Information Theoretic Analysis of Proactive Routing Overhead in Mobile Ad Hoc Networks

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Abstract

This paper considers basic bounds on the overhead of link-state protocols in mobile ad hoc networks. Hierarchical protocols are known for their good scalability properties, and hence this paper considers a two-level hierarchical protocol. In such protocols, nodes need to keep track of shortest path information, link states and cluster membership. Two types of overheads are considered; the memory needed to store routing-related information, including link-states and cluster membership, and the messages that need to be exchanged to keep track of the changes in the network. Memory overhead is important practically for dimensioning network nodes, while message overhead is important since it reduces the effective capacity of the network to carry user data (vis-a-vis control data). The scalability properties of the routing overhead are analyzed for different modes of network scaling. Practical implications, such as optimal cluster size, average/fixed memory requirement and routing protocol parameter selections are discussed.

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1 Introduction

We view the problem of *routing* as a problem of *maintaining state information* about the network so that packets can be forwarded within an acceptable quality metric. Different classes of routing protocols can be viewed within that premise. For example, proactive routing protocols maintain up to date link state information periodically, while reactive protocols maintain link state information on an as-needed basis. Geographic routing protocols maintain geographic state (location) information.

For dynamic networks - almost every network is dynamic - significant routing overhead may be needed in order to maintain state in the network, and, in our opinion, such an overhead has yet to be fully understood. But the real effective throughput achievable (per node) in a network depends not only on how much a node can send, but also how much of the sent information is actual user data vis-*à*-vis control packets.

It is difficult to compare the overhead of various routing protocols since as yet there exists no absolute bounds on the minimum overhead incurred by any of the protocol classes. It would be useful to fill in this gap by developing some general theory, which ideally would precisely relate routing overhead of a class of protocols to the other relevant network parameters such as node mobility.

The key departure point of the new framework proposed here is to treat the state (e.g. link state or node location, or a hybrid of both, etc.) as a random variable that exhibits random changes. The minimum protocol overhead is related to the minimum amount of information (i.e. effort) needed to identify the current state of the system, possibly within a distortion bound. The various state changes are derived from the probability distribution describing the cause of dynamism e.g. the nodes mobility pattern. Thus the protocol overhead can be brought into an information theoretic framework, and terms like entropy rate now yield new practical design guidelines and basic limits.

This paper (and its earlier conference versions) represents the first step in this journey by considering the class of hierarchical link-state routing protocols. The extension to multiple levels is straightforward, but we do not include the results here for length considerations. We have also extended the results to reactive protocols in [1] as well as geographic routing protocols in [2].

In this paper, we derive lower bounds on the minimum routing overhead (bits per unit time) and memory (bits) associated with a proactive routing protocol in an ad hoc network of mobile nodes as a function of the network parameters. The topology of an ad hoc network is randomly changing due to random node movement as well as random link-state changes (due to fading for example). To maintain up to date topology information, proactive routing requires that nodes of a network exchange control messages containing the new topology or topology change information. The memory requirement is related to the amount of state information stored or processed. The routing overhead is related to the product of the message size and the number of hops the message travels.

In order to derive lower bounds on memory requirement and routing overhead, the first step is to find lower bounds on the sizes of exchanged control messages, and then relate this to the rate of exchange through the parameters of the mobility model. The minimum complexity of the control message sizes is intuitively a function of the complexity of the network. We capture this dependence through the information-theoretic measure called the *Minimum Expected Codeword Length (MCL)* (see for example [3] Section 5.4) which is the minimum number of bits required to describe (i.e. encode) a change. Because of the hierarchical (clustered) structure, three different topology levels (types) are analyzed, where each topology level represents different granularity of the network topology. Three methods are used to derive expressions for the MCLs; topology cardinality (Method 1), topology probability distribution (Method 2) and topology prediction (Method 3).

To our knowledge, there is no previous work that bounds routing overhead using such information theoretic measure – related analytical work focuses on *modeling* (rather than bounding) routing overhead [4, 5]. Gallager [6] analyzes the protocol overhead in a pure information theoretic manner. His paper uses an entropy measure to determine the basic limitations on the amount of protocol information that must be transmitted in a stationary (i.e. without mobility) data communication network to keep track of source and receiver addresses and of the starting and stopping of messages. Topology changes as it applies to routing was not considered in [6].

Gavoille in [7] defines routing as a distributed algorithm over a static undirected random graph, and proposes multiple open research problems based on the structures of these graphs and the quality of service requirements. One of the interesting open questions is the trade-off between the size of memory used to store topology information in the nodes of a network and the accuracy of finding shortest paths. The problem somehow is similar to our problem of finding out the trade-off between memory requirement and communication routing overhead.

The main contributions of this paper are (i) to derive analytic expressions of the MCLs for three information theoretic techniques for each topology level; (ii) bound the average communication routing overhead by the entropy rates of the sequences of topologies; (iii) bound the average/fixed memory requirement; (iv) provide a scalability analysis of average memory requirement and communication routing overhead with the number of nodes and number of clusters for three different modes of scaling; and (v) provide an analysis of the optimal number of clusters that asymptotically minimizes the communication routing overhead.

The paper is organized as follows. Section 2 introduces the network model (including the routing protocol, topology definitions and mobility model) and the notations used in the remainder of the paper. Section 3 analyzes the minimum expected codeword lengths (MCLs). Section 4 discusses the relationships between the entropy rates, the topology evolution, and the routing overhead. Section 5 derives lower bounds on the memory requirement and routing overhead. Section 6 studies the scalability of the overheads as a function of the network size. Section 7 presents practical implications deduced from applying the theoretical results. Finally, Section 8 concludes the paper and outlines future research directions.

2 Network Model

2.1 Topology

We consider a fixed number N of distinguishable nodes that move within a bounded region. Each node has a unique identifier - denoted as NUI (from 1 to N). The bounded region is divided into fixed number M of sub-regions, labelled by a unique sub-region index, denoted as RUI (from 1 to M). There is a maximum of $K > 1$ neighboring sub-regions for any given sub-region. Furthermore, we assume that the mobility patterns of the nodes are statistically independent and identically distributed (i.i.d).

All the nodes within a sub-region form a cluster. A *cluster head* is selected for each non-empty sub-region, which is randomly chosen from the nodes within the sub-region. Let the integer valued random variable $\mathcal{N}$ denote the number of nodes within a given sub-region. The range of $\mathcal{N}$ is $0, 1, \ldots, N$. Let n denote the number of nodes in a given sub-region at some time instant (i.e. $0 \leq n \leq N$). It is possible that a sub-region becomes empty and thus will not have a cluster or a cluster head. A node that is not a cluster head is called *regular node*. A cluster head *owns* a regular node if the two nodes belong to the same sub-region. Any node that does not belong to the same sub-region is called *exterior node*.

We understand that the current cluster model does not capture the need of supporting dynamic cluster formation and elimination for ad hoc networks. But our work in this paper can be extended to include the dynamic cluster formation and elimination by including the cluster formation policies and dynamic distribution of this information.

The underlying physical topology of an ad hoc network is represented by a connected,

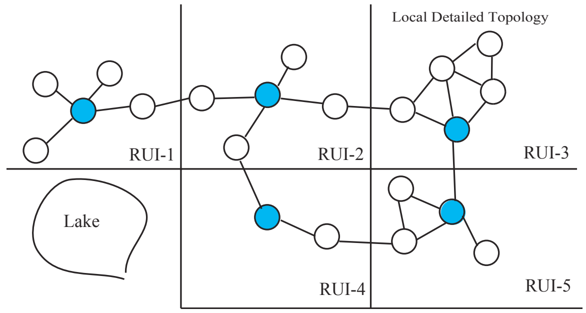

Figure 1: A snapshot of the network topology. Blue (dark) nodes are cluster heads. There is one cluster head in every sub-region. Sub-regions do not have to be rectangular or identical.

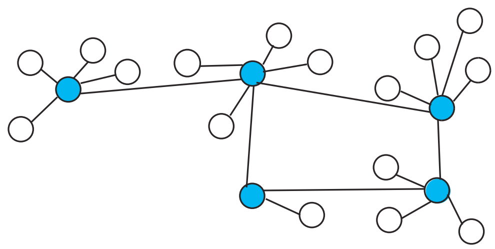

Figure 2: The corresponding global ownership topology. Blue (dark) nodes are cluster heads.

undirected randomly changing graph, $G = (V, E)$, where V is the set of graph nodes and E is the set of edges. An example of the physical topology is depicted in Figure 1. Each mobile node is represented by a graph node using mobile node's *NUI*. An edge exists between two arbitrary graph nodes if single-hop communication between those two corresponding nodes is admissible. In this paper, we assume that the transmission and reception ranges of a node are equal and hence edges are bi-directional.

Three different topology types are deduced from the physical topology in this paper. Each topology type is represented by a graph. The first topology type is *local detailed topology*, which is a sub-graph of the physical topology (see Figure 1, sub-region RUI-3). Each local detailed topology only has the nodes of a given sub-region, and it inherits the edges from the physical topology. There are M instances of local detailed topologies (sub-graphs) at any given moment. The second topology type is a simplified version of local detailed topology named *local ownership topology* which only specifies the cluster head and the *NUI* list of the regular nodes of the cluster omitting the detailed knowledge of the physical connectivity of the nodes. In a graph representing a local ownership topology, each regular node has an edge to its cluster head. The third topology type is *global ownership topology*, which is the aggregation of all local ownership topologies (see Figure 2). In a graph representing the global ownership topology, an edge exists between each regular node to its cluster head to reflect the ownership relationship, and an edge exists between two cluster heads of neighboring sub-regions to reflect the neighborhood relationship.

2.2 Proactive Routing Protocol

The model of routing between two mobile nodes is as follows. Each regular node maintains a shortest path to the cluster head. When a source node needs to communicate to a destination node of the same sub-region, the source node first sends a route query to its cluster head following the shortest path. Upon receiving the route query, the cluster head computes the shortest path between the source and the destination nodes, and sends the shortest path information to the source node. The packets between the source and destination nodes will follow the shortest path. When a source node needs to communicate to a destination node in a remote sub-region, the source node sends the packets to its cluster head. The cluster head forwards the packets to the remote cluster head of the remote sub-region where the destination node is located. The remote cluster head further forwards these packets to the destination node.

To support the above routing model, we assume a generic two-level hierarchal proactive routing protocol. Each cluster head maintains the knowledge (i.e. the up to date status) of the *local detailed topology* of its sub-region and the global ownership topology of the whole network.

How do cluster heads and regular nodes update the topology knowledge as the network topologies change? There is a need to have a mechanism at the routing layer to detect, collect and distribute the network topology changes. We assume that each node has a clock that is not required to be synchronized. Each node notifies its existence via a periodic transmission of "HELLO" messages to its neighboring nodes, and detects the link changes with its neighboring nodes by listening to the transmission of HELLO messages from its neighboring nodes. In addition each regular node periodically sends the link status changes (if any) to its cluster head. In this protocol, we select the two time periods to be the same. The update message of link status change is sent out at discrete constant time intervals, such as for example by incorporating the information in the HELLO messages. The time period is the *interior update time* τ_i. We assume that τ_i is a constant for all the nodes. The assumption is helpful for technical reasons. It also matches several routing protocol designs due to practical reasons. For example, in OLSR proactive protocol [8], the parameter "HELLO_INTERVAL" can be viewed as τ_i in our protocol model. The transmission of HELLO messages is widely used in the wireless communication protocols to detect the physical/MAC layer link status changes, which is called the "HELLO" protocol [5]. Upon receiving the link status change message from any of its regular nodes, a cluster head updates its local detailed topology. If the cluster head finds the shortest path of a regular node has broken, it will notify the regular node of a new shortest path.

Similarly, each cluster head periodically broadcasts its local ownership topology changes to other cluster heads. The time period is *exterior update time* τ_e. Upon receiving an updated message from another cluster head, a cluster head updates the global ownership topology.

The process of detecting, collecting and distributing the topology changes produces a routing overhead $\mathbb{R}_t$. We separate the routing overhead into two parts. The first part is the *interior routing overhead* $\mathbb{R}_i$ which is the bit rate needed to maintain the local detailed topology of a sub-region. This part includes the overhead of detecting the link status changes by sending HELLO message, updating the cluster head knowledge of the link status changes, and maintaining the shortest paths between the regular nodes to their cluster head. The second part is the *exterior routing overhead* $\mathbb{R}_e$ which is the bit rate needed to maintain the global ownership topology, which includes the overheads of distributing local ownership topologies among the cluster heads. Hence $\mathbb{R}_t = \mathbb{R}_i + \mathbb{R}_e$.

Notice that a cluster head will need to maintain information about which regular nodes have links to which neighboring cluster heads, so that it can use those nodes to forward packets to the corresponding neighboring clusters. A regular node that can communicate with a neighboring node that belongs to another cluster needs to include this information in the link state update message to the cluster head. However, for simplicity, we neglect this additional overhead in the current analysis compared to the other overheads quantified in this paper. This simplifies the analysis of the MCL for local ownership topology.

In order to support the routing protocol operation, each regular node requires $\mathbb{M}_r$ bits and each cluster head requires $\mathbb{M}_c$ bits of memory.

Finally, we make an important (though well known) observation that aids in the routing overhead analysis. The portion between an intermediate node and the cluster-head along the shortest path between a regular node and the cluster head is also a shortest path between the intermediate node and the cluster head. Hence, we assume in the routing protocol that this portion of the path is used by the intermediate node as the shortest path between itself and the cluster head. Therefore, each node needs only to remember its next neighboring node along its shortest path.

2.3 Mobility

Nodes are assumed to be capable of moving freely within the region of interest, and hence to leave a cluster and to join another cluster of another sub-region. Furthermore, we assume that if a node leaves a sub-region, it has equal chance to join any cluster of its neighboring sub-regions.

Let the random variable $X_j \in \{0, 1\}$ denote the link status between any two nodes - a and b of a sub-region, where 1(0) represents the link exists (does not exist). The index j denotes the discretized time when a sends the update message of the link status change to its cluster head. The sequence of $\{X_1, X_2, ..., X_j, ...\} = \{X_j\}_{j=1}^{\infty}$ forms a random process.

Similarly, let $Y_j \in \{0, 1\}$ denote the ownership status of a node for a given sub-region, where 0(1) denotes the node is outside (inside) the sub-region. The index j denotes the discretized time when the cluster head broadcasts its local ownership topology change to other cluster heads. The sequence of ownership status $\{Y_1, Y_2, ..., Y_j, ...\} = \{Y_j\}_{j=1}^{\infty}$ forms another random process.

The random processes $\{X_j\}_{j=1}^{\infty}$ and $\{Y_j\}_{j=1}^{\infty}$ are modeled as Markov chains. The mobility model of the network is specified by the following three parameters. The first is the conditional probability p_{00} that two nodes in the same sub-region that are not directly connected remain unconnected within the interval of the interior update time τ_i. The second is the conditional probability p_{11} that a direct link between two nodes of the same sub-region remains connected within the interval τ_i. The third is the probability q_0 that a node stays at the same sub-region within a time interval τ_e. We assume that all the above three probabilities are constants for the network (which is true for a homogeneous mobility model).

2.4 Summary of Notations

Tables 1 and 2 summarize the main notations in this paper, where Table 1 contains the network parameters used in the abstract network and Table 2 contains the quantities deduced in this paper.

In the analysis of the effect of mobility, we will refer to the entropy function $H(\cdot)$ defined as

$$H(p) = -p \log p - (1 - p) \log(1 - p) \, ; 0 < p < 1.$$

Table 1: List of notations for the network abstraction

Notation	Description
N	Total number of nodes
M	Total number of sub-regions
K	Maximum number of neighboring sub-regions
$\mathcal{N}$	Number of nodes in a given sub-region
n	Number of nodes in a given sub-region at some time instant
τ_i	*Interior update time*
τ_e	*Exterior update time*
p_{00}	Probability that two nodes remain unconnected within τ_i
p_{11}	Probability that two nodes remain connected within τ_i
q_0	Probability of a node staying in same sub-region within τ_e
l_r	Average path length between a regular node and its cluster head
l_c	Average path length between two neighboring cluster heads

3 Minimum Expected Codeword Length

In this section, we derive the MCLs for the topologies described in Section 2.1. The MCL is computed using the entropy as defined by Shannon [9]. In this paper, all logarithms are base 2.

Three different information-theoretic methods are used to deduce the MCLs for each topology type. The first method is to deduce the MCL according to the cardinality of the topology type without any prior knowledge of the probability distribution of topologies. The second is to deduce the MCL according to the probability distribution of the topologies. The third is to deduce the MCL according to the conditional probability distribution based on the knowledge of previous topology at the previous time step and the mobility pattern.

3.1 Global Ownership Topology

3.1.1 Cardinality

Theorem 3.1 The total number of possible topologies is given by

$$G = \sum_{i=1}^{\min(N,M)} \frac{N!M!}{i!(M-i)!(N-i)!} i^{N-i} \tag{1}$$

Table 2: List of notations for the derived quantities

Notation	Description
G	Cardinality of global ownership topology
I_G^C	MCL for global ownership topology based on cardinality
I_G^P	MCL for global ownership topology based on topology stationary probability distribution
I_G^M	MCL for global ownership topology based on prediction using previous topology knowledge
L	Cardinality of local ownership topology
I_L^C	MCL for local ownership topology based on cardinality
I_L^P	MCL for local ownership topology based on topology stationary probability distribution
I_L^M	MCL for local ownership topology based on prediction using previous topology knowledge
D	Cardinality of local detailed topology
I_D^C	MCL for local detailed topology based on cardinality
I_D^P	MCL for local detailed topology based on topology stationary probability distribution
I_D^M	MCL for local detailed topology based on prediction using previous topology knowledge
R_i	Interior routing overhead within time interval τ_i
R_e	Exterior routing overhead within time interval τ_e
$\mathbb{R}_i$	Interior routing overhead in bit/second
$\mathbb{R}_e$	Exterior routing overhead in bit/second
$\mathbb{R}_t$	Total routing overhead in bit/second
$\mathbb{M}_r$	Memory required for a regular node
$\mathbb{M}_c$	Memory required for a cluster head

Proof: First, consider the case $N \geq M$. Let $\underline{n} = (n_1, n_2, \ldots, n_M)$ denote a specific organization of the nodes over the sub-regions. Then the possible topologies for this case is

$$\frac{N!}{n_1!n_2!\ldots n_M!}\prod_{i=1}^{M} g(n_i)$$

where $g(\cdot)$ is defined as

$$g(x) = \begin{cases} x & \text{for } x \geq 1; \\ 1 & \text{for } x = 0. \end{cases} \quad (2)$$

$g(x)$ is the number of ways of selecting a cluster head for a sub-region with x nodes. Let R denote the set of all possible organizations of the nodes. Let

$$f(x_1, x_2, \ldots, x_M) = (x_1 + x_2 + \cdots + x_M)^N = \sum_{\underline{n}\in R} \frac{N!}{n_1!n_2!\ldots n_M!} x_1^{n_1} x_2^{n_2} \ldots x_M^{n_M} \quad (3)$$

The total number of topologies in which there are no empty sub-regions can be calculated by taking the partial derivative of (3) w.r.t x_i and then setting $x_i = 1$, which yields

$$\left(\frac{N!}{(N-M)!}\right) M^{N-M} \quad (4)$$

Consider now that there is a single empty sub-region s. Then the total number of topologies can be calculated by taking the partial derivative of (3) w.r.t x_i and then set $x_i = 1 \forall i \neq s$ and $x_s = 0$, which yields

$$\left(\frac{N!}{(N-M+1)!}\right)(M-1)^{N-M+1} \quad (5)$$

And since there are $\binom{M}{1}$ ways of having a single empty sub-region, the total number of topologies with a single empty sub-region is thus

$$\left(\frac{M!}{1!(M-1)!}\right)\left(\frac{N!}{(N-M+1)!}\right)(M-1)^{N-M+1} \quad (6)$$

By induction, the number of topologies with exactly i empty sub-regions is

$$\left(\frac{M!}{i!(M-i)!}\right)\left(\frac{N!}{(N-M+i)!}\right)(M-i)^{N-M+i} \quad (7)$$

where $0 \leq i \leq (M-1)$. Summing over all i yields the result. Similar derivation applies for the case $N < M$ (but notice in this case that the number of empty sub-regions will range from $(M-N)$ to $(M-1)$). ■

3.1.2 MCL based on Cardinality

Let I_G^C denote the MCL based on cardinality. From (1),

$$I_G^C = \log(G) \tag{8}$$

From the proof of Theorem 3.1, we have shown that for a distribution $\underline{n} = (n_1, n_2, \ldots, n_M)$ of nodes for each sub-region, the total number of possible topologies is

$$\frac{N!}{n_1!n_2!\ldots n_M!}\prod_{i=1}^{M} g(n_i) \text{ with} \sum_{i=1}^{M} n_i = N \tag{9}$$

$\prod_{i=1}^{M} g(n_i)$ reaches its maximum value when $\underline{n} = (n_1, n_2, \ldots, n_M)$ has a uniform distribution, so the above expression is upper bounded by

$$\frac{N!}{n_1!n_2!\ldots n_M!}\left\lceil\frac{N}{M}\right\rceil^M \tag{10}$$

It is easy to show after some algebraic manipulations that

$$M^N \leq G \leq \left\lceil\frac{N}{M}\right\rceil^M M^N \tag{11}$$

and equivalently

$$N\log M \leq I_G^C \leq N\log M + M\log\left\lceil\frac{N}{M}\right\rceil \tag{12}$$

The result (12) can be interpreted as follows. The lower bound in (12) is the minimum number of bits needed to describe a global ownership topology when omitting the information about which nodes are the cluster heads. (12) means that the introduction of cluster heads at most increases the MCL by $M\log\lceil\frac{N}{M}\rceil$. For large N,

$$\lim_{N\to\infty}\frac{N\log M + M\log\lceil\frac{N}{M}\rceil}{N\log M} = 1 \tag{13}$$

and hence

$$I_G^C \approx N\log M \quad ; N \gg 1 \tag{14}$$

From (14), I_G^C is proportional to the total number of nodes N when N is large enough. The interpretation is that as the total number of nodes becomes large, the information amount ($M\log\lceil\frac{N}{M}\rceil$) specifying the cluster heads becomes a small portion of total information amount for the global ownership topology. The information is $\log M$ for each node. Therefore, if we use small clusters, we need more bits to describe the global ownership topology.

3.1.3 MCL based on Topology Stationary Probability Distribution

Let x represent a specific way of assigning nodes into sub-clusters. Let X represent any of the possible assignments. Let y represent a specific assignment of cluster heads to the sub-regions, and let Y represent any of the possible cluster head assignments. Then, the event of having a global ownership topology v can be viewed as the joint event of x and y. Based on the conditional entropy property (page 16, (2.14) of [10]),

$$\begin{aligned} I_G^P &= H(X,Y) = H(X) + H(Y|X) = \\ &- \sum_{x\in X} p(x)\log p(x) - \sum_{x\in X} p(x) \sum_{y\in Y} p(y|x)\log p(y|x) \end{aligned} \tag{15}$$

It is easy to know that there is a total N^M possible assignments in X. Based on the assumption that mobile nodes are identical, distinguishable, and able to freely move within the region, we can conclude that X has a uniform probability distribution. The probability of having a particular assignment x is $\frac{1}{N^M}$, and hence $H(X) = N\log M$.

Let $\underline{n} = (n_1, n_2, \ldots, n_M)$ denote the number of nodes in the sub-regions for a given assignment x. There is a total $\prod_{i=1}^M g(n_i)$ ways of specifying cluster heads, where $g(\cdot)$ is defined in (2). Each way of specifying cluster heads has the same probability $\frac{1}{\prod_{i=1}^M g(n_i)}$. Therefore,

$$H(Y|X) = \frac{1}{N^M}\sum_{x\in X}\log\left(\prod_{i=1}^M g(n_i)\right) \tag{16}$$

Finally,

$$I_G^P = N\log M + \frac{1}{N^M}\sum_{x\in X}\log\left(\prod_{i=1}^M g(n_i)\right) \tag{17}$$

Using the same bounding method in (10), I_G^C can be bound as

$$N\log M \le I_G^P \le I_G^C \le N\log M + M\log\left\lceil\frac{N}{M}\right\rceil \tag{18}$$

Usually, $N\log M \gg M\log\lceil\frac{N}{M}\rceil$, therefore,

$$I_G^P \approx I_G^C \tag{19}$$

i.e. the MCL based on the topology probability distribution is almost equal to the MCL based on cardinality.

It might be also worth mentioning that the probability of having a global ownership topology v can be deduced easily based on the conditional probability property

$$p(v) = p(x,y) = p(x)p(x|y) = \frac{1}{N^M\prod_{i=1}^M g(n_i)} \tag{20}$$

3.1.4 MCL based on Prediction Using Previous Topology Knowledge

Given the global ownership topology at the previous update time instant, the information needed to update the new ownership of a node is

$$i_G = -1\left(q_0 \log q_0 + (1 - q_0)\log(\frac{1 - q_0}{K})\right) \tag{21}$$

Since the nodes mobility patterns are $i.i.d$, the total information required to describe the ownership of all the nodes is Ni_G. Similar to the discussions of (12), the introduction of cluster heads increases the MCL by at most $M \log\lceil\frac{N}{M}\rceil$.

Let I_G^M denote the MCL required to describe the topology based on prediction using previous topology knowledge, then

$$Ni_G \leq I_G^M \leq Ni_G + M\log\lceil\frac{N}{M}\rceil \tag{22}$$

i_G in (21) reaches its maximum value when $q_0 = 1/(K+1)$, i.e. for this case, the probability that a node stays at the current sub-region is equal to the probability that the node moves to any of the neighboring sub-regions. Therefore (for worst case mobility),

$$I_G^M \leq N\log(K+1) + M\log\lceil\frac{N}{M}\rceil \tag{23}$$

For large N, the second term of the right hand side of the previous equation can be ignored.

From (14) and (23),

$$\frac{I_G^M}{I_G^C} \leq \frac{\log(K+1)}{\log M}\ ; N >> 1 \tag{24}$$

with equality when $q_0 = \frac{1}{K+1}$.

From (14) and (23),

$$\frac{I_G^M}{I_G^C} \leq \frac{\log(K+1)}{\log M} \leq 1\ ; N >> 1 \tag{25}$$

with equality when $(K+1) = M$.

The above result can be explained as follows. In the case of prediction, the number of bits needed to specify the cluster of a node at the next time slot scales as $\log(K+1)$; while in non-prediction case, it scales as $\log M$ since the prediction case assumes the knowledge of current cluster and there are a maximum of $(K+1)$ possible clusters at the next time slot.

Equation 25 says that the MCL (I_G^M) based on prediction is smaller than the MCL (I_G^C) based on cardinality, even if the mobility parameter is chosen to maximize i_G (21) and I_G^M (i.e. when $q_0 = \frac{1}{K+1}$).

For many practical scenarios, even the largest number of neighboring sub-regions is usually less than the total number of sub-regions. In this case, for large N, the previous results state that using prediction to update the topology information will result in large savings in the MCL.

3.2 Local Ownership Topology

3.2.1 Cardinality

Theorem 3.2 The total number of possible local ownership topologies for a given sub-region is

$$L = (2^{N-1})N + 1 \tag{26}$$

Proof: For a non-empty cluster, there are N ways of specifying the cluster heads. For the remaining $N-1$ nodes, each has two possibilities (is or is not a member of the cluster). Therefore we have the first term of (26). We add 1 because there is a trivial case when there are no nodes in the sub-region.

■

3.2.2 MCL based on Cardinality

Let I_L^C denote the MCL based on the cardinality. Then

$$I_L^C = \log(2^{N-1}N + 1) \tag{27}$$

For large N,

$$\lim_{N\to\infty} \frac{\log(2^{N-1}N + 1)}{N} = 1 \tag{28}$$

and hence

$$I_L^C \approx N \quad ; N \gg 1 \tag{29}$$

From (14) and (8),

$$\frac{MI_L^C}{I_G^C} \approx \frac{M}{\log M} \quad ; N \gg 1 \tag{30}$$

where the factor "M" comes from the fact that there are M local ownership topologies.

The result of (30) says that the sum of the MCLs required to describe the local ownership topologies individually is larger than the MCL required to describe the global ownership topology based on the cardinalities. The reason is that I_L^C computes the MCL of local ownership topology information locally even though the local ownership topologies of different sub-regions are related. This is the overhead due to the fragmentation of a global ownership topology into multiple local ownership topologies; and the overhead (30) is increasing with M.

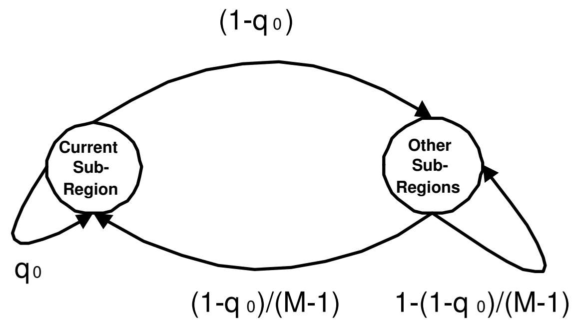

Figure 3: The state transfer diagram of a node moving into and away from a given sub-region.

3.2.3 MCL based on Topology Stationary Probability Distribution

Each node is equally likely to belong to any of the sub-regions with probability $\frac{1}{M}$. For each sub-region, the maximum number of bits required to specify a cluster head is $\log N$ using NUI. Let I_L^P denote the MCL based on the stationary probability distribution.

$$I_L^P \leq -N\left(\frac{1}{M}\log\frac{1}{M} + (1-\frac{1}{M})\log(1-\frac{1}{M})\right) + \log N \tag{31}$$

For large M,

$$I_L^P \leq \frac{N\log M + M\log N}{M} \; ; M \gg 1 \tag{32}$$

From (29) and (32),

$$\frac{I_L^P}{I_L^C} \leq \frac{N\log M + M\log N}{NM} \; ; M \gg 1 \; ; N \gg 1 \tag{33}$$

3.2.4 MCL based on Prediction Using Previous Topology Knowledge

Assuming that each node is equally likely to move to any of its neighboring sub-regions, the probability of a node moving into a given sub-region within τ_e is $\frac{1-q_0}{M-1}$. Figure 3 depicts the state transition diagram of a node ownership for a given sub-region. For a given sub-region, the ownership of a node is defined as the status of whether the node belongs to the sub-region or not. Given that the node is owned by the sub-region, the information amount to describe ownership change at the next time step is $H(q_0)$. Similarly, given that the node is not owned by the sub-region, the information amount to describe ownership change at the next time step is $H(\frac{1-q_0}{M-1})$.

The events that could change the local ownership topology of a given sub-region are: 1) the change of the ownership of some nodes due to nodes joining from other sub-regions or moving out of the sub-region; and/or 2) the possible need of re-selecting a cluster head due to the departure of the existing cluster head or the change of an empty sub-region to a non-empty sub-region.

For a sub-region with n nodes, the first part could be updated by

$$i_1(n) = nH(q_0) + (N-n)H(\frac{1-q_0}{M-1}) \tag{34}$$

bits.

The probability that a new cluster head needs to be selected at next time step is $(1-q_0)$ if $n > 0$, or 1 if $n = 0$. If there is a need of selecting a cluster head, the information amount to specify a new cluster head is $\log N$. Therefore the second part of information amount is $(1-q_0)\log N$ bits.

Finally, the information amount to describe the topology change is

$$i(n) = nH(q_0) + (N-n)H\left(\frac{1-q_0}{M-1}\right) + (1-q_0)\log N \tag{35}$$

When considering the mobility of nodes among different sub-regions, the number of nodes itself is a random number. Since the probability that a node belongs to a given sub-region is $\frac{1}{M}$, the probability distribution for $\mathcal{N}$ is binomial,

$$P_{\mathcal{N}}(n) = \frac{N!}{n!(N-n)!}\left(1-\frac{1}{M}\right)^{(N-n)}\left(\frac{1}{M}\right)^n ; 0 \leq n \leq N, \tag{36}$$

and the average number of nodes in a sub-region is thus

$$E[\mathcal{N}] = \frac{N}{M}. \tag{37}$$

The variance of the cluster-size is

$$VAR[\mathcal{N}] = N\frac{1}{M}\left(1-\frac{1}{M}\right) = \frac{N(M-1)}{M^2} \tag{38}$$

Let I_L^M denote the MCL based on previous local ownership topology knowledge. Taking the average over $P_{\mathcal{N}}(n)$,

$$I_L^M = (1-q_0)\log N + \sum_{n=0}^{N}\left[\frac{N!}{n!(N-n)!}\left(1-\frac{1}{M}\right)^{(N-n)}\left(\frac{1}{M}\right)^n\right] i_1(n) \tag{39}$$

Define

$$i_L^M \triangleq \frac{1}{M}H(q_0) + \frac{(M-1)}{M}H(\frac{1-q_0}{M-1}) \tag{40}$$

i_L^M can be viewed as the MCL for a node ownership status change. It is easy to show from (40), (39) and (36) that

$$\begin{aligned} I_L^M &= \tfrac{N}{M}H(q_0) + \tfrac{(M-1)N}{M}H(\tfrac{1-q_0}{M-1}) + (1-q_0)\log N \\ &= Ni_L^M + (1-q_0)\log N \end{aligned} \tag{41}$$

$I_L^M = 0$ if $q_0 = 1$ (i.e. if the node does not leave the current sub-region).

Theorem 3.3

$$i_L^M \leq H(\frac{1}{M}) \leq 1 \tag{42}$$

Proof: The entropy function $H(p)$ is a concave function [3]. Therefore, for $0 \leq \alpha \leq 1$, and $0 \leq x, y \leq 1$

$$H(\alpha x + (1-\alpha)y) \geq \alpha H(x) + (1-\alpha)H(y) \tag{43}$$

Select $\alpha = \frac{1}{M}$, $x = q_0$, and $y = \frac{1-q_0}{M-1}$, then,

$$\begin{aligned} &H(\alpha x + (1-\alpha)y) \\ &= H(\tfrac{q_0}{M} + \tfrac{M-1}{M}\tfrac{1-q_0}{M-1}) \\ &= H(\tfrac{1}{M}) \end{aligned} \tag{44}$$

and

$$\begin{aligned} &H(\tfrac{1}{M}) = H(\alpha x + (1-\alpha)y) \\ &\geq \alpha H(x) + (1-\alpha)H(y) \\ &= \tfrac{1}{M}H(q_0) + \tfrac{M-1}{M}H(\tfrac{1-q_0}{M-1}) \\ &= i_L^M \end{aligned} \tag{45}$$

■

3.3 Local Detailed Topology

The analysis of local detailed topology focuses on the interior topology change within a cluster, which is analyzed by the link changes among the nodes within a cluster. The analysis of the overhead of nodes moving away and into a cluster has already been considered in the analysis of local ownership topology, therefore, the analysis here considers that both the cluster node and regular nodes do not change.

3.3.1 Cardinality

The total number of possible local detailed topologies in a sub-region with n nodes with fixed cluster head is

$$D = 2^{\frac{n(n-1)}{2}} \tag{46}$$

This can be easily deduced from the adjacency matrix of the graph representation of local ownership topology. The matrix of any instance of this random graph is an $(n \times n)$ symmetric matrix with the diagonal element values equal to 1. There are only $\frac{n(n-1)}{2}$ independent elements. Each element reflects a link status between two nodes of the sub-region. Notice that if the cluster head is allowed to change, the number is (46) is multiplied by n.

3.3.2 MCL Based on Cardinality

Let I_D^C denote the MCL based on cardinality.

$$I_D^C = \frac{n(n-1)}{2} \tag{47}$$

The MCL can be viewed as a binary bit string with length $\frac{n(n-1)}{2}$. Each bit is either 1 or 0 to reflect whether the direct link exists or not. I_D^C scales as $\Theta(n^2)$.

3.3.3 MCL Based on Topology Stationary Probability Distribution

As addressed in Section 2.3, the link status between two nodes of a given sub-region is a random variable. Let p_1 denote the probability that the random variable equals 1, i.e. the two given nodes have a direct link. Then $H(p_1)$ is the MCL. There is a total of $n(n-1)/2$ possible links for the sub-region. Therefore, the MCL I_D^P based on the probability distribution is

$$\begin{aligned} I_D^P &= \tfrac{n(n-1)}{2}(-p_1 \log p_1 - (1-p_1)\log(1-p_1)) \\ &= \tfrac{n(n-1)}{2} H(p_1) \end{aligned} \tag{48}$$

I_D^P also scales as $\Theta(n^2)$. Compared with (47), I_D^P is less than I_D^C. The ratio of the two MCLs is $H(p_1)$. In the next section, we provide an expression for p_1 as a function of p_{00} and p_{11}.

3.3.4 MCL Based on Prediction Using Previous Topology Knowledge

As addressed in Section 2.3, the change of an individual link is modeled as a Markov process as depicted in Figure 4. When the Markov process reaches its steady state,

$$\begin{pmatrix} 1-p_1 \\ p_1 \end{pmatrix} = \begin{pmatrix} p_{00} & 1-p_{11} \\ 1-p_{00} & p_{11} \end{pmatrix} \begin{pmatrix} 1-p_1 \\ p_1 \end{pmatrix} \tag{49}$$

then,

$$p_1 = \frac{1-p_{00}}{2-p_{11}-p_{00}} \tag{50}$$

Given that two nodes are not directly connected, the information amount to describe the link status at the next time step is $H(p_{00})$. Similarly, given that two given nodes are directly connected, the information amount to describe the link status at the next time step is $H(p_{11})$.

Let s denote the total number of possible direct links of a sub-region,

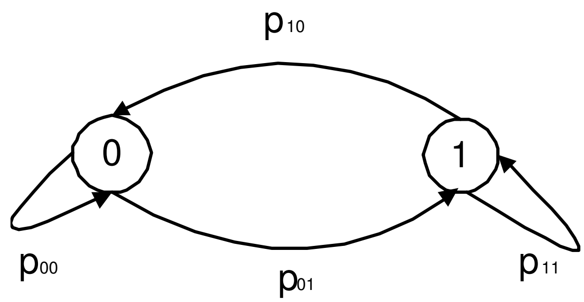

Figure 4: The state transfer diagram of a Markov process for the status of a given link.

$$s = \frac{n(n-1)}{2} \tag{51}$$

For a given topology with l $(0 \leq l \leq s)$ directly connected links, the information amount to describe the change of the topology is

$$i(l) = (s-l)H(p_{00}) + l * H(p_{11}) \tag{52}$$

In this analytic derivation, we assume that the random variables representing the link status are independent and identically distributed (i.i.d). The argument is as follows. Let us randomly select two possible links, say variable X for link status between node a and node b and variable Y for link status between node c and node d. If all four nodes are distinct, clearly X and Y are independent of each other. Other situation is that one of the nodes is common, say a and d are the same. From the assumption that nodes have an i.i.d mobility pattern, we can infer that the nodes also have i.i.d mobility patterns if observed from node a. Therefore, X and Y are still independent from each other. Let Z denote the link status between node b and node c. Unfortunately, Z is not independent from X and Y. For example, if both X and Y equal 1, i.e. there is direct link between a and b and between a and c, then there is a higher probability that there is direct link between b and c. This conditional probability enters into the equations with a factor of p_1^2 since X and Y are independent form each other. Since in typical situations, p_1 itself is a relatively small number, the effect of this conditional probability is small.

From the above discussion, the probability distribution of L, the number of links in a local detailed topology is

$$p_L(l) = \binom{s}{l} p_1^l (1-p_1)^{(s-l)} \; ; 0 \leq l \leq s \tag{53}$$

Taking the expectation of $i(l)$ using (53) and after some algebraic manipulations

$$I_D^M = \frac{n(n-1)}{2}((1-p_1)H(p_{00}) + p_1 H(p_{11})) = \frac{n(n-1)}{2} i_D^M \tag{54}$$

where

$$i_D^M = (1-p_1)H(p_{00}) + p_1 H(p_{11}) \tag{55}$$

.

I_D^M is the MCL based on the knowledge of the local detailed topology at the previous time instant. Similar to I_D^C and I_D^P, I_D^M also scales as $\Theta(n^2)$. i_D^M can be interpreted as the MCL for each link given the previous link status. Compared with (48), the ratio of the two MCLs I_D^M/I_D^P is $i_D^M/H(p_1)$.

Theorem 3.4 The MCL based on the probability distribution is larger or equal to the MCL based on prediction based on previous local detailed topology knowledge, i.e.,

$$I_D^M \leq I_D^P \leq \frac{n(n-1)}{2} \tag{56}$$

and

$$i_D^M \leq H(p_1) \leq 1 \tag{57}$$

Furthermore, the two MCLs are equal if and only if $(p_{00} + p_{11}) = 1$. (A numerical example is plotted in Figure 5.)

Proof: The entropy function $H(p)$ is a concave function [3]. Therefore, for $0 \leq \alpha \leq 1$, and $0 \leq x, y \leq 1$

$$H(\alpha x + (1-\alpha)y) \geq \alpha H(x) + (1-\alpha)H(y). \tag{58}$$

Select $\alpha = 1 - p_1$, $x = p_{00}$, and $y = (1 - p_{11})$, then

$$\alpha = 1 - p_1 = 1 - \frac{1-p_{00}}{2-p_{00}-p_{11}} = \frac{1-p_{11}}{2-p_{00}-p_{11}} \tag{59}$$

and

$$1 - \alpha = p_1 = \frac{1-p_{00}}{2-p_{00}-p_{11}}. \tag{60}$$

Finally,

$$\begin{aligned} &H(\alpha x + (1-\alpha)y) = H(\tfrac{(1-p_{11})p_{00} + (1-p_{00})(1-p_{11})}{2-p_{00}-p_{11}}) \\ &= H(\tfrac{1-p_{11}}{2-p_{00}-p_{11}}) \\ &= H(1-p_1) = H(p_1) \end{aligned} \tag{61}$$

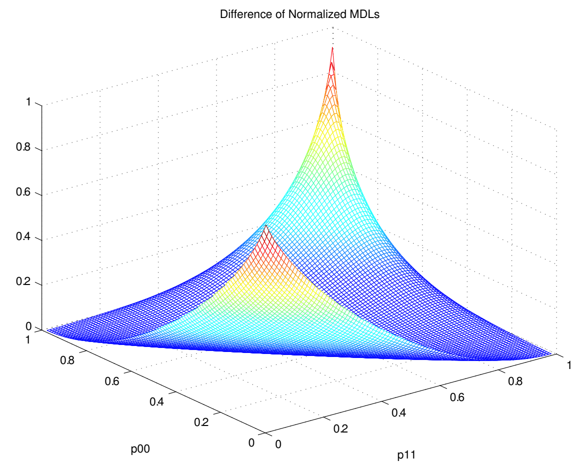

Figure 5: The difference between I_D^P and I_D^M without the factor s (i.e. $(I_D^P - I_D^M)/s$).

and

$$\begin{aligned} H(p_1) &= H(\alpha x + (1-\alpha)y) \\ &\geq (1-p_1)H(p_{00}) + p_1 H(1-p_{11}) \\ &= (1-p_1)H(p_{00}) + p_1 H(p_{11}) \\ &= i_D^M. \end{aligned} \tag{62}$$

where we use the relation result of $H(1-p_{11}) = H(p_{11})$. By multiplying the two sides of the equation by $n(n-1)/2$, we prove that $I_D^M \leq I_D^P$.

If $p_{00} + p_{11} = 1$, then $H(p_{00}) = H(p_{11})$. From (50), $p_1 = p_{11}$. Then,

$$i_D^M = p_{00}H(p_{00}) + p_{11}H(p_{11}) = H(p_1).$$

■

It is worth to point out that the I_D^M is computed using the conditional entropy of the state transition probability distribution. If we only use the link status change without using the conditional probability distribution, the result is different from I_D^M. We now consider this case. Define a random variable $X \in \{0, 1\}$, where 1(0) means the link status changed (not changed). The probability p_x of a link status not changed is

$$p_x = (1-p_1)p_{00} + p_1 p_{11} \tag{63}$$

Let I_D^X denote the MCL based on the probability distribution of X, then

$$I_D^X = \frac{n(n-1)}{2} H(p_x) \tag{64}$$

The difference between I_D^X and I_D^M is plotted in Figure 6.

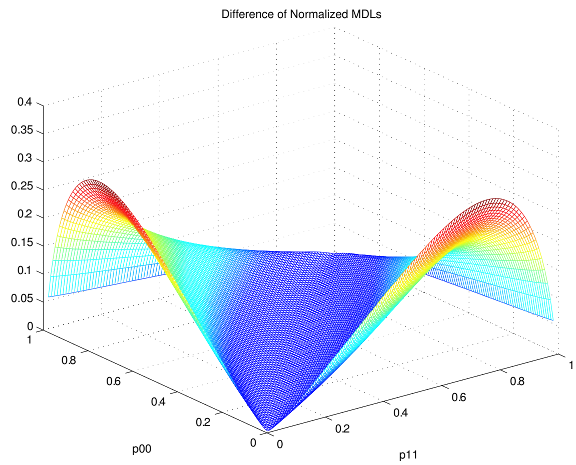

Figure 6: The difference between I_D^X and I_D^M without the factor s (i.e. $(I_D^X - I_D^M)/s$).

Theorem 3.5

$$I_D^X \geq I_D^M \tag{65}$$

Proof: Entropy function $H(p)$ is a concave function, therefore,

$$\begin{aligned} i_D^M &= (1-p_1)H(p_{00}) + p_1 H(p_{11}) \\ &\geq H((1-p_1)p_{00} + p_1 p_{11}) = H(p_s) \end{aligned} \tag{66}$$

By multiplying both sides of the equation by $n(n-1)/2$, we prove the theorem. ■

The result of this section shows that the most efficient way of updating the local detailed topology is only updating the topology change based on topology prediction using the state transition probability distribution.

4 Topology Evolution and Entropy Rate

Entropy rate [3] is defined as the rate of increase of the entropy of a sequence of random variables as the length of the sequence n increases. For $n \to \infty$, this becomes the entropy rate of a stochastic process. The entropy rate E of a stochastic process R_i is given by

$$E = \lim_{n\to\infty} \frac{1}{n} H(R_1, R_2, ..., R_n) \tag{67}$$

For a stationary Markov sequence $(R_1, R_2, ..., R_n, ...)$ with m states,

$$H = -\sum_{i=0}^{m} p_i \sum_{j=1}^{m} p_{j|i} \log p_{j|i} \tag{68}$$

where $(p_1, ..., p_m)$ is the steady state distribution, and $p_{j|i}$ is the state transition probability from state i to state j. This is called the "Markov Entropy."

In the following, we analyze two random processes. The first is the sequence of the local ownership topologies at the time instants that are multiples of τ_e. The second is the sequence of local detailed topologies at time instants that are multiples of τ_i. We demonstrate that both of the random processes are Markov. The entropy rates of the two random processes are the lower bounds on the rate of update of the MCLs of the topology changes. Finally, the MCLs of (41) and (54) are used to estimate the entropy rates for the two random processes. The entropy rates are functions of the parameters of the mobility model q_0, p_{00} and p_{11}.

4.1 Local Ownership Topology

The sequence of ownership status $\{Y_j\}_{j=1}^{\infty}$ forms a Markov process with a state transition diagram depicted in Figure 3. Let V_i denote a vectored random variable formed by the N random variables of ownership status of all the nodes for the sub-region. The sequence $\{V_j\}_{j=1}^{\infty}$ forms a random process. Clearly, $\{V_j\}_{j=1}^{\infty}$ is also a Markov process. Let the random variable Z_j denote the RUI of the cluster head of the sub-region, and let $L_j = (V_j, Z_j)$. L_j represents the local ownership topology at time step i. Because the probability distribution of Z_{j+1} (next cluster head) at next time step $(j+1)$ is uniquely determined by V_j and Z_j, the sequence of $\{L_j\}_{j=1}^{\infty}$ also forms a Markov process $\mathbb{L}$.

Let E_L denote the entropy rate of the random process $\mathbb{L}$. From (68), the entropy rate is the average conditional entropy given the stationary probability distribution of the random variable. Since I_L^M (given by 41) is the MCL based on the previous local ownership topology, then

$$E_L = I_L^M = \frac{N}{M}H(q_0) + \frac{(M-1)N}{M}H(\frac{1-q_0}{M-1}) + (1-q_0)\log N \tag{69}$$

4.2 Local Detailed Topology

Similarly, the evolution of the local detailed topology for a given sub-region is a Markov process. As discussed in Section 3.3, we can use a random graph to represent the local detailed topology. The adjacency matrix of the random graph has $\frac{n(n-1)}{2}$ independent elements. The value of each element is a random variable representing a link status of two nodes of the sub-region. The link status evolution is a Markov process depicted in Figure 4. The $\frac{n(n-1)}{2}$ random variables are independent of each other. Let D_j denote the joint $\frac{n(n-1)}{2}$-tuple random variables. D_j represents the local detail topology at the time step i. The sequence $\{D_j\}_{j=1}^{\infty}$ forms a Markov process $\mathbb{D}$. Let E_D denote the entropy rate of this random process, then, from (54),

$$E_D = I_D^M = \frac{n(n-1)}{2}((1-p_1)H(p_{00}) + p_1 H(p_{11})). \tag{70}$$

5 Memory and Routing Message Overheads

In this section, we derive lower bounds on the average memory requirement and communication routing overhead using the results from previous sections. For the memory overhead, we give both (a) lower bounds on the average memory requirements and (b) fixed memory requirements for regular nodes and cluster heads sufficient to store the routing information. The latter quantity is useful for practical reasons, e.g. when dimensioning the nodes with a fixed dedicated memory for routing functions. We note that the mobility parameters (i.e. q_0, p_{00}, and p_{11}) impact the average memory requirement, while the fixed memory requirements are functions of the network parameters N and M only. For the fixed memory case, we also present possible feasible encoding implementation schemes for routing information storage.

It will be shown that, compared with the average memory requirement, the fixed memory requirement is much higher. The advantage of using an encoding scheme (that achieves the average) rather than a fixed memory is that the encoding scheme has a lower average memory requirement, which means that the node will have more memory left on the average to dedicate to other non-routing related processes. Thus the average lower bounds are also useful practically.

For the message routing overhead, we give lower bounds on the average routing overhead using prediction-based MCL. Bounds using other methods can also be derived and a discussion is included in Section 7.3.

5.1 Memory Requirement

In this section we derive lower bounds on the memory requirement, in bits, for regular nodes and cluster heads, to support routing. Notice that each node has to maintain a list of the identifiers of the nodes in the network, which is a constant $N \log N$.

For each topology type, the entropy of the stationary probability distribution is the lower bound on the memory required to store the topology, and the entropy rate is the lower bound on the additional memory required to store the topology change. The methodology of finding the memory requirement is summarized in Figure 7.

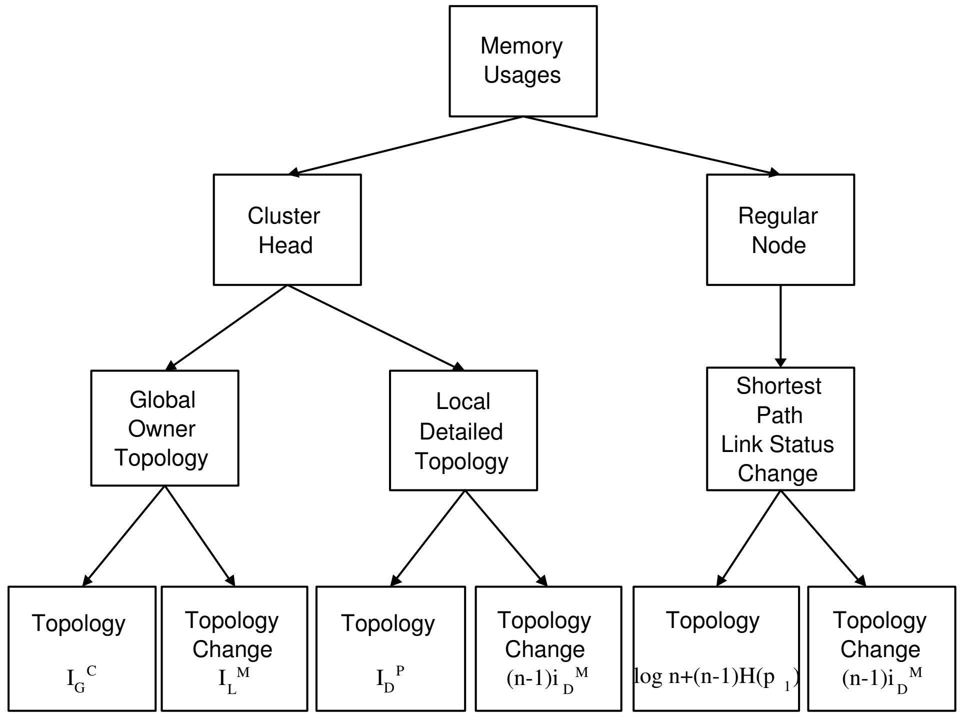

Figure 7: Average memory requirement for cluster head and regular node.

5.1.1 Regular Node

(A) Average Memory Requirement

As discussed in Section 2.2 (last paragraph), a regular node only needs to remember the next node along its shortest path to maintain a shortest path to its cluster. The regular node also needs to store the link status with other nodes of the same sub-region, and to store the link status change. The total memory requirement becomes

$$\mathbb{M}_r = \log n + (n-1)\left(H(p_1) + i_D^M\right) \tag{71}$$

The memory requirement is simply $\log n$ for a sub-region with n nodes. Here $\log n$ is the lower bound on the memory requirement (the quantity becomes $\log N$ if the *NUI* of the next node is used). The factor $(n-1)$ comes from the fact that there are $(n-1)$ possible links for a regular node. The $(n-1)H(p_1)$ and $(n-1)i_D^M$ are the lower bounds on memory requirement to store the link status and the link status change.

$$\mathbb{M}_r \leq \log N + 2(N-1) \tag{72}$$

A tighter bound on the average memory requirement may be computed numerically from (71) by averaging given n has a binomial distribution as given in (36).

(B) Fixed Memory Requirement

A simple encoding scheme exists that uses a fixed memory of $\log N + 2N$ can be implemented as follows. The next-hop node NUI requires at most $\log N$. The link state to all nodes in the same sub-region requires a string of binary bits of length no more than $N-1$.

5.1.2 Cluster Head

(A) Average Memory Requirement

A cluster head needs to store both the global ownership topology and the local detailed topology. Let $\mathbb{M}_{cg}$ denote the memory requirement to store the global ownership topology and the corresponding topology change, and let $\mathbb{M}_{cd}$ denote the memory requirement for storing the local detailed topology and the corresponding topology change. For global ownership topology, the memory requirement for is I_G^P, and the memory requirement for topology changes is I_L^M. For local detailed topology, the memory requirement is I_D^P, and the memory required for local topology changes is $(n-1)i_D^M$. Hence,

$$\mathbb{M}_{cg} = I_G^P + I_L^M = I_G^C + I_L^M \tag{73}$$

Here, we use the result $I_G^P = I_G^C$ from (48).

$$\mathbb{M}_{cd} = I_D^P + (n-1)i_D^M \tag{74}$$

Finally,

$$\mathbb{M}_c = \mathbb{M}_{cg} + \mathbb{M}_{cd} = I_G^C + I_L^M + I_D^P + (n-1)i_D^M \tag{75}$$

The expressions for I_G^C, I_L^M and I_D^P, i_D^M are given in (8), (39), and (48), (55).

The information regarding maintaining a routing path among neighboring cluster heads belongs to the local detailed topology, i.e. the local detailed topology helps to route a packet to another cluster head. Certain nodes belong to two neighboring clusters. Their identifiers are remembered by the corresponding cluster heads.

From (41), we have

$$I_L^M = N i_L^M + (1-q_0)\log N \leq N + \log N \tag{76}$$

Combined with the result of (14), we have

$$\mathbb{M}_{cg} = I_G^P + I_L^M \leq N(1+\log M) + \log N \tag{77}$$

For the local detailed topology, from (48), we know that

$$I_D^P = \frac{n(n-1)}{2} H(p_1) \leq \frac{n(n-1)}{2} \leq \frac{N(N-1)}{2} \tag{78}$$

Also,

$$(n-1) i_D^M \leq (n-1) \leq N \tag{79}$$

Finally, the total average memory requirement is bounded by

$$\mathbb{M}_c \leq N(1 + \log M) + \log N + \frac{N(N-1)}{2} + N \tag{80}$$

(B) Fixed Memory Requirement

Similar to the analysis for the case of a regular node, a simple encoding scheme exists that uses a fixed memory. This scheme, which is simple but not efficient in terms of average memory use, is provided here as a practical example. Assume an ordered list of NUI's. For the global topology, we need to store both the ownership of each node to the sub-region RUI, and identify the cluster head of each cluster. To identify an RUI requires $\log M$ bits. By knowing the ownership of nodes to a cluster, identifying the cluster head is the same as identifying whether a node is a head or not, which requires only one bit. Therefore, to identify all the cluster heads requires an N bit binary string. For the local detailed topology, there are at most $\frac{N(N-1)}{2}$ links, so a binary string of length $\frac{N(N-1)}{2}$ bits can be used to store a local topology. A string of N bits suffices to store the link status change of neighboring nodes. Thus we have the total fixed memory requirement as $N \log M + \log N + \frac{N(N+3)}{2}$.

5.2 Routing Message Overhead

In this section, we derive lower bounds on the routing overhead. The bound on the routing overhead is computed as the product of routing message length (i.e. the MCL) and the average number of hops the message travels (assuming shortest paths). In this section, the prediction-based encoding is used to estimate the message routing overhead. The reason is that the MCL messages usually requires traveling multiple hops, therefore, the most compressed message is used to compute the lower bound. It is worth pointing out that this analysis does not account for the routing overhead incurred to "hand-over" the information from a cluster head that moves out of a given sub-region to the node that takes over as a new cluster head.

5.2.1 Exterior Routing

Each cluster head distributes its local ownership topology changes to other cluster heads. The average message length is lower bounded by I_L^M (41). We assume that each cluster

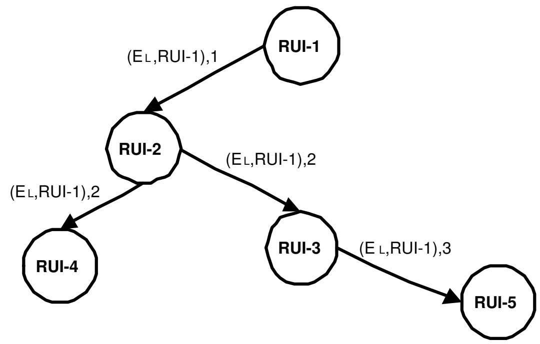

Figure 8: The span tree T(i) of RUI-1 based on the network topology given in Figure 1.

forms a spanning tree to distribute the update message in order to avoid redundant message transmissions. Then the number of hops the update message travels is $(M-1)l_c$. Therefore, the minimum average routing overhead (in bits) during a period of time of τ_e is

$$R_e = M(M-1)I_L^M l_c \tag{81}$$

where the factor M comes from the fact that there are M sub-regions.

We now address how the spanning tree is created and the incurred overhead. For a sub-region with RUI i, let $\mathbb{T}(i)$ denote the spanning tree with $(M-1)$ edges. Each edge connects two neighboring sub-regions (Figure 8). The nodes of $\mathbb{T}(i)$ represent the sub-regions identified by RUIs. The root of $\mathbb{T}(i)$ is the sub-region i. Each sub-region is assigned a level number that is the minimum number of links from the graph node (a sub-region) to the graph root (sub-region i) of $\mathbb{T}(i)$.

5.2.2 Interior Routing

As addressed in Section 2.2, the interior routing overhead consists of detecting the link status changes, maintaining the local detailed topology, and notifying/informing regular nodes of the new shortest paths.

Let R_h denote the number of bits required to detect the link status changes within a time unit τ_i. Then

$$R_h = n \log n \tag{82}$$

where $\log n$ is the number of bits required to identify itself to its neighboring nodes.

Let R_d denote the number of bits required to maintain the local detailed topology within a time period τ_i. During this time period, each regular node sends one update message of link status change to its cluster head. The cluster head infers the topology change from the update messages received from its regular nodes. The lower bound on the length of the update message is $(n-1)i_D^M$ for a regular node. There are $(n-1)$ regular nodes. Therefore,

$$R_d = (n-1)(n-1)l_r i_D^M \approx 2 l_r I_D^M \tag{83}$$

where l_r is the average path length from a regular node to its cluster head.

Let R_p denote the number of bits required to send the new shortest path information (i.e. information about the shortest paths between each node in the cluster and the cluster head) from the cluster head to the regular nodes at the end of a time slot τ_i. As a consequence of the observation made in Section 2.2 (last paragraph), an update message for the shortest path for a regular node also updates all the shortest paths of the intermediate nodes along the path. It is worth to point out that these shortest paths from the regular nodes to their cluster heads form a shortest path tree rooted at the cluster. The reason that it is a tree can be explained as following. First, each regular node has a path to its cluster. Second, the regular node only needs to remember one path to its cluster head. Thus no cyclic path can be formed using the shortest paths.

Let $\mathbb{P}$ denote the set of all shortest paths (between each regular node and the cluster head) used by the regular nodes of a sub-region, and $\mathbb{L}$ denote the set of all the links extracted from the shortest paths of $\mathbb{P}$. The number of links in $\mathbb{L}$ is the same as the number of regular nodes i.e. $(n-1)$. As the local detailed topology evolves, only the event(s) of link(s) in $\mathbb{L}$ becoming unconnected will cause one or more regular nodes to lose their shortest path(s). These events trigger the cluster head to send new shortest path information. We start from a simplest case with assumption that the shortest paths of $\mathbb{P}$ do not overlap. Therefore, each link belongs to one and only one shortest path. For a link $l \in \mathbb{L}$, let h denote the number of hops from the link to the cluster head following the shortest path identified by link l. Then $h = l_r$. If link l is broken, one or more regular node(s) could lose their shortest path(s). The position of the link could be at any place along the path. Therefore, the average number of nodes n losing their shortest paths due to link l breakage is

$$\sum_{i=1}^{h} \frac{h-i+1}{h} = \frac{h+1}{2} \approx \frac{l_r}{2} \tag{84}$$

The overhead for updating a new shortest path for a regular node is derived as follows. The number of hops a path has is l_r. Therefore the length of message to encode (i.e. uniquely specify) the path is $l_r \log n$. The message travels l_r hops to reach the destination regular node. The routing overhead is $l_r^2 \log n$ to update a new path. There are $(n-1)$ possible links in $\mathbb{L}$, and the probability of a link brokage is $(1-p_{11})$. Therefore,

$$\begin{aligned} R_p &= (1-p_{11})(n-1)\left(\tfrac{l_r}{2}\right) l_r^2 \log n \\ &\approx \tfrac{(1-p_{11})(n-1)l_r^3 \log n}{2} \end{aligned} \tag{85}$$

Now consider the general case where the shortest paths of $\mathbb{P}$ could overlap, the number of regular nodes that lose their shortest paths should be higher than the result given in (84) by a certain factor larger than 1. R_p still is the lower bound on the routing overhead. The interior routing overhead is the sum of the three above components, hence,

$$R_i = R_h + R_d + R_p \approx N \log n + 2I_D^M l_r + R_p \tag{86}$$

5.3 Total Routing Overhead

Let $\mathbb{R}_h$ denote the minimum bit rate required to detect the link status change, $\mathbb{R}_d$ denote the minimum bit rate required to track the changes of the local detailed topology for all the sub-regions, and $\mathbb{R}_p$ denote the minimum bit rate required to send the new shortest paths information to the regular nodes for all the sub-regions, then

$$\mathbb{R}_h = \frac{N \log n}{\tau_i} \tag{87}$$

$$\mathbb{R}_d = \frac{2MI_D^M l_r}{\tau_i} \tag{88}$$

$$\mathbb{R}_p = \frac{M * R_p}{\tau_i} \tag{89}$$

and

$$\mathbb{R}_i = \mathbb{R}_h + \mathbb{R}_d + \mathbb{R}_p = \frac{N \log n + 2MI_D^M l_r}{\tau_i} + \frac{MR_p}{\tau_i} \tag{90}$$

$$\mathbb{R}_e = \frac{M(M-1)I_L^M l_c}{\tau_e} \tag{91}$$

$$\mathbb{R}_t = \mathbb{R}_e + \mathbb{R}_i = \frac{M(M-1)I_L^M l_c}{\tau_e} + \frac{N \log n + M(I_D^M l_r + R_p)}{\tau_i} \tag{92}$$

The composition of different components of routing overhead is summarized in Fig. 9.

5.4 Routing Overhead for Individual Nodes

5.4.1 Cluster Head

For a cluster head, the routing overhead has two parts. The first part comes from the interior routing overhead. Within a time interval of τ_i, $\log n$ is the overhead of sending HELLO messages; the overhead for updating shortest paths is R_p/l_r as given in (85).

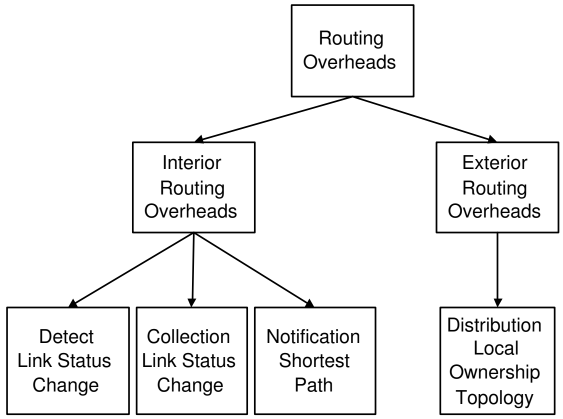

Figure 9: Routing overhead composition.

The second part comes from the exterior routing overhead. Within the time interval of τ_e, the number of bits sent out from a cluster head is I_L^M. Therefore, the routing overhead for a cluster head is

$$\mathbb{R}_c = \frac{\log n + \frac{R_p}{l_r}}{\tau_i} + \frac{I_L^M}{\tau_e} \tag{93}$$

5.4.2 Regular Node

Similarly, the routing overhead for a regular node also has two parts. The first part comes from interior routing overhead. Within a time interval of τ_i, the number of bits sent out by a regular node is $(\log n + (n-1)l_r i_D^M)$ for detecting and distributing the link status change. Also, a regular node needs to help distribute the shortest path information. This is $R_p(l_n - 1)/l_r(n-1)$ within a time interval of τ_i.

The second part comes from exterior routing overhead. A regular node needs to help distribute the local ownership topology information. For a given regular node, the average number of bits sent out is $\frac{R_e(l_c-1)}{(N-M)l_c}$ within a time interval of τ_e. The explanation for this is as follows. For a given time interval of τ_e, only a limited number of regular nodes participate in forwarding the exterior routing messages - in fact this number is $M(M-1)(l_c-1)$ (see (91),notice that $(l_c - 1)$ comes from the fact that a cluster head also forwards the routing message from one cluster head to another cluster once) - which is significantly smaller than the total number of regular nodes $(N - M)$. The total overhead due to external routing per τ_e is given by $M(M-1)(l_c-1)I_L^M$ (see (91)). The total number of regular nodes in the network is $N - M$, some of them may or may not participate in routing for a given time slot, so the average external overhead per node per unit time is $\frac{M(M-1)(l_c-1)I_L^M}{(N-M)\tau_e}$ which is (from

Table 3: Memory Overhead

	Topology	Topology Change	Ratio	Scale
$\mathbb{M}_r$	$\log n +$ $(n-1)H(p_1)$	$(n-1)i_D^M$	$\frac{i_D^M}{H(p_1)}$	$O\left(\frac{N}{M}\right)$
$\mathbb{M}_{cd}$	I_D^P	$(n-1)i_D^M$	$\frac{2i_D^M}{nH(p_1)}$	$O\left(\frac{N^2}{M^2}\right)$
$\mathbb{M}_{cg}$	I_G^C	I_L^M	$\frac{i_L^M}{\log M}$	$O(N \log M)$

(91)) also equal to $\frac{R_e(l_c-1)}{(N-M)l_c\tau_e}$.

Thus,

$$\mathbb{R}_r = \frac{\log n + (n-1)l_r i_D^M}{\tau_i} + \frac{R_p(l_n-1)}{l_r(n-1)\tau_i} + \frac{R_e(l_c-1)}{(N-M)l_c\tau_e} \tag{94}$$

6 Scalability Analysis

In this section, we summarize our results of routing memory and message overheads, and analyze the impacts of number of nodes N, number of sub-regions M, and the mobility parameters on the scaling of these overheads. In the following discussion, we use $\left(\frac{N}{M}\right)$ for the asymptotic value of n, the number of nodes in a sub-region.

6.1 Scalability Analysis of Memory Overhead

Table 3 gives a summary of the memory requirement results from the previous section. The forth column is the ratio between the third column over second column. The last column of the table is how $\mathbb{M}_r$, $\mathbb{M}_{cd}$ and $\mathbb{M}_{cg}$ scale with N and M.

For $\mathbb{M}_r$, the ratio is $\frac{i_D^M}{H(p_1)}$, which is always less than or equal to 1 based on (62). $\mathbb{M}_r$ scales with the number of nodes n in the sub-region. For $\mathbb{M}_{cd}$, the ratio is $\frac{1}{n}$. $\mathbb{M}_{cd}$ scales with n^2. The major component of $\mathbb{M}_{cd}$ is the memory required to store the current local detailed topology I_D^P. For $\mathbb{M}_{cg}$, the ratio is $I_L^M/I_G^C = \frac{i_L^M}{\log M}$. i_L^M is the memory overhead associated with node ownership change, and $\log M$ is the memory overhead associated with node ownership information. From (42), the ratio is smaller than $\frac{1}{\log M}$. $\mathbb{M}_{cg}$ scales with $N \log M$. The major component of $\mathbb{M}_{cg}$ is the memory required to store current global ownership topology I_G^C.

The ratio between the total memory requirement for regular node and cluster head is

$$\frac{\mathbb{M}_c}{\mathbb{M}_r} = \frac{I_G^C + I_D^P}{\log(n) + (n-1)(H(p_1) + i_D^M)} \approx \frac{N \log M + \frac{n(n-1)H(p_1)}{2}}{(n-1)(H(p_1) + i_D^M)} \approx \frac{2M^2 \log M + NH(p_1)}{2M(H(p_1) + i_D^M)} \tag{95}$$

The result shows that the role of a cluster head requires much larger memory.

6.2 Average Number of Hops Traversed by a Routing Message

In order to study the scalability of the routing overhead, we first derive expressions for l_r and l_c. We make the following approximation to estimate l_c from l_r,

$$l_c \approx 2l_r \tag{96}$$

The argument is as follows. To send a message from a cluster head to one of its neighbor cluster heads, a message has to travel an average of l_r to reach the boundary of the neighboring cluster and another l_r to reach from the boundary to the head of the neighboring cluster. Thus, what remains is to derive an expression for l_r.

Let d_r denote the physical distance between a regular node and its cluster head, A denote the physical area covered by the network, and d_0 denote the communication radius of a node. We have

$$d_r = \Theta\left(\sqrt{\frac{A}{M}}\right) \tag{97}$$

and

$$l_r = \Theta\left(\frac{d_r}{d_0}\right) = \Theta\left(\frac{\sqrt{A/M}}{d_0}\right) \tag{98}$$

In the following, we will consider three different cases for selecting d_0 and provide an expression for l_r for each case.

6.2.1 Model 1: Keep average node degree g constant

Define the average node degree g as the average number of nodes within the direct communication radius of a given node. Let g denote the average node degree, then,

$$g = \pi d_0^2 (\frac{N}{A}) \tag{99}$$

and

$$d_0 = \sqrt{\frac{gA}{\pi N}} \tag{100}$$

If g is kept constant,

$$l_r = \Theta\left(\frac{\sqrt{A/M}}{d_0}\right) = \Theta\left(\sqrt{\frac{N}{M}}\right) = \beta\sqrt{\frac{N}{M}} \tag{101}$$

6.2.2 Model 2: Keep network connected

From [11], g should be $\Theta(\log N)$ to keep the network asymptotically connected. Then,

$$g = \pi d_0^2(\frac{N}{A}) = \Theta(\log N) \tag{102}$$

and,

$$d_0 = \sqrt{\frac{gA}{\pi N}} = \Theta\left(\sqrt{\frac{\log N}{N}}\right) \tag{103}$$

Finally,

$$l_r = \Theta\left(\sqrt{\frac{N}{M\log N}}\right) \tag{104}$$

6.2.3 Model 3: Keep communication radius constant

If d_0 is kept constant,

$$l_r = \Theta\left(\frac{\sqrt{A/M}}{d_0}\right) = \Theta\left(\sqrt{\frac{1}{M}}\right) \tag{105}$$

6.3 Scalability Analysis of Routing Overhead

Table 4 summarizes the results of routing overhead analysis from Section 5.2 in the first column. The next three columns summarize how the corresponding components of routing message overhead scale according to the three different scaling modes presented. The details of derivations are skipped but they are generally straightforward. Notice that all the routing overhead components in Table 4 are linear with either l_r or l_c, except $\mathbb{R}_h$ since a HELLO message travels over one hop only.

From (98), (104) and (105), we know how l_c and l_r scale with N. From (41), we know how I_L^M scales with N. Finally, from (37), we know how n scales with N. Combining these results yields the results in Table 4.

Table 4: Scalability of Routing Overhead with N

	Expression	Model 1	Model 2	Model 3
$\mathbb{R}_e$	$\frac{M(M-1)I_L^M}{\tau_e} l_c$	$N^{\frac{3}{2}}$	$\frac{N^{\frac{3}{2}}}{\sqrt{\log N}}$	N
$\mathbb{R}_h$	$\frac{N \log n}{\tau_i}$	$N \log N$	$N \log N$	$N \log N$
$\mathbb{R}_d$	$\frac{Mn(n-1)i_D^M}{\tau_i} l_r$	$N^{\frac{5}{2}}$	$\frac{N^{\frac{5}{2}}}{\sqrt{\log N}}$	N^2
$\mathbb{R}_p$	$\frac{M(1-p_{11})(n-1)\log n}{2\tau_i} l_r^3$	$N^{\frac{5}{2}} \log N$	$\frac{N^{\frac{5}{2}}}{\sqrt{\log N}}$	$N \log N$

7 Practical Implications

In this paper, we utilized three information theoretic techniques to derive lower bounds on the routing overhead, in terms of number of bits per unit time and memory requirement in bits, for hierarchical proactive routing in an ad hoc network of mobile nodes. We have shown that the most efficient way of maintaining up to date topology information is to encode the changes based on prediction. In this section, we will apply the theoretical results of previous sections to answer some of the questions that a designer or engineer of an ad hoc network may be interested in. The goal of the section is to provide some general guidelines to the engineers in selecting encoding techniques and network parameters based on the design priorities.

7.1 Cluster Size and Memory Requirement

7.1.1 Cluster Size Minimizing the Memory Requirement of Cluster Heads

The memory for global ownership topology is $N \log M$ (14). The memory requirement for local detailed topology is $\frac{N^2}{2M^2} H(p_1)$ (48) or $\frac{N^2}{2M^2}$ (47), depending on the encoding technique.

In the following, we use $\frac{N^2}{2M^2}$ instead of $\frac{N^2}{2M^2} H(p_1)$ as the memory requirement to store the interior topology of a cluster (local detailed topology). There are two reasons for doing this. First, p_1 is a parameter related to the mobility, and usually it is not a design parameter. Second, in a practical scenario, it is more reasonable to use a method not requiring the knowledge of mobility parameter p_1 to encode/decode and store the local detailed topology. Now, the total memory requirement for cluster head is

$$\mathbb{M}_c = N \log M + \frac{N^2}{2M^2} \tag{106}$$

Taking the derivative with respect to M , the cluster size that asymptotically minimizes the memory requirement for each cluster head is

$$M_{optm} = \sqrt{(\ln 2)N} \tag{107}$$

7.1.2 The Number of Clusters Achieving Balanced Memory

Let M_{bal} denote the number of clusters at which the memory required to maintain the interior topology of a cluster becomes equal to the memory required to maintain the exterior topology of the cluster. When the number of clusters is smaller than M_{bal}, there is a need for more memory to maintain the interior topology of each cluster (on the average); otherwise, there is a need for more memory to maintain the exterior topology of a cluster.

Setting the right-hand side of the two equations (106) equal,

$$M_{bal}^2 \log M_{bal} = \frac{N}{2} \tag{108}$$

7.1.3 Cluster Size Minimizing the Ratio of Memory Requirements for Cluster Head and Regular Node

Widely different memory requirements for the roles of cluster head and regular node could be a practically unattractive feature. For example, since nodes switch roles, it will be necessary to equip all nodes with memory equal to the larger of the two quantities, and hence incur extra hardware cost. In the following, we derive the ratio between the memory requirements of a cluster head and a regular node. The lower bound on memory requirement for a regular node is given in (71) using the prediction-based encoding technique. Using the same argument as for calculating the memory requirement for cluster head (106), we use encoding technique based on cardinality to distribute the link status change. The memory requirement becomes $\mathbb{M}_r = \log n + (n-1) \approx \frac{N}{M}$. Here, we neglect the $\log n$ term, and approximate $(n-1)$ by n for simplicity. Finally,

$$r_m = \frac{\mathbb{M}_c}{\mathbb{M}_r} = M \log M + \frac{N}{2M} \tag{109}$$

The ratio is only a function of N and M, and it reaches its minimal value when $M = M_{ratio}$ where M_{ratio} satisfies

$$2M_{ratio}^2(1 + \ln M_{ratio}) = (\ln 2)N \tag{110}$$

Equations 107 and 110 tell us that in general the optimal cluster size (M_{optm}) for memory requirement and the optimal cluster size (M_{ratio}) for the memory ratio are not necessarily the same (in fact, they are usually not equal).

Table 5: Scalability of Routing Overheads with M (Model 1)

	Expression	Scale
$\mathbb{R}_e$	$\frac{2M(M-1)N}{\tau_e}\left(\beta\sqrt{\frac{N}{M}}\right)$	$M^{\frac{3}{2}}$
$\mathbb{R}_h$	$N\log N$	1
$\mathbb{R}_d$	$\frac{N^2}{\tau_i M}\left(\beta\sqrt{\frac{N}{M}}\right)$	$\frac{1}{M^{\frac{3}{2}}}$
$\mathbb{R}_p$	$\frac{(1-p_{11})N\log N}{2\tau_i}\left(\beta\sqrt{\frac{N}{M}}\right)^3$	$\frac{1}{M^{\frac{3}{2}}}$

7.2 Cluster Size Minimizing the Routing Message Overhead

An important design question is how the cluster size impacts the message routing overhead i.e. is there an optimal cluster size M_{optr} that asymptotically minimizes the routing message overhead?

In the following, we derive the optimal cluster size M_{optr} for the physical scaling Model 1 (Section 6.2.1) and use the encoding technique based on cardinality to distribute routing messages for maintaining both exterior topology and interior topology. The routing overhead for each component is summarized in Table 5.

Taking the derivative of the sum of the terms in Table 5 with respect to M and setting the derivative to zero, and after some algebraic manipulations,

$$M_{optr} = \sqrt[3]{\frac{\tau_e\left(2N + (1-p_{11})\beta^2 N\log N\right)}{4\tau_i}} \tag{111}$$

The optimal cluster sizes for the other two physical scaling models can also be deduced in a similar manner.

7.3 Impacts of Selection of Encoding Technique

We have presented three encoding techniques for the control messages, based on cardinality, probability distribution (of the topology), or (mobility) prediction. The impact of selection of the encoding technique is investigated in this section.

Impact on Exterior Routing Overhead

First, observe the following. The lower bound of exterior routing overhead is given in (81) in terms of I_L^M (41) assuming prediction based encoding. The result for the other two techniques can be computed by replacing I_L^M in (81) by I_L^C (from (27)) or I_L^P (from (31)).

Comparing these quantities, amounts to comparing N, $NH(\frac{1}{M})$ and Ni_L^M. Theorem 3.3 (42) shows that $i_L^M \leq H(\frac{1}{M}) \leq 1$.

Let us first compare between the prediction based technique, on one hand, versus the other two non-prediction based techniques, on the other hand. For the case that there is not much mobility ($q_0 \approx 1$), using the prediction based encoding technique will be a better choice. This also agrees with the intuition since with high q_0, there is not much benefit of trying to predict the next location of the node.

Now consider the two non-prediction based techniques. The ratio of the overheads between cardinality-based and the probability distribution-based techniques is determined by the value of $H(\frac{1}{M})$. Hence, it is straightforward to conclude that the probability distribution-based technique is especially beneficial in the case of large M, which is also intuitively reasonable. For example, if there are two clusters, there is not much difference in terms of message overhead between using the two encoding techniques. But when there are four clusters, using the probability distribution-based encoding technique results in reducing the overhead by 19%, according to the results, and by 46% when there are eight clusters. The percentage is computed using the value of $(1 - H(\frac{1}{M}))$.

Impact on Interior Routing Overhead

The interior routing overhead is given in Table 4, where only the $\mathbb{R}_d$ term is related to the encoding technique used. The impact of selection of the encoding technique on $\mathbb{R}_d$ can be quantified by comparing 1, $H(p_1)$ and i_D^M. From Theorem 3.4 (56), we have $i_D^M \leq H(p_1) \leq 1$.

The discussion here parallels the one for exterior routing overhead. For the case that there is not much mobility ($p_{00} \approx 1$ and $p_{11} \approx 1$) to change the link status (the change of local detailed topology) for nodes within τ_i, using the prediction based encoding technique will be a better choice. The overhead comparison between cardinality-based and distribution-based techniques is determined by $H(p_1)$.

7.4 Selection of the Update Intervals τ_e and τ_i

The bounds on the routing overheads depend on the routing protocol parameters τ_i and τ_e, which control how frequently do the local ownership and detailed topology information get updated, respectively. Larger values of τ_i and τ_e result in lower control message overhead. However, larger values also result in deviation between the actual state of the network and the recorded topology information at the nodes. This deviation may result in loss of packets, since packets may be forwarded along links that no longer exist.

Thus, in practice, a protocol designer would choose the maximum values of τ_i and τ_e that keep the deviation between the actual and the recorded topologies within a certain bound. Notice that the deviation is quantified by the MCLs. Thus,

for local ownership topology, we have:

$$I_L^M \leq c_e \tag{112}$$

and for local detailed topology, we have

$$I_D^M \leq c_i \tag{113}$$

Here, c_e and c_i are acceptable topology inaccuracies (or deviations) for local ownership topology and local detailed topology in the unit of bits. When we treat τ_e as variable, we have:

$$q_0 = q_0(\tau_e) \tag{114}$$

Using (41) and (112),

$$\frac{N}{M}H(q_0) + \frac{(M-1)N}{M}H(\frac{1-q_0}{M-1}) + (1-q_0)\log N \leq c_e \tag{115}$$

Notice that q_0 is a function [2] of τ_e, and hence (115) provides the maximum τ_e that results in the lowest message overhead while keeping the topology distortion bounded by c_e.

A similar analysis follows for τ_i. We can relate the parameters of p_{00} and p_{11} to τ_i for a given mobility model. Using (113), we can find the maximum τ_i that keeps the topology deviation bounded by c_i.

In general, we can also pose and solve another problem: find τ_e and τ_i that minimize the overall routing overheads (including the extra costs discussed and the routing overheads given in Table 4). However, the solution for this problem has to be conducted numerically.

8 Conclusion and Future Research

In this paper, we use information theory to build a generic methodology to analyze the lower bounds on memory requirement and routing overhead for proactive routing in mobile ad hoc networks. The procedure of applying this methodology is summarized in Figure 10.

[2]The relationship between τ_e and q_0 is determined by the underlying node mobility model. For example, for two dimensional random walk mobility model, the probability of the location of a node moving away from its original position can be estimated as a two-dimensional Gaussian distribution with standard deviation proportional to τ_e. Since q_0 is the probability that a node stays at the same sub-region within a time interval τ_e, we can derive q_0 as a function of τ_e by considering the geographic size of a sub-region.

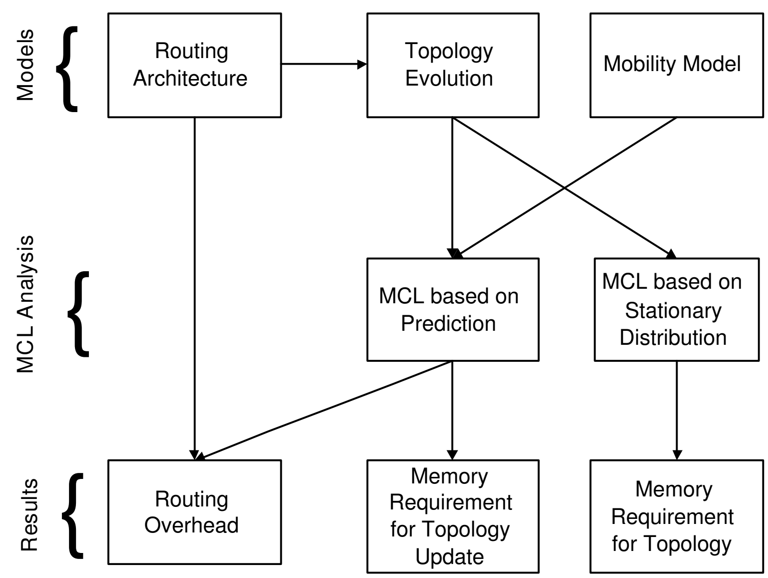

Figure 10: Methodology of finding lower bounds on memory requirement and routing overhead.

The methodology can be regarded as being composed of two main steps. First, the concept of MCL is used to derive bounds on the message length (in bits) required to inform nodes about topology changes, for each type of topology of interest. The analytic expressions for the MCL are derived using three methods, each of which has practical implications which are discussed in Section 7.3.

The second step is to derive lower bounds on routing overhead by analyzing the architecture of the routing protocol. The routing overhead in bits per unit time is derived from the MCL results of the first step. Both internal routing (within a cluster) and external routing (across clusters) are derived as "optimal codes traveling over shortest paths," where optimality is in the sense of expected length of the codewords that encode the source (the topologies) (see for example [3] Chapter 5 for a discussion of optimal codes). The memory requirement of the routing protocol is also considered, and bounds are derived on the minimum memory requirement.

Many avenues of future work can build on this work. First the same methodology could be applied to multiple-level proactive protocols. Second, different routing protocol architectures, in terms of topology information aggregation and routing message distribution schemes could be considered, as a variation of the distributed scheme addressed in this paper. Third, the results could be extended to capture the re-clustering overhead for networks with dynamic number of clusters. Forth, we can apply the methodology to analyze reactive routing protocols, where the routing overhead depends on the traffic pattern. Finally, the model can be applied to other models of networks that have variable topology over time, such as computation grids and peer-to-peer overlay networks.

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