$$\begin{aligned}(R_{s,n})_{n=1}^{S} = \pi_{(PT,PR)}\text{conv}(I'(PT,PR)) + \pi_{(PT,ST_1)}\text{conv}(I'(PT,ST_1)) + \\ \pi_{(ST_1,PR)}\text{conv}(I'(ST_1,PR)) + \pi_0\text{conv}(I'_0)\end{aligned} \tag{66}$$

Now since the set of rational numbers is a dense subset of the set of real numbers, arbitrarily small numbers δ_1,$\hat{\epsilon} > 0$ can be found such that $\hat{\lambda}_p = \lambda_p + \delta_1 \in Q$, $\hat{\lambda}_p < \frac{1}{3}$ and there exists variables $\hat{\pi}_{(PT,PR)}$, $\hat{\pi}_{(PT,ST_1)}$,$\hat{\pi}_{(ST_1,PR)}$, $\hat{\pi}_0$ such that

$$\hat{\lambda}_p = \hat{\pi}_{(PT,PR)}\frac{1}{3} + \hat{\pi}_{(PT,ST_1)} \tag{67}$$

$$\hat{\pi}_{(PT,PR)}, \hat{\pi}_{(PT,ST_1)} \geq 0 \tag{68}$$

$$\hat{\pi}_{(PT,ST_1)} = \hat{\pi}_{(ST_1,PR)} \tag{69}$$

$$\hat{\pi}_0 + \hat{\pi}_{(PT,PR)} + \hat{\pi}_{(PT,ST_1)} + \hat{\pi}_{(ST_1,PR)} = 1 \tag{70}$$

$$\lambda_{s,n} + \hat{\epsilon} \leq \hat{R}_{s,n} \quad \forall n = 1, 2, .., S \text{ where} \tag{71}$$

$$\begin{aligned}(\hat{R}_{s,n})_{n=1}^{S} = \hat{\pi}_{(PT,PR)}\text{conv}(I'(PT,PR)) + \hat{\pi}_{(PT,ST_1)}\text{conv}(I'(PT,ST_1)) +\\ \hat{\pi}_{(ST_1,PR)}\text{conv}(I'(ST_1,PR)) + \hat{\pi}_0\text{conv}(I'_0)\end{aligned} \tag{72}$$

Therefore $(\lambda_{s,1}, \lambda_{s,2}, .., \lambda_{s,S})^T \in$ Interior($\Lambda(\hat{\lambda}_p)$). Since $\hat{\lambda}_p \in Q$ it can be written as $\hat{\lambda}_p = \frac{\hat{M}}{\hat{N}}$ where $\hat{M}$ and $\hat{N}$ are integers coprime to each other; $\hat{N}$ denotes the length of the smallest period of the primary packet arrival process when the primary data arrival rate is $\hat{\lambda}_p L_p$ bits per slot, $\hat{M}$ denotes the number of primary packet arrivals in a period of length N slots.

We select an integer Y s.t. $Y\delta_1 N > 3$. Then we partition the time-line by selecting T to be $\hat{\zeta}(1)$ where $\hat{\zeta}(1)$ is the arrival instant of the first primary packet when the primary data-arrival rate is $\hat{\lambda}_p L_p$ bits per slot. Instead of using renewal frames of length N slots as was done in prior sections we define a *superframe* consisting of $Y\hat{N}$ contiguous slots with the first superframe beginning at slot $T+1$. For r=1,2,.. let m_r and $\hat{m}_r$ denotes the number of primary packets that are transmitted (partly or completely) during the r-th superframe if the primary data arrival rate are $\lambda_p L_p$ and $\hat{\lambda}_p L_p$ bits per slot respectively. We reuse the symbol $t_{r,i}$ (where $1 \leq i \leq YN$) to indicate the i-th slot in r-th superframe. Then

$$m_r \leq 2 + [\lambda_p Y N] \tag{73}$$

$$< (\lambda_p + \delta_1) Y N \tag{74}$$

$$< \hat{m}_r \tag{75}$$

For any $\rho \in R$ $[\rho]$ indicates the greatest integer less than or equal to ρ. (73) follows because if the primary data arrival rate is $\lambda_p L_p$ bits per slot then number of primary packets whose transmission times (at least partly) include slots in r-th superframe can at most be equal to the number of packets formed from data received during the superframe (including left-over bits at $t_{r,1}$) and possibly one primary packet transmission that began in $(r-1)$-th superframe and includes some slots in r-th superframe. Now since $\hat{\lambda}_p < \frac{1}{3}$ and $K_{(PT,PR)} = 3$ we have $3\hat{m}_r < YN$ for all $r = 1, 2, ...$ If the primary data-arrival rate is $\lambda_p L_p$ bits per slot, we index the m_r primary packets that are transmitted (at least partly) during r-th superframe as $x_{r,1}, x_{r,2}, .., x_{r,m_r}$.If the primary data-arrival rate is $\hat{\lambda}_p L_p$ bits per slot, we index the $\hat{m}_r$ primary packets that are transmitted (at least partly) during r-th superframe as $\hat{x}_{r,1}, \hat{x}_{r,2}, .., \hat{x}_{r,\hat{m}_r}$.

We assume policy SCRP is used for both systems with primary data arrival rates of $\lambda_p L_p$ and $\hat{\lambda}_p L_p$ bit per slot respectively. Let $w_{r,i}$ (where $2 \leq i \leq m_r - 1$) denote the time-slots when $x_{r,i}$'s transmission begins. Since $\lambda_p < \frac{1}{3}$ and $K_{(PT,PR)} = 3$ therefore $w_{r,i}, w_{r,i}+1, w_{r,i}+2$ belong to r-th superframe when $2 \leq i \leq m_r - 1$.

We assume $x_{r,1}$ is directly transmitted in slots $t_{r,1}-1$,$t_{r,1}$ and $t_{r,1}+1$. Since $3m_r < 3\hat{m}_r < YN$ therefore there exists an idle slot $y_{r,1}$ in r-th superframe s.t. $y_{r,1} \notin \{t_{r,1}, t_{r,1}+1, w_{r,i}, w_{r,i}+1, w_{r,i}+2 \text{ where } 2 \leq i \leq m_r - 1\}$. Assume x_{r,m_r} is directly transmitted starting from slot $t_{r,1}+YN-2$. Then since only two of the transmission slots for x_{r,m_r} belong to r-th superframe and $3m_r < YN$ there exists an idle slot y_{r,m_r} in r-th superframe such that $y_{r,m_r} \notin \{y_{r,1}, t_{r,1}, t_{r,1}+1, w_{r,i}, w_{r,i}+1, w_{r,i}+2 \text{ where } 2 \leq i \leq m_r - 1\}$. We denote the time-slot when $\hat{x}_{r,j}$'s (where $1 \leq j \leq \hat{m}_r$) transmission begins by $\hat{w}_{r,j}$.

We assume that at some $t_{r,1}$ the two systems with different primary data arrival rates have the same secondary queue-length vector $(U_{s,n}(t_{r,1}))_{n=1}^{S}$. Let $\mu_{s,n}(\tau)$ and $\hat{\mu}_{s,i}(\tau)$ denote the transmission rate offered to ST_n (where $1 \leq n \leq S$) at time-slot τ for the system with primary data arrival rates of $\lambda_p L_p$ and $\hat{\lambda}_p L_p$ bit per slot respectively (we drop the superscript corresponding to the policy identifier in the notation of offered transmission-rates because both systems use policy SCRP). Then, using same procedure as used to prove Lemma 4, it can be shown that

$$\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+1} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}(\tau) + \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}(y_{r,1}) \quad \geq \quad \sum_{\tau=\hat{w}_{r,1}}^{\tau=\hat{w}_{r,1}+2} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\hat{\mu}_{s,n}(\tau) - \hat{B}_1 \tag{76}$$

$$\sum_{\tau=w_{r,j}}^{\tau=w_{r,j}+2} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}(\tau) \quad \geq \quad \sum_{\tau=\hat{w}_{r,j}}^{\tau=\hat{w}_{r,j}+2} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\hat{\mu}_{s,n}(\tau) - \hat{B}_2 \text{ where } 2 \leq j \leq m_r - 1 \tag{77}$$

$$\sum_{\tau=t_{r,1}+YN-2}^{\tau=t_{r,1}+YN-1} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}(\tau) + \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}(y_{r,m_r}) \geq \sum_{\tau=\hat{w}_{r,m_r}}^{\tau=\hat{w}_{r,m_r}+2} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\hat{\mu}_{s,n}(\tau) - \hat{B}_3 \tag{78}$$

where $\hat{B}_1$,$\hat{B}_2$,$\hat{B}_3 > 0$ are finite constants.

Now for the system with primary data-arrival rate of $\lambda_p L_p$ bits per slot consider the set of slots $\beta_r = \{ \tau : \tau \in r\text{-th super-frame}, \tau \notin \{y_{r,1}, y_{r,m_r}, t_{r,1}, t_{r,1}+1, w_{r,i}, w_{r,i}+1, w_{r,i}+2, \text{ where } 2 \leq i \leq m_r - 1\}\}$ and $\hat{\beta}_r = \{ \tau : \tau \in r\text{-th super-frame}, \tau \notin \{\hat{w}_{r,i}, \hat{w}_{r,i}+1, \hat{w}_{r,i}+2, \text{ where } 1 \leq i \leq m_r\}\}$. Then since $m_r < \hat{m}_r$, all slots in β_r are idle slots while some slots in $\hat{\beta}_r$ are busy slots used to transmit ($\hat{m}_r$-m_r) primary packets. Therefore it can be shown, following same procedure as in proof of Lemma 4, that

$$\sum_{\tau \in \beta_r} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}(\tau) \geq \sum_{\tau \in \hat{\beta}_r} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\hat{\mu}_{s,n}(\tau) - \hat{K_4} \tag{79}$$

where $\hat{K_4} > 0$ is a finite constant. Therefore using (76)- (79) we obtain

$$\sum_{\tau=t_{r,1}}^{t_{r,1}+YN-1} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}(\tau) \geq \sum_{\tau=t_{r,1}}^{t_{r,1}+YN-1} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\hat{\mu}_{s,n}(\tau) - YN \max(\hat{B}_1, \hat{B}_2, \hat{B}_3, \hat{B}_4) \tag{80}$$

(80) can be shown to be true in general when $x_{r,1}$ and x_{r,m_r} are either transmitted directly or relayed such that some of the corresponding transmission slots belong to r-th superframe. Next we proceed to the proof of Lemma 7.

Proof of Lemma 7: Under SCRP and for any time-slot t, $\lambda_p \leq \frac{1}{3}$, $U_p(t) \leq M$. Therefore $U_p(t)$ is strongly stable.

For $n = 1, 2, ..., S$ we consider the secondary queue-lengths at the beginning of r-th super-frame ($r = 1, 2, ...$),

$Z_{s,n}(t_r) \overset{\triangle}{=} U_{s,n}(t_{r,1})$. We denote the vector $(Z_{s,1}(t_r), Z_{s,2}(t_r), .., Z_{s,S}(t_r))^T$ by $(Z_{s,n}(t_r))_{n=1}^S$. We define a Lyapunov function $L((Z_{s,n}(t_r))_{n=1}^S) \overset{\triangle}{=} \sum_{n=1}^S Z_{s,n}^2(t_r)$. The conditional drift $\triangle(t_r)$ is defined as

$$\triangle(t_r) \overset{\triangle}{=} E[L((Z_{s,n}(t_{r+1}))) - L((Z_{s,n}(t_r)))|(Z_{s,n}(t_r))_{n=1}^S] \tag{81}$$

Now for $n = 1, 2, ..., S$,

$$U_{s,n}(t_{r,1} + YN) \leq \max[U_{s,n}(t_{r,1}) - \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+YN-1} \mu_{s,n}(\tau), 0] + \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+YN-1} A_{s,n}(\tau) \tag{82}$$

Since maximum arrival or transmission rate of secondary packets is less than or equal to 1 secondary packet per slot (as maximum transmission rate of secondary nodes is 1 secondary packet per slot, if the system is stable the arrival rate of secondary packets cannot be greater than 1 secondary packet per slot) when $\lambda_p < f_0$,

$$\triangle(t_r) \leq S.(YN)^2.(1+1) - 2\sum_{n=1}^{S} Z_{s,n}(t_r)E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+YN-1} (\mu_{s,n}(\tau) - A_{s,n}(\tau))|(Z_{s,n}(t_r))_{n=1}^S] \tag{83}$$

$$= 2S(YN)^2 - 2E[\sum_{n=1}^{S} Z_{s,n}(t_r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+YN-1} \mu_{s,n}(\tau)|(Z_{s,n}(t_r))_{n=1}^S] + 2\sum_{n=1}^{S} Z_{s,n}(t_r)E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+YN-1} A_{s,n}(\tau)] \tag{84}$$

$$\leq 2S(YN)^2 + 2YN\max(\hat{B}_1, \hat{B}_2, \hat{B}_3, \hat{B}_4) - 2E[\sum_{n=1}^{S} Z_{s,n}(t_r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+YN-1} \hat{\mu}_{s,n}(\tau)|(Z_{s,n}(t_r))_{n=1}^S] + 2\sum_{n=1}^{S} Z_{s,n}(t_r)E[\sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+YN-1} A_{s,n}(\tau)] \tag{85}$$

(85) follows from (80). Now since $(\lambda_{s,1}, \lambda_{s,2}, ..., \lambda_{s,S})^T \in$Interior$(\Lambda(\hat{\lambda}_p))$ therefore, following the same techniques as used in proof of Theorem 1, we can show there exists $\hat{\epsilon}$,$\hat{K_5} > 0$, s.t.

$$-E[\sum_{n=1}^{S} Z_{s,n}(t_r) \sum_{\tau=t_{r,1}}^{\tau=t_{r,1}+YN-1} (\hat{\mu}_{s,n}(\tau) - A_{s,n}(\tau))|(Z_{s,n}(t_r))_{n=1}^S] \leq \hat{K_5} - \sum_{n=1}^{S} YN\hat{\epsilon}Z_{s,n}(t_{r,1}) \tag{86}$$

Substituting (86) in (85) we obtain

$$\triangle(t_r) \leq 2\hat{K_5} + 2S(YN)^2 + 2YN\max(\hat{B}_1, \hat{B}_2, \hat{B}_3, \hat{B}_4) - 2\sum_{n=1}^{S} YN\hat{\epsilon}Z_{s,n}(t_{r,1}) \tag{87}$$

Therefore by Theorm 4.1 in [16] and due to boundedness of the secondary packet arrival and departure processes, the secondary queue-length processes are stable. ■

VII. CONCLUSION

In this work we studied the problem of opportunistic cooperation in a cognitive network where some nodes may benefit from cooperative relaying while others may suffer loss of transmission of opportunities. Assuming a deterministic periodic primary packet arrival process a scheduling and relaying algorithm is developed for this network using Lyapunov drift techniques. For any primary data arrival-rate that can be supported by network in absence of cooperation the set of secondary arrival-rate vectors for which the network is stable under this algorithm includes the region (minus its boundary) composed of all secondary arrival-rate vectors for which the network can be stabilized without any relay. For any primary data arrival-rate greater than what can be supported through only direct transmission but less than what can be supported maximally with relay, the algorithm stabilizes the network for a non-empty set of secondary arrival-rate vectors. The set of secondary arrival-rate vectors for which the network can be stabilized is therefore expanded under this cooperative scheduling algorithm.

In future we seek to extend this analysis to a more general network where the service-time of packets in different links are stochastic (which is a more realistic model of wireless networks) and cases involving multiple primary s-d pairs. For those cases, using a similar approach as used in this work, we hope to find algorithms that stabilize the network for almost all secondary-arrival rate vectors for which the network can be stabilized without any relay and for a non-empty set of secondary arrival-rate vectors for which the network cannot be stabilized without cooperative relay.

APPENDIX A

PROOF OF LEMMA 4

Proof: Since when $\lambda_p \leq f_0$, the inter-arrival time between two primary packets is greater than or equal to $K_{(PT,PR)}$ slots and each primary packet can be transmitted within $K_{(PT,PR)}$ time-slots, therefore under both ALT1 and ALT2 policies, transmission of a primary packet begins at the same slot in each frame. The transmission of j-th primary packet begins at slot $\zeta(j)+1$. We partition slots in r-th frame ($r \in 1, 2, 3...$) into a collection of candidate busy time-slots $\tilde{T}_r \overset{\triangle}{=} \{\zeta(j)+1, \zeta(j)+2, ..., \zeta(j)+K_{(PT,PR)} : 1+rM > j \geq 1+(r-1)M\}$ and the remaining slots that are always idle, $\hat{T}_r \overset{\triangle}{=} \{t \in [t_{r,1}, t_{r,1}+(N-1)] : t \notin \tilde{T}_r\}$. Let $U^{\phi}_{s,n}(\tau)$ (where $n = 1, 2, .., S$ and $t_{r,1} \leq \tau \leq t_{r,1}+(N-1)$) denote the transmission queue-length of ST_n at time-slot τ under any policy ϕ given that $U^{\phi}_{s,n}(t_{r,1}) = U_{s,n}(t_{r,1})$. Similarly we denote the vector $v^*_0(\tau)$, $v^*_{(l_1,l_2)}(\tau)$ (where $(l_1, l_2) \in$L$_p$, $t_{r,1} \leq \tau \leq t_{r,1}+(N-1)$) under some policy ϕ by $v^*_{0,\phi}(\tau)$ and $v^*_{(l_1,l_2),\phi}(\tau)$ respectively. In the rest of the proof we abuse the notations slightly and use $\mu^{\phi}_{s,n}(\tau)$ (where $n = 1, 2, .., S$ and $t_{r,1} \leq \tau \leq t_{r,1}+(N-1)$) to denote the transmission rate offered, under policy ϕ, to ST_n at time-slot τ for a given secondary queue-length vector $(U_{s,1}(t_{r,1}), U_{s,2}(t_{r,1}), ..., U_{s,S}(t_{r,1}))^T$ at the beginning of r-th frame. This allows us to write $\psi^{\phi}(t_{r,1})$ simply as $\psi^{\phi}(t_{r,1}) = \sum_{i=1}^{S} U_{s,i}(t_{r,1}) E[\sum_{t=t_{r,1}}^{t_{r,1}+N-1} \mu^{\phi}_{s,i}(t)]$.

For any two integers h_1 and h_2 we define δ_{h_1,h_2} as $\delta_{h_1,h_2} \overset{\triangle}{=} |h_1 - h_2|$. Since arrival rate and maximum transmission rate of secondary packets per slot are both no greater than 1, we notice that for any two time-slots t_1 and t_2 , $t_2 > t_1$, $n = 1, 2, .., S$

$$U_{s,n}(t_2) \geq U_{s,n}(t_1) - \delta_{t_1,t_2} \tag{88}$$

$$U_{s,n}(t_1) \geq U_{s,n}(t_2) - \delta_{t_1,t_2} \tag{89}$$

Next we consider slot $\zeta(j)+1$ that is in T_r for some $r = 1, 2, ...$ i.e. $1+rM > j \geq 1+(r-1)M$. Lets assume that the j-th primary packet is relayed via ST_{θ_1} while under ALT2 it is relayed via ST_{θ_2}. ($1 \leq \theta_1, \theta_2 \leq S_{rel}$, $1+rM > j \geq 1+(r-1)M$).

For convenience, we abbreviate (PT, PR), (PT, ST_{θ_1}), (ST_{θ_1}, PR), (PT, ST_{θ_2}), (ST_{θ_2}, PR) as z_0, z_1,z_2,z_3 and

z_4 respectively.

$$\sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_0}} \sum_{i=1}^{S} U_{s,i}(t_{r,1})\mu_{s,i}^{ALT2}(\tau)$$

$$= \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_3}} \sum_{i=1}^{S} U_{s,i}(t_{r,1})\mu_{s,i}^{ALT2}(\tau) + \sum_{\tau=\zeta(j)+K_{z_3}+1}^{\zeta(j)+K_{z_3}+K_{z_4}} \sum_{i=1}^{S} U_{s,i}(t_{r,1})\mu_{s,i}^{ALT2}(\tau) + \sum_{\tau=\zeta(j)+K_{z_3}+K_{z_4}+1}^{\zeta(j)+K_{z_0}} \sum_{i=1}^{S} U_{s,i}(t_{r,1})\mu_{s,i}^{ALT2}(\tau) \tag{90}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_3}} \sum_{i=1}^{S} (U_{s,i}^{ALT2}(\tau) - \delta_{\tau,t_{r,1}})\mu_{s,i}^{ALT2}(\tau) + \sum_{\tau=\zeta(j)+K_{z_3}+1}^{\zeta(j)+K_{z_3}+K_{z_4}} \sum_{i=1}^{S} (U_{s,i}^{ALT2}(\tau) - \delta_{\tau,t_{r,1}})\mu_{s,i}^{ALT2}(\tau)$$

$$+ \sum_{\tau=\zeta(j)+K_{z_3}+K_{z_4}+1}^{\zeta(j)+K_{z_0}} \sum_{i=1}^{S} U_{s,i}(t_{r,1})\mu_{s,i}^{ALT2}(\tau) \tag{91}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_3}} \sum_{i=1}^{S} U_{s,i}^{ALT2}(\tau)\mu_{s,i}^{ALT2}(\tau) + \sum_{\tau=\zeta(j)+K_{z_3}+1}^{\zeta(j)+K_{z_3}+K_{z_4}} \sum_{i=1}^{S} U_{s,i}^{ALT2}(\tau)\mu_{s,i}^{ALT2}(\tau)$$

$$+ \sum_{\tau=\zeta(j)+K_{z_3}+K_{z_4}+1}^{\zeta(j)+K_{z_0}} ((U_{s,i}(t_{r,1}))_{i=1}^{S})^T v_{0,ALT2}^{*}(\zeta(j)+1) - 2N^2S \tag{92}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_3}} ((U_{s,i}^{ALT2}(\tau))_{i=1}^{S})^T v_{z_3,ALT2}^{*}(\tau) + \sum_{\tau=\zeta(j)+K_{z_3}+1}^{\zeta(j)+K_{z_3}+K_{z_4}} ((U_{s,i}^{ALT2}(\tau))_{i=1}^{S})^T v_{z_4,ALT2}^{*}(\tau)$$

$$+ \sum_{\tau=\zeta(j)+K_{z_3}+K_{z_4}+1}^{\zeta(j)+K_{z_0}} ((U_{s,i}^{ALT2}(\zeta(j)+1) - \delta_{t_{r,1},\zeta(j)+1})_{i=1}^{S})^T v_{0,ALT2}^{*}(\zeta(j)+1) - 2N^2S \tag{93}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_3}} ((U_{s,i}^{ALT2}(\tau))_{i=1}^{S})^T v_{z_3,ALT2}^{*}(\tau) + \sum_{\tau=\zeta(j)+K_{z_3}+1}^{\zeta(j)+K_{z_3}+K_{z_4}} ((U_{s,i}^{ALT2}(\tau))_{i=1}^{S})^T v_{z_4,ALT2}^{*}(\tau)$$

$$+ \sum_{\tau=\zeta(j)+K_{z_3}+K_{z_4}+1}^{\zeta(j)+K_{z_0}} ((U_{s,i}^{ALT2}(\zeta(j)+1))_{i=1}^{S})^T v_{0,ALT2}^{*}(\zeta(j)+1) - 3N^2S \tag{94}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_3}} ((U_{s,i}^{ALT2}(\tau))_{i=1}^{S})^T v_{z_3,ALT2}^{*}(\zeta(j)+1) + \sum_{\tau=\zeta(j)+K_{z_3}+1}^{\zeta(j)+K_{z_3}+K_{z_4}} ((U_{s,i}^{ALT2}(\tau))_{i=1}^{S})^T v_{z_4,ALT2}^{*}(\zeta(j)+1)$$

$$+ \sum_{\tau=\zeta(j)+K_{z_3}+K_{z_4}+1}^{\zeta(j)+K_{z_0}} ((U_{s,i}^{ALT2}(\zeta(j)+1))_{i=1}^{S})^T v_{0,ALT2}^{*}(\zeta(j)+1) - 3N^2S \tag{95}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_3}} ((U_{s,i}^{ALT2}(\zeta(j)+1) - \delta_{\zeta(j)+1,\tau})_{i=1}^{S})^T v_{z_3,ALT2}^{*}(\zeta(j)+1) + \sum_{\tau=\zeta(j)+K_{z_3}+1}^{\zeta(j)+K_{z_3}+K_{z_4}} \{((U_{s,i}^{ALT2}(\zeta(j)+1) - \delta_{\zeta(j)+1,\tau})_{i=1}^{S})^T$$

$$v_{z_4,ALT2}^{*}(\zeta(j)+1)\} + \sum_{\tau=\zeta(j)+K_{z_3}+K_{z_4}+1}^{\zeta(j)+K_{z_0}} ((U_{s,i}^{ALT2}(\zeta(j)+1))_{i=1}^{S})^T v_{0,ALT2}^{*}(\zeta(j)+1) - 3N^2S \tag{96}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_3}} ((U_{s,i}^{ALT2}(\zeta(j)+1))_{i=1}^{S})^T v_{z_3,ALT2}^{*}(\zeta(j)+1) + \sum_{\tau=\zeta(j)+K_{z_3}+1}^{\zeta(j)+K_{z_3}+K_{z_4}} ((U_{s,i}^{ALT2}(\zeta(j)+1))_{i=1}^{S})^T v_{z_4,ALT2}^{*}(\zeta(j)+1)$$

$$+\sum_{\tau=\zeta(j)+K_{z_3}+K_{z_4}+1}^{\zeta(j)+K_{z_0}}((U_{s,i}^{ALT2}(\zeta(j)+1))_{i=1}^{S})^T v_{0,ALT2}^*(\zeta(j)+1)-5N^2S \tag{97}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_1}}((U_{s,i}^{ALT2}(\zeta(j)+1))_{i=1}^{S})^T v_{z_1}^*(t_{r,1})+\sum_{\tau=\zeta(j)+K_{z_1}+1}^{\zeta(j)+K_{z_1}+K_{z_2}}((U_{s,i}^{ALT2}(\zeta(j)+1))_{i=1}^{S})^T v_{z_2}^*(t_{r,1})$$

$$+\sum_{\tau=\zeta(j)+K_{z_1}+K_{z_2}+1}^{\zeta(j)+K_{z_0}}((U_{s,i}^{ALT2}(\zeta(j)+1))_{i=1}^{S})^T v_{0}^*(t_{r,1})-5N^2S \tag{98}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_1}}\sum_{i=1}^{S}(U_{s,i}(t_{r,1})-\delta_{t_{r,1},\zeta(j)+1})\mu_{s,i}^{ALT1}(\tau)+\sum_{\tau=\zeta(j)+K_{z_1}+1}^{\zeta(j)+K_{z_1}+K_{z_2}}\sum_{i=1}^{S}(U_{s,i}(t_{r,1})-\delta_{t_{r,1},\zeta(j)+1})\mu_{s,i}^{ALT1}(\tau)$$

$$+\sum_{\tau=\zeta(j)+K_{z_1}+K_{z_2}+1}^{\zeta(j)+K_{z_0}}\sum_{i=1}^{S}(U_{s,i}(t_{r,1})-\delta_{t_{r,1},\zeta(j)+1})\mu_{s,i}^{ALT1}(\tau)-5N^2S \tag{99}$$

$$\geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_1}}\sum_{i=1}^{S}U_{s,i}(t_{r,1})\mu_{s,i}^{ALT1}(\tau)+\sum_{\tau=\zeta(j)+K_{z_1}+1}^{\zeta(j)+K_{z_1}+K_{z_2}}\sum_{i=1}^{S}U_{s,i}(t_{r,1})\mu_{s,i}^{ALT1}(\tau)$$

$$+\sum_{\tau=\zeta(j)+K_{z_1}+K_{z_2}+1}^{\zeta(j)+K_{z_0}}\sum_{i=1}^{S}U_{s,i}(t_{r,1})\mu_{s,i}^{ALT1}(\tau)-8N^2S \tag{100}$$

$$=\sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{z_0}}\sum_{i=1}^{S}U_{s,i}(t_{r,1})\mu_{s,i}^{ALT1}(\tau)-8N^2S \tag{101}$$

In (91), (93), (99) we used (88). (92), (94), (97), (100) follows because $\delta_{\tau,t_{r,1}}, K_{z_0}, K_{z_1}, K_{z_2}K_{z_3}, K_{z_4} \leq N$ if $\tau \in [t_{r,1}, t_{r,1}+N-1]$ where $r=1,2,...$ (95), (98) follows because of the way ALT2 is defined. In (96) we used (89).

Similarly for the case when ALT1 directly transmit the j-th primary packet while ALT2 uses some relay to t transmit he same and vice-versa or if both ALT1 and ALT2 directly transmit the j-th primary packet (where $1+rM > j \geq 1+(r-1)M$) we can show that

$$\sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{(PT,PR)}}\sum_{i=1}^{S}U_{s,i}(t_{r,1})\mu_{s,i}^{ALT2}(\tau) \geq \sum_{\tau=1+\zeta(j)}^{\zeta(j)+K_{(PT,PR)}}\sum_{i=1}^{S}U_{s,i}(t_{r,1})\mu_{s,i}^{ALT1}(\tau)-B_a \tag{102}$$

where $B_a > 0$ is a finite constant. Therefore

$$\psi^{ALT2}(t_{r,1}) = \sum_{j=1+(r-1)M}^{j=rM}\sum_{n=1}^{S}U_{s,n}(t_{r,1})E[(\sum_{\tau=\zeta(j)+1}^{\tau=\zeta(j)+K_{(PT,PR)}}\mu_{s,n}^{ALT2}(\tau))]+\sum_{\tau\in\hat{T}_r}\sum_{n=1}^{S}U_{s,n}(t_{r,1})E[(\mu_{s,n}^{ALT2}(\tau))] \tag{103}$$

$$= \sum_{j=1+(r-1)M}^{j=rM}\sum_{n=1}^{S}U_{s,n}(t_{r,1})E[(\sum_{\tau=\zeta(j)+1}^{\tau=\zeta(j)+K_{(PT,PR)}}\mu_{s,n}^{ALT2}(\tau))]+\sum_{\tau\in\hat{T}_r}\sum_{n=1}^{S}U_{s,n}(t_{r,1})E[(\mu_{s,n}^{ALT1}(\tau))] \tag{104}$$

$$\geq \{\sum_{j=1+(r-1)M}^{j=rM}\sum_{n=1}^{S}U_{s,n}(t_{r,1})E[(\sum_{\tau=\zeta(j)+1}^{\tau=\zeta(j)+K_{(PT,PR)}}\mu_{s,n}^{ALT1}(\tau))]+\sum_{\tau\in\hat{T}_r}\sum_{n=1}^{S}U_{s,n}(t_{r,1})E[(\mu_{s,n}^{ALT1}(\tau))]\}$$

$$-M\max(B_a, 8N^2S) \tag{105}$$

$$= \psi^{ALT1}(t_{r,1})-B_1 \tag{106}$$

where $B_1 = M \max(B_a, 8N^2S) > 0$ is a finite constant. (104) follows because ALT1 and ALT2 has same scheduling policies at idle-slots $\tau \in \hat{T}_r$. (105) follows from (102) and (101). ■

APPENDIX B
PROOF OF LEMMA 5

Proof: We note that SCRP is almost similar to ALT2 in that both result in same relaying decisions and hence the location of idle slots is same under both policies. However at any idle slot w in r-th frame $(r = 1, 2, ...)$ ALT2 selects the transmission rate vector that maximizes $(U_{s,1}(t_{r,1}), U_{s,2}(t_{r,1}), ..., U_{s,S}(t_{r,1}))v$ over $v \in I_0'$ while SCRP selects $v \in I_0'$ that maximizes $(U_{s,1}(w), U_{s,2}(w), .., U_{s,S}(w))v$. Therefore following similar procedure as in Lemma 4, we can show that there exists finite constant $B_2 \geq 0$ s.t.

$$\psi^{SCRP}(t_{r,1}) \geq \psi^{ALT2}(t_{r,1}) - B_2 \tag{107}$$

■

APPENDIX C
PROOF OF LEMMA 6

Proof: For any λ_p such that $f_0 \leq \frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} \leq f_{S_{rel}}$ (where k_1 is a positive integer), there are one of 4 possibilities:

1) $\tilde{F}_{j-1} < \frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} < \tilde{F}_j$ for some j such that $2 \leq j \leq |\tilde{F}|$.
2) $\tilde{F}_{j-1} = \frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} < \tilde{F}_j$ for some j such that $2 \leq j \leq |\tilde{F}|$.
3) $\tilde{F}_{j-1} < \frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} = \tilde{F}_j$ for some j such that $2 \leq j \leq |\tilde{F}|$.
4) $\tilde{F}_{j-1} = \frac{1}{k_1+1} \leq \lambda_p < \frac{1}{k_1} = \tilde{F}_j$ for some j such that $2 \leq j \leq |\tilde{F}|$.

We prove Lemma 6 for λ_p satisfying case 4. For λ_p corresponding to other cases, the Lemma can be proved similarly.

For λ_p corresponding to case 4, any primary packet with inter-arrival time of $(k_1 + 1)$ slots can be transmitted in as many slots using direct transmission or via some secondary transmitter. Similarly, any primary packet with inter-arrival time of (k_1) slots can also be transmitted in as many slots via some secondary transmitter.

We index the $\kappa_1(\lambda_p)$ and $\kappa_2(\lambda_p)$ primary packets that arrive in r-th frame (where $r = 1, 2, ...$) as $x_{r,1}, x_{r,2}, ..., x_{r,\kappa_1(\lambda_p)}$ and $\grave{x}_{r,1}, \grave{x}_{r,2}, ..., \grave{x}_{r,\kappa_2(\lambda_p)}$ respectively. For all i s.t. $1 \leq i \leq \kappa_1(\lambda_p)$ let $w^{\phi}_{r,i}$ denote the time-slot when primary packet $x_{r,i}$'s transmission begins under policy ϕ. If it is transmitted using a relay ST_{n_1} (where $1 \leq n_1 \leq S_{rel}$) for which $\frac{1}{f_{n_1}} < k_1 + 1$ then it creates $(k_1 + 1) - \frac{1}{f_{n_1}}$ idle slots at times denoted as $y^{n_1,\phi}_{r,i,1}, y^{n_1,\phi}_{r,i,2}, ..., y^{n_1,\phi}_{r,i,(k_1+1)-\frac{1}{f_{n_1}}}$ respectively. Similarly for all $\grave{i}$ s.t. $1 \leq \grave{i} \leq \kappa_2(\lambda_p)$ let $\grave{w}^{\phi}_{r,\grave{i}}$ denote the time-slot when primary packet $\grave{x}_{r,\grave{i}}$'s transmission begins under policy ϕ. If it is transmitted using a relay ST_{n_2} (where $1 \leq n_2 \leq S_{rel}$) for which $\frac{1}{f_{n_2}} < k_1$ then it creates $k_1 - \frac{1}{f_{n_2}}$ idle slots at times denoted as $\grave{y}^{n_2,\phi}_{r,\grave{i},1}, \grave{y}^{n_2,\phi}_{r,\grave{i},2}, ..., \grave{y}^{n_2,\phi}_{r,\grave{i},(k_1)-\frac{1}{f_{n_2}}}$ respectively.

Next assume $x_{r,i}$ is relayed via ST_{θ_1} under ALT3 and by ST_{θ_2} under SCRP (where $1 \leq \theta_1, \theta_2 \leq S_{rel}$ and $\frac{1}{f_{\theta_1}}, \frac{1}{f_{\theta_2}} \leq (k_1 + 1)$. Assume the queue-length vector $(U_{s,n}(t_{r,1}))_{n=1}^{S}$ is same under both ALT3 and SCRP for some $r = 1, 2, ...$ In the rest of the proof we abuse the notations slightly and use $\mu^{\phi}_{s,n}(\tau)$ (where $n = 1, 2, .., S$ and $t_{r,1} \leq \tau \leq t_{r,1} + (N-1)$) to denote the transmission rate offered, under policy ϕ, to ST_n at time-slot τ for a given secondary queue-length vector $(U_{s,1}(t_{r,1}), U_{s,2}(t_{r,1}), ..., U_{s,S}(t_{r,1}))^T$ at the beginning of r-th frame. This allows us to write $\psi^{\phi}(t_{r,1})$ simply as $\psi^{\phi}(t_{r,1}) = \sum_{i=1}^{S} U_{s,i}(t_{r,1}) E[\sum_{t=t_{r,1}}^{t_{r,1}+N-1} \mu^{\phi}_{s,i}(t)]$.

For convenience, we abbreviate (PT, PR), (PT, ST_{θ_1}), (ST_{θ_1}, PR), (PT, ST_{θ_2}), (ST_{θ_2}, PR) as z_0, z_1,z_2,z_3 and z_4 respectively. Then

$$\sum_{\tau=w_{r,i}^{SCRP}}^{w_{r,i}^{SCRP}+\frac{1}{f_{\theta_2}}-1} \sum_{n=1}^{S} U_{s,n}(t_{r,1}) \mu^{SCRP}_{s,n}(\tau) + \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} \sum_{n=1}^{S} U_{s,n}(t_{r,1}) \mu^{SCRP}_{s,n}(y^{\theta_2,SCRP}_{r,i,\hat{n}})$$

$$\geq \sum_{\tau=w_{r,i}^{SCRP}}^{w_{r,i}^{SCRP}+K_{z_3}-1} \sum_{n=1}^{S} (U^{SCRP}_{s,n}(\tau) - \delta_{\tau,t_{r,1}}) \mu^{SCRP}_{s,n}(\tau) + \sum_{\tau=w_{r,i}^{SCRP}+K_{z_3}}^{w_{r,i}^{SCRP}+\frac{1}{f_{\theta_2}}-1} \sum_{n=1}^{S} (U^{SCRP}_{s,n}(\tau) - \delta_{\tau,t_{r,1}}) \mu^{SCRP}_{s,n}(\tau)$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} \sum_{n=1}^{S} (U^{SCRP}_{s,n}(y^{\theta_2,SCRP}_{r,i,\hat{n}}) - \delta_{y^{\theta_2,SCRP}_{r,i,\hat{n}},t_{r,1}}) \mu^{SCRP}_{s,n}(y^{\theta_2,SCRP}_{r,i,\hat{n}}) \tag{108}$$

$$\geq \sum_{\tau=w_{r,i}^{SCRP}}^{w_{r,i}^{SCRP}+K_{z_3}-1} \sum_{n=1}^{S} U^{SCRP}_{s,n}(\tau) \mu^{SCRP}_{s,n}(\tau) + \sum_{\tau=w_{r,i}^{SCRP}+K_{z_3}}^{w_{r,i}^{SCRP}+\frac{1}{f_{\theta_2}}-1} \sum_{n=1}^{S} U^{SCRP}_{s,n}(\tau) \mu^{SCRP}_{s,n}(\tau)$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} \sum_{n=1}^{S} U^{SCRP}_{s,n}(y^{\theta_2,SCRP}_{r,i,\hat{n}}) \mu^{SCRP}_{s,n}(y^{\theta_2,SCRP}_{r,i,\hat{n}}) - 3N^2S \tag{109}$$

$$\geq \sum_{\tau=w_{r,i}^{SCRP}}^{w_{r,i}^{SCRP}+K_{z_3}-1} ((U^{SCRP}_{s,n}(\tau))_{n=1}^{S})^T v^{*}_{z_3,SCRP}(w_{r,i}^{SCRP}) + \sum_{\tau=w_{r,i}^{SCRP}+K_{z_3}}^{w_{r,i}^{SCRP}+\frac{1}{f_{\theta_2}}-1} ((U^{SCRP}_{s,n}(\tau))_{n=1}^{S})^T v^{*}_{z_4,SCRP}(w_{r,i}^{SCRP})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} ((U^{SCRP}_{s,n}(y^{\theta_2,SCRP}_{r,i,\hat{n}}))_{n=1}^{S})^T v^{*}_{0,SCRP}(w_{r,i}^{SCRP}) - 3N^2S \tag{110}$$

$$\geq \sum_{\tau=w_{r,i}^{SCRP}}^{w_{r,i}^{SCRP}+K_{z_3}-1} ((U^{SCRP}_{s,n}(w_{r,i}^{SCRP}))_{n=1}^{S})^T v^{*}_{z_3,SCRP}(w_{r,i}^{SCRP}) + \sum_{\tau=w_{r,i}^{SCRP}+K_{z_3}}^{w_{r,i}^{SCRP}+\frac{1}{f_{\theta_2}}-1} ((U^{SCRP}_{s,n}(w_{r,i}^{SCRP}))_{n=1}^{S})^T v^{*}_{z_4,SCRP}(w_{r,i}^{SCRP})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} ((U^{SCRP}_{s,n}(w_{r,i}^{SCRP}))_{n=1}^{S})^T v^{*}_{0,SCRP}(w_{r,i}^{SCRP}) - 6N^2S \tag{111}$$

$$\geq \sum_{\tau=w_{r,i}^{SCRP}}^{w_{r,i}^{SCRP}+K_{z_1}-1} ((U^{SCRP}_{s,n}(w_{r,i}^{SCRP}))_{n=1}^{S})^T v^{*}_{z_1,ALT3}(w_{r,i}^{ALT3}) + \sum_{\tau=w_{r,i}^{SCRP}+K_{z_1}}^{w_{r,i}^{SCRP}+\frac{1}{f_{\theta_1}}-1} ((U^{SCRP}_{s,n}(w_{r,i}^{SCRP}))_{n=1}^{S})^T v^{*}_{z_1,ALT3}(w_{r,i}^{ALT3})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} ((U_{s,n}^{SCRP}(w_{r,i}^{SCRP}))_{n=1}^{S})^T v_{0,ALT3}^{*}(w_{r,i}^{ALT3}) - 6N^2S \tag{112}$$

$$\geq \sum_{\tau=w_{r,i}^{SCRP}}^{w_{r,i}^{SCRP}+K_{z_1}-1} ((U_{s,n}(t_{r,1}))_{n=1}^{S})^T v_{z_1,ALT3}^{*}(t_{r,1}) + \sum_{\tau=w_{r,i}^{SCRP}+K_{z_1}}^{w_{r,i}^{SCRP}+\frac{1}{f_{\theta_1}}-1} ((U_{s,n}(t_{r,1}))_{n=1}^{S})^T v_{z_2,ALT3}^{*}(t_{r,1})$$

$$+ \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} ((U_{s,n}(t_{r,1}))_{n=1}^{S})^T v_{0,ALT3}^{*}(t_{r,1}) - 9N^2S \tag{113}$$

$$= \sum_{\tau=w_{r,i}^{ALT3}}^{w_{r,i}^{ALT3}+\frac{1}{f_{\theta_1}}-1} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{ALT3}(\tau) + \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{ALT3}(y_{r,i,\hat{n}}^{\theta_1,ALT3}) - 9N^2S \tag{114}$$

(108) follows from (88). (109) follows because $k_1+1, K_{z3}, K_{z4}, \delta_{\tau,t_{r,1}} < N$ when $r = 1, 2, .., \tau \in [t_{r,1}, t_{r,1}+N-1]$. (110), (112) follows from the way SCRP is defined. (111) follows from (89) and because $k_1+1, K_{z3}, K_{z4}, \delta_{\tau,t_{r,1}} < N$ when $r = 1, 2, .., \tau \in [t_{r,1}, t_{r,1}+N-1]$. (113) follows from (88) and because $k_1+1, K_{z1}, K_{z2}, \delta_{\tau,t_{r,1}} < N$ when $r = 1, 2, .., \tau \in [t_{r,1}, t_{r,1}+N-1]$. Similarly we can repeat the above analysis for the cases where one of the algorithms use direct transmission and while other uses a relay to transmit $x_{r,i}$ or both use direct transmission and obtain

$$\sum_{\tau=w_{r,i}^{SCRP}}^{w_{r,i}^{SCRP}+\frac{1}{f_{\theta_2}}-1} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{SCRP}(\tau) + \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_2}}} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{SCRP}(y_{r,i,\hat{n}}^{\theta_2,SCRP}) \tag{115}$$

$$\geq \sum_{\tau=w_{r,i}^{ALT3}}^{w_{r,i}^{ALT3}+\frac{1}{f_{\theta_1}}-1} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{ALT3}(\tau) + \sum_{\hat{n}=1}^{(k_1+1)-\frac{1}{f_{\theta_1}}} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{ALT3}(y_{r,i,\hat{n}}^{\theta_1,ALT3}) - B_b \tag{116}$$

where $B_b > 0$ is a finite constant. Similarly repeating the above procedure for $\grave{x}_{r,\grave{i}}$s and assuming packet $\grave{x}_{r,\grave{i}}$ is relayed via $ST_{\grave{\theta}_1}$ under ALT3 and by $ST_{\grave{\theta}_2}$ under SCRP (where $1 \leq \grave{\theta}_1, \grave{\theta}_2 \leq S_{rel}$ and $\frac{1}{f_{\grave{\theta}_1}}, \frac{1}{f_{\grave{\theta}_2}} \leq k_1$.

$$\sum_{\tau=\grave{w}_{r,\grave{i}}^{SCRP}}^{\grave{w}_{r,\grave{i}}^{SCRP}+\frac{1}{f_{\grave{\theta}_2}}-1} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{SCRP}(\tau) + \sum_{\hat{n}=1}^{(k_1)-\frac{1}{f_{\grave{\theta}_2}}} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{SCRP}(\grave{y}_{r,\grave{i},\hat{n}}^{\grave{\theta}_2,SCRP}) \tag{117}$$

$$\geq \sum_{\tau=\grave{w}_{r,\grave{i}}^{ALT3}}^{\grave{w}_{r,\grave{i}}^{ALT3}+\frac{1}{f_{\grave{\theta}_2}}-1} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{ALT3}(\tau) + \sum_{\hat{n}=1}^{(k_1)-\frac{1}{f_{\grave{\theta}_1}}} \sum_{n=1}^{S} U_{s,n}(t_{r,1})\mu_{s,n}^{ALT3}(\grave{y}_{r,\grave{i},\hat{n}}^{\grave{\theta}_1,ALT3}) - B_c \tag{118}$$

where $B_c > 0$ is a constant. Using the results from the above cases and adding them up for all $x_{r,i}$ and $\grave{x}_{r,\grave{i}}$ we obtain

$$\psi^{SCRP}(t_{r,1}) \geq \psi^{ALT3}(t_{r,1}) - B_3 \tag{119}$$

where $B_3 = M\max(B_b, B_c, 9N^2S) > 0$ is a finite constant. ■

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