# Delay and Throughput in Random Access Wireless Mesh Networks

Nabhendra Bisnik, Alhussein Abouzeid
Rensselaer Polytechnic Institute
Troy, NY 12180
bisnin@rpi.edu, abouzeid@ecse.rpi.edu

***Abstract*—Wireless mesh networks (WMNs) are emerging as a popular means of providing connectivity to communities in both affluent and poor parts of the world. The presence of backbone mesh routers and the use of multiple channels and interfaces allow mesh networks to have better capacity than infrastructure-less multihop ad hoc networks. In this paper we characterize the average delay and capacity in WMNs that utilize random medium access (MAC). We model residential area WMNs as open G/G/1 queuing networks. The analytical model takes into account the density of the mesh clients and mesh routers, the random packet arrival process, the degree of locality of traffic and the collision avoidance mechanism of random access MAC. The diffusion approximation method is used to obtain closed form expressions for (a) end-to-end packet delay and (b) maximum achievable per-node throughput. The analytical results describe how the performance of WMNs scales with the number of mesh routers and clients. The results obtained from simulations agree closely with the analytical results. For the asymptotic case (as the network size grows indefinitely), we discuss how the results obtained using the proposed queuing network framework compare against previous well known results on asymptotic capacity of infrastructure-less ad hoc networks.**

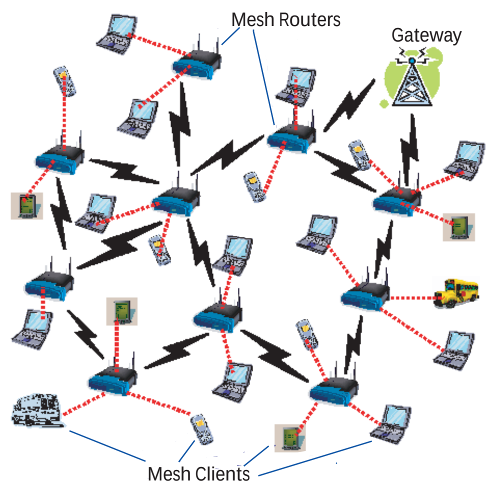



Fig. 1. Wireless mesh network.

## I. Introduction

The deployment of wireless mesh networks (WMNs) in order to provide connectivity among communities is becoming increasingly popular [1], [2], [3], [4], [5]. A typical WMN architecture is shown in Figure 1. A WMN consists of *mesh routers* and *mesh clients* [6]. The mesh clients are the wireless devices to which the WMN provides connectivity. The mesh routers form the backbone of a WMN. The mesh routers are stationary nodes, generally mounted on high visibility points like rooftops. The mesh routers have enhanced capabilities in comparison to mesh clients such as higher transmit power, multiple receive/transmit antenna, unlimited power supply etc. If a mesh client needs to communicate with a node (another mesh client or a gateway) that is not within its communication range, the mesh client forwards the packet to its nearest mesh router. The packet is then forwarded over the mesh router backbone over multiple hops, according to the underlying routing protocol, until it reaches a mesh router that can forward the packet to the destination node. A popular way of organizing WMNs is to use separate standards for inter-router and router-client communication so that the traffic on the mesh backbone is not effected by the interference due to mesh clients. For example IEEE 802.11a may be used for communication over backbone links while IEEE 802.11b may be used for communication between mesh clients and routers [4]. WMNs are projected to be the networking solution of the future, specially in poorer parts of the world where it not economically feasible to provide broadband through cable or DSL.

While designing a WMN, it is important to understand how the delay and capacity of a WMN scale with the number of clients and mesh routers. The design of a WMN would depend on various factors such as mesh client density, the available budget, required bit rate and the expected traffic pattern. The size and budget of a WMN may vary from a few clients and very low budget, as in neighborhood networks like MIT Roofnet [2] to city wide meshes with thousands of clients and multi-million dollar budgets [5]. So it is important to be able to answer questions like what bit rate would be available to $n$ clients if the budget allows $m$ mesh routers with $l$ available channels, or how many mesh clients can be served with a given bit rate if the budget allows deployment of $m$ mesh routers with $l$ available channels over a given area?

In this paper we provide some answers to the above questions by characterizing the average delay and maximum achievable throughput in random access MAC based WMNs in terms of network parameters. We restrict our analysis to intra-mesh communication scenarios i.e. cases where mesh clients communicate with each other using the mesh router backbone. We propose a very general, yet analytically tractable routing and MAC model for random access MAC based WMNs. The MAC model takes into account the collision avoidance and back-off mechanisms of random access MAC protocols like IEEE $802.11$. The model is used to develop an open G/G/1 queuing network model for the delay analysis of WMNs. Each mesh router is a station in the equivalent queuing network representation. The average delay is the expectation of packet delay over all packets and all possible network topologies. The diffusion approximation method [10] is used to solve the queuing network. It provides closed form expressions of end-to-end delay and maximum achievable throughput per client in terms of the number of mesh clients, number of mesh routers, number of available channels and the traffic pattern of a WMN. We present a brief discussion on how our results compare with the well known results on asymptotic capacity of wireless ad hoc networks.

The rest of the paper is organized as follows. In the next section we present a brief review of related work. In section III we describe the well known diffusion approximation method for solving open G/G/1 queuing networks. The network model is described in detail in section IV followed by the delay analysis in section V. We discuss the analytical results and various trade-offs in section V-C. The simulation results and concluding remarks are presented in section VI and VII respectively.

## II. Related work

The asymptotic capacity of multihop wireless networks is studied in [13], [18], [19], [12]. In [13] it is shown that for a network with $n$ stationary nodes, the per-node capacity scales as $\Theta(W/\sqrt{n\log n})$. In [18], the authors use simulations in order to study the dependence of per-node capacity on IEEE $802.11$ MAC interactions and traffic pattern for various topologies like single cell, chain, uniform lattice and random network. An estimate of the expression for one-hop capacity and upper bound of per-node throughput is obtained using the simulation results. Extensive simulations are used in order to study the effects of variation of various network parameters, like number of nodes and path length, on network throughput in [19] and the simulations results agree closely with [13]. [12] shows that for mobile networks with loose delay constraints, the per-node capacity is $\Theta(1)$.

Most of the research effort for WMNs has been focused on developing efficient strategies for routing, channel assignment and scheduling in order to maximize throughput [11], [16], [21], [22], [15], [7]. The scalability properties of WMNs have so far not been sufficiently studied. In [14], the authors propose a collision domain based method in order to calculate the per-node throughput for a given WMN topology and gateway location. A linear program for verifying the feasibility of a rate is developed in [15] and is used to obtain upper and lower bounds on capacity. In [17] the author study the effect of number of channels and interfaces on the capacity of multihop wireless network. They found that in general if the number of available channels is greater than the number of interfaces then the capacity of the wireless network degrades by a factor that depends on the ratio of number of interfaces to number of available channels. However in some cases, where number of available channels is $O(\log n)$, there is no degradation in the capacity. Some recent papers have focused on measurement based performance evaluation of WMNs [8], [20].

## III. Diffusion approximation method

In this section we briefly describe how the diffusion approximation method is used to solve an open G/G/1 queuing network. (Please see [10] for details). The diffusion approximation method allows us to evaluate closed form expressions for the average end-to-end delay. Suppose we have an open queuing network with $n$ service stations, numbered from $1$ to $n$. The external arrival of jobs (packets in the case of a communication network) is a renewal process with an average inter-arrival time of $1/\lambda_e$ i.e. new packets arrive in the network at the rate $\lambda_e$. The squared coefficient of variance of inter-arrival time of new packets equals $c_A$. The mean and coefficient of variance of the service time at a station $i$ are denoted by $1/\mu_i$ and $c_{Bi}$, respectively.

The *visit ratio* of a station in a queuing network is defined as the average number of times a packet is forwarded by (i.e. visits) the station. The visit ratio of station $i$, denoted by $e_i$, is given by

$$e_i = p_{0i}(n) + \sum_{j=1}^{j=n} p_{ji}(n) \cdot e_j \tag{1}$$

where $p_{0i}$ denotes the probability that a packet enters the queuing network from station $i$ and $p_{ji}$ denotes the the probability that a packet is routed to station $i$ after completing its service at station $j$.

There are two sources of packet arrivals at a station: the packets that are generated at the station and the packets that are forwarded to the station by other stations. The resulting arrival rate is termed the *effective arrival rate* at a station. The effective arrival rate at the station $i$, denoted by $\lambda_i$ is given by

$$\lambda_i = \lambda_e e_i \tag{2}$$

The *utilization factor* of station $i$, denoted by $\rho_i$, is given by

$$\rho_i = \frac{\lambda_i}{\mu_i} \tag{3}$$

The squared coefficient of variance of the inter-arrival time at a station $i$, denoted by $c^2_{Ai}$, is approximated using

$$c^2_{Ai} = 1 + \sum_{j=0}^{n} (c^2_{Bj} - 1) \cdot p^2_{ji} \cdot e_j \cdot e_i^{-1} \tag{4}$$

where $c^2_{B0} = c^2_A$.

According to the diffusion approximation, the approximate expression for the probability that the number of packets at station $i$ equals $k$, denoted by $\pi_i(k)$, is

$$\pi_i(k) = \begin{cases} 1-\rho_i & k=0 \\ \rho_i(1-\hat{\rho}_i)\hat{\rho}_i^{k-1} & k>0 \end{cases} \tag{5}$$

where

$$\hat{\rho}_i = \exp\left(-\frac{2(1-\rho_i)}{c^2_{Ai}\cdot\rho_i + c^2_{Bi}}\right) \tag{6}$$

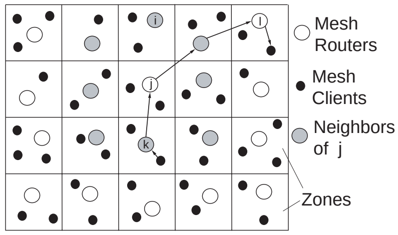



Fig. 2. WMN model with square zones of area $a(n)$ each.

The mean number of packets at a station $i$, denoted by $\overline{K_i}$, is

$$\overline{K_i} \quad = \quad \frac{\rho_i}{1-\hat{\rho}_i} \tag{7}$$

## IV. Queuing network model

In this paper we consider community WMNs deployed in residential neighborhoods. In such a WMN the mesh routers would typically be mounted on rooftops of houses at a regular interval so as to maintain sufficient connectivity, such as equipping one house per block with a mesh router. Each mesh router is responsible for serving clients (laptops, PDAs etc) present in its vicinity. The area covered by a mesh router is referred to as the *zone* of the router. For a large installation in a residential neighborhood the zones of the routers would be pretty regular, e.g. a mesh router might be responsible for covering a block. We consider delay and capacity of such WMNs for intra-mesh communication scenario involving applications such at voice/video chat, security camera streams, computer games and community portals.

In this section we present a network model for such a WMN and illustrate that how an equivalent queuing network model can be constructed. We then calculate some important parameters of the queuing network that would be used

### A. The network model

The network consists of $n$ mesh clients, also referred as nodes, distributed uniformly and independently over a torus of unit area. We assume a torus area so as to avoid complexities introduced in the analysis due to edge effects. The torus is divided into non overlapping zones of area $a(n)$ each. $a(n)$ is chosen such that $1/a(n)$ is an integer so that there are $1/a(n)$ zones in the network. Each zone has a mesh router that is responsible for serving clients with in the zone. The mesh routers are numbered 1 through $1/a(n)$. Two mesh routers are said to be *neighbors* if their zones share a common point. The set of neighbors of router $i$ is represented by $N(i)$. The number of neighbors of each router is a constant and equals $\kappa$. Figure 2 illustrates the WMN model.

The traffic model may be described as follows. Each node may be a source and destination of packets. We assume that packets of size $L$ bits are generated by each node according to an i.i.d. Poisson process [1] with rate $\lambda$. As soon as a packet is generated by a node, it is transmitted to the mesh router of its zone. After that mesh routers relay the packet over the backbone until it reaches the zone where the destination client is located. The probability that a packet received by a mesh router is destined to a node within its zone is $p(n)$. We refer to $p(n)$ as the "zone absorption probability." The probability that a packet received by a mesh router is forwarded to a neighboring router is $(1-p(n))$. If a packet is not absorbed by nodes of a zone then all the neighboring mesh routers are equally likely to be the next hop of the packet The parameter $p(n)$ of the traffic model characterizes the degree of locality of the traffic. The traffic is highly localized for large $p(n)$ while small $p(n)$ implies unlocalized traffic. Thus it is easy to quantify the dependence of delay and capacity on average path length.

[1]It should however be noted that our analysis can be easily extended to any arbitrary packet arrival process since the diffusion approximation method applies to non-Poisson arrival process as well but the SCVs and other expressions would become more complex.

The interference model may be described as follows. If a mesh router $i$ transmits a packet to router $j$ then the transmission will be successful only if (i) $j$ is a neighbor of $i$ and (ii) no other neighboring mesh router of $j$ transmits simultaneously with $i$. The client-router and router-router communication takes place on a separate band. Also the level of contention between clients in a zone would be less than that between mesh routers because of the volume of traffic involved. So we assume that a client may transfer a packet to its mesh router as soon as it is generated. Therefore the delay between generation of a packet at a node and its transfer to the mesh router is assumed to be negligible.

Each mesh router is assumed to have infinite buffers and thus no packets are dropped in the network. The packets are served by the routers in FCFS manner.

WMNs can be modeled as a queuing network as shown in Figure 3(a). The stations of the queuing network represent the routers of a WMN. The forwarding probabilities $p_{ij}$ of the queuing network equals the probability that a packet is transmitted from the queue at mesh router $i$ to the queue at router $j$. Figure 3(b) shows a representation of a mesh router as a station in the queuing network.

### B. Parameters of the queuing network model

In this subsection we will derive expressions for the parameters of the queuing network model.

*Lemma 1:* The probability that a packet is forwarded from the queue of a mesh router $i$ to the queue of mesh router $j$, denoted by $p_{ij}(n)$ equals

$$p_{ij}(n) = \begin{cases} \frac{1-p(n)}{\kappa} & j \in N(i) \\ 0 & otherwise \end{cases} \tag{8}$$

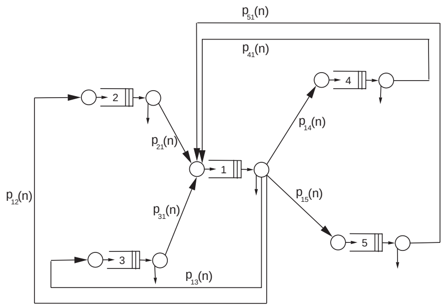
*Proof:*

$$p_{ij}(n) = P[\text{packet is not absorbed by } i] \times$$
$$P[i \text{ transmits the packet to } j | \text{ packet not absorbed}]$$

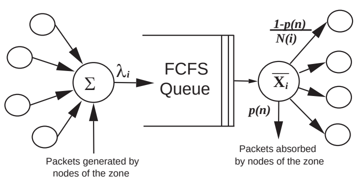
Now $P[\text{packet is not absorbed by } i] = (1-p(n))$ and $P[i \text{ transmits the packet to } j| \text{ packet not absorbed}] = \frac{1}{\kappa}$ if $j \in N(i)$ and equals 0 otherwise. This leads to (8). ■

*Lemma 2:* The visit ratio of a mesh router $i$, denoted by $e_i$, equals $\frac{a(n)}{p(n)}$.

*Proof:* Since the packets arrive at each node according i.i.d. Poisson process, the probability that a new packet




(a) A WMN represented as queuing network.




(b) A router of a WMN represented as station of the queuing network.

Fig. 3. Queuing model of WMN.

entering the network enters the network through zone $i$ (i.e. probability that the node that generates the new packet is in zone $i$) equals $a(n)$ . Substituting $p_{0i}(n)$ and $p_{ij}$ in (1), the visit ratio of mesh router $i$ can be expressed as

$$e_i = a(n) + \sum_{j \in N(i)} \frac{1-p(n)}{\kappa} e_j$$

From symmetry $e_i = e_j \; \forall \; 1 \leq i; j \leq \frac{1}{a(n)}$. Using symmetry and rearranging the above equation, we get

$$e_i = \frac{a(n)}{p(n)} \tag{9}$$

■

*Lemma 3:* The effective packet arrival rate at a mesh router $i$, denoted by $\lambda_i$, equals $na(n)\frac{\lambda}{p(n)}$.

*Proof:* Since the packet generation process at each mesh client is an i.i.d. Poisson process with rate $\lambda$, new packets arrive in the network at rate $\lambda_e = n\lambda$. Using $\lambda_i = \lambda_e e_i$, and substituting $e_i$ from (9), we get

$$\lambda_i = na(n)\frac{\lambda}{p(n)} \tag{10}$$

■

*Lemma 4:* The number of hops traversed by a packet in a WMN, denoted by $\overline{s}$, equals $\frac{1}{p(n)}$.

*Proof:* Let $s$ denote the number of mesh routers that forward a packet before it reaches the destination. Then

$$P[s=k] = (1-p(n))^{k-1}p(n) \quad k \geq 1$$

Thus,

$$\overline{s} = E[s] = \sum_{k=1}^{\infty} k \cdot (1-p(n))^{k-1} p(n) = \frac{1}{p(n)} \tag{11}$$

■

## V. Delay analysis of WMNs

In this section we present the delay analysis of the WMN model described in Section IV. We first present the medium access protocol model followed by analysis and discussions.

### A. The random access MAC model

According to the interference model, transmission of a mesh router is successful only if none of the neighbors of the receiver transmit concurrently on the same channel. Therefore transmission of a router would be guaranteed to be successful if all the none of its one and two hop neighbors that use the same channel transmit when the router is transmitting. The one and two hop neighbors of a mesh router that transmit on the same channel as the router are referred to as *interfering neighbors*. Let $I$ denote the number of interfering neighbors of a mesh router. For a WMN with square zones, as shown in Figure 2, $I = 24$ if all the routers transmit over the same communication channel. In general, the number of interfering neighbors is inversely proportional to the number of available channels. The random access MAC model ensures that no two interfering neighbors transmit at the same time.

The random access MAC model used in our analysis is as follows. Before transmitting each packet a mesh router counts down a random back off timer. The duration of the back off timer is exponentially distributed with mean $1/\xi$. The back-off timer is frozen each time an interfering neighbor starts transmitting and it is resumed when the transmission of the interfering neighbor ends. The router starts transmitting the packet when its back off timer expires. The back-off timers of all interfering neighbors are immediately frozen. This ensures that the packet is correctly received by desired receiver. This model is similar back-off based random MAC protocols like IEEE $802.11$ DCF and is still analytically tractable. Neglecting the time required for exchange of control packets (RTS, CTS and ACK), the time required for transmission of a packet equals $L/W$ seconds.

### B. Delay analysis

The end-to-end delay in a WMN is defined as the sum of the queuing and transmission delays at the intermediate mesh routers. In order to evaluate the end-to-end delay we first prove the following two lemmas. In Lemma 5 we find the average number of *active* interfering neighbors of a mesh router. A router is said to be active if it has a packet to send. Lemma 6 characterizes the number of times the back-off timer a router is frozen during a transmission epoch.

*Lemma 5:* Let $H_i$ denote the number of active interfering neighbors of a mesh router $i$. Then

$$E[H_i] = I\rho \tag{12}$$

$$E[H_i^2] = I\rho(1+(I-1)\rho) \tag{13}$$

where $\rho$ is the utilization factor of the mesh routers.

*Proof:* From symmetry the utilization factor of every mesh router is the same and equals $\rho$. Thus $H_i$ has binomial distribution with parameters $(I, \rho)$. (12) and (13) directly follow. ■

*Lemma 6:* Let $M_i$ denote the number of times the timer of a mesh router $i$ freezes during a transmission epoch. Then

$$E[M_i] = I\rho \quad (14)$$
$$E[M_i^2] = 2I\rho(1+(I-1)\rho) + I\rho \quad (15)$$

*Proof:* Let $T_i$ denote the duration of the back off timer of the mesh router $i$. The number of times the timer is frozen equals the number of times the back off timers of the interfering neighbors expire while router $i$ counts down its back-off timer. We assume that $H_i$ remains constant during the transmission epoch. Since all the back-off timers have exponential distribution, the number of timer expirations in the duration $t$ would have Poisson distribution with mean $\xi ht$. Thus the probability that $M_i$ equals $m$, given that $T_i = t$ and $H_i = h$, is

$$P[M_i = m | T_i = t, H_i = h] = \frac{e^{-\xi ht}(\xi ht)^m}{m!}$$

Hence

$$E[M_i | T_i = t, H_i = h] = h\xi t \quad (16)$$
$$E[M_i | H_i = h] = E_{T_i}[E[h\xi t | T_i = t]] = h$$

Taking expectation of both sides w.r.t $H_i$ we get (14). Similarly

$$E[M_i^2 | T_i = t, H_i = h] = h^2\xi^2 t^2 + h\xi t \quad (17)$$

$$E[M_i^2 | H_i = h] = h^2\xi^2 E[t^2] + h\xi E[t] = 2h^2 + h$$

Taking expectation of both sides w.r.t $H_i$ and substituting (12) and (13) we get (15). ■

*Theorem 1:* Let $\overline{X_i}$ and $\overline{X_i^2}$ denote the mean and second moment of the service time of a mesh router $i$. Then

$$\overline{X_i} = E[X_i] = \frac{\frac{1}{\xi} + \frac{L}{W}}{1 - \frac{L}{W}\lambda_i I} \quad (18)$$

$$\overline{X_i^2} = (2\overline{H^2} + 3\overline{H} + 1)\frac{L^2}{W^2} + 2(2\overline{H} + 1)\frac{L}{W}\frac{1}{\xi} + \frac{2}{\xi^2} \quad (19)$$

where $\overline{H} = E[H_i]$ (eqn. (12)) and $\overline{H^2} = E[H_i^2]$ (eqn. (13)).

*Proof:* The total service time is the sum of (i) the duration of back-off timer ($T_i$), (ii) the time for which the timer remains frozen ($M_i L/W$) and (iii) the transmission time ($L/W$). Thus we have

$$X_i = T_i + M_i\frac{L}{W} + \frac{L}{W}$$

Taking expectation of both sides we get,

$$E[X_i] = \overline{X_i} = \frac{1}{\xi} + I\rho\frac{L}{W} + \frac{L}{W} = \frac{1}{\xi} + I\lambda_i\overline{X_i}\frac{L}{W} + \frac{L}{W}$$

Rearranging, we get (18).
Also,

$$X_i^2 = (T_i + M_i\frac{L}{W} + \frac{L}{W})^2$$

Using (16) and (17), the expected value of $X_i^2$ given that $T_i = t$ and $H_i = h$ is given by

$$E[X_i^2 | T_i = t, H_i = h] = (\frac{L}{W})^2(1 + 3h\xi t + h^2\xi^2 t^2) + (\frac{L}{W})(2h\xi t^2 + 2t) + t^2$$

Taking expectation of both sides w.r.t $T_i$ we get

$$E[X_i^2 | H_i = h] = (\frac{L}{W})^2(1 + 3h + 2h^2) + (\frac{L}{W})(4h\frac{1}{\xi} + 2\frac{1}{\xi}) \quad (20)$$

Taking expectation of both sides w.r.t to $H_i$ and substituting (12) and (13), we get (19). ■

Using (18) and (19) the SCV of the service time of a mesh router is given by

$$c_{Bi} = (\overline{X_i^2} - \overline{X_i}^2)/\overline{X_i}^2$$

Using expressions for $c_{Bi}$, $\rho$ and $p_{ij}$, the SCV of the inter arrival time at a mesh router, $c_{Ai}$, and $\hat{\rho}$ can be determined using (4) and (6).

*Theorem 2:* For the random access MAC model described in Section V-A, the average end to end delay, denoted by $D(n)$ in a WMN is

$$D(n) = \frac{\rho}{na(n)\lambda \cdot (1 - \hat{\rho})} \quad (21)$$

*Proof:* Let $\overline{D_i}$ denote the average packet delay at mesh router $i$. According to Little's law, $\overline{D_i} = \overline{K_i}/\lambda_i$, where $\overline{K_i}$ is the average number of packets in the queue of router $i$. Substituting $\overline{K_i}$ and $\lambda_i$ from (7) and (10) we get

$$\overline{D_i} = \frac{p(n)\rho}{na(n)\lambda(1 - \hat{\rho})}$$

By symmetry, the average packet delay at all mesh routers is the same, therefore the average end-to-end delay equals $\overline{D_i}$ times the average number of hops between the source and destination mesh router i.e. $D(n) = \overline{s}D_i$ which leads to (21) ■

### C. Maximum achievable throughput

In this subsection we derive the expression for maximum achievable throughput in a WMN and compare the obtained result with the maximum achievable throughput in infrastructure-less wireless ad hoc networks.

The *maximum achievable throughput* ($\lambda_{\max}$) is the maximum value of the packet arrival rate $\lambda$ at the mesh clients for which the average end-to-end delay remains finite.

*Corollary 1:* The maximum achievable throughput for a WMN, denoted by $\lambda_{\max}$, is given by

$$\lambda_{\max} = \frac{p(n)}{na(n)(c + \frac{L}{W}I)} \quad (22)$$

where $c = \frac{1}{\xi} + \frac{L}{W}$. Also from (22), $\lambda_{\max} = \Theta\left(\frac{1}{\overline{s}na(n)}\right)$

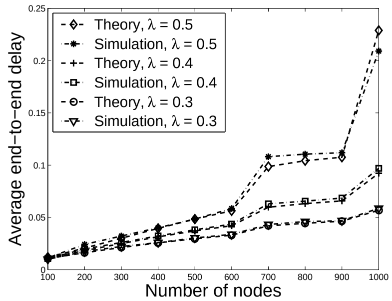



Fig. 4. Average end-to-end delay in WMN vs. number of clients for varying packet arrival rates.

The Corollary 1 follows from the the fact that in order to have finite delay, $\lambda_i \overline{X_i} < 1$.

We now compare the result of Corollary 1 on maximum achievable throughput against the Gupta-Kumar result [13] for the asymptotic case where $n \to \infty$. It is obvious that $\lambda_{\max}$ increases with decrease in $a(n)$. $a(n) = \log n/n$ would ensure, in asymptotic case, that each zone has at least one node with high probability. It would therefore be a good choice for zone areas in a large installation in order to prevent under-utilization. For $a(n) = \log n/n$, the number of hops between an arbitrary source and destination would be $O(\sqrt{n/\log n})$ [9]. Thus for this case we may fix the absorption probability $p(n)$ to be $\sqrt{\log n/n}$. Replacing these values of $a(n)$ and $p(n)$ in (22) we get $\lambda_{\max} = \frac{1}{\sqrt{n \log n}(c+IL/W)} = \Theta\left(\frac{1}{\sqrt{n \log n}}\right)$. This result is similar to the asymptotic capacity of multihop wireless ad hoc networks result in [13].

This similarity in the results can be explained in the following manner. In order to derive the throughput capacity result in [13], the surface is divided into cells using Voronoi tessellation. It is shown that asymptotically the traffic served by a cell is less than $k\lambda\sqrt{n \log n}$ with high probability, where $k$ is a constant. The fact that the number of interfering neighbors is bounded by a constant yields the throughput capacity result. The traffic served by a cell in Kumar-Gupta model is analogous to the effective packet arrival rate ($\lambda_i$) at a mesh router in our model. For $a(n) = \log n/n$ and $p(n) = \sqrt{\log n/n}$, $\lambda_i$ at each router is equal to $\lambda\sqrt{n \log n}$ which has the same scaling properties as the traffic served by each cell in [13]. The MAC model ensures proportional fairness among the $I$ interfering mesh routers by using i.i.d. random backoff timers and eliminating collisions. Thus, for packet size $L$ bits, each router gets at least $\frac{1}{1/\xi+(I+1)L/W}$ fraction of the available bandwidth. Therefore for the WMN model the maximum achievable throughput also scales as $1/(\sqrt{n \log n})$.

## VI. Simulations

In this section we compare the simulation results with the analytical results in order to verify the validity of the assumptions made in our analysis and the accuracy of the diffusion approximation method.

The simulation setting for the WMN is the following. The network consists of $n$ nodes that are uniformly and independently distributed over a torus of unit area. The torus is partitioned into $\langle \frac{1}{\sqrt{\log n/n}} \rangle^2$ non-overlapping square zones of equal area, where the operator $\langle \cdot \rangle$ rounds off the operand to the closest integer value. All the mesh routers transmit over the same channel, so each mesh router has $24$ interfering neighbors. The zone absorption probability for all the simulations is $\sqrt{\frac{\log n}{n}}$. The values of parameters $a(n)$ and $p(n)$ are chosen according to the discussions in section V-C. The results for average delay are obtained by averaging the delay results over several topologies. Figure 4 shows the comparison between the average end-to-end delay obtained from the simulations and the analytical results. The analytical results agree closely with the simulation results.

## VII. Conclusion and Future Work

The scalability properties of random access MAC based WMNs are not completely understood. In this paper we presented delay analysis of random access MAC based WMN. We found the closed form expressions for the average end-to-end delay and maximum achievable throughput. For appropriate network parameters, the result on maximum achievable throughput agrees with the well established information-theoretic results [13]. Extension of the delay analysis and characterization of the maximum achievable throughput in many-to-one (or many-to-few) communication scenarios, that occur when mesh clients connect to Internets through few gateways, would be the focus of further research.

## References

[1] Bay Area Wireless User Group. http://www.bawug.org/.
[2] MIT Roofnet. http://www.pdos.lcs.mit.edu/roofnet.
[3] Seattle Wireless. http://www.seattlewireless.net.
[4] SoCal Free Net Homepage. http://socalfreenet.org.
[5] Wireless Philadelphia Homepage. http://www.phila.gov/wireless.
[6] I. F. Akyildiz, X. Wang, and W. Wang. Wireless mesh networks: a survey. *Computer Networks and ISDN Systems*, 47(4):445–487, 2005.
[7] M. Alicherry, R. Bhatia, and L. E. Li. Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks. In *MobiCom '05*, pages 58–72, 2005.
[8] J. Bicket, D. Aguayo, S. Biswas, and R. Morris. Architecture and evaluation of an unplanned 802.11b mesh network. In *MobiCom '05*, pages 31–42, 2005.
[9] N. Bisnik and A. Abouzeid. Queuing network models for delay analysis of multihop wireless ad hoc networks. Technical Report, ECSE Department, http://www.rpi.edu/~bisnin/qmodels.pdf, January 2005.
[10] G. Bolch, S. Greiner, H. de Meer, and K. S. Trivedi. *Queuing Networks and Markov Chains*, chapter 10, pages 423–430. John Wiley and Sons, 1998.
[11] R. Draves, J. Padhye, and B. Zill. Routing in multi-radio, multi-hop wireless mesh networks. In *MobiCom '04*, pages 114–128, 2004.
[12] M. Grossglauser and D. N. C. Tse. Mobility increases the capacity of ad hoc wireless networks. *IEEE Trans. on Net.*, 10(4):477–486, 2002.
[13] P. Gupta and P. R. Kumar. Capacity of wireless networks. *IEEE Trans. on Information Theory*, pages 388–404, March 2000.
[14] J. Jun and M. L. Sichitiu. The nominal capacity of wireless mesh networks. *IEEE Wireless Communications*, Oct 2003.
[15] M. Kodialam and T. Nandagopal. Characterizing the capacity region in multi-radio multi-channel wireless mesh networks. In *MobiCom '05*, pages 73–87, 2005.
[16] P. Kyasanur and N. Vaidya. Routing and interface assignment in multi-channel multi-interface wireless networks. In *WCNC*, 2005.
[17] P. Kyasanur and N. H. Vaidya. Capacity of multi-channel wireless networks: impact of number of channels and interfaces. In *MobiCom '05*, pages 43–57, 2005.
[18] J. Li, C. Blake, D. S. J. D. Couto, H. I. Lee, and R. Morris. Capacity of ad hoc wireless networks. In *Mob. Comp. and Net.*, pages 61–69, 2001.
[19] G. Nemeth, Z. R. Turanyi, and A. Valko. Throughput of ideally routed wireless ad hoc networks. *ACM SIGMOBILE Mobile Computing and Communications Review*, 5(4):40–46, 2001.
[20] B. Raman and K. Chebrolu. Design and evaluation of a new mac protocol for long-distance 802.11 mesh networks. In *MobiCom '05*, pages 156–169, 2005.
[21] A. Raniwala and T. Chiueh. Architecture and algorithms for an IEEE 802.11 based multi-channel wireless mesh network. In *INFOCOM'05*.
[22] A. Raniwala, K. Gopalan, and T. cker Chiueh. Centralized channel assignment and routing algorithms for multi-channel wireless mesh networks. *SIGMOBILE Mob. Comp. Comm. Rev.*, 8(2):50–65, 2004.