Delay Analysis of Multihop Cognitive Radio Networks Using Network of Virtual Priority Queues

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Abstract

In this paper, we characterize the average end-to-end delay and maximum achievable per-node throughput in an opportunistic secondary cognitive radio network co-existing with a primary network where both networks consist of static nodes that use random medium access schemes. Assuming an ideal sensing mechanism, we first model the secondary network as a two-class priority queuing network and use queuing approximation techniques to obtain a set of relations involving the mean and second moments of the inter-arrival time and service-time of packets at a secondary node. Then, utilizing these parameters in an equivalent open G/G/1 queuing network, we obtain closed form expressions for average end-to-end delay of a packet in the secondary network and the maximum achievable throughput of a secondary node. The results are validated against extensive simulations.

1. Introduction

The rise in the number of wireless devices with increasing capabilities has increased the demand for frequency spectrum. However, the licensed spectrum is often under-utilized at a given time. This has led to an interest in the study of cognitive radio networks that opportunistically access the frequency spectrum when it is not being used by any licensed user. The licensed users are also known as the primary users (PUs) who enjoy higher priority for spectrum access over the un-licensed users who are known as secondary users (SUs).

A multi-hop wireless ad-hoc network is a decentralized network consisting of nodes that co-ordinate among themselves without any pre-existing infrastructure. Every node with a packet to transmit has to access the channel according to some distributed medium access control (MAC) protocol so that its transmission does not collide with that from a neighboring node. A node can be either a source or a relay for every packet that it transmits. The end-to-end delay of a packet is defined as the time taken by a packet, after its generation, to reach its destination. The average end-to-end delay is the end-to-end delay *averaged over all packets and topologies* and it depends on the traffic pattern, number of nodes, MAC-scheme etc.

In this paper, we investigate the average end-to-end delay and maximum achievable throughput in a multi-hop secondary network using random access based MAC co-existing with another multi-hop primary network using similar random access based MAC. To the best of our knowledge, no previous work addressed the average end-to-end delay in such a system. It is assumed that both networks share a single channel and use backoff and collision avoidance schemes similar to IEEE 802.11 random access MAC. We assume infinitesimally small sensing intervals and an ideal sensing process.

The main results of this paper are:

1) We obtain closed-form expressions for the average end-to-end delay and maximum achievable throughput in the secondary network.
2) We show that for the case, when the parameters are comparable to ones used for the stand-alone ad-hoc wireless network in [1] and the secondary network is denser than the primary, the maximum achievable throughput of a secondary node, $\lambda_{max}^{(s)} = o(\frac{W}{\sqrt{n^{(s)}log(n^{(s)})}})$ where W is the transmission bandwidth and $n^{(s)}+1$ is the number of secondary nodes in the network.

There exist several related works on queuing delay in cognitive networks. In [2], the queuing delay *in a single-hop* network of multiple SUs in presence of multiple primary channels that uses random access is analyzed by using continuous fluid-queue approximation to characterize the queue dynamics. In [3], the delay performance of one SU in the presence of other PUs sharing the same channel is considered. A time-threshold scheme for SU-packet transmission is proposed by developing a Markovian model wherein each state is the number of SU packets at the beginning of each idle time-slot. However, this scheme neither takes into account the contention effects nor does it use a multi-hop scheme. In [4], the authors characterize the minimum multi-hop delay and connectivity of the secondary network as a function of SU and PU densities. However, this work also does not address the scenario where different secondary nodes are contending for the channel. In [5], the authors use pre-emptive priority queuing system to evaluate the average waiting time of delay-sensitive and delay-insensitive packets for two cases- (a) multiple PUs and a single SU where the SU senses only at the beginning of a time-slot and (b) a single PU and a single SU where the SU senses the channel continuously. Other works such as [6], [7] etc. also analyze delay for single-hop SUs by using a priority queue model for channel access. While we use a similar priority queue-model as [5]- [7], in our case the service-time process of an SU is interrupted due to transmission process of nearby PUs and SUs and the latter in turn depends on their own respective service-time processes. As a result, our scenario is different from that in previous works.

In [8] the authors obtain closed form expression for average delay in a multi-hop network with uniformly distributed nodes that are using IEEE 802.11 based random-access scheme and a probabilistic routing protocol. The authors first obtained exact expressions for the mean and second moments of the effective service-time of nodes and then used a diffusion approximation for single-class G/G/1 systems to obtain closed form expressions for average delay. In contrast, this work considers two co-existing and interacting networks (primary and secondary) where nodes from one network (i.e. primary) have higher priority in accessing the channel than the nodes from the second network (i.e. secondary). This coupling of the behavior of the queues in the two networks introduced new modeling challenges, which are analyzed by applying new approximation techniques that has not been used before in this context.

We first model the secondary network as an open network of G/G/1 pre-emptive resume service First Come First Serve (FCFS) priority queues and use certain queuing approximation techniques from [9] to find relations involving the effective service-time and inter-arrival time of packets at a secondary node. We then model the secondary network as a collection of G/G/1 (non-priority) queues for which the effective service time and the inter-arrival time of a job at any station satisfies the relations obtained in the first step. This enables the derivation of closed form expressions for the maximum achievable throughput and average end-to-end delay using diffusion approximation for an open queuing network consisting of G/G/1 stations as given in [10]. The simulation results show the validity of the results for a range of practical network parameters.

In the next section, we describe our multi-hop cognitive radio network model. Section 3 briefly summarizes the theoretical queuing network results on diffusion approximation for priority queues [9] and the diffusion approximation for non-priority single-class G/G/1 systems [10], which are used in later sections. In Section 4, we briefly describe the multi-hop wireless ad-hoc network model in [8] on which our network model is based and mention some key results that we will be using in our work. In Sections 5 and 6, we derive the expressions for the delay and maximum achievable throughput for the network described in Section 2. In Section 7, we compare the analytical and simulation results and find that they closely match for wide range of channel utilization scenarios. Section 8 concludes the paper.

2. Network model

We consider two networks- a primary and a secondary that co-exist together and share a single channel[1]. The primary and secondary networks consist of $n^{(p)}+1$ and $n^{(s)}+1$ nodes respectively that are distributed uniformly and independently over a torus of unit area. The transmission radius of a primary and a secondary node are given by $r^{(p)}(n^{(p)})$ and $r^{(s)}(n^{(s)})$ respectively. All secondary (or primary) nodes located within distance $r^{(s)}(n^{(s)})$(or $r^{(p)}(n^{(p)})$)of a given secondary (or primary) node are neighbors to that node as they can communicate to each other directly. All secondary (or primary) nodes located within distance $2r^{(s)}(n^{(s)})$(or $2r^{(p)}(n^{(p)})$)of a given secondary (or primary) node are interfering neighbors to that node as the secondary (or primary) node should freeze its back-off timers or any on-going transmissions every time such a node starts transmitting. Similarly, all primary nodes located within a distance of $r^{(p)}(n^{(p)})+r^{(s)}(n^{(s)})$ act as the interfering primary neighbors to a secondary

1. The network model is based on that developed in [8], extended here to the case of primary-secondary network.

node. A protocol model of interference for both networks is assumed, wherein a primary (or secondary) node i can successfully transmit to another primary (or secondary) node j iff j is located within its transmission radius and no other interfering neighbor of j, neither primary nor secondary, is transmitting at the same time.

Each node can act as a source/destination/relay of a packet. A primary and a secondary node generates packets at rate $\lambda^{(p)}$ and $\lambda^{(s)}$ packets/second respectively. The size of a primary or secondary packet is constant and both are equal to L bits. On receiving a packet from its neighbors, a primary (or secondary) node absorbs it with probability $q^{(p)}(n^{(p)})$ (or $q^{(s)}(n^{(s)})$) or forwards it to another primary (or secondary) neighbor with probability $1 - q^{(p)}(n^{(p)})$ (or $1 - q^{(s)}(n^{(s)})$). Nodes in both networks use random access MAC with exponential back-off timers. The mean durations of the back-off timer are $\frac{1}{\xi^{(p)}}$ and $\frac{1}{\xi^{(s)}}$ seconds for the primary and the secondary nodes respectively. We assume an ideal sensing process, i.e. the secondary nodes can sense the transmission activities of a primary neighboring node almost instantaneously and pause any of their ongoing transmission processes. A given primary node listening for channel activity can differentiate between the channel usage by a neighboring primary or a secondary node. If a primary node senses that the channel is being used by a secondary node, it treats the channel as if it is idle i.e. the back-off timer is updated as if the channel is idle. This is because secondary traffic has lower priority that primary. This ensures that the primary network remains unaffected by secondary network activity. In addition we make the following assumptions about the network model:

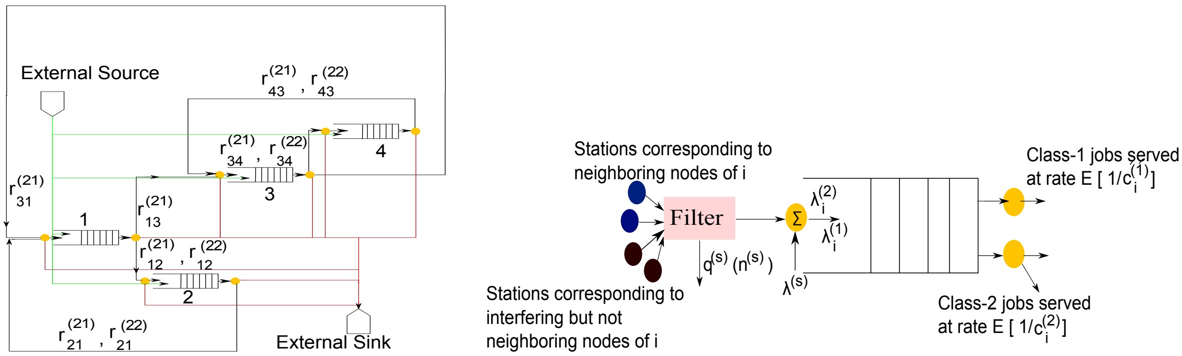
(A1) If a secondary transmitter is active, this implies that no other primary or secondary node is active within the secondary transmitter's interference range.

(A2) The packet generation process is an iid Poisson process.

In general two or more primary or secondary interfering neighbors of a secondary node can simultaneously transmit. The *logical union* of such transmission processes is the channel activity process as observed by the secondary node. However due to A1 we assume, while a secondary node with a packet to transmit is counting down its backoff timer, the number of times its timer is frozen is equal to the *sum* of the number of times any interfering neighboring primary or secondary node's timer expires before that of the given secondary node.

The queuing network model needs to account for the interruption caused due to the transmission activity of interfering primary nodes which forces any secondary node to pause its on-going service of a packet (includes the durations of both the back-off timer and the transmission time). Since the primary network does not care about the presence of secondary nodes, they always have access to the channel while the secondary nodes can access it opportunistically. We therefore model the secondary network as an open queuing network consisting of G/G/1 FCFS pre-emptive resume service priority queues with 2-classes of jobs. Each station of this queuing network corresponds to a secondary node. We refer to the queue associated with such a station or a node as a secondary queue. The arrival of a high-priority job (a transmission activity) should cause a secondary station to pre-empt any ongoing service of a lower priority job (a real packet). Since, the duration of any interruption is equal to the packet transmission time $\frac{L}{W}$, the service-time of any high-priority job is $\frac{L}{W}$.The model can be summarized as follows:

1) The transmission processes from all interfering neighboring primary and secondary nodes constitute an



(a) Representation of multi-hop secondary network as a priority queuing network. Stations 1,2 and 3,4 correspond to pairs of neighboring nodes respectively; nodes corresponding to stations 1,3 are interfering neighbors to each other but not neighboring nodes; pair of nodes corresponding to stations 1,4 and 2,4 respectively are not interfering neighbors to each other. Green (red) lines reflect routes from (to) external source (sink).

(b) Representation of a node i in the secondary network as a station in the priority queuing network. Stations corresponding to neighboring nodes route only class-2 jobs to i, stations corresponding to other interfering nodes route both class-1 and class-2 jobs to i. The filter block absorbs class-2 jobs with probability $q^{(s)}(n^{(s)})$. Served class-1 jobs are routed to an external sink, others to neighbors of i with equal probability. $\lambda_i^{(m)}$ and $c_i^{(m)}$ denote the arrival rate and effective-service time respectively of a class-m (m=1,2) job at i.

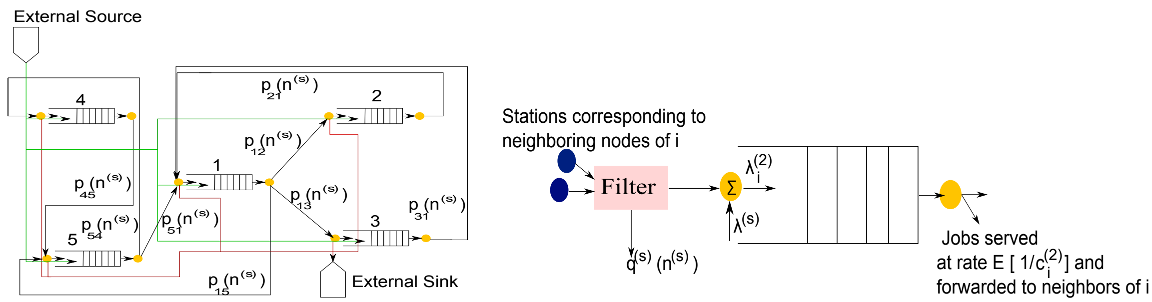
Figure 1. Priority-queuing network model representation of a secondary network. $r_{ij}^{(21)}$=1 $\forall i,j$ that are interfering nodes to each other. $\forall i,j$ that are neighboring nodes, $\frac{1}{r_{ij}^{(22)}}$ is the number of neighbors of i.

virtual arrival process of higher priority class-1 jobs at a given secondary queue. Because of A1, the arrival process of class-1 jobs at a secondary queue can be considered as the sum of the transmission processes by interfering primary and secondary nodes. These processed jobs are then forwarded to a sink with probability 1.

2) The packets generated by a given secondary node (modeled as external arrival process of class-2 jobs at the queue) and those received from neighboring secondary nodes, but not absorbed, constitute the arrival process of lower priority class-2 jobs at the corresponding secondary queue. The processed jobs are then forwarded to all interfering secondary neighbors of the given node as a class-1 job with probability 1 and to each secondary neighbor as a class-2 job with probability equal to the reciprocal of the number of neighboring nodes.

The priority queuing network representation of the secondary network is shown in Fig 1.

The novelty of the model is in the introduction of virtual jobs and sinks to model the effect of both PU and



(a) Representation of multi-hop secondary network as a G/G/1 queuing network. Each station forwards a job only to a station corresponding to a neighboring node. Green (red) lines reflect routes from (to) external source (sink).

(b) Representation of a node i in the secondary network as a station in the G/G/1 queuing network. The filter block absorbs class-2 packets with probability $q^{(s)}(n^{(s)})$.

Figure 2. G/G/1 queuing network model representation of a secondary network. $\forall i, j$ that are neighboring nodes, $\frac{1}{p_{ij}(n^{(s)})}$ is the number of neighbors of i.

SU traffic on a node in the secondary network. The virtual jobs do not correspond to real packets that ought to be forwarded to another node; they are immediately transferred to an external sink after processing.

Similar to [8], once the overall service-time process is known, the secondary network can also be modeled as a G/G/1 queuing network as shown in Fig 2, where each station corresponds to a secondary node.

3. Diffusion approximation for queuing networks

In this section, we give a brief overview of the diffusion approximation techniques used in our work. In the next sub-section, we review the diffusion approximation for priority queues [9]. Following that, we review the diffusion approximation for a general G/G/1-FCFS network (non-priority) [10].

3.1. Diffusion approximation for priority queues

In [9], the authors consider a pre-emptive resume G/G/1-FCFS priority queuing network with K classes of jobs indexed as $(1, 2, ...K)$ where a job of the k-th class $(1 \leq k < K)$ has higher priority than a job belonging to $(k+1)$-th class. For a network with a single class of jobs, the sum of jobs entering station j in a time-interval t is assumed to be normally distributed with mean

$$\lambda_j t = \{\sum_{i=1}^{M} \lambda_i r_{ij} + \lambda_{0j}\}t \tag{1}$$

and variance

$$\sigma_{Aj}^2 t = \{\sum_{i=1}^{M} C_{Di}^2 \lambda_i r_{ij} + C_{0j}^2 \lambda_{0j}\} t \tag{2}$$

where r_{ij} is the probability that a job served by station i is transferred to station j, λ_{0j} denotes the arrival rate of jobs from a station external to the network, M denotes the number of stations in the network; C_{Di} and C_{0j} denote the coefficient of variation of inter-departure time of jobs from station i and a station external to the network respectively (this assumption can be extended to multiclass networks with similar expressions for the mean and variance). For a station i in such a network, the co-efficient of variation of inter-departure time of a job (C_{Di}) is approximately related to the coefficient of variation of service-time (C_{Bi}), the coefficient of variation of inter-arrival time of a job (C_{Ai}) and utilization-factor (ρ_i) of station i as,

$$C_{Di}^2 = \rho_i^2 C_{Bi}^2 + C_{Ai}^2 (1-\rho_i) + \rho_i (1-\rho_i) \tag{3}$$

Let $a^{(k)}(t)$, $b^{(k)}(t)$ and $c^{(k)}(t)$ denote the inter-arrival time, service-time and effective service-time or completion-time (defined as the time-period between the beginning and end of service of a job; it is same as service-time for the highest priority-class, is greater than service-time for other classes because the on-going service of a job of a given class can be interrupted with the arrival of a job of higher priority class) respectively of a k-th class job in a station in a multi-class network. Let $\lambda^{(k)}$ and $C_A^{(k)}$ denote the arrival rate and co-efficient of variation of arrival process of k-thclass jobs while $\mu^{(k)}$ and $C_B^{(k)}$ denote the service-rate and co-efficient of variation of service-time of a k-th class job in a station in the network respectively. The change in total number of jobs of class $1, 2, .., K$ at any station during the time-period $[0, t]$ is approximated to be normally distributed with mean $\beta^{(K)} t$ and variance $\alpha^{(K)} t$ where,

$$\beta^{(K)} = \sum_{k=1}^{K} \lambda^{(k)} - \sum_{k=1}^{K} \frac{\rho^{(k)}}{R^{(k)}} \mu^{(k)} \tag{4}$$

$$\alpha^{(K)} = \sum_{k=1}^{K} \lambda^{(k)} C_A^{(k)^2} + \sum_{k=1}^{K} \frac{\rho^{(k)}}{R^{(k)}} \mu^{(k)} C_B^{(k)^2}, \tag{5}$$

where $R^{(k)} = \sum_{l=1}^{k} \rho^{(l)}$, $\rho^{(l)} = \frac{\lambda^{(l)}}{\mu^{(l)}}$. When the server is busy processing a k-th class job, the service maybe interrupted with the arrival of a higher priority job. If there are n such breaks within the service time T of a class-k job, then n is assumed to be approximately normally distributed with mean $\sum_{l=1}^{k-1} \lambda^{(l)} T$ and variance $\sum_{l=1}^{k-1} \lambda^{(l)} C_A^{(l)^2} T$ and each such break is distributed according to the distribution $\gamma^{(k-1)}(t)$ of the system serving jobs of classes $1, ...k-1$. The total duration of such breaks in T has the pdf

$$\psi^{(k)}(t|T) = \sum_{n=0}^{\infty} P_{n|T} \gamma^{(k-1)*n}(t), \tag{6}$$

where $P_{n|T}$ is the probability of n breaks in T. The moment and the second moment of the busy-time are approximately given as below,

$$E[\gamma^{(k)}(t)] = -\frac{1}{\beta^{(k)}} \tag{7}$$

$$E[\gamma^{(k)^2}(t)] = -\frac{\alpha^{(k)}}{\beta^{(k)^3}} + \frac{1}{\beta^{(k)^2}} \tag{8}$$

The pdf of the completion time is,

$$f_{c^{(k)}(x)}(t) = \int_0^\infty f_{b^{(k)}(x)}(t)\psi^{(k)}(t-T|T)1(t-T)\mathrm{d}T \tag{9}$$

where $1(t) = 0$ for $t < 0$ and $f_{g(x)}(t)$ denote the pdf of a function $g(t)$. From (9), the authors derive the moment and second moment of $c^{(k)}(t)$.

Let $d^{(k)}(t)$ denote the inter-departure time in the stream of class-k jobs. Then approximately,

$$\begin{aligned} f_{d^{(k)}(x)}(t) = \frac{\rho^{(k)}}{1-R^{(k-1)}} f_{c^{(k)}(x)}(t) + (1 - \frac{\rho^{(k)}}{1-R^{(k-1)}}) \\ [(1-R^{(k-1)}) f_{a^{(k)}(x)}(t) * f_{c^{(k)}(x)}(t) + \\ R^{(k-1)} f_{a^{(k)}(x)}(t) * \gamma^{(k-1)}(t) * f_{c^{(k)}(x)}(t)] \end{aligned} \tag{10}$$

Using (10), the squared coefficient of variation of inter-departure time of a k-th class job from a station j ($C_{Dj}^{(k)^2}$) can be expressed as a function of coefficient of variation of inter-arrival time of a l-th class job at a station j ($C_{Aj}^{(l)^2}$) ($l = 1, ...k$). Also, $C_{Aj}^{(l)^2}$ can be expressed as a function of inter-departure times of jobs that are being routed to station j,

$$\begin{aligned} C_{Aj}^{(l)^2} &= \frac{\sum_{i=1}^{M}\sum_{k=1}^{K} r_{ij}^{(kl)}\lambda_i^{(k)}[(C_{Di}^{(k)^2}-1)r_{ij}^{(kl)}+1]}{\lambda_j^{(l)}} \\ &+ \frac{C_{0j}^{(l)^2}\lambda_{0j}^{l}}{\lambda_j^{(l)}}, \end{aligned} \tag{11}$$

where $r_{ij}^{(kl)}$ is the probability that a class k-job leaving station i goes directly to station j as a class l-job, M denotes the number of stations in the network, $\lambda_j^{(l)}$ denotes the arrival rate of l-th class jobs at a station j; λ_{0j}^{l} and $C_{0j}^{(l)^2}$ denotes the average arrival rate and coefficient of variation of arrival time respectively of a job of class l at station j from a station external to the network. Solving (10) and (11), $C_{Aj}^{(l)^2}$ and $C_{Dj}^{(l)^2}$ can be computed and subsequently used to obtain average packet length at any station.

3.2. Diffusion approximation for G/G/1- queuing network

A diffusion approximation for a general G/G/1-FCFS network is described in [10] and [11]. Consider a network consisting of n stations with G/G/1- queues, where K_i denote the number of jobs at station i. The coefficient of variation of inter-arrival times of jobs at station i is denoted by C_{Ai}^2. The external arrival process at each station has mean inter-arrival time $\frac{1}{\lambda}$ and coefficient of variation C_A. The service-time at station i has mean $\frac{1}{\mu_i}$ and coefficient of variation C_{B_i}. The visit-ratio (e_i) of station i is defined as the mean number of visits of a job to i-th station and is given by,

$$e_i = p_{0i} + \sum_{j=1}^{n} p_{ji}(n)e_j, \tag{12}$$

where p_{0i} denotes the probability that a job entering the network from outside first enters the i-th station, p_{ji} denotes the probability that a job completed by station j is transferred to station i. Using diffusion approximation, the marginal probability of queue length of station i is given by,

$$\hat{\pi}_i(k_i) = \begin{cases} 1-\rho_i & k_i = 0, \\ \\ \rho_i(1-\hat{\rho}_i)\hat{\rho}_i^{\,k_i-1} & k_i \geq 1. \end{cases} \tag{13}$$

where

$$\rho_i = \frac{\lambda e_i}{\mu_i} \tag{14}$$

$$\hat{\rho}_i = exp(-\frac{2(1-\rho_i)}{C_{Ai}^2 . \rho_i + C_{Bi}^2}) \tag{15}$$

The coefficient of variation of inter-arrival times at station i is given by,

$$C_{Ai}^2 = 1 + \sum_{j=0}^{n}(C_{Bj}^2 - 1)p_{ji}^2 e_j e_i^{-1} \tag{16}$$

where $C_{B0}^2 = C_A^2$. The mean number of jobs at station i is given by,

$$\bar{K}_i = \frac{\rho_i}{1-\hat{\rho}_i} \tag{17}$$

4. Diffusion approximation in multi-hop wireless networks

In this section we briefly review the work in [8] because we base our network model on that in [8]. We also mention some results from the paper which we use in our work.

Let a wireless network consist of $n+1$ nodes with each node having equal transmission range denoted by $r(n)$. If the distance between any two nodes is less than $r(n)$ and $2r(n)$, they are said to be neighbors and interfering neighbors respectively. If $N(i)$ denotes the number of nodes that are interfering neighbors to a node i, then

$$\bar{N}(i) = E[N(i)] = 4nA(n), \tag{18}$$

where $A(n) = \pi r^2(n)$ and the average is taken over all possible network topologies. Each node is assumed to generate packets of constant size of L bits with average rate λ packets per second. Each node absorbs a packet received from its neighbors with a probability $p(n)$ and transmits it to any of its neighbors with probability $1-p(n)$. All the neighbors to the node are equally likely to receive such a transmitted packet. Then the probability that a packet transmitted by node i enters the queue of node j is,

$$p_{ij} = \frac{1-p(n)}{n} \tag{19}$$

The average number of hops traversed by a packet before being absorbed is $\frac{1}{p(n)}$. A Random Access-MAC protocol with exponential back-off parameter ξ is assumed. If X_i denote the time required by node i to serve a packet of length L bits, then

$$\bar{X}_i = E[X_i] = \frac{\frac{1}{\xi} + \frac{L}{W}}{1 - 4nA(n)\lambda_i \frac{L}{W}}, \tag{20}$$

where

$$\lambda_i = \frac{\lambda}{p(n)} \tag{21}$$

is the effective expected packet arrival rate at i and W denotes the bandwidth of the channel. λ_i is also the average transmission rate of packets from i. The standard deviation of service-time of node i, denoted by σ^2_{Xi} is shown to be

$$\sigma^2_{Xi} = \frac{L^2}{W^2}(\bar{m} + \bar{m^2} + \sigma^2_m) + 2(2\bar{m}+1)\frac{L}{W}\frac{1}{\xi} + \frac{1}{\xi^2}, \tag{22}$$

where

$$\bar{m} = \rho 4nA(n) \tag{23}$$

$$\bar{m^2} = \rho^2 4nA(n)(1 + 4(n-1)A(n)) + (1-\rho)\rho 4nA(n) \tag{24}$$

$$\rho = \lambda_i \bar{X}_i \tag{25}$$

$$\sigma^2_m = \bar{m^2} - (\bar{m})^2 \tag{26}$$

The squared coefficient of variation of service-time of node i (C^2_{Bi}) is,

$$C^2_{Bi} = \frac{\sigma^2_{Xi}}{\bar{X_i}^2} \tag{27}$$

The squared coefficient of variation of inter-arrival time at node i is given by

$$C^2_{Ai} = 1 + (C^2_{Bi} - 1)\frac{(1-p(n))^2}{n} \tag{28}$$

The maximum achievable throughput is found to be

$$\lambda_{max} = \frac{p(n)}{\frac{1}{\xi} + \frac{L}{W} + 4nA(n)\frac{L}{W}} \tag{29}$$

The Gupta-Kumar model [1] characterizes the throughput capacity for a stand-alone wireless network where the nodes are distributed uniformly over a sphere of unit surface area and the source-destination pairs are chosen randomly. In [8] the achievable throughput of a single-class queuing network obtained using diffusion approximation was compared with that from the Gupta-Kumar model by selecting $p(n) = \sqrt{\frac{log(n)}{n}}$ and $r(n) = \sqrt{\frac{log(n)}{n}}$ so that the expected number of hops between source and destination and the transmission range respectively are comparable. The maximum achievable throughput for this case is shown to be,

$$\lambda_{max} = \frac{\frac{1}{4\pi}\frac{W}{\sqrt{nlog(n)}L}}{1 + \frac{c}{4\pi log(n)\frac{L}{W}}} = o(\frac{W}{\sqrt{nlog(n)}}) \tag{30}$$

where $c = \frac{1}{\xi} + \frac{L}{W}$.

5. Delay analysis

In this section, we find the average end-to-end delay for the network described in Section 2.

Let i denote a secondary node. Let $N_i^{(p)}$ (or $N_i^{(s)}$) denote the number of primary (or secondary) nodes that are interfering neighbors to i. Let $M_{i,j}^{(p)}$ $(1 \leq j \leq N_i^{(p)})$ and $M_{i,k}^{(s)}$ $(1 \leq k \leq N_i^{(s)})$ denote the primary and secondary nodes respectively that are interfering neighbors to i and let $\lambda_{i,j}^{(p)}$ and $\lambda_{i,k}^{(s)}$ denote their corresponding average transmission rate in packets/second. Let $r_{iv}^{(22)}$ denote the probability that a packet (equivalently a class-2 job in the priority-queue representation) is forwarded from secondary node i to another secondary node v and it enters the queue of v. Let $C_{A^{(p)}}$ and $C_{D^{(p)}}$ denote the coefficient of variation of inter-arrival time and inter-departure time of packets at a primary node respectively; let $\rho^{(p)}$ denote the utilization at a primary station (from the symmetry of the nodes, those terms are equal for all primary nodes and hence the node index is dropped). Let $\bar{X_l}^{(p)}$ and $\sigma^2_{X_l^{(p)}}$ denote the mean and standard deviation of effective service-time at a primary node l.

Our primary network model is exactly the same as in [8] and the secondary network is different from the primary only with regards to priority of channel access. Then following results, which are similar to the ones proved for a multi-hop ad-hoc network in [8], are true for our network (also refer to Section 4).

Lemma 1: For all secondary nodes i, v and primary node l,

(i) $\bar{N}_i^{(p)} = n^{(p)} A_{r^{(p)},r^{(s)}}(n^{(p)}, n^{(s)})$, $\bar{N}_i^{(s)} = 4n^{(s)} A_{r^{(s)}}(n^{(s)})$ where $A_R(n) = \pi R^2(n)$ and $A_{R,r}(n_1, n_2) = \pi(R(n_1) + r(n_2))^2$.

(ii) $\lambda_{i,j}^{(p)} = \frac{\lambda^{(p)}}{q^{(p)}(n^{(p)})}$ and $\lambda_{i,k}^{(s)} = \frac{\lambda^{(s)}}{q^{(s)}(n^{(s)})}$ where $1 \leq j \leq N_i^{(p)}$ and $1 \leq k \leq N_i^{(s)}$.

(iii) $r_{iv}^{(22)} = \frac{(1-q^{(s)}(n^{(s)}))}{n^{(s)}}$.

(iv) Average number of hops traversed by a secondary packet before being absorbed is $\frac{1}{q^{(s)}(n^{(s)})}$.

(v) $\bar{X}_l^{(p)} = \frac{\frac{1}{\xi^{(p)}} + \frac{L}{W}}{1 - 4n^{(p)} A_{r^{(p)}}(n^{(p)}) \frac{\lambda^{(p)}}{q^{(p)}(n^{(p)})} \frac{L}{W}}$.

(vi) $\sigma^2_{X_l^{(p)}} = \frac{L^2}{W^2}(\bar{m} + \bar{m^2} + \sigma_m^2) + 2(2\bar{m} + 1)\frac{L}{W}\frac{1}{\xi^{(p)}} + \frac{1}{\xi^{(p)2}}$

where

$$\bar{m} = \rho^{(p)} 4n^{(p)} A_{r^{(p)}}(n^{(p)}). \tag{31}$$

$$\bar{m^2} = \rho^{(p)2} 4n^{(p)} A_{r^{(p)}}(n^{(p)})(1 + 4(n^{(p)} - 1) A_{r^{(p)}}(n^{(p)})) + (1 - \rho^{(p)})\rho^{(p)} 4n^{(p)} A_{r^{(p)}}(n^{(p)}) \tag{32}$$

$$\rho^{(p)} = \frac{\lambda^{(p)}}{q^{(p)}(n^{(p)})} \bar{X}_l^{(p)} \tag{33}$$

$$\sigma_m^2 = \bar{m^2} - (\bar{m})^2 \tag{34}$$

Proof: The proof is omitted. □

Theorem 1: For the random-access network, the average end-to-end delay of a secondary packet, $D(n^{(s)}, n^{(p)})$ is given as

$$D(n^{(s)}, n^{(p)}) = \bar{D}_i \frac{1}{q^{(s)}(n^{(s)})}, \tag{35}$$

where $\bar{D}_i$ is a function whose closed-form expression can be found in terms of $n^{(s)}$, $n^{(p)}$, $\lambda^{(s)}$,$\lambda^{(p)}$, $q^{(s)}(n^{(s)})$, $q^{(p)}(n^{(p)})$,$r^{(p)}(n^{(p)})$ and $r^{(s)}(n^{(s)})$.

Proof: (Outline of proof: We consider the 2-class priority queuing network model of the secondary network. Like the analysis in [9] we approximate the number of arrivals of class-1 jobs at a secondary station within a given interval as being normally distributed with its mean and variance a function of transmission rate and inter-departure time of other jobs in the network. Similarly, we also approximate the number of times the service of a class-2 job being interrupted due to the arrival of a class-1 job, as being normally distributed with certain assumptions about its mean and variance. Similar to [9] we then approximate the pdf of the completion and inter-departure time of class-2 jobs in the secondary queuing network. Using the above approximations, Lemma 1 and due to symmetry of the nodes we obtain a set of polynomial equations involving mean and second moments of the inter-arrival time and completion-time of class-2 jobs at a secondary station. Then using the diffusion approximation from [10] and the previously obtained relations we first calculate exact closed form expressions for the second moments of inter-arrival time and completion-time in terms of known parameters and use them to obtain the average number of packets at any secondary node. Using Little's Theorem, we obtain the average system delay at any secondary node which when multiplied by the average number of hops give us the closed form expression for average end-to-end multihop delay.)

We consider the 2-class priority queuing network model of the secondary network. From Lemma 1, the average number of class-2 jobs transmitted by station i (corresponding to secondary node i) per second is given by,

$$\lambda_i^{(2)} = \frac{\lambda^{(s)}}{q^{(s)}(n^{(s)})} \tag{36}$$

The mean of service-time of a class-1 job ($b_i^{(1)}(t)$) at i is given by

$$E[b_i^{(1)}(t)] = \frac{L}{W} = \frac{1}{\mu_i^{(1)}} \tag{37}$$

The mean and second moment of service-time of a class-2 job ($b_i^{(2)}(t)$) at i is given by

$$E[b_i^{(2)}(t)] = \frac{1}{\xi^{(s)}} + \frac{L}{W} = \frac{1}{\mu_i^{(2)}} \tag{38}$$

$$E[b_i^{(2)^2}(t)] = \frac{2}{\xi^{(s)^2}} + \frac{L^2}{W^2} + 2\frac{L}{W\xi^{(s)}}, \tag{39}$$

Let $a_i^{(k)}(t)$ denote the inter-arrival time of a class-k (k=1,2) job at i. As in [9], we assume that the number of arrival of class-1 jobs at i in time-interval t is normally distributed with mean $\lambda_i^{(1)}t$ and coeffiecient of variation of inter-arrival times $C_{A,i}^{(1)}$ whose expressions are obtained from Section 3 as,

$$\begin{aligned} \lambda_i^{(1)} &= \bar{N}_i^{(p)}\lambda_{i,j}^{(p)} + \bar{N}_i^{(s)}\lambda_{i,k}^{(s)} \\ &= \bar{N}_i^{(p)}\frac{\lambda^{(p)}}{q^{(p)}(n^{(p)})} + \bar{N}_i^{(s)}\frac{\lambda^{(s)}}{q^{(s)}(n^{(s)})} \\ &= n^{(p)}A_{r^{(p)},r^{(s)}}(n^{(p)},n^{(s)})\frac{\lambda^{(p)}}{q^{(p)}(n^{(p)})} + \\ &\quad 4n^{(s)}A_{r^{(s)}}(n^{(s)})\frac{\lambda^{(s)}}{q^{(s)}(n^{(s)})} \quad \text{and} \end{aligned} \tag{40}$$

$$
\begin{aligned}
C_{A,i}^{(1)^2} &= \frac{1}{\lambda_i^{(1)}}\{\sum_{j=1}^{\bar{N}_i^{(s)}} 1.\lambda_{i,M_{i,j}^{(s)}}^{(s)}[(C_{D,M_{i,j}^{(s)}}^{(2)^2}-1).1+1]+ \\
&\sum_{k=1}^{\bar{N}_i^{(p)}} 1.\lambda_{i,M_{i,k}^{(p)}}^{(p)}[(C_{D,M_{i,k}^{(p)}}^{(p)^2}-1).1+1]\} \qquad (41)
\end{aligned}
$$

where $C_{D,M_{i,k}^{(p)}}^{(p)^2}$, and $C_{D,M_{i,j}^{(s)}}^{(s)^2}$ denote the squared coefficient of variation of inter-departure times of packets transmitted by primary node $M_{i,k}^{(p)}$ and a secondary node $M_{i,j}^{(s)}$ respectively. We assume that the number of arrivals of class-1 jobs, $n_i^{(2)}$ that occur within a service-time $b_i^{(2)}(t)$ of a class-2 job is normally distributed with mean $\lambda_i^{(1)}b_i^{(2)}(t)$ and variance $\lambda_i^{(1)}C_{A,i}^{(1)^2}b_i^{(2)}(t)$. Using (7) and (8) the moment and second moment of the busy-time $(\gamma_i^{(1)}(t))$ corresponding to a class-1 job at station i are given by,

$$
E[\gamma_i^{(1)}(t)] = -\frac{1}{\beta_i^{(1)}} \qquad (42)
$$

$$
E[\gamma_i^{(1)^2}(t)] = -\frac{\alpha_i^{(1)}}{\beta_i^{(1)^3}} + \frac{1}{\beta_i^{(1)^2}} \qquad (43)
$$

where

$$
\beta_i^{(1)} = \lambda_i^{(1)} - \mu_i^{(1)} \qquad (44)
$$

$$
\alpha_i^{(1)} = \lambda_i^{(1)}(C_{A,i}^{(1)^2}) \qquad (45)
$$

The last equality follows because the service-time of a class-1 job is a constant. Since all nodes in the network are similar, by symmetry $C_{A,i}^{(1)^2} = C_{A,j}^{(1)^2}$, $E[\gamma_i^{(1)}(t)] = E[\gamma_j^{(1)}(t)]$, $E[\gamma_i^{(1)^2}(t)] = E[\gamma_j^{(1)^2}(t)]$, $E[b_i^{(2)^2}(t)] = E[b_j^{(2)^2}(t)]$, $E[b_i^{(2)}(t)] = E[b_j^{(2)}(t)]\ \forall i,j$. Next we approximate the pdf of the completion-time $c_i^{(2)}(t)$ and inter-departure time $d_i^{(2)}(t)$ of a class-2 job at i using (9) and (10) respectively. Using (9), the mean of $c_i^{(2)}(t)$ is given by,

$$
\begin{aligned}
E[c_i^{(2)}(t)] &= \{E[\gamma_i^{(1)}(t)]\lambda_i^{(1)}+1\}\frac{1}{\mu_i^{(2)}} \\
&= \{\frac{1}{1-\frac{\lambda_i^{(1)}}{\mu_i^{(1)}}}\}\frac{1}{\mu_i^{(2)}} \qquad (46)
\end{aligned}
$$

Using (9) and (43), the second moment of $c_i^{(2)}(t)$ is given by,

$$
\begin{aligned}
E[c_i^{(2)^2}(t)] &= E[b_i^{(2)^2}(t)](1+E[\gamma_i^{(1)}(t)]\lambda_i^{(1)})^2 + E[b_i^{(2)}(t)](E[\gamma_i^{(1)}(t)])^2C_{A,i}^{(1)^2}\lambda_i^{(1)} - \lambda_i^{(1)}(E[\gamma_i^{(1)}(t)])^2 + E[\gamma_i^{(1)^2}(t)]\lambda_i^{(1)}\} \\
&= g_{i,1} + g_{i,2}C_{A,i}^{(1)^2} \qquad (47)
\end{aligned}
$$

where

$$
g_{i,1} = E[b_i^{(2)^2}(t)](1+E[\gamma_i^{(1)}(t)]\lambda_i^{(1)})^2 + E[b_i^{(2)}(t)]\{-\lambda_i^{(1)}(E[\gamma_i^{(1)}(t)])^2 + \frac{\lambda_i^{(1)}}{\beta_i^{(1)^2}}\} \qquad (48)
$$

$$
g_{i,2} = E[b_i^{(2)}(t)](E[\gamma_i^{(1)}(t)])^2\lambda_i^{(1)} - \frac{(\lambda_i^{(1)})^2E[b_i^{(2)}(t)]}{\beta_i^{(1)^3}} \qquad (49)
$$

From the symmetry of nodes, $g_{i,1} = g_{j,1}$, $g_{i,2} = g_{j,2}$, $E[c_i^{(2)}(t)] = E[c_j^{(2)}(t)]$ and $E[c_i^{(2)^2}(t)] = E[c_j^{(2)^2}(t)]$ $\forall\ i, j$. From (10), the mean of the inter-departure time of a class-2 job from station i is given by

$$\begin{aligned} E[d_i^{(2)}(t)] &= E[c_i^{(2)}(t)] + (1 - \frac{\rho_i^{(2)}}{1-\rho_i^{(1)}})(E[a_i^{(2)}(t)]) \\ &\quad +\rho_i^{(1)}(1 - \frac{\rho_i^{(2)}}{1-\rho_i^{(1)}})E[\gamma_i^{(1)}(t)], \end{aligned} \tag{50}$$

where

$$\rho_i^{(k)} = \frac{\lambda_i^{(k)}}{\mu_i^{(k)}} = \rho_j^{(k)}, \tag{51}$$

$$E[a_i^{(2)}(t)] = \frac{1}{\lambda_i^{(2)}} = E[a_j^{(2)}(t)] \tag{52}$$

$\forall\ i, j$ $(k = 1, 2)$. From (10) the second moment of the inter-departure time of a class-2 job from station i is given by,

$$\begin{aligned} E[d_i^{(2)^2}(t)] = E[c_i^{(2)^2}(t)] + (1 - \tfrac{\rho_i^{(2)}}{1-\rho_i^{(1)}})(E[a_i^{(2)^2}(t)] \\ +2E[c_i^{(2)}(t)]E[a_i^{(2)}(t)]) + \rho_i^{(1)}(1 - \tfrac{\rho_i^{(2)}}{1-\rho_i^{(1)}})\{E[\gamma_i^{(1)^2}(t)]+ \\ 2E[\gamma_i^{(1)}(t)](E[c_i^{(2)}(t)] + E[a_i^{(2)}(t)])\} \end{aligned} \tag{53}$$

Again from the symmetry of nodes, $E[d_i^{(2)}(t)] = E[d_j^{(2)}(t)]$ and $E[d_i^{(2)^2}(t)] = E[d_j^{(2)^2}(t)]$ $\forall i, j$. Hence, the coefficient of variation of inter-departure times of class-2 jobs,

$$C_{D,i}^{(2)^2} = \frac{E[d_i^{(2)^2}(t)]}{(E[d_i^{(2)}(t)])^2} - 1 = C_{D,j}^{(2)^2}$$

$\forall\ i, j$ $(k = 1, 2)$. We can re-write (41) as,

$$C_{A,i}^{(1)^2} = v_1 C_{D,i}^{(2)^2} + v_2, \tag{54}$$

where v_1=$\bar{N}_i^{(s)}\frac{\lambda^{(s)}}{\lambda_i^{(1)}q^{(s)}(n^{(s)})}$ and v_2=$\bar{N}_i^{(p)}\frac{\lambda^{(p)}}{q^{(p)}(n^{(p)})\lambda_i^{(1)}}C_D^{(p)^2}$.
$C_D^{(p)^2}$ is obtained by using (3) that relates the co-efficient of variation of service-time ($C_B^{(p)}$) of a primary node, $C_A^{(p)^2}$ and $\rho^{(p)}$

$$C_D^{(p)^2} = \rho^{(p)^2}C_B^{(p)^2} + C_A^{(p)^2}(1-\rho^{(p)}) + \rho^{(p)}(1-\rho^{(p)}) \tag{55}$$

.$\rho^{(p)}$, $C_A^{(p)^2}$ and $C_B^{(p)^2}$ can be computed using Lemma 1, (28) and (27) respectively. Now, using (43) and substituting the expressions of $E[c_i^{(2)^2}(t)]$ and $C_{A,i}^{(1)^2}$ from (47) and (54) respectively in (53), we have,

$$\begin{aligned} E[d_i^{(2)^2}(t)] &= h_1 + h_2 C_{A,i}^{(1)^2} + h_3 C_{A,i}^{(2)^2} \\ &= h_1 + h_2(v_1 C_{D,i}^{(2)^2} + v_2) + h_3 C_{A,i}^{(2)^2} \\ &= h_1' + h_2' C_{D,i}^{(2)^2} + h_3 C_{A,i}^{(2)^2}, \end{aligned} \tag{56}$$

where

$$h_1 = g_{i,1} + (1 - \frac{\rho_i^{(2)}}{1-\rho_i^{(1)}})(2E[c_i^{(2)}(t)]E[a_i^{(2)}(t)] + \frac{1}{\lambda_i^{(2)^2}}) + \rho_i^{(1)}(1 - \frac{\rho_i^{(2)}}{1-\rho_i^{(1)}})\{\frac{1}{\beta_i^{(1)^2}} + 2E[\gamma_i^{(1)}(t)](E[c_i^{(2)}(t)] + E[a_i^{(2)}(t)])\} \quad (57)$$

$$h_2 = g_{i,2} - \frac{\lambda_i^{(1)}\rho_i^{(1)}(1 - \frac{\rho_i^{(2)}}{1-\rho_i^{(1)}})}{\beta_i^{(1)^3}} \quad (58)$$

$$h_3 = \frac{(1 - \frac{\rho_i^{(2)}}{1-\rho_i^{(1)}})}{\lambda_i^{(2)^2}}, h'_1 = h_1 + v_2 h_2, \quad h'_2 = v_1 h_2 \quad (59)$$

Now we apply the diffusion approximation from [10] to a network of G/G/1 queues where the inter-arrival time, effective-service time and inter-departure time of jobs from any station i are equal to the random variables $a_i^{(2)}(t)$, $c_i^{(2)}(t)$ and $d_i^{(2)}(t)$ respectively. Then using (28) we get

$$C_{A,i}^{(2)^2} = 1 + \sum_{j=0}^{n^{(s)}}\{\frac{E[c_i^{(2)^2}(t)] - (E[c_i^{(2)}(t)])^2}{(E[c_i^{(2)}(t)])^2} - 1\}r_{ji}^{(22)^2}e_j e_i^{-1}$$
$$= k_3 + k_4 C_{A,i}^{(1)^2}, \quad (60)$$

where k_3=$1 + \frac{(1-q^{(s)}(n^{(s)}))^2}{n^{(s)}}\frac{1}{(E[c_i^{(2)}(t)])^2}(g_{i,1} - 2(E[c_i^{(2)}(t)])^2)$, k_4=$\frac{(1-q^{(s)}(n^{(s)}))^2}{n^{(s)}}\frac{1}{(E[c_i^{(2)}(t)])^2}g_{i,2}$. Substituting the expression of $C_{A,i}^{(2)^2}$ obtained from (60) in (56), then replacing $C_{A,i}^{(1)^2}$ using (54) and equating both sides, we get,

$$C_{D,i}^{(2)^2} = \frac{(\frac{1}{(E[d_i^{(2)}(t)])^2}h'_1 - 1) + \frac{1}{(E[d_i^{(2)}(t)])^2}h'_3(k_3 + k_4 v_2)}{1 - \frac{1}{(E[d_i^{(2)}(t)])^2}(h'_3 k_4 v_1 + h'_2)} \quad (61)$$

Substituting the value of $C_{D,i}^{(2)^2}$ as obtained from (61) in (54), we obtain $C_{A,i}^{(1)^2}$ from which in turn we obtain $C_{A,i}^{(2)^2}$ using (60).

Now that we know the mean and coefficient of variation of the effective service-time and inter-arrival time of a packet at any secondary node i, we can compute the average number of secondary packets in its queue, $\bar{K}_i$ using (17). Then, using Little's theorem, the mean of the system delay D_i at node i, is given by

$$\bar{D}_i = \frac{\bar{K}_i}{\lambda_i^{(2)}} \quad (62)$$

From Lemma 1, the average number of hops traversed by a secondary packet before being absorbed by another secondary node is $\frac{1}{q^{(s)}(n^{(s)})}$. Therefore, the average end-to-end delay is given by,

$$D(n^{(s)}, n^{(p)}) = \bar{D}_i \frac{1}{q^{(s)}(n^{(s)})} \quad (63)$$

□

6. Maximum achievable throughput

The absorption probabilities $q^{(s)}(n^{(s)})$ and $q^{(p)}(n^{(p)})$ and the transmission radius $r^{(s)}(n^{(s)})$ and $r^{(p)}(n^{(p)})$ can be chosen such that in both the primary and secondary networks, the transmission radius and the average number of hops traversed by a packet prior to absorption are comparable to the corresponding parameters in the Gupta-Kumar model for a stand-alone wireless multihop network with same number of nodes. Specifically, choosing $q^{(s)}(n^{(s)})$=$\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}$, $r^{(s)}(n^{(s)})$=$\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}$, $q^{(p)}(n^{(p)})$=$\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$, $r^{(p)}(n^{(p)})$ =$\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$ gives us the comparable parameters. We show that if the secondary network is denser than the primary network, the maximum achievable throughput of a secondary node asymptotically approaches that of a stand-alone multi-hop wireless network with comparable transmission radius and average number of hops between source and destination of a packet.

Corrolary 1: When $n^{(s)} = {n^{(p)}}^{\beta}(\beta{>}1)$, $q^{(s)}(n^{(s)}) = \sqrt{\frac{log(n^{(s)})}{n^{(s)}}}$, $r^{(s)}(n^{(s)})$=$\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}$, $q^{(p)}(n^{(p)}) = \sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$, $r^{(p)}(n^{(p)})$ =$\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$, the maximum achievable throughput of a secondary node, $\lambda^{(s)}_{max}$=$o(\frac{W}{\sqrt{n^{(s)}log(n^{(s)})}})$.

Proof: Substituting the expressions of $\lambda_i^{(1)}$, $\mu_i^{(1)}$ and $\mu_i^{(2)}$ from (40), (37) and (38) respectively into (46), we get the average completion time or the effective service time of a secondary packet by secondary node i as,

$$E[c_i^{(2)}(t)] = \frac{\frac{1}{\xi^{(s)}} + \frac{L}{W}}{1 - (\frac{n^{(p)}A_{r^{(p)},r^{(s)}}(n^{(p)},n^{(s)})\lambda^{(p)}}{q^{(p)}(n^{(p)})} + \frac{4n^{(s)}A_{r^{(s)}}(n^{(s)})\lambda^{(s)}}{q^{(s)}(n^{(s)})})\frac{L}{W}} \tag{64}$$

The utilization factor of the secondary node i is obtained as,

$$\begin{aligned} \rho_i &= \lambda_i^{(2)} E[c_i^{(2)}(t)] \\ &= \frac{\frac{\lambda^{(s)}}{q^{(s)}(n^{(s)})}(\frac{1}{\xi^{(s)}} + \frac{L}{W})}{1 - (\frac{n^{(p)}A_{r^{(p)},r^{(s)}}(n^{(p)},n^{(s)})\lambda^{(p)}}{q^{(p)}(n^{(p)})} + \frac{4n^{(s)}A_{r^{(s)}}(n^{(s)})\lambda^{(s)}}{q^{(s)}(n^{(s)})})\frac{L}{W}} \end{aligned} \tag{65}$$

For a finite delay, $\rho_i < 1$. Then using (65) and (36), we get a bound on $\lambda^{(s)}$ and hence the maximum achievable throughput as,

$$\lambda^{(s)}_{max} = \frac{q^{(s)}(n^{(s)})\{1 - (\frac{n^{(p)}A_{r^{(p)},r^{(s)}}(n^{(p)},n^{(s)})\lambda^{(p)}}{q^{(p)}(n^{(p)})})\frac{L}{W}\}}{\frac{1}{\xi^{(s)}} + \frac{L}{W} + 4n^{(s)}A_{r^{(s)}}(n^{(s)})\frac{L}{W}} \tag{66}$$

Substituting $q^{(s)}(n^{(s)})$=$\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}$, $r^{(s)}(n^{(s)})$=$\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}$, $q^{(p)}(n^{(p)})$=$\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$, $r^{(p)}(n^{(p)})$ =$\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$, and defining $c^{(s)} = \frac{1}{\xi^{(s)}} + \frac{L}{W}$ and $c^{(p)} = \frac{1}{\xi^{(p)}} + \frac{L}{W}$, we have

$$\lambda^{(p)} = \lambda^{(p)}_{max} = \frac{\frac{1}{4\pi}\frac{W}{\sqrt{n^{(p)}log(n^{(p)})}L}}{1 + \frac{c^{(p)}}{4\pi log(n^{(p)})\frac{L}{W}}}, \tag{67}$$

$$\begin{aligned}\lambda^{(s)}_{max} &= \frac{q^{(s)}(n^{(s)})\{1-(\frac{n^{(p)}A_{r^{(p)},r^{(s)}}(n^{(p)},n^{(s)})\lambda^{(p)}}{q^{(p)}(n^{(p)})})\frac{L}{W}\}}{\frac{1}{\xi^{(s)}}+\frac{L}{W}+4n^{(s)}A_{r^{(s)}}(n^{(s)})\frac{L}{W}} \\ &= \frac{\frac{1}{4\pi}\frac{W}{\sqrt{n^{(s)}log(n^{(s)})}L}}{1+\frac{c^{(s)}}{4\pi log(n^{(s)})\frac{L}{W}}} - \frac{1}{c^{(s)}+4n^{(s)}\pi\frac{log(n^{(s)})}{n^{(s)}}\frac{L}{W}}\{\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}(\sqrt{\frac{log(n^{(s)})}{n^{(s)}}} \\ &+ \sqrt{\frac{log(n^{(p)})}{n^{(p)}}})^2\frac{n^{(p)}\pi L\lambda^{(p)}}{W\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}}\} \end{aligned} \tag{68}$$

If $n^{(p)} < \infty$ is a constant and $n^{(s)} \to \infty$, (i.e. $\beta \to \infty$), then

$$\lim_{n^{(s)}\to\infty}\frac{log(n^{(s)})}{n^{(s)}} = \lim_{n^{(s)}\to\infty}\frac{1}{n^{(s)}} = 0 << \frac{log(n^{(p)})}{n^{(p)}} \tag{69}$$

If $1 < \beta < \infty$ is a constant and $n^{(p)} \to \infty$,

$$\frac{\frac{log(n^{(s)})}{n^{(s)}}}{\frac{log(n^{(p)})}{n^{(p)}}} = \frac{\beta log(n^{(p)})n^{(p)}}{n^{(p)\beta}log(n^{(p)})} = \beta n^{(p)^{(1-\beta)}} \to 0 \tag{70}$$

From (69) and (70), we conclude $\frac{log(n^{(s)})}{n^{(s)}} << \frac{log(n^{(p)})}{n^{(p)}}$ when $n^{(s)} \to \infty$ and $n^{(s)} > n^{(p)}$. Therefore,

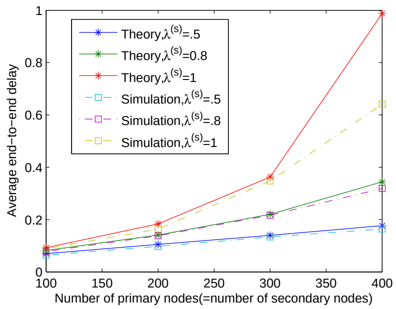
$$\lim_{n^{(s)}\to\infty}\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}+\sqrt{\frac{log(n^{(p)})}{n^{(p)}}} \approx \sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$$

So,

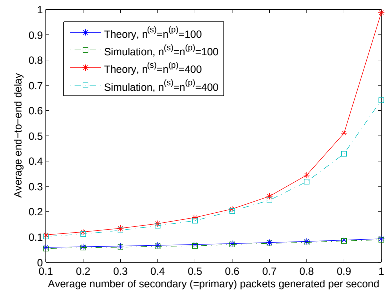
$$\begin{aligned} &\lim_{n^{(s)}\to\infty}\frac{\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}\{(\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}+\sqrt{\frac{log(n^{(p)})}{n^{(p)}}})^2\frac{\lambda^{(p)}n^{(p)}\pi L}{\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}W}\}}{c^{(s)}+4n^{(s)}\pi\frac{log(n^{(s)})}{n^{(s)}}\frac{L}{W}} \\ \approx &\lim_{n^{(s)}\to\infty}\frac{\frac{1}{4\pi log(n^{(s)})\frac{L}{W}}\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}\frac{n^{(p)}\pi\lambda^{(p)}L}{W}\frac{log(n^{(p)})}{n^{(p)}}\sqrt{\frac{n^{(p)}}{log(n^{(p)})}}}{1+\frac{c^{(s)}}{4\pi log(n^{(s)})\frac{L}{W}}} \\ = &\lim_{n^{(s)}\to\infty}\frac{\frac{1}{4}\sqrt{\frac{n^{(p)}log(n^{(p)})}{n^{(s)}log(n^{(s)})}}\frac{\frac{1}{4\pi}\frac{W}{\sqrt{n^{(p)}log(n^{(p)})}L}}{1+\frac{c^{(p)}}{4\pi log(n^{(p)})\frac{L}{W}}}}{1+\frac{c^{(s)}}{4\pi log(n^{(s)})\frac{L}{W}}} \\ = &\lim_{n^{(s)}\to\infty}\frac{\frac{1}{4\pi}\frac{W}{\sqrt{n^{(s)}log(n^{(s)})}L}}{1+\frac{c^{(s)}}{4\pi log(n^{(s)})\frac{L}{W}}}\{\frac{1}{4(1+\frac{c^{(p)}}{4\pi log(n^{(p)})\frac{L}{W}})}\} \\ << &\lim_{n^{(s)}\to\infty}\frac{\frac{1}{4\pi}\frac{W}{\sqrt{n^{(s)}log(n^{(s)})}L}}{1+\frac{c^{(s)}}{4\pi log(n^{(s)})\frac{L}{W}}} \end{aligned} \tag{71}$$

From (68) and (71), we conclude, $\lambda^{(s)}_{max}\to\frac{\frac{W}{4\pi\sqrt{n^{(s)}log(n^{(s)})}L}}{1+\frac{c^{(s)}}{4\pi log(n^{(s)})\frac{L}{W}}}=o(\frac{W}{\sqrt{n^{(s)}log(n^{(s)})}})$ when $n^{(s)}\to\infty$ and $n^{(s)}>n^{(p)}$. □

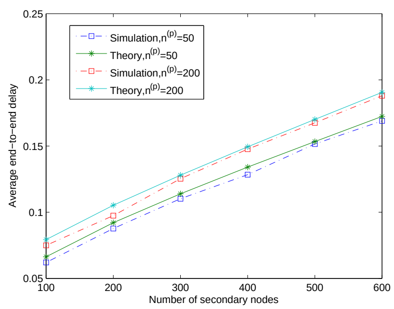
It is to be noted that in [12] and [13], the authors have shown that a secondary network with more nodes than a primary network can achieve the same throughput scaling as a stand-alone wireless network. However, the authors in [12] and [13] assumed nodes (both primary and secondary) that are distributed inside a unit square according to



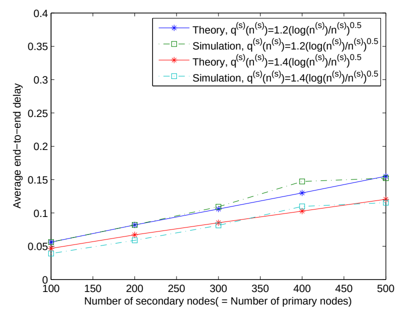
(a) Average end-to-end delay versus number of primary (= number of secondary) nodes.



(b) Average end-to-end delay versus secondary (= primary) packet generation rate.



(c) Average end-to-end delay versus number of secondary nodes with fixed number of primary nodes, $n^{(p)}$.



(d) Average end-to-end delay versus number of secondary(= number of primary) nodes. A shortest hop routing protocol is used in simulation.

Figure 3. Comparison of the analytical and simulation results.

a Poisson process, while we considered nodes distributed according to a uniform distribution over a torus of unit area. It is to be noted that in [12] and [13], the authors have shown that a secondary network with more nodes than a primary network can achieve the same throughput scaling as a stand-alone wireless network. However, the authors in [12] and [13] assumed nodes (both primary and secondary) that are distributed inside a unit square according to a Poisson process, while we considered nodes distributed according to a uniform distribution over a torus of unit area.

7. Simulations

We compare our analytical results with those obtained through simulation in C-programming language so as to verify the validity of our assumptions. The simulation setting consists of $n^{(p)}$ primary and $n^{(s)}$ secondary nodes that are uniformly distributed over a torus of unit area. The transmission radius of a primary and a secondary node are chosen as $r^{(p)}(n^{(p)}) = 0.8\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$ and $r^{(s)}(n^{(s)}) = 0.8\sqrt{\frac{log(n^{(s)})}{n^{(s)}}}$ respectively which are sufficient to ensure connectivity of the two networks. The length (L) of a primary or a secondary packet is 1Kbits. Each primary (or secondary) node produces packets at the rate of $\lambda^{(p)}$ (or $\lambda^{(s)}$) packets/second. The transmission bandwidth of the channel is $W = 10^6$ bits/sec. The back-off timers for both the primary and secondary nodes are assumed to be exponentially distributed with mean back-off duration of 0.01 seconds. The probabilistic routing and MAC protocol as described in Section 2 are used in Fig 3a- 3c. For Fig 3d we assume a more realistic *shortest hop routing* protocol where every primary (secondary) node transmits packets, along the shortest path, to exactly one destination primary (secondary) node that is located approximately (i.e. the integer closest to) $q^{(p)}(n^{(p)})$ ($q^{(s)}(n^{(s)})$) hops away respectively; we still assume ideal sensing and use the MAC protocol from Section 2. Given a topology, we obtain the average end-to-end delay of a secondary packet by averaging the end-to-end delay for every secondary packet that has been absorbed within the run-time of the simulation.

In Fig 3a, we plot average end-to-end delay with $\lambda^{(s)} = \lambda^{(p)} = 0.5, 0.8$ and 1 respectively while $n^{(s)}(= n^{(p)})$ is varied from 100 to 400 at steps of 100. Fig 3b shows the variation of average end-to-end delay when $n^{(s)} = n^{(p)}$= 100 and 400, while $\lambda^{(s)}(= \lambda^{(p)})$ is varied from 0.1 to 1. Fig 3c shows variation of average end-to-end delay with $n^{(s)}$ when $n^{(p)}$=50 and 200 respectively with $\lambda^{(s)} = \lambda^{(p)} = 0.5$. In Fig 3a, 3b and 3c, we use $q^{(p)}(n^{(p)}) = \sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$ and $q^{(s)}(n^{(s)}) = \sqrt{\frac{log(n^{(s)})}{n^{(s)}}}$. In Fig 3d, we use $q^{(p)}(n^{(p)}) = q^{(s)}(n^{(s)}) = 1.2\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$ and $1.4\sqrt{\frac{log(n^{(p)})}{n^{(p)}}}$ respectively and plot average end-to-end delay versus $n^{(s)}(= n^{(p)})$ with $\lambda^{(s)} = \lambda^{(p)} = 0.5$.

From the simulation results, it is observed that the network model is reasonably accurate for a wide range of utilization of channel local to a secondary node (an increasing function of $\lambda^{(s)}$ and $\lambda^{(p)}$) except for very high range. From the simulations, it is measured that for low and medium channel utilization up to about 0.65, the model works reliably well. For low absorption probabilities and high $\lambda^{(s)}$ or $\lambda^{(p)}$, the assumption A1 is no longer valid as there are significant number of instances when two or more interfering neighboring nodes are simultaneously transmitting. In this case, by approximating the arrival process of class-1 jobs at a secondary queue as a sum of individual transmission processes, we over-estimate the number of events when the service of a class-2 job is interrupted due to arrival of a class-1 job. As a result, the average end-to-end delay calculated is higher than that obtained from simulation results. This can be observed in Fig 3a when $\lambda^{(s)} = 1$ and $n^{(s)} = 400$, and in Fig 3b when $n^{(s)} = n^{(p)} = 400$ and $\lambda^{(s)} = \lambda^{(p)} > 0.9$. However, since random access MAC is not recommended for cases of high channel utilizations, our model can be successfully applied for more practical scenarios that use random access where the channel utilization is low or medium.

8. Conclusions

In this paper we considered two multi-hop ad-hoc networks using IEEE 802.11 based MAC protocol with each network having different priority of channel access. Using a simple probabilistic routing protocol and assuming ideal sensing process, we obtained closed form expressions for the average end-to-end delay and maximum achievable throughput for the lower priority secondary network. We also showed that when the secondary network is denser than the higher priority primary network, the maximum achievable throughput for any node in the former network asymptotically approaches that of a stand-alone wireless network. We verified that our theoretical results match with the simulation results for low to moderate channel utilizations. Future work will extend the analysis to the cases of multiple channels and using non-ideal sensing mechanism.

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