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Auction-Based Spectrum Sharing in Cognitive Radio Networks with Heterogeneous Channels

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***Abstract*—Cognitive radio is a novel communication paradigm that can significantly improve spectrum utilization by allowing the cognitive radio users to dynamically utilize the licensed spectrum. To achieve this, studying efficient spectrum allocation mechanisms is imperative. In this paper, we consider a cognitive radio network consisting of a primary spectrum owner (PO), multiple primary users (PU) and multiple secondary users (SU). We design an auction-based spectrum sharing mechanism where the SUs bid to buy spectrum bands from the PO who acts as the auctioneer, selling idle spectrum bands to make a profit. Existing auction mechanisms assume that all the channels are identical. However, we consider a more general and more realistic case where channels have different qualities. Also, we allow SUs to express their preferences for each channel separately. That is, each SU submits a vector of bids, one for each channel. The proposed auction mechanism results in efficient allocation that maximizes SUs' valuations, and it has desired economic properties that we formally prove in the analysis. In addition, numerical results show performance improvements in terms of social welfare, SUs' utilities and PO's revenue, compared to the case of identical channels.**

I. Introduction

With the ever-increasing demand for wireless communications, spectrum scarcity and efficient use of wireless spectrum is becoming a major challenge. The Federal Communications Commission (FCC) has reported that the conventional fixed spectrum assignment is no longer capable of meeting today's wireless spectrum requirements. Also, according to the spectrum usage measurements by the FCC's Spectrum Policy Task Force, many of the allocated spectrum bands are idle most of the times or not used in some areas [1]. That calls for better spectrum management techniques and policies.

A promising approach to improve spectrum utilization is dynamic spectrum sharing which is realized by cognitive radio networks [2]. In dynamic spectrum sharing, unlicensed secondary users (SU) are allowed to utilize the radio spectrum owned by a primary owner (PO). For this purpose, designing a spectrum sharing mechanism that can efficiently allocate the spectrum bands to SUs, seems imperative. It is necessary for the mechanism to provide sufficient incentives for both PO and SUs to participate in spectrum sharing.

In a simple spectrum auction scenario, the POs act as auctioneers and sell their idle spectrum bands to SUs to make a profit, and the SUs act as bidders who want to buy spectrum bands. In such a setting, auction-based mechanisms appear to be the most appropriate approach because they can capture many of the key features of the spectrum sharing problem. First, in an auction, it is possible to consider situations where the seller is not assumed to know any prior information about the valuation of items to the buyers. This aspect can not be easily taken into account in pricing-based or other conventional market-based mechanisms. Second, auctions can be designed to allocate items to the buyers with highest valuations, thus making an efficient allocation. Third, auctions require minimum interactions between seller and buyers, because the buyers just need to submit their bids over the items. This makes the implementation of the mechanisms easier and more practical compared to the other market mechanisms.

In this paper, we consider a cognitive radio network consisting of a PO, multiple primary users (PUs) and multiple (SUs). The PO acts as the auctioneer, selling idle spectrum bands to SUs. Unlike existing auction mechanisms that assume identical channels, we consider a more general and more realistic case where channels have different qualities. Also, SUs are allowed to express their preferences for each channel separately. That is, each SU submits a vector of bids, one for each channel. This model provides much more flexibility for SUs and is more practical compared to the existing spectrum auctions. We design an auction mechanism to allocate the channels efficiently, maximizing SUs' valuations and also satisfying desired economic properties.

Technically, this problem can be modeled by a non-identical multiple items auction mechanism where each bidder has a different view of the available items. An auction is described by a pair of functions, namely the allocation function and the payment function. Also, it is desired for an auction mechanism to have some economic properties that we summarize here. *Individual Rationality*; a mechanism is called individually rational if the utility of each bidder is always nonnegative, otherwise, they may choose not to participate. *Incentive Compatibility*; in an incentive compatible mechanism, no bidder has incentive to lie about his valuations. *No Positive Transfers*; in an auction with no positive transfers, the payments are nonnegative – this means the mechanism should not pay bidders [3].

In order to determine the allocation rule of the auction, we transform the problem into the problem of finding a maximum weight matching in a bipartite graph. This matching provides us with the efficient allocation that maximizes SUs' valuations. For the payment function, we use the general case of the Vickrey Clarke Groves (VCG) mechanism [3] where a bidder pays the externality he causes. This auction mechanism runs in polynomial time and satisfies all the Incentive Compatibility, Individual Rationality, and No Positive Transfers conditions.

The main contributions of this paper are as follows. We consider a cognitive radio network with heterogeneous channel qualities which is more realistic compared to prior work. Second, we also give SUs the flexibility to express their preferences for channels separately. In this way, an SU can submit a different bid for each available channel. We then propose an auction-based spectrum sharing mechanism that results in efficient spectrum allocation that maximizes SUs' valuations. The proposed auction has desired economic properties (Incentive Compatibility, Individual Rationality, and No Positive Transfers) that we formally prove in the analysis. Also, we provide numerical results that show performance improvements in terms of social welfare, SUs' utilities and PO's revenue, compared to the case of identical channels.

The rest of this paper is organized as follows. In Section II we review and discuss related work. Section III presents the system model used in this paper. In Section IV, we present the auction-based algorithm and prove its properties. Numerical results are presented in Section V. Finally, Section VI concludes the paper and outlines possible future work.

II. Related Work

Game theory and auction design have been recently used for wireless spectrum allocation and management [4]–[15]. Here we summarize some of the most relevant results.

In [4], an auction-based spectrum management scheme for cognitive radio networks has been presented. The network consists of a primary base station and several primary and secondary users. The service provider determines the number of channels to be sold and holds the auction among the secondary users. Since the channels are assumed to be identical, the Vickrey auction determines the winners and payments. A similar network topology has been considered in [5], however channels are assumed to be different. The model is based on the contract theory in which the PO acts like a monopolist and determines the qualities and prices for spectrum bands with the objective of maximizing his own revenue. However, in this approach SUs cannot submit bids and the PO needs some prior information about SUs' valuations.

In [6], the idea of having multiple auctioneers, i.e. multiple POs, has been presented. In this setting, each PO gradually raises the trading price and each SU chooses one auctioneer for bidding. After several bidding/asking rounds, the mechanism converges to an equilibrium where no PO and SU would like to change his decision. Also, [7] considers two wireless service providers, and the authors study the optimal pricing for service providers and optimal service provider selection for SUs. They show that the equilibrium price and its uniqueness depend on the SUs' geographical density and spectrum propagation characteristics. In [8], the authors study the dynamics of spectrum sharing and pricing in a competitive environment where multiple POs try to sell spectrum bands to multiple SUs. They use evolutionary game theory to model the evolution and the dynamic behavior of SUs. The competition among POs has been modeled as a noncooperative game, and an iterative algorithm has been presented to find the Nash equilibrium.

In [9], Zhou et al. proposed TRUST, a general framework for truthful double spectrum auctions. This framework aims to provide spectrum reuse while achieving truthfulness and other desired economic properties. TRUST takes any reusability-driven spectrum allocation method as an input, and applies its own winner determination and pricing policy. There is an external auctioneer with complete information that holds the auction between POs and SUs.

The authors in [10] consider a setting in which SUs have flexibility to bid for a bundle of frequencies at different times. In fact, the spectrum opportunity is divided by frequency and time, so that SUs can bid for a combination of them. This flexibility, however, brings computational complexity. Since the general problem falls into the combinatorial auctions category, obtaining the efficient allocation is NP-hard, and only approximate solutions can be achieved. In [11], the authors study the effect of interference created among different agents who may obtain the right to use the same spectrum at nearby locations. This interference results in complementarities among the traded spectrum bands, which brings computational complexity to the design of efficient mechanisms. Since finding the efficient allocation is NP-hard, some constant factor approximations have been discussed.

Recently, a group of researchers considered two-tier market models for dynamic spectrum access. In tier-1, SUs buy the spectrum from the POs in a large time scale, and in tier-2, SUs trade the obtained spectrum among themselves in a small time scale. In [12], for example, the authors use Nash bargain games to derive the equilibrium prices for each tier. However, each tier is studied independently and the connection of tiers has not been explored yet.

Despite all the previous work, the problem of designing an auction-based spectrum sharing mechanism with heterogeneous channels and expressive bidding capability for SUs has not been addressed. Here, we tackle this problem.

III. System Model

In this paper, we consider a cognitive radio network consisting of a primary network and a secondary network. There is a primary spectrum owner (PO) (a base station or an access point) and a set of primary users (PU) in the primary network. The PO has some idle spectrum bands (or channels) that are not used by PUs. The secondary network consists of a set of secondary users (SUs), where each SU refers to a pair of secondary transmitter and receiver. The PO is willing to sell his idle channels to the SUs to obtain some profit, and SUs are willing to buy channels for their services. An example of cognitive radio network is depicted in Fig. 1.

We model the spectrum trading process as an auction in which the PO is the auctioneer and the SUs are the bidders. In our model, we consider heterogeneous channels, that is, channels have different qualities. The quality of channel j is defined as the maximum allowable transmission power on it, and is denoted by q_j.

In our setting, each SU has a different view of the available channels. We allow SUs to express their preferences over each channel separately. Thus, each SU submits a vector of bids; one for each channel. Let m denote the number of

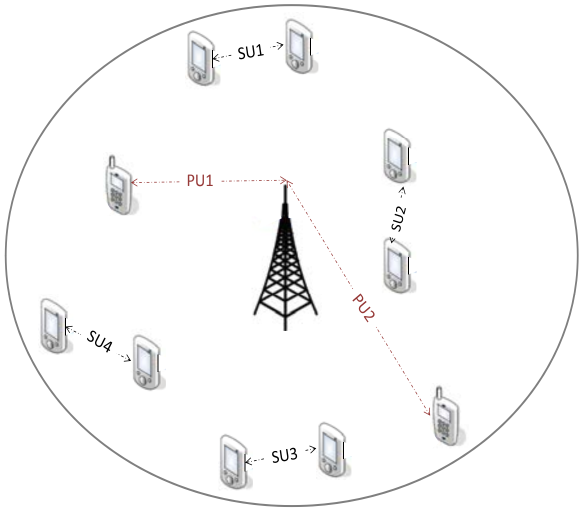

Fig. 1. A cognitive radio network with one primary owner, two primary users, and four secondary users.

available channels and n denote the number of SUs. Then, $V_i = (v_{i1}, v_{i2}, \dots, v_{im})$ is the vector of bids submitted by SU i, consisting of m values for the available channels. The valuation matrix submitted to the PO will be of the following form:

$$W = \begin{pmatrix} V_1 \\ V_2 \\ \dots \\ V_n \end{pmatrix}$$

For an SU, the valuation for a channel is defined as the benefit of obtaining that channel. We assume that SUs prefer channels with higher capacities. Therefore, SUs' valuation for a channel is related to the channel capacity, which is a function of channel quality, the interference coming from PUs, and the path loss factor between the secondary transmitter and receiver. We define SU i's valuation for channel j as:

$$v_{ij} = B \log_2 \big(1 + q_j \frac{G_i}{I_i + \sigma^2}\big) \tag{1}$$

where B is the channel bandwidth, σ^2 is the noise variance, G_i is the channel gain between the SU i's transmitter and receiver, I_i is the interference coming from the PO and PUs. Without loss of generality, we assume that σ^2 is the same for all SUs.

It should be noted that valuations are private information of SUs, and it is not reasonable to assume that this information is known by other SUs or the PO. In fact, this is one reason that we use auction mechanisms. In this way, SUs declare their valuations, and by designing an Incentive Compatible (IC) auction we make sure that SUs do not have incentives to lie about their valuations.

We assume that each channel can only be used by one SU at a time. Also, each SU can only use one channel at a time. Let p_i denote the payment that SU i has to make if he gets a channel. Then, the utility of SU i, denoted by u_i, is defined as the difference between his valuation for the obtained channel, say channel j, and the price he has to pay, i.e. $u_i = v_{ij} - p_i$. Also, $u_i = 0$ if SU i does not get any channel. Another essential assumption in designing a truthful (or IC) auction is the rationality of bidders. That means that they want to maximize their own utilities. Therefore, an SU tries to obtain a channel with a price lower than his valuation for the channel.

IV. The Auction Based Mechanism

In this section, we derive an auction based mechanism for spectrum sharing with certain guaranteed properties that we prove in this section. From the bidding perspective, the proposed auction is a one-shot auction where SUs submit their bids to the PO simultaneously, and the PO holds the auction, taking into account the bids collectively. SUs compute their valuations according to (1) after the PO announces the qualities of the available channels.

The auction mechanism takes the valuation matrix, W, as input and determines two outputs; the channel allocation and the payments. The channel allocation is represented by an $n \times m$ matrix, denoted by X. Each element of the allocation matrix $x_{ij} \in \{0, 1\}$ indicates whether the channel j is allocated to SU i or not. That is, $x_{ij} = 1$ shows that the SU i has obtained the right to access channel j and $x_{ij} = 0$ otherwise. As mentioned in the system model, we assume that each channel can only be used by one SU at a time, and each SU can only use one channel at a time. Therefore, we impose the following constraints for a feasible allocation: 1) $\sum_j x_{ij} \le 1$, and 2) $\sum_i x_{ij} \le 1$. The auction should also determine the payments for each SU. We represent the payments by a payment vector $P = (p_1, p_2, \dots, p_n)$ where p_i denotes the price that SU i has to pay.

Auctions can be designed with different objectives. One common goal is to optimize the *social welfare*. The social welfare of an allocation $X = \{x_{ij}\}_{n \times m}$ is the sum of the valuations of all the SUs for this allocation. Formally, it can be written as:

$$S = \sum_i \sum_j x_{ij} \cdot v_{ij} \tag{2}$$

The allocation that maximizes the social welfare is referred to as an efficient allocation. Formally, the efficient channel allocation problem can be written as:

$$X^* = \quad \arg\max_X S = \arg\max_X \sum_i \sum_j x_{ij} \cdot v_{ij} \tag{3}$$

$s.t.$

$$\sum_j x_{ij} \le 1, \forall i$$
$$\sum_i x_{ij} \le 1, \forall j$$
$$x_{ij} \in \{0, 1\}, \forall i, \forall j$$

where the constraints in the above formulation are the feasibility constraints that we discussed earlier. In the next subsection, we present a method to achieve an efficient allocation.

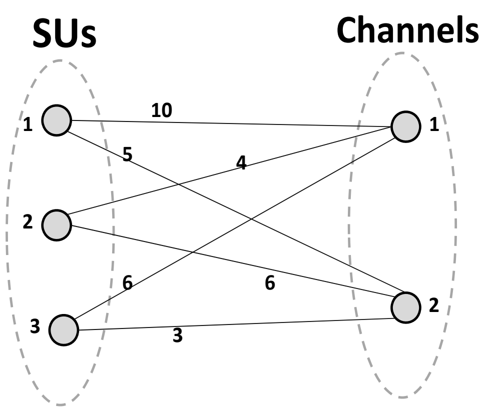

Fig. 2. A weighted complete bipartite graph with two channels and three SUs.

A. Efficient Channel Allocation

Now we transform the problem of efficient channel allocation, i.e. (3), into a maximum weight matching problem in graph theory [16]. Then, the problem can be solved using Kuhn-Munkres algorithm (also known as Hungarian algorithm) [17]. We first review some basic concepts from graph theory and the matching problem.

A *bipartite graph* is a graph whose vertices can be divided into two disjoint sets V_1 and V_2, such that every edge in the graph connects a vertex in V_1 to one in V_2. A *complete bipartite graph* is a bipartite graph such that for any two vertices $i \in V_1$ and $j \in V_2$, ij is an edge in the graph. A *weighted graph* is a graph whose edges are associated with weights, usually a real number. The weight of the edge connecting vertices i and j is denoted by w_{ij}. In a bipartite graph, a *matching* is a subset of edges such that they do not share an endpoint. In other words, a matching is a subset of edges such that for each vertex, there is at most one edge in the matching that is incident upon this vertex.

Now, given a weighted complete bipartite graph, the problem of maximum weight matching is to find a matching with maximum weight. This is a well-studied problem in graph theory and it can be solved by the Kuhn-Munkres algorithm (or Hungarian algorithm) in polynomial time [17]. We do not present the details of the Kuhn-Munkres algorithm in this paper. Instead, we transform the original channel allocation problem, i.e. (3), into a maximum weight matching problem, and we show that these two problems are equivalent.

We can easily build a complete bipartite graph $G(V_1, V_2)$ by letting V_1 be the set of SUs and V_2 be the set of available channels. The edges in this graph represent bids of SUs for the channels. Since each SU submits a bid for each available channel, the graph is a complete bipartite graph. The weight of the edge ij is defined as the valuation of the SU i for the channel j, i.e. v_{ij}. A sample graph is depicted in Fig. 2 with two channels and three SUs.

We claim that X is an efficient channel allocation matrix if and only if M is a maximum weight matching in the constructed graph G. First, suppose there is an efficient channel allocation matrix X. Then each nonzero element of X corresponds to an edge in the maximum weight matching M. For example, $x_{ij} = 1$ means that channel j is allocated to SU i, so the edge ij will be in the matching. It should be noted that this set of edges form a matching, because each channel can only be allocated to one SU and each SU can only use one channel at a time (feasibility constraints for the allocation). Also, this is a maximum weight matching since we have an efficient allocation that maximizes summation of SUs' valuations that correspond to edge weights in the graph.

Conversely, suppose that we have a maximum weight matching M in graph G, then the channel allocation matrix $X = \{x_{ij}\}_{n\times m}$ can be formed easily. For each edge ij in M, set its corresponding element in X to 1, i.e. $x_{ij} = 1$, and set all the other elements to zero. This results in an efficient channel allocation matrix. First, according to the definition of a matching, the resulting matrix satisfies the feasibility constraints. Second, since edge weights in the graph represent SUs' valuations and M is a maximum weight matching, the resulting allocation matrix is efficient.

In addition to the allocation rule, the proposed auction should specify the payment rule, i.e. the price each SU has to pay. In the next subsection, we provide details on the payment rule of the proposed auction.

B. The Payment Rule

The goal is to find a payment rule for the efficient allocation that satisfies some desired economic properties (Incentive Compatibility, Individual Rationality and No Positive Transfers). We present the payment rule in this subsection and we discuss the economic properties in the next subsection.

We use the well-known Vickrey Clarke Groves (VCG) mechanism with Clarke pivot payments [3]. Based on this payment rule, SU i pays the externality he causes. In other words, SU i pays the difference between the social welfare of the others with and without his participation. Let $X = \{x_{ij}\}_{n\times m}$ and $Y = \{y_{ij}\}_{n\times m}$ be efficient channel allocation matrices with and without SU i's participation, respectively. (In order to exclude SU i, we set the ith row of Y to zero.) Then, the payment for SU i is calculated by the following formula:

$$p_i = \sum_{j\neq i}\sum_{k} y_{jk}\cdot v_{jk} - \sum_{j\neq i}\sum_{k} x_{jk}\cdot v_{jk} \tag{4}$$

As an example, consider the graph in Fig. 2 with two channels and three SUs. SUs' valuations are $V_1 = (10, 5)$, $V_2 = (4, 6)$ and $V_3 = (6, 3)$. The efficient allocation matrix X obtained by the mechanism is:

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

That is, SU 1 gets channel 1 and SU 2 gets channel 2. To calculate p_1, we need to find the efficient allocation without SU 1's participation, denoted by matrix Y:

$$Y = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Now, using equation 4, $p_1 = 12 - 6 = 6$. Similarly, we can find Y for SU 2 and calculate $p_2 = 13 - 10 = 3$.

It is worth noting that in equation 4, valuations of SU i are excluded in the summations and SU i does not have any control over his payment. This makes the mechanism robust against SUs' strategic behaviors. In the next subsection, we discuss the economic properties of the proposed auction.

C. Desired Economic Properties

It is desired for an auction to have certain economic properties. First, we formally define these properties, then we show that the proposed auction satisfies the desired economic properties.

- **Incentive Compatibility**; Let V_i be user i's true valuation vector and V_{-i} be the valuation vectors of all other users (excluding i). Let the utility of i be $u_i = \sum_j x_{ij} \cdot v_{ij} - p_i$ when V_i and V_{-i} are declared, and be $u_i' = \sum_j x_{ij}' \cdot v_{ij} - p_i'$ when V_i' and V_{-i} are declared. An auction is called incentive compatible if for every user i, every V_i and every V_i' we have $u_i \geq u_i'$. This is sometimes referred to as truthfulness, and states that the dominant strategy for users is to declare their true valuations regardless of what other users do.
- **Individual Rationality**; An auction is individually rational if for every user i, we have $u_i \geq 0$. That means, users do not suffer as a result of participating in the auction and the winners do not pay more than their valuations.
- **No Positive Transfers**; In an auction with no positive transfers we have $p_i \geq 0$, for every user i. This property prevents the auctioneer from having to pay agents.

Theorem 1: The proposed auction mechanism is incentive compatible, individually rational and has no positive transfers.

Proof: We first prove incentive compatibility. Using the payment rule, i.e. equation 4, utility of user i, when declaring V_i and V_{-i}, is $u_i = \sum_j x_{ij} \cdot v_{ij} + \sum_{j \neq i} \sum_k x_{jk} \cdot v_{jk} - \sum_{j \neq i} \sum_k y_{jk} \cdot v_{jk}$, but when declaring V_i' and V_{-i}, is $u_i' = \sum_j x_{ij}' \cdot v_{ij} + \sum_{j \neq i} \sum_k x_{jk}' \cdot v_{jk} - \sum_{j \neq i} \sum_k y_{jk} \cdot v_{jk}$. Since X maximizes social welfare among all the possible allocations, we have this inequality: $\sum_j x_{ij} \cdot v_{ij} + \sum_{j \neq i} \sum_k x_{jk} \cdot v_{jk} \geq \sum_j x_{ij}' \cdot v_{ij} + \sum_{j \neq i} \sum_k x_{jk}' \cdot v_{jk}$. Now, by subtracting the term $\sum_{j \neq i} \sum_k y_{jk} \cdot v_{jk}$ from both sides of the inequality, we get $u_i \geq u_i'$. Which is the incentive compatibility property.

Let $X = \{x_{ij}\}_{n \times m}$ and $Y = \{y_{ij}\}_{n \times m}$ be social welfare maximizing allocations with and without SU i's participation, respectively. To show individual rationality, consider the utility of user i:

$$
\begin{aligned}
u_i &= \sum_j x_{ij} \cdot v_{ij} + \sum_{j \neq i} \sum_k x_{jk} \cdot v_{jk} - \sum_{j \neq i} \sum_k y_{jk} \cdot v_{jk} \\
&\geq \sum_j \sum_k x_{jk} \cdot v_{jk} - \sum_j \sum_k y_{jk} \cdot v_{jk} \\
&\geq 0
\end{aligned}
$$

The first inequality holds since $\sum_j y_{ij} \cdot v_{ij} \geq 0$. The second inequality holds because $X = \{x_{ij}\}_{n \times m}$ is the allocation that maximizes the social welfare, $\sum_j \sum_k x_{jk} \cdot v_{jk}$.

Showing no positive transfers is quite easy. Using the payment rule, equation 4, we have $p_i = \sum_{j \neq i} \sum_k y_{jk} \cdot v_{jk} - \sum_{j \neq i} \sum_k x_{jk} \cdot v_{jk} \geq 0$, since $Y = \{y_{ij}\}_{n \times m}$ maximizes the social welfare without i's participation, $\sum_{j \neq i} \sum_k y_{jk} \cdot v_{jk}$.

■

V. Numerical Results

In this section, we evaluate the performance of the proposed auction mechanism in different network scenarios. The number of SUs and number of available channels are set to be {5,10,15,20,25,30,35,40} and {3,6,9,12,15,18}, respectively. Noise variance is chosen to be $\sigma^2 = 10^{-5}$ and channel bandwidths equal $B = 1$. Also, channel qualities (i.e. maximum allowable transmission powers) are randomly drawn from Uniform distribution U[0.01,1]. We run each setting 1000 times in MATLAB. At first, SUs compute their valuations according to the equation 1. Then, a bipartite graph is formed and the Hungarian algorithm is used to determine channel allocations. knowing the allocations, we determine payments using equation 4.

We consider two settings; first, number of SUs changes while number of channels is fixed at 3, second, we fix number of SUs at 25 and change the number of channels. The performance of the proposed auction is compared with the case of identical channels where all the channel qualities are set to a mean value. Social welfare, average payment of SUs, average utility of SUs, and revenue of the PO are considered as performance metrics, where revenue of the PO is defined as the sum of SU payments $\sum_i p_i$.

As can be seen in Fig. 3, social welfare increases with number of SUs. With more SUs participating in the auction, we have wider range of valuations, and since the auction favors SUs with high valuations, the winners have higher valuations that leads to higher social welfare.

The average payment of SUs is depicted in Fig. 4. We observe that as the number of SUs increases and channel access becomes more competitive, payments increase. This is because with more competition, winning SUs cause more externality, and consequently they have to pay more. This competition also benefits the PO, since its revenue increases, as shown in Fig. 5. However, this competitive environment is not favorable for SUs. Fig. 6 shows that the average utility of SUs decreases

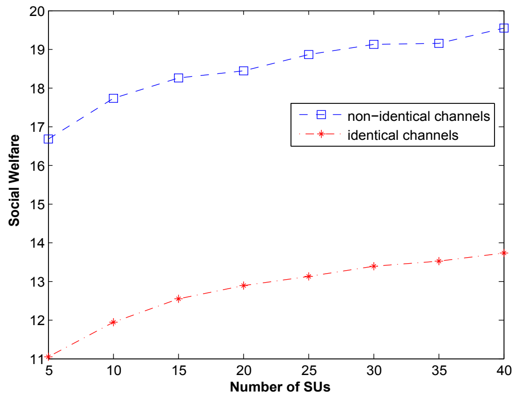

Fig. 3. Social Welfare (equation 2) versus the number of secondary users, with fixed number of channels m=3.

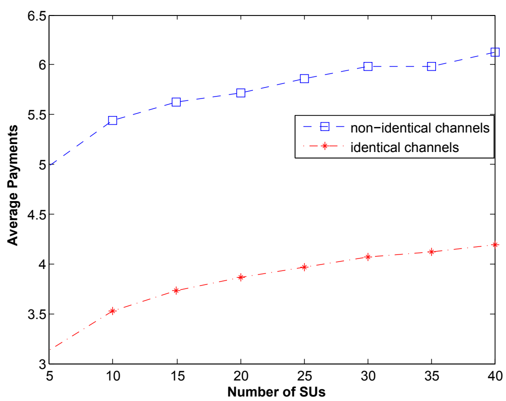

Fig. 4. Average payments versus the number of secondary users, with fixed number of channels m=3.

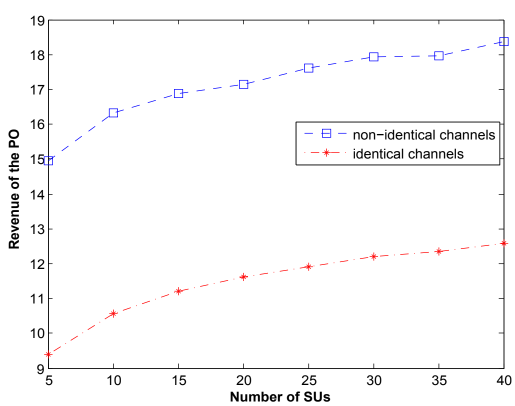

Fig. 5. Revenue of the PO versus the number of secondary users, with fixed number of channels m=3.

with the number of SUs. That happens because with more competition, SUs have to pay more, resulting in lower utilities.

Now we consider the case of fixed number of SUs, and variable number of channels. As shown in Fig. 7, social welfare increases with the number of channels. This is clearly because with more channels available, we are adding more positive terms to the social welfare (see equation 2). Fig. 8 depicts

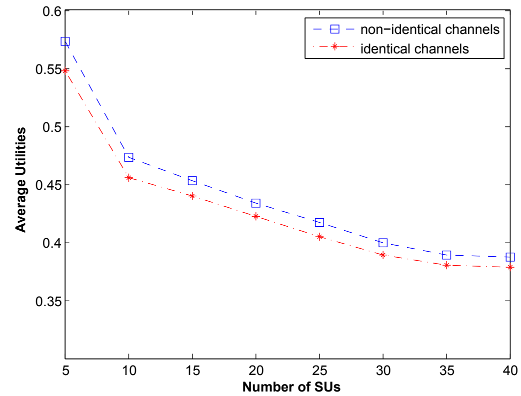

Fig. 6. Average utilities versus the number of secondary users, with fixed number of channels m=3.

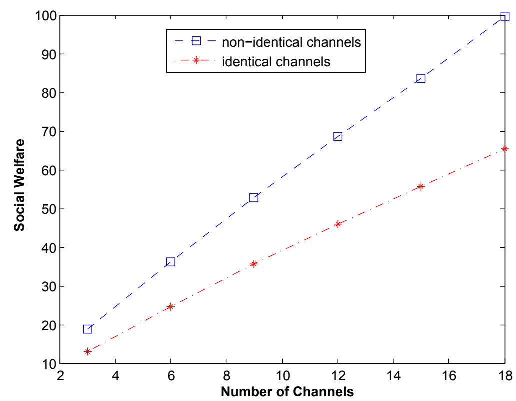

Fig. 7. Social Welfare (equation 2) versus the number of channels, with fixed number of SUs n=25.

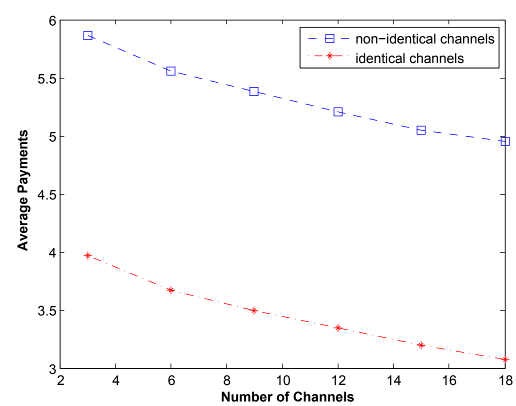

Fig. 8. Average payments versus the number of channels, with fixed number of SUs n=25.

the average payment of SUs when the number of channels increases. As can be seen, payments slightly decrease with the number of channels. With more channels available, there is less competition among SUs. Therefore, winning SUs cause less externality and pay less.

Although average payment of SUs slightly decreases, revenue of the PO increases with number of channels, as shown in

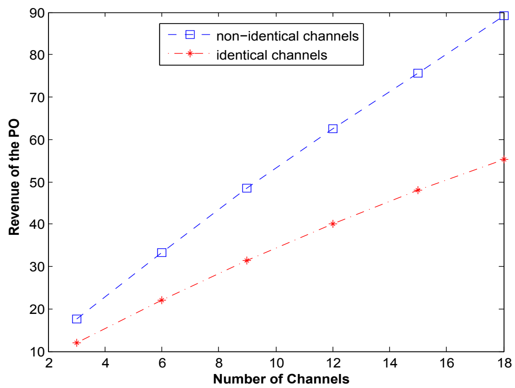

Fig. 9. Revenue of the PO versus the number of channels, with fixed number of SUs n=25.

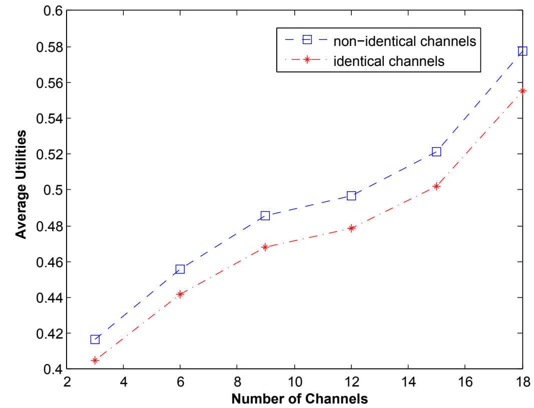

Fig. 10. Average utilities versus the number of channels, with fixed number of SUs n=25.

Fig. 9. That is because the PO sells more items (channels) and gets higher revenue, even though each item's price is slightly lowered. It can be seen from Fig. 10 that average utility of SUs increases with number of channels. Since winning SUs pay less when the number of channels increases, we witness an increase in utilities.

From the above numerical comparisons, we observe that by designing an auction for non-identical channels (i.e. channels with different qualities) performance of the mechanism can be considerably improved compared to the case of identical channels case. With non-identical channels, SUs can better express their needs and we get a wide range of valuations. Since the auction favors SUs with higher valuations, winners in the non-identical channels case have higher valuations compared to the winners in the identical channels case. Therefore, by a similar argument as mentioned earlier, non-identical channels auction results in higher social welfare, higher payments and consequently higher revenue for the PO. Also, the utility of SUs is improved by considering non-identical channels.

VI. Conclusion

In this paper, we studied the problem of spectrum sharing in cognitive radio networks. An auction-based mechanism has been proposed where the SUs bid to buy spectrum bands from the PO who acts as the auctioneer. Unlike existing auction mechanisms that assume identical channels, we have considered a more general case where channels have different qualities. Also, in our setting, SUs are allowed to express their preferences for each channel separately. The proposed auction results in efficient allocation that maximizes SUs' valuations, and it has desired proven economic properties. Simulation results have shown performance improvements in terms of social welfare, SUs' utilities and PO's revenue, compared to the case of identical channels. Here we have assumed that SUs' valuations are related to channel capacities. A possible direction for future work is to consider other forms of valuation functions, and study its effect on utility of SUs and other performance metrics. Other valuation functions might depend on SU's service type or queue length.

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