Queuing Network Models for Delay Analysis of Multihop Wireless Ad Hoc Networks

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***Abstract*— In this paper we focus on characterizing the average end-to-end delay and maximum achievable per-node throughput in random access multihop wireless ad hoc networks with stationary nodes. We present an analytical model that takes into account the number of nodes, the random packet arrival process, the extent of locality of traffic, and the back off and collision avoidance mechanisms of random access MAC. We model random access multihop wireless networks as open G/G/1 queuing networks and use the diffusion approximation in order to evaluate closed form expressions for the average end-to-end delay. The mean service time of nodes is evaluated and used to obtain the maximum achievable per-node throughput. The analytical results obtained here from the queuing network analysis are discussed with regard to similarities and differences from the well established information-theoretic results on throughput and delay scaling laws in ad hoc networks. We perform extensive simulations and verify that the analytical results closely match the results obtained from simulations.**

I. Introduction

A multihop wireless ad hoc network is a collection of nodes that communicate with each other without any established infrastructure or centralized control. The transmission power of a node is limited, thus the packets may have to be forwarded by several intermediate nodes before they reach their destinations. So each node may be a source, destination and relay. The wireless medium is shared and scarce, therefore ad hoc networks require an efficient MAC protocol [1]. Since ad hoc networks lack infrastructure and centralized control, the MAC protocols for ad hoc networks should be distributed. The random access MAC protocols, e.g. MACA [5] and MACAW [1], are thus ideal choice for ad hoc networks. The delay and throughput of wireless ad hoc networks depend on the number of nodes, transmission range of the nodes, traffic pattern and behavior of MAC protocol [4], [3].

In this paper we investigate how the end-to-end delay and maximum achievable throughput in a random access based MAC multihop wireless network with stationary nodes depend on the number of nodes, transmission range and traffic pattern. We propose a queuing network model for delay analysis of random access multihop wireless ad hoc networks. The queuing network model proposed in this paper is unique in that it allows us to derive closed form expressions for the average end-to-end delay and maximum achievable throughput. The packet delay is defined as the time taken by a packet to reach its destination node after it is generated. The average end-to-end delay is the expectation of the packet delay over all packets and all possible network topologies. Our analysis takes into account the queuing delays at source and intermediate nodes. The packets are assumed to have a fixed size and random arrival process. Moreover we also characterize how the average end-to-end delay and maximum achievable throughput vary with the degree of locality of traffic. The purpose of this study is not to accurately model the performance of standard protocols (like IEEE 802.11 MAC) but to gain insights into the queuing delays and maximum achievable throughput in random access multihop wireless ad hoc networks.

Several studies have focused on finding the maximum achievable throughput and characterizing capacity-delay trade-offs in wireless ad hoc networks [4], [7], [3], [8]. In [4] it is shown that for a wireless network with n stationary nodes, the per-node capacity scales as $\Theta(W/\sqrt{n \log n})$. In [7], the authors use simulations in order to study the dependence of per-node capacity on IEEE 802.11 MAC interactions and traffic pattern for various topologies like single cell, chain, uniform lattice and random network. An estimate of the expressions for one-hop capacity and upper bound of per-node throughput is obtained using the simulation results.

In [3], the authors characterize the delay-throughput trade-offs in wireless networks with stationary and mobile nodes. It is shown that for a network with stationary nodes, the average delay and throughput are related by $D(n) = \Theta(nT(n))$, where $D(n)$ and $T(n)$ are the average end-to-end delay and throughput respectively. However the delay is defined as the time taken by a packet to reach the destination *after it has left the source*. Also, according to the network model, the packet size scales with throughput. Under these assumptions the delay is simply proportional to the average number of hops between a source destination pair. *i.e. in their model, there is no delay due to queuing*. If, more realistically, the packet size is assumed to be constant and the delay is defined as time taken by a packet to reach the destination *after its arrival at the source*, there would be queuing delays at the source and intermediate nodes.

Several recent studies have proposed queuing models for performance evaluation of the IEEE 802.11 MAC. A finite queuing model is proposed and used in [13] for evaluating the packet blocking probability and MAC queuing delays in a Basic Service Set with N nodes. A queuing model for performance evaluation of IEEE 802.11 MAC based WLAN in the presence of HTTP traffic is proposed in [9]. In [10] the service time of a node, in IEEE 802.11 MAC based wireless ad hoc network, is modeled as a Markov modulated general arrival process. The resulting M/MMGI/1/K queuing model is used for delay analysis over a single hop in the network. An analytical model for evaluating closed form expression for the average queuing delay over *a single hop* in IEEE 802.11 based wireless networks is presented in [12]. In [11], the authors use queuing theoretic approach in order to calculate the mean packet delay, maximum throughput and collision probability for an elementary four node network with hidden nodes and extend the results to *linear* wireless networks. It is worth noting that none of the prior works [13], [9], [10], [12], [11] extends to a general *two dimensional wireless network*.

We propose a detailed analytic model for multihop wireless ad hoc networks based on open G/G/1 queuing networks. We first evaluate the mean and second moment of service time over a single hop by taking into account the back off and collision avoidance mechanisms of IEEE 802.11 MAC. We then use the diffusion approximation for solving open queuing networks in order to evaluate the closed form expression for the average end-to-end packet delay. Using the average service time of the nodes we obtain an expression for the maximum achievable throughput. We present detailed discussions on how the maximum achievable throughput obtained from our model compares with the per-node capacity of Gupta-Kumar's model. The analytical results are verified against extensive simulations and numerical computations.

The rest of the paper is organized as follows. In Section II we briefly describe the well known diffusion approximation for solving open G/G/1 queuing networks. Detailed description of the network model is presented in Section III. The delay and throughput analysis of multihop wireless networks is presented in Section IV. The comparison of the analytical and simulation results is presented in Section V. Finally we present concluding remarks in Section VI.

II. Diffusion Approximation Method

The diffusion approximation [2] can be used for solving an open G/G/1 queuing network provided that all the nodes in the network are single server with first-come first-serve (FCFS) service strategy. The advantage of using the diffusion approximation in this work is that it allows us to obtain closed form expressions for the average end-to-end delay.

In this section we briefly describe how the diffusion approximation is used to solve an open G/G/1 queuing network. (Please see [2] for details). Suppose we have an open queuing network with n service stations, numbered from 1 to n. The external arrival of a job is a renewal process with an average inter-arrival time of $1/\lambda_e$ and the coefficient of variance of inter-arrival time equals c_A. The mean and coefficient of variance of the service time at a station i are denoted by $1/\mu_i$ and c_{Bi}, respectively.

The *visit ratio* of a station in a queuing network is defined as the average number of times a packet is forwarded by (i.e. visits) the station. The visit ratio of station i, denoted by e_i, is given by

$$e_i = p_{0i}(n) + \sum_{j=1}^{j=n} p_{ji}(n) \cdot e_j \quad (1)$$

where p_{0i} denotes the probability that a job enters the queuing network from station i and p_{ji} denotes the the probability that a job is routed to station i after completing its service at station j.

There are two sources of packet arrivals at a station: the jobs that are generated at the station and the jobs that are forwarded to the station by other stations. The resulting arrival rate is termed the *effective arrival rate* at a station. The effective arrival rate at the station i, denoted by λ_i is given by

$$\lambda_i = \lambda_e e_i \quad (2)$$

The *utilization factor* of station i, denoted by ρ_i, is given by

$$\rho_i = \lambda_i/\mu_i \quad (3)$$

The squared coefficient of variance of the inter-arrival time at a station i, denoted by c_{Ai}^2, is approximated using

$$c_{Ai}^2 = 1 + \sum_{j=0}^{n} (c_{Bj}^2 - 1) \cdot p_{ji}^2 \cdot e_j \cdot e_i^{-1} \quad (4)$$

where $c_{B0}^2 = c_A^2$.

According to the diffusion approximation, the approximate expression for the probability that the number of jobs at station i equals k, denoted by $\hat{\pi}_i(k)$, is

$$\hat{\pi}_i(k) = \begin{cases} 1-\rho_i & k=0 \\ \rho_i(1-\hat{\rho}_i)\hat{\rho}_i^{k-1} & k>0 \end{cases} \quad (5)$$

where

$$\hat{\rho}_i = \exp\left(-\frac{2(1-\rho_i)}{c_{Ai}^2 \cdot \rho_i + c_{Bi}^2}\right) \quad (6)$$

The mean number of jobs at a station i, denoted by $\overline{K_i}$, is

$$\overline{K_i} = \rho_i/(1-\hat{\rho}_i) \quad (7)$$

III. Queuing Network Model

In this section we present the network model and develop a queuing network model for multihop wireless networks. We also derive expressions for the parameters of the queuing network model.

A. The network model

The network consists of $n+1$ nodes, numbered 1 to $n+1$, that are distributed uniformly and independently over a torus of unit area. We assume a torus area in order to avoid complications in the analysis caused by the edge effects. Each node is assumed to have an equal transmission range, denoted by $r(n)$. Let r_{ij} denote the distance between nodes i and j. Nodes i and j are said to be *neighbors* if they can directly communicate with each other, i.e. if $r_{ij} \leq r(n)$. Let $N(i)$ denote the set of nodes that are neighbors of node i. All the neighbors of a node lie on a disc of area $A(n) = \pi r(n)^2$ centered at the node. The area $A(n)$ is termed the *"communication area"* of a node. The communication area is chosen such that the network is connected which ensures that $N(i) \neq \phi \ \forall \ i$. The transmission rate of each node equals W bits/second.

We use a special case of the Protocol Model of interference described in [4]. If node i transmits to node j then the transmission will be successful only if (i) $r_{ij} \leq r(n)$ and (ii) $r_{kj} > r(n)$ for every other node $k \neq i, j$ that transmits simultaneously with node i. In other words, node i can successfully transmit a packet to node j only if i is a neighbor of j and no other neighbor of j is transmitting concurrently with i. (This is equivalent to setting Δ = 0 in the Protocol Model in [4]).

The traffic model for the network may be described as follows. Each node in the network could be a source, destination and/or relay of packets. Each node generates packets with rate λ packets/sec. The delay analysis is possible for any packet generation process as long as the mean and SCV of packet inter arrival time is known. For the sake of simplicity, we assume in our model that the packet generation process at each node is an i.i.d. Poisson process. The size of each packet is constant and equals L bits. When a node receives a packet from any of its neighbors, it either forwards the packet to its neighbors with probability $(1 - p(n))$ or absorbs the packet with probability $p(n)$. The probability $p(n)$ is referred to as "absorption probability". In other words, the absorption probability is the probability that a node is the destination of a packet given that the node has received the packet from its neighbors. When a node forwards a packet, each of its neighbors is equally likely to receive the packet. The advantage of such a model is that we can control the locality of the traffic by varying the parameter $p(n)$. The traffic is highly localized if $p(n)$ is large while a small value of $p(n)$ implies unlocalized traffic. This would help us to characterize the effect of the locality of the traffic on the average delay and maximum achievable throughput.

We assume that each node in the network has infinite buffers which means that no packets are dropped in the network. The packets are served by the nodes on first come first serve basis.

Multihop wireless ad hoc networks can be modeled as a queuing network as shown in Figure 1(a). The stations of the queuing network correspond to the nodes of the wireless network. The forwarding probabilities in the queuing network, denoted by p_{ij}, correspond to the probability that a packet that is transmitted by node i enters the node j's queue. Figure 1(b) shows a representation of a node in the ad hoc network as a station in the queuing network.

The end-to-end delay in a wireless network equals the sum of queuing and transmission delays at source and intermediate nodes. We will use the queuing network model shown in Figures 1(a) and 1(b) in order to mathematically analyze the end-to-end delay.

B. Parameters of the queuing network model

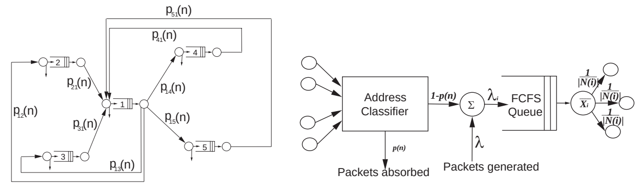
In this section we derive expressions for the parameters of the queuing network model of multihop wireless networks.

Lemma 1: The expected probability that a packet is forwarded from node i to node j, denoted by $\overline{p_{ij}}(n)$, is

$$\overline{p_{ij}}(n) = \begin{cases} \frac{1-p(n)}{n}(1-(1-A(n))^n) & i \neq j \\ 0 & i = j \end{cases} \tag{8}$$

Proof: Let $P[i \rightarrow j]$ denote the probability that a packet forwarded by node i enters the queue at node j. We define $\beta_{ij}^{j,k} = P[i \rightarrow j | j \in N(i), |N(i)| = k]$, $\beta_{ij}^{j} = P[i \rightarrow j | j \in N(i)]$ and $\alpha_i^{j,k} = P[|N(i)| = k | j \in N(i)]$. Thus

$$\beta_{ij}^{j,k} = \frac{1}{k}(1 - P[j \text{ absorbs the packet}]) = \frac{1-p(n)}{k}$$



(a) Representation of multihop wireless ad hoc network as a queuing network.

(b) Representation of a node of multihop wireless ad hoc network as a station in the queuing network.

Fig. 1. Queuing network model for multihop wireless ad hoc network.

Since the nodes are uniformly and independently distributed over a unit area, the probability that a node is in neighborhood of the node i equals $A(n)$. Hence $P[j \in N(i)] = A(n)$ and

$$\alpha_i^{j,k} = \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} (1-A(n))^{n-k} A(n)^{k-1}$$

Therefore,

$$\overline{\beta_{ij}^{j}} = E[\beta_{ij}^{j}] = \sum_{k=1}^{n} \beta_{ij}^{j,k} \alpha_i^{j,k} = \frac{1-p(n)}{nA(n)}(1-(1-A(n))^n)$$

Also according to the model node i cannot forward a packet to node j unless $j \in N(i)$. Hence $E[P[i \rightarrow j] | j \notin N(i)] = 0$. So the expected forwarding probability is given by

$$\overline{p_{ij}}(n) = \overline{\beta_{ij}^{j}} P[j \in N(i)] = \frac{1-p(n)}{n}(1-(1-A(n))^n)$$

■

Lemma 2: The expected visit ratio of node i, denoted by $\overline{e_i}$, is given by

$$\overline{e_i} = \frac{1}{(n+1)p(n)} \ \forall \ i \tag{9}$$

Proof: The visit ratio of a node in the queuing network is given by (1). Taking expectation of both sides of the equation we have,

$$\overline{e_i} = \frac{1}{n+1} + \sum_{j=1}^{j=n+1} \overline{p_{ji}}(n)\overline{e_j}$$

Each node of the wireless network is similar, thus from symmetry $\overline{e_i} = \overline{e_j} \ \forall \ i; j$. Also $\overline{p_{ij}} = \frac{1-p(n)}{n}(1-(1-A(n))^n)$. Since in our model $A(n)$ is chosen such that the network is connected with high probability, therefore $(1-(1-A(n))^n) \approx 1$ and hence $\overline{p_{ij}}(n) \approx \frac{1-p(n)}{n}$. From symmetry

$$\overline{e_i} = \frac{1}{n+1} + \sum_{j=1, j\neq i}^{j=n+1} \frac{1-p(n)}{n}\overline{e_i}$$

By rearranging the above equation we get (9). ■

Lemmas 1 and 2 (equations 8-9) indicate that the nodes visit ratio and the forwarding probabilities averaged over all possible instances of the topologies are similar to the visit ratios and forwarding probabilities of an average topology where each node has a number of neighbors equal to the average case. Thus, as a result of these two lemmas, one may derive the remaining set of model parameters (effective packet arrival rate and number of packets traversed) by considering the average case topology. Applying these results in the diffusion model will provide expressions for the average end to end delay, defined as the expectation of the packet delay over all packets and all possible networks.

Lemma 3: The effective packet arrival rate at a node i, denoted by λ_i, is

$$\lambda_i = \lambda / p(n) \tag{10}$$

Proof: The packet arrival process at each node is an i.i.d. Poisson process with rate λ. So the total external arrival rate, denoted by λ_e, equals $(n+1)\lambda$. According to (2), $\lambda_i = \lambda_e e_i$. Substituting $\overline{e_i}$ from (9) and λ_e we get (10). ■

Lemma 4: The expected number of hops traversed by a packet between its source and destination, denoted by $\overline{s}$, equals $\frac{1}{p(n)}$.

Proof: Let s denote the number of hops between a source and destination, then $P[s = k] = (1 - p(n))^{k-1} p(n)\ k \geq 1$. Thus,

$$\overline{s} = E[s] = \sum_{k=1}^{\infty} k \cdot (1 - p(n))^{k-1} p(n) = \frac{1}{p(n)} \tag{11}$$

■

The average queuing delay depends upon the service time distribution of the nodes. The service time distribution depends on the MAC protocol used by the nodes.

IV. Queuing Analysis

In this section we first present a model for a random access MAC that accounts for the back off and collision avoidance mechanisms of IEEE 802.11 MAC. We then present the delay analysis of multihop wireless ad hoc networks by integrating the MAC model with the queuing network model developed in Section III.

A. The MAC model

1) Interfering neighbors: Two nodes are said to be *interfering neighbors* if they lie within a distance of $2r(n)$ of each other. The transmission of a node would be successful if none of the interfering neighbors of the node transmits concurrently. Also two nodes may successfully transmit at the same time if they are not interfering neighbors of each other. The definition of interfering neighbors is similar to the definition given in [4].

2) The random access MAC model: Before transmitting each packet the nodes count down a random back-off timer. The duration of the timer is exponentially distributed with mean $1/\xi$. As in IEEE 802.11, the timer of a node freezes each time an interfering neighbor starts transmitting a packet. When the back off timer of a node expires, it starts transmitting the packet and the back off timers of all its interfering neighbors are immediately frozen. The timers of the interfering neighbors are resumed as soon as the transmission of the packet is complete. The time required to transmit a packet from a node to its neighbor is $L/W + T_o$, where T_o is the time required for the exchange of RTS, CTS and ACK packets. We assume that T_o is negligible compared to L/W, so in our analysis we assume that the time required to transmit a packet is L/W. The model is mathematically tractable and at the same time captures the behavior of IEEE 802.11 MAC protocol.

B. Delay analysis

With the help of the following three lemmas we determine the mean and second moments of the service time of nodes using the random access MAC model. We then present the result for end-to-end delay in multihop wireless networks.

Lemma 5: Let H_i denote the number of interfering neighbors of a node i. Then

$$E[H_i] = 4nA(n) \tag{12}$$

$$E[H_i^2] = 4nA(n)(1 + 4(n-1)A(n)) \tag{13}$$

where $A(n) = \pi \cdot r(n)^2$.

Proof: Since the nodes are uniformly distributed over a unit area, the probability that a node is an interfering neighbor of node i equals $\pi(2r(n))^2$. Thus the probability that $H_i = h$ is given by

$$P[H_i = h] = \binom{n}{h} (4\pi r(n)^2)^h \cdot (1 - 4\pi r(n)^2)^{(n-h)}$$

Thus H_i has a binomial distribution. (12) and (13) are the first and second moment of the binomial distribution. ■

Lemma 6: Let M_i denote the number of interfering neighbors of a node i that have at least one packet to transmit. Then under steady state,

$$E[M_i] = \rho 4nA(n) \tag{14}$$

$$E[M_i^2] = \rho^2 \cdot 4nA(n)(1 + 4(n-1)A(n)) + (1-\rho)\rho 4nA(n) \tag{15}$$

where ρ is the utilization factor of the nodes.

Proof: Let the number of interfering neighbors of node i be H_i. Let $Y_j, 1 \leq j \leq H_i$, be an indicator random variable associated with node j, indicating whether under steady state node j has a packet to transmit or not. ($Y_j = 1$ if node j has a packet to transmit, $Y_j = 0$ if node j has no packet to transmit). Using (5) $P(Y_j = 1) = \rho_j$, where ρ_j is the utilization factor of node j. By symmetry $\rho_j = \rho\ \forall\ j$. M_i is equal to $\sum_{j=1}^{H_i} Y_j$. The expected value of M_i equals

$$E[M_i] = E_{H_i}[E[M_i | H_i = h]] = E_{H_i}[\sum_{j=1}^{h} E[Y_j]] = \rho E[H_i]$$

Substituting (12), we get (14).
Similarly the expected value of M_i^2, given $H_i = h$, is given by

$$E[M_i^2 | H_i = h] = E[(\sum_{j=1}^{h} Y_j)(\sum_{k=1}^{h} Y_k)]$$

Since Y_j is independent of Y_k, we get

$$\begin{aligned} E[M_i^2 | H_i = h] &= \sum_{j=1}^{h} \sum_{k=1, k \neq j}^{h} E[Y_j] E[Y_k] + \sum_{j=1}^{h} E[Y_j^2] \\ \Rightarrow E[M_i^2] &= \rho^2 E[H_i^2] + (1-\rho)\rho E[H_i] \end{aligned}$$

Substituting (12) and (13), we get (15). ■

Lemma 7: Let Z_i denote the number of times the timer of a node i is frozen before its expiration. Then

$$E[Z_i] = 4 \cdot \rho nA(n) \tag{16}$$

Proof: Let T_i denote the duration of the back off timer of node i. During a transmission epoch M_i may not remain constant. In order to simplify the analysis we assume that M_i remains constant throughout a transmission epoch of node i. The timer of node i is frozen each time a timer of any of the interfering neighbors of i expires. The timer of each node has an exponential distribution. Thus the probability that $Z_i = z$, given that $M_i = m$ and $T_i = t_i$, is

$$P[Z_i = z | T_i = t_i, M_i = m] = e^{-m \cdot \xi \cdot t_i} \cdot (m \cdot \xi \cdot t_i)^z / z!$$

$$\Rightarrow E[Z_i|T_i = t_i, M_i = m] = m\xi t_i \quad (17)$$
$$\Rightarrow E[Z_i|M_i = m] = m\xi E[t_i] = m \Rightarrow E[Z_i] = E[M_i]$$

Substituting $E[M_i]$ from (14), we get (16). ∎

Theorem 1: Let $\overline{X_i}$ and $\overline{X_i^2}$ denote the mean and second moment of service time required to serve a packet by a node i. Then

$$\overline{X_i} = E[X_i] = \frac{\frac{1}{\xi} + \frac{L}{W}}{1 - 4nA(n)\lambda_i \frac{L}{W}} \quad (18)$$

$$\overline{X_i^2} = E[X_i^2](1 + 3\overline{m} + 2\overline{m^2})\frac{L^2}{W^2} + 2(2\overline{m} + 1)\frac{L}{W}\frac{1}{\xi} + \frac{2}{\xi^2} \quad (19)$$

where $\overline{m} = E[M_i]$ (eqn. (14)) and $\overline{m^2} = E[M_i^2]$ (eqn. (15)).

Proof: The time taken by node i to serve a packet, denoted by X_i, is the sum of three terms: (i) the duration of the random back off timer (t_i), (ii) the duration for which the timer remains frozen ($Z_i L/W$), and (iii) the transmission time (L/W). Thus

$$X_i = t_i + Z_i\frac{L}{W} + \frac{L}{W} \quad (20)$$

Taking expectation of both sides we get,

$$E[X_i] = E[t_i] + E[Z_i]\cdot\frac{L}{W} + \frac{L}{W} = \frac{1}{\xi} + 4\rho nA(n)\frac{L}{W} + \frac{L}{W}$$

Substituting $\rho = \lambda_i\overline{X_i}$ and by rearranging, we get (18).

Also from (20) we have $X_i^2 = (t_i + Z_i + \frac{L}{W})^2$ Given $M_i = m$ and $T_i = t_i$, Z_i has a Poisson distribution. So $E[Z_i^2|M_i = m, T_i = t_i] = m\xi t_i + (m\xi t_i)^2$. Using this and (17), we get

$$E[X_i^2|T_i = t_i, M_i = m] = (1 + \frac{L^2}{W^2}m^2\xi^2)t_i^2 + (\frac{2L}{W} + \frac{3L^2}{W^2}m\xi)t_i + \frac{L^2}{W^2}$$

Taking expectation with respect to t_i we get

$$E[X_i^2|M_i = m] = E_{T_i}[E[X_i^2|T_i = t_i, M_i = m]] =$$
$$(1 + 3m + 2m^2)\frac{L^2}{W^2} + 2(2m + 1)\frac{L}{W}\frac{1}{\xi} + \frac{2}{\xi^2}$$

Taking expectation of the RHS w.r.t m, we get (19). ∎

Corollary 1: The standard deviation of service time of a node i, denoted by $\sigma_{X_i}^2$, is given by

$$\sigma_{X_i}^2 = \frac{L^2}{W^2}(\overline{m} + \overline{m^2} + \sigma_m^2) + 2(2\overline{m} + 1)\frac{L}{W}\frac{1}{\xi} + \frac{1}{\xi^2} \quad (21)$$

where $\sigma_m^2 = \overline{m^2} - \overline{m}^2$.

The squared coefficient of variance of the service time at a node i, denoted by c_{Bi}^2 is given by $\sigma_{X_i}^2/\overline{X_i}^2$. Using (4), the squared coefficient of variance of the inter arrival time at a node i, denoted by c_{Ai}^2, is given by

$$c_{Ai}^2 = 1 + \sum_{j=1, j\neq i}^{n+1}(c_{Bi}^2 - 1)\frac{1 - p(n)}{n} = 1 + (c_{Bi}^2 - 1)(1 - p(n))$$

With the knowledge of c_{Ai}^2, c_{Bi}^2 and ρ, we can find the parameter $\hat{\rho}$ as given in (6).

Theorem 2: For the random access MAC model the average end-to-end delay in a multihop wireless network, denoted by $D(n)$, is given by

$$D(n) = \frac{\rho_i}{\lambda\cdot(1 - \hat{\rho})} \quad (22)$$

Proof: Let $\overline{D_i}$ denote the average delay at a node i. According to Little's Law, $\overline{D_i} = \overline{K_i}/\lambda_i$, where $\overline{K_i}$ is the average number of packets in the queue of node i. Substituting $\overline{K_i}$ from (7) we get

$$\overline{D_i} = \overline{K_i}/\lambda_i = \rho/(\lambda_i(1 - \hat{\rho}))$$

By symmetry the average delay at each node is same. Thus the average end-to-end delay equals the product of the average number of hops traversed by a packet and the average delay at each node. Hence $D(n) = \overline{s}\cdot\overline{D_i}$ which leads to (22). ∎

C. Maximum achievable throughput

The *maximum achievable throughput*, denoted by λ_{max}, is defined as the maximum packet arrival rate at each node for which the average end-to-end delay remains finite. If the packet arrival rate exceeds λ_{max}, the delay would tend to infinity. The following corollary, that follows from Theorem 1, yields a relationship between the maximum achievable throughput and the network parameters.

Corollary 2: For a multihop wireless network the maximum achievable throughput λ_{max} is

$$\lambda_{max} = \frac{p(n)}{\frac{1}{\xi} + \frac{L}{W} + 4nA(n)\frac{L}{W}} \quad (23)$$

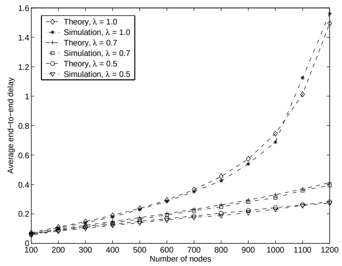
Also from (23), $\lambda_{max} = o\left(1/\overline{s}nA(n)\right)$.
Corollary 2 directly follows from $\rho_i = \lambda_i\overline{X_i} < 1$.

The result of Corollary 2 re-emphasizes the importance of carefully choosing the transmission ranges of nodes. λ_{max} increases with decrease in $r(n)$. However if $r(n)$ is too small then the network would become disconnected. According to [6], the network is asymptotically connected for $r(n) = \omega(\sqrt{\log n/n})$. So for a connected network $A(n) = \omega(\log n/n)$ and $\lambda_{max} = o(\frac{p(n)}{c+4\log(n)(L/W)})$.

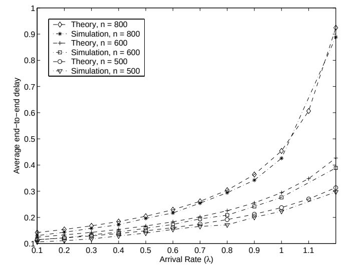
Another interesting observation is the dependence of λ_{max} on the traffic pattern. λ_{max} is directly proportional to $p(n)$. From (11) the expected number of hops traversed by a packet equals $1/p(n)$. Thus another way of interpreting the result is that λ_{max} is inversely proportional to the expected number of hops between a source-destination pair.

We further investigate how our result on the maximum achievable throughput compares with the result by Gupta-Kumar on throughput capacity. According to the Gupta-Kumar model, the nodes are distributed uniformly and independently over a sphere of unit surface area and each source chooses a random destination. Therefore the expected distance between a source and the corresponding destination equals the expected distance between two points uniformly and independently distributed on a sphere. Thus the expected distance between a source destination pair in Gupta-Kumar's model is a constant (i.e. does not vary with n), say $\overline{s_{GK}}$. The transmission range in their model is $\omega(\sqrt{\log n/n})$. Thus the expected number of hops between a source-destination pair in Gupta-Kumar model is $o(\sqrt{n/\log n})$.

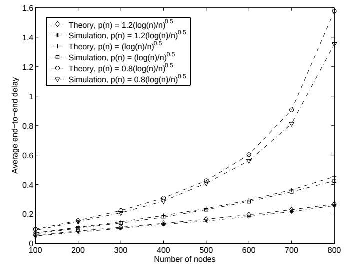
In order to compare our results with Gupta-Kumar's results we choose our model parameters such that we have comparable average number of hops between a source-destination pair and comparable transmission range. In our model if we choose $p(n) = \sqrt{\log n/n}$, then the expected number of hops between a source and destination node is $\overline{s} = 1/p(n) = \sqrt{n/\log n}$, which is comparable to the Gupta-Kumar model. Also $r(n) = \sqrt{\log n/n}$ or $A(n) = \pi\log n/n$ makes the transmission range of our model comparable to that of the Gupta-Kumar model. So for the model parameters that are comparable to the Gupta-Kumar model, the maximum achievable throughput is



(a) Average end-to-end delay vs. number of nodes.



(b) Average end-to-end delay vs. arrival rate.



(c) Average end-to-end delay vs. number of nodes.

Fig. 2. Comparison of the analytical results with simulation results.

$$\lambda_{max} = \frac{\frac{1}{4\pi}\frac{W}{\sqrt{n\log n}L}}{1+\frac{c}{4\pi\log n(L/W)}} \tag{24}$$

or $\lambda_{max} = o(W/\sqrt{n\log n})$.

The above discussion implies that for the similar values of parameters of the network model we get a bound similar to the Gupta-Kumar's bound on throughput capacity, but for our model the bound is not achievable. The reason for the bound not being achievable is that in our model we consider a random access MAC protocol rather than a perfect deterministic scheduling. Thus the bound becomes unachievable because some channel capacity is wasted by the nodes during contention for the channel.

V. Simulations

In this section we compare the simulation results with the analytical results. The aim of the comparison is to verify the validity of the assumptions made in our analysis and the accuracy of the diffusion approximation method.

The simulation setting is the following. The network topology for the simulations consists of n nodes scattered randomly over a torus of unit surface area. Each node can communicate with the nodes within a distance $r(n) = \sqrt{\log n/n}$. The random access MAC protocol used by the nodes is the same as described in $IV - B$. Each node produces packets of size $L = 1$ Kbits at the rate of λ packets/sec. The transmission rate of each node is $W = 10^6$ bits/sec. The probabilistic routing described in Section III is used for the simulations. The average delay for a particular topology is obtained by averaging the end-to-end delay of all packets produced during the simulation. In order to average out the effect of topology, each simulation is repeated over several topologies. The average end-to-end delay is obtained by averaging the average delay for all topologies.

Figure 2(a) shows how the average end to end delay, as obtained from the simulations, varies with the number of nodes for $\lambda = 0.5$, $\lambda = 0.7$ and $\lambda = 1.0$ with $p(n) = \sqrt{\log n/n}$. Figure 2(b) shows how the average end to end delay varies with the arrival rate (λ) for $n = 500$, 600 and 800 with $p(n) = \sqrt{\log n/n}$. Figure 2(c) shows how the average end to end delay varies with the number of nodes for various values of absorption probability with $\lambda = 1$ packets/sec. The theoretical values of the average end-to-end delay as obtained from the analytical results are plotted alongside the simulation results in Figures 2(a), 2(b) and 2(c). It is observed that the simulation results agree closely with the theoretical values.

VI. Conclusion and future work

Characterization of capacity and delay in ad hoc networks has been focus of considerable research. However capacity and delay of networks based on random access MAC, like IEEE 802.11, have not been substantially studied. In this paper we presented delay analysis of random access MAC multihop wireless ad hoc networks. We derived closed form expressions for the average end-to-end delay and maximum achievable throughput. We showed that, for comparable network parameters, the upper bound on maximum achievable throughput is of the same order as the Gupta-Kumar's bound. However for the random access MAC the bound is not achievable. The analytical results are verified using simulations.

The results and framework presented in this paper leads to several venues for future research. Our current directions include the delay analysis and characterization of the maximum achievable throughput for hierarchical networks, many to one communication scenarios, wireless networks with sleeping nodes and wireless networks with other medium access control algorithms.

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