

# On the Cost of Knowledge of Mobility in Dynamic Networks

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**Abstract**—In this paper, an information-theoretic framework is developed for characterizing the minimum cost, in bits per second, of tracking the motion state information, such as locations and velocities, of nodes in dynamic networks. The minimum-cost motion-tracking problem is formulated as a rate-distortion problem, where the minimum cost is the minimum rate of information required to identify the network state at a sequence of tracking time instants within a certain distortion bound. The formulation is general in that it can be applied to a variety of mobility models, distortion criteria, and stochastic sequences of tracking time instants. Under the Gauss-Markov mobility model, lower bounds on the information rate of tracking the motion state information of nodes in dynamic networks are derived, where the motion state of a node is 1) the node’s locations only, or 2) both its locations and velocities. The results are then used to analyze the protocol overhead of geographic routing protocols in mobile ad hoc networks. The minimum overhead incurred by maintaining the geographic information of the nodes is characterized in terms of node mobility, packet arrival process and distortion bounds. This leads to precise mathematical description of the observation that, given certain state-distortion allowance, protocols aimed at tracking motion state information (such as geographic routing protocols) may not scale beyond a certain level of node mobility.

## I. INTRODUCTION

In this paper, we study communication networks whose state keeps changing over time. Here the state information of a network may be composed of link states, node locations, velocity of nodes, etc. In many cases, keeping track of the state information of a dynamic network so as to maintain a timely view of the network is a crucial task. As networks continue to grow in size and become more dynamic, the network state information may change rapidly over time and hence significant control overhead may be incurred by tracking the network state. Such an overhead, in our opinion, has yet to be fully understood.

The premise of this work is that, instead of trying to find solutions that produce the least overhead under certain conditions for each of the aforementioned examples and other network scenarios, we aim to develop a general framework that may bound the overhead while satisfying a constraint that the network state information is maintained within some targeted accuracy. In this paper, we consider dynamic networks with mobile nodes where the state information being tracked is the *motion state* of nodes. Here the motion state of a node can be (any combinations of) the location, velocity and/or acceleration of the node. The key departure point of this framework is to treat a node’s motion state as a random (vector) process that exhibits random changes. The minimum

overhead is the minimum amount of state information rate needed such that the current state of nodes of the network can be identified within a certain distortion bound. This motivates the use of information theory and rate-distortion theory as a tool for developing lower bounds on such an overhead.

We consider the scenario where the motion state of nodes of a dynamic network is tracked by a network master at a sequence of time instants. Here the network master could be any node of the network and is also called ‘the master node’. In our analysis, the *actual* motion state of a node and the motion state *perceived* by the master node are both treated as random processes. The discrepancy between the actual and perceived state is evaluated under some predefined distortion measure, for example, the mean-squared distortion measure. Then, under the given distortion measure, the information rate of tracking the motion state of nodes in a dynamic network is the bit rate at which the master node must receive information such that the distortion evaluated at the time instants of interest is bounded. We formulate the problems of finding the minimum value of these bit rates as rate-distortion problems. Using the rate-distortion formulation and under the Gauss-Markov mobility model [1], we obtain lower bounds on the minimum bit rate of tracking the motion state of dynamic networks, where the motion state of a node is, either 1) the node’s location information only, or 2) the node’s location and velocity information.

Further we show how the derived results can be applied to analyzing the protocol overhead of geographic routing protocols in packet-switched wireless networks. We characterize the protocol overhead incurred in terms of node mobility, packet arrival process, and distortion bound  $\epsilon$ . We then connect the results obtained here with the results on the throughput capacity of stationary multi-hop wireless networks evaluated in [2], in order to characterize the *effective* throughput capacity available for users. It is shown that, under the Gauss-Markov mobility model, in order to prevent the total traffic from being overwhelmed by protocol overhead, the average speed of nodes in the network must scale down as the total number of nodes grows. This implies that, within a certain state-distortion bound, protocols aimed at tracking motion state information may not be scalable beyond a certain level of node mobility.

The contributions of this paper are summarized as follows:

- 1) We present an information-theoretic formulation for evaluating the overhead incurred by tracking the motion state information of dynamic networks. The formulation is general

in that it can be applied to any stochastic sequences of tracking time instants, any distortion measures, and any mobility models, as shown in the analysis.

2) For the Gauss-Markov mobility model and *arbitrary* tracking time distributions, we derive lower bounds for minimum information rate at which a master node must receive motion state information of nodes in the network such that the errors in nodes' motion state are bounded (Theorem 1 and Corollary 2).

3) The obtained results are applicable to the analysis of protocol overhead of geographic routing protocols in mobile ad hoc networks. We derive the minimum overhead incurred by maintaining the geographic information of nodes in terms of node mobility, packet arrival process and distortion bounds. It can be observed that given certain state-distortion allowance, protocols aimed at tracking motion state information (such as geographic routing protocols) are not scalable beyond a certain level of node mobility (Theorem 3 and Corollary 4).

The rest of the paper is organized as follows. A brief overview of related work and the relations with this paper are presented in Section II. The problem definition and its mathematical formulation is presented Section III. The evaluation of lower bounds on the information rates of tracking motion state information of dynamic networks is presented in Section IV. Applications to analyzing the protocol overhead of geographic routing protocols in mobile ad hoc networks is introduced in Section V. We present conclusions and future research directions in Section VI.

## II. RELATED WORK

Several recent studies have used information theory to understand network protocols or applications, and we review the most related results below.

An information-theoretic approach is used in [3] to study the complexity of tracking a mobile user in a cellular environment and a position update and paging scheme is proposed. Distributed information discovery in P2P systems is studied from an information-theoretic perspective in [4], through which the overhead induced by the task of information discovery is bounded. Authors in [5] studies the scheduling problem in a TDMA-based network and uses rate-distortion theory to characterize the basic limits on the amount of network information that should be transmitted in order to achieve a given level of network performance. An information-theoretic approach is proposed in [6] in order to characterize the minimum routing overhead and memory requirements of a specific two-level hierarchical routing protocols for ad hoc networks. The overhead of generic link state routing protocols is studied using a rate-distortion approach in [7]. The authors in [8] study the family of geographic routing protocols for mobile wireless networks and rate-distortion lower bounds for the geographic routing overheads are evaluated under Brownian-motion mobility model.

While our work is in some sense along the lines with [6]–[8], there are some major differences that underline the contributions of this paper and are summarized as follows:

1) In this paper we study the network state tracking problem where the network state is the *motion state* of nodes and hence has a broader generality and applicability than the location state information studied in [8].

2) While the previous work [8] relies solely on the use of Brownian motion mobility model to deliver mathematically tractable results, in this paper we extend the analysis to the Gauss-Markov mobility model, which is considered to be more realistic, and more widely used in the literature. This allows us, when studying the protocol overhead of geographic routing protocols, to derive some interesting new results, e.g. the maximum scaling of the average speed (in terms of total number of nodes) that can be supported by geographic routing in mobile ad hoc networks (Corollary 4).

## III. PROBLEM FORMULATION

We study a general motion-tracking problem described as follows. A number of  $n$  mobile nodes are deployed over a 2-dimensional plane. For each node  $i$  let  $S_i(t)$  denote its motion state at time  $t$ , where  $S_i(t)$  can be node  $i$ 's location  $X_i(t)$ , its velocity  $V_i(t)$ , its acceleration  $A_i(t)$ , or any possible combinations of these three, all at time  $t$ . Assume node  $i$  encodes its motion state information and updates to a master node so that the master node can track node  $i$ 's motion at time instants  $\{T_j : j = 1, 2, \dots\}$  with tracking rate  $\lambda := \lim_{N \rightarrow \infty} \frac{N}{E[T_N]}$  such that the actual motion states  $\{S_i(T_j) : j = 1, 2, \dots\}$  of node  $i$  are perceived as  $\{\hat{S}_i(T_j) : j = 1, 2, \dots\}$  by the master node. Now we want to find the minimum information rate at which node  $i$  must update its motion state information so that the information  $\{\hat{S}_i(T_j) : j = 1, 2, \dots\}$  decoded by the master node is within a certain accuracy as compared to the actual motion state information  $\{S_i(T_j) : j = 1, 2, \dots\}$ . A strict mathematical formulation of this problem is presented through the following definitions.

*Definition 1:*  $d(S_i(t), \hat{S}_i(t))$  is the distortion measure between node  $i$ 's actual motion state and the state available at the master node at time  $t$ .

*Definition 2:*  $S_i^N = [S_i(T_1), S_i(T_2), \dots, S_i(T_N)]$  is the vector of motion states of node  $i$  at time instants  $T_j$ ,  $1 \leq j \leq N$ . Similarly,  $\hat{S}_i^N = [\hat{S}_i(T_1), \hat{S}_i(T_2), \dots, \hat{S}_i(T_N)]$  is the vector of motion states of node  $i$  perceived by the master node at time instants  $T_k$ ,  $1 \leq k \leq N$ .

*Definition 3:*  $T^N = [T_1, T_2, \dots, T_N]$  is the vector of the first  $N$  time instants that node  $i$ 's motion state information is tracked by the master node.

*Definition 4:*  $\mathcal{S}_i^N$ ,  $\hat{\mathcal{S}}_i^N$  and  $\mathcal{T}^N$  are defined as the sets of all possible vectors  $S_i^N$ ,  $\hat{S}_i^N$  and  $T^N$ , respectively.

*Definition 5:*  $P_N$  denote the probability measure on the sample space  $\mathcal{S}_i^N \times \hat{\mathcal{S}}_i^N \times \mathcal{T}^N$ . Then the distortion between  $S_i^N$  and  $\hat{S}_i^N$ , denoted by  $D_N(S_i^N, \hat{S}_i^N)$ , is defined by

$$D_N(S_i^N, \hat{S}_i^N) = \frac{1}{N} \sum_{k=1}^N E[d(S_i(T_k), \hat{S}_i(T_k))] \quad (1)$$

where the expectation is taken with respect to the probability measure  $P_N$ .

**Definition 6:** Let  $W_N \in \mathcal{W}_N$  denote the encoded version of the actual motion states  $S_i^N$ , where  $\mathcal{W}_N$  is the alphabet of  $W_N$  with cardinality  $|\mathcal{W}_N| = 2^{NR}$ .

Without loss of generality we assume that  $\mathcal{W}_N = \{1, 2, 3, \dots, 2^{NR}\}$ . Note that  $T^N$  is statistically correlated with  $S_i^N$ , therefore  $S_i^N$  is a useful piece of information for the master node to decode/reconstruct  $S^N$ . By the terminology of information theory,  $T^N$  is known as *side information*.

Now we can present this problem formulation based on rate-distortion theory to find the minimum information rate at which node  $i$  must update its motion state information such that  $D_N(S_i^N, \hat{S}_i^N) \leq \epsilon$ .

**Definition 7:** The  $N^{\text{th}}$ -order rate-distortion function with side information  $T^N$ ,  $R_N(\epsilon)$ , is defined as the minimum rate required to achieve distortion  $\epsilon$  if the side information  $T^N$  is available to the decoder. Precisely,  $R_N(\epsilon)$  is the infimum of all rates  $R$  such that there exists maps  $f_N : \mathcal{S}_i^N \rightarrow \{1, 2, \dots, 2^{NR}\}$ ,  $g_N : \mathcal{T}^N \times \{1, 2, \dots, 2^{NR}\} \rightarrow \hat{\mathcal{S}}_i^N$  such that

$$D_N(S_i^N, g_N(T^N, f_N(S_i^N))) \leq \epsilon \quad (2)$$

Over a long period of time, the minimum information rate  $U(\epsilon)$  (in bits per second) at which the master node must receive such that the distortion between the actual and perceived motion state information is bounded by  $\epsilon$ , is given by

$$U(\epsilon) = \lambda \lim_{N \rightarrow \infty} R_N(\epsilon) \quad (3)$$

Note that the problem formulation presented above is general in that it is not restricted to any particular mobility models or any particular distortion measures. In this paper, the mobility models under study is the Gauss-Markov mobility model defined as follows.

Let  $V_i(t) = \{V_{i1}(t), V_{i2}(t)\}$  denote the velocity of a node  $i$  at time  $t$ . Hence the node  $i$ 's location at time  $t$  is given by

$$X_i(t) = X_i(0) + \int_0^t V_i(s) ds \quad (4)$$

Let  $\widetilde{W}_1(t)$  and  $\widetilde{W}_2(t)$  denote two independent Wiener processes (without drift) with unit variance. Then Node  $i$  is said to be moving according to the Gauss-Markov mobility model with drift velocity  $\mu = \{\mu_1, \mu_2\}$ , velocity variance  $\eta^2$ , and relaxation time  $\tau$ , if and only if  $V_{i1}(t)$  and  $V_{i2}(t)$  are two independent Ornstein-Uhlenbeck processes with drift velocities  $\mu_1$  and  $\mu_2$  respectively, and satisfying the following stochastic differential equations

$$dV_{ij}(t) = -\frac{1}{\tau} V_{ij}(t) dt + \frac{\eta}{\sqrt{\tau}} d\widetilde{W}_j(t), \quad j = 1, 2. \quad (5)$$

Here the parameter  $\tau$  in the Gauss-Markov mobility model indicates how strong the current velocity of a node is correlated to the past, with a large  $\tau$  indicating a strong correlation.

Note that, while the mobility models used in this paper are all *time-continuous*, the Gauss-Markov mobility model first introduced in [1] and used by many others are *time-discrete*, which is sampled from the time-continuous model with some time interval  $\Delta t$ . The discretized Gauss-Markov model used

in [1] has parameters  $\{\mu, \eta^2, \alpha\}$ , in which the correlation parameter has the relation  $\alpha = e^{-\Delta t/\tau}$ , while other parameters still hold the same meanings as those in our model.

#### IV. INFORMATION RATES OF MOTION TRACKING

In this section we study the motion-tracking problem the motion state of interest is composed of a node's location information and its velocity information, i.e., the motion state of a node  $i$  at time  $t$  is described by  $S_i(t) = \{X_i(t), V_i(t)\}$ . We use the mean-squared error as the the distortion measure, i.e.,

$$d(S_i(t), \hat{S}_i(t)) = \{d_1(S_i(t), \hat{S}_i(t)), d_2(S_i(t), \hat{S}_i(t))\} \quad (6)$$

where

$$\begin{cases} d_1(S_i(t), \hat{S}_i(t)) := \sum_{j=1}^2 (X_{ij}(t) - \hat{X}_{ij}(t))^2 \\ d_2(S_i(t), \hat{S}_i(t)) := \sum_{j=1}^2 (V_{ij}(t) - \hat{V}_{ij}(t))^2 \end{cases} \quad (7)$$

A lower bound on the minimum information rate of tracking a node's location and velocity information jointly under mean-squared distortion measure is given in the following theorem.

**Theorem 1:** Assume a node  $i$  is moving according to the Gauss-Markov mobility model with drift velocity  $\mu$ , velocity variance  $\eta^2$  and relaxation time  $\tau$ . Then the lower bound on the minimum information rate  $U(\epsilon)$  of joint tracking a node's location and velocity information, such that the mean-squared distortion between the actual and perceived motion information at time instants  $\{T_j\}_{j=1}^\infty$  is bounded by  $\epsilon = \{\epsilon_1, \epsilon_2\}$ , is given by

$$U(\epsilon) \geq \max\{L_1, L_2\} \quad (8)$$

where

$$\begin{cases} L_1(\epsilon) = \lambda \left( E \left[ \log_2 \left( \frac{2\tilde{Y}}{\tau} \left( 1 + e^{-\tilde{Y}/\tau} \right) - 4 \left( 1 - e^{-\tilde{Y}/\tau} \right) \right) \right] \right. \\ \quad \left. + E \left[ \log_2 \left( 1 - e^{-\tilde{Y}/\tau} \right) \right] + \log_2 \frac{\eta^2 \tau^2}{\epsilon_1 \epsilon_2} \right) \\ L_2(\epsilon_1) = \lambda \left( E \left[ \log_2 \left( \frac{2\tilde{Y}}{\tau} \left( 1 + e^{-\tilde{Y}/\tau} \right) - 4 \left( 1 - e^{-\tilde{Y}/\tau} \right) \right) \right] \right. \\ \quad \left. - E \left[ \log_2 \left( 1 + e^{-\tilde{Y}/\tau} \right) \right] + \log_2 \frac{\eta^2 \tau^2}{\epsilon_1} \right) \end{cases}$$

where  $\tilde{Y}$  is a random variable following the mixture density  $f_{Y_j}(t)$  of tracking time intervals  $Y_j := T_j - T_{j-1}$ ,  $j = 1, 2, \dots$

*Proof:* Please see [9]. ■

From Theorem 1 we note that, the lower bound  $L_2$  on  $U(\epsilon)$  is dependent on the location error  $\epsilon_1$  and is independent on the velocity error  $\epsilon_2$ . In fact, it is actually the lower bound for the minimum rate of tracking a node's location information only when the node's movement is governed by the Gauss-Markov mobility model. Therefore we have the following corollary.

**Corollary 2:** Assume a node  $i$  is moving according to the Gauss-Markov mobility model with drift velocity  $\mu$ , velocity variance  $\eta^2$  and relaxation time  $\tau$ . Then the lower bound on the minimum information rate  $U(\epsilon)$  of tracking a node's location information only, such that the mean-squared distortion between the actual and perceived location information at time instants  $\{T_j\}_{j=1}^\infty$  is bounded by  $\epsilon$ , is given by

$$U(\epsilon) \geq L_2(\epsilon) \quad (9)$$

where  $L_2$  is from (8).

## V. APPLICATION: ANALYZING THE OVERHEAD OF GEOGRAPHIC ROUTING PROTOCOLS

In this section, we present how the theories developed in the above sections can be applied to analyze the overhead incurred by geographic routing protocols over mobile ad hoc networks. First we present a brief introduction of geographic routing protocols in Section V-A. Then in Section V-B we characterize the overhead of geographic routing protocols for mobile wireless networks and investigate how the protocol overhead interacts with the data traffic.

### A. Geographic Routing Protocols

Geographic routing is a routing technique that uses the position of destination nodes in order to make routing decisions [11], and requires nodes to know their locations. This may be accomplished using GPS [12] or other mechanisms [13]. Also geographic routing requires a distributed location service which allows source nodes to collect the location information of the destination nodes [14]. All geographic routing protocols function in the following manner. When a new packet arrives at a source node, it queries the location service in order to discover the current position of the destination. The position of the destination is added to packet headers and the source and intermediate nodes forward the packet to a neighbor that is closer to the destination.

It is clear that, in geographic routing, the process of collecting the destinations' location information by the source nodes produces an overhead. Such a process can be abstracted as follows. For any source-destination pair  $(k, i)$ , destination node  $i$  will encode its location information and send out as control packets, which may be received by the source node  $k$  such that it can make routing decisions upon the arrivals of data packets at node  $k$ . Therefore, the *control message overhead associated with S-D pair  $(k, i)$*  is formed by the bit streams of all such control packets received by node  $k$ . In the following subsection we will apply the results derived in previous sections to bound such an overhead.

### B. Overhead-Throughput Analysis of Mobile Ad Hoc Networks using Geographic Routing

Consider a mobile ad hoc network of  $n$  nodes that are randomly distributed over a torus with surface area of  $A(n) = \Theta(n)$ . The movement of each node is governed by the Gauss-Markov mobility model with drift velocity  $\mu(n)$ , velocity variance  $\eta^2(n)$  and relaxation time  $\tau(n)$ . The transmission range of each node is assumed to be fixed and is denoted by  $r(n)$ . We pick uniformly at random a matching of source-destination pairs, so that each node is the destination of exactly one source. The source node  $k$  is constantly generating data packets (with packet size  $c$  bits) destined to  $i$  at rate  $\lambda(n)$  packets/sec such that the  $j^{th}$  packet destined to destination  $i$  is generated at time  $T_j = j/\lambda(n)$ ,  $\forall j \geq 0$ .

We assume that both the drift velocity  $\mu(n)$  and the velocity variance  $\eta^2(n)$  of a node can only scale as  $O(1)$ . Since the network radius scales as  $\Theta(n)$ , this ensures that the nodes do not wrap around the surface during the time scale

corresponding to packet inter-arrival times. So if we look at the movement of a node during a small time interval, then with high probability the motion is similar to the Gauss-Markov model on an infinite 2-D plane. Therefore we study the movement of nodes over the torus as if they are moving over an infinite plane and hence results from previous sections can be applied here.

We also assume that the relaxation time  $\tau(n)$  scales as  $\Theta(1)$ . This is because, either  $\tau(n) = o(1)$  or  $\tau(n) = \omega(1)$  will make the Gauss-Markov mobility model become trivial: 1) when  $\tau(n) \rightarrow 0$ , nodes are performing Brownian motion with zero location variance; 2) when  $\tau(n) \rightarrow \infty$ , nodes are just moving with constant velocity  $\mu(n)$ .

According Corollary 2, in order to have node  $i$ 's location error bounded by  $\epsilon(n)$  at node  $k$ , the minimum bit rate at which node  $k$  must receive node  $i$ 's location information is

$$U(\epsilon) \geq \lambda(n) \left( \log_2 \left( \frac{2}{\lambda(n)\tau} \left( 1 + e^{-\frac{1}{\lambda(n)\tau}} \right) - 4 \left( 1 - e^{-\frac{1}{\lambda(n)\tau}} \right) \right) \right. \\ \left. - \log_2 \left( 1 + e^{-\frac{1}{\lambda(n)\tau}} \right) + \log_2 \frac{\eta^2(n)\tau^2}{2\epsilon(n)} \right)$$

Note that the maximum per-node throughput of a random multi-hop wireless network scales as  $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$  [2], and it is indicated by Corollary 2 that in mobile wireless networks, the control message overhead produced by geographic routing protocols reduces the per-node throughput at least by  $U(\epsilon(n))$ . Therefore, the bit rate of transmitting data packets from a source to its destination,  $c\lambda(n)$ , which is termed as *effective per-node throughput*, can only scale as  $O\left(\frac{1}{\sqrt{n \log n}}\right)$ , and it remains to be seen whether the effective per-node throughput  $c\lambda(n)$  can actually attain the order  $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$ . This question is answered in the following theorem.

*Theorem 3:* Consider a mobile ad hoc network where node movement follows the Gauss-Markov mobility model. Then in order to have the effective per-node throughput  $c\lambda(n)$  achieve the optimal scaling  $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$ , the velocity variance of a node  $\eta^2(n)$  must scale as  $O\left(\frac{\epsilon(n)}{\sqrt{n \log n}}\right)$ .

*Proof:* Please see [9]. ■

The results of Theorem 3 determines the level of node mobility (in terms of total number of nodes in the network) that geographic routing protocols can ultimately support. The exact order of node mobility, which is for this case the velocity variance  $\eta^2(n)$ , however, is still not explicit and is dependent on the location error  $\epsilon(n)$ . It is clear that the location error  $\epsilon(n)$  cannot be too large since otherwise the routing protocols will not function properly. Since the determination of location error  $\epsilon(n)$  in order to achieve satisfactory network performance (e.g. high delivery ratio) is out of the scope of this paper, we need to borrow some results of such a study in the literature, e.g. [19] and [20]. It is observed in [19] that for geographic routing protocols using greedy forwarding, 20% error in node's location information (relative to its transmission range  $r$ ) may cause substantial loss in performance. An improvement, comprising of maintaining two hop neighborhood information, is proposed

which makes geographic routing tolerant to 40% error in location. Similar observations regarding loss in performance due to location errors induced by increased mobility and beacon interval are made in [20]. Based on these observations, for the mean-squared location error  $\epsilon(n)$  we use the order  $\epsilon(n) = O(r^2(n))$  as a rule of thumb to determine the location error that a geographic routing protocol can tolerate. In this case we have the following corollary.

*Corollary 4:* Assume a geographic routing protocol can only tolerate a mean-squared location error  $\epsilon(n) = O(r^2(n))$ , and let the drift velocity  $\mu(n)$  in the Gauss-Markov model for each node to be 0. Then in order to have the effective per-node throughput  $c\lambda(n)$  achieve the optimal scaling  $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$ , the average speed of a node  $\overline{|V(n)|}$  must scale as  $O\left(\sqrt{\frac{\log n}{n}}\right)$ . For the case  $\overline{|V(n)|} = \Theta(1)$ , the effective per-node throughput  $c\lambda(n)$  can only scale as  $O\left(\frac{1}{\sqrt{n \log^{1.5}(n)}}\right)$ .

*Proof:* Please see [9]. ■

The results of Corollary 4 indicate that, under Gauss-Markov mobility model without drift, in order to have the data rate of each S-D pair achieving the maximum per-node throughput, the average speed of each node must scale down with the increase of the number of nodes  $n$ . If we let the average speed of each node stay constant, then, as compared to the maximum per-node throughput scaling  $\Theta\left(\frac{1}{\sqrt{n \log n}}\right)$  that can be achieved in static multi-hop wireless networks, the data rate of each S-D pair must be reduced by a factor of  $\log n$  in order to let the network accommodate the protocol overhead incurred by geographic routing protocols.

It is worth noting that, under the same network setting, results in [21] indicate that, for topology-based proactive routing protocols (e.g. DBF [15] and OLSR [16]), the maximum mobility degree (measured in node's average speed) can be supported by the network is  $\overline{|V(n)|} = O\left(\frac{1}{(n \log n)^{3/2}}\right)$ , while for reactive routing protocols that maintain the route state or vector state of a network (e.g. DSR [18] and AODV [17]), the maximum mobility degree must scale as  $\overline{|V(n)|} = O\left(\frac{1}{n^2}\right)$ . By comparison we understand that, asymptotically, geographic routing protocols may tolerate a relatively higher degree of mobility and hence have better scalability than the routing protocols examined in [21] that maintain other types of state information of the network.

## VI. CONCLUSION

In this paper, we present an information-theoretic framework for bounding the motion-tracking cost in dynamic networks. In this problem, a tracking node wants to maintain the motion state information, such as locations and velocities, of other nodes in the network. The objective is to find the minimum amount of information rate (i.e. the overhead) required by the tracking node such that the distortion between actual and perceived motion state information of other nodes is bounded. We formulate this minimum overhead problem as a rate-distortion problem with side information. As compared to previous work [8], such a formulation not only helps to

yield better lower bounds on the information rate (protocol overhead), but also extend the analysis to more realistic mobility models, and incorporate more general stochastic models for the sequence of tracking time instants. It is shown that our theoretical analysis can be applied to analyze the protocol overhead of geographic routing protocols in mobile ad hoc networks. The generality of our proposed framework leads to produce some interesting results for large scale mobile ad hoc networks, including finding the maximum scaling of average node speed that can be supported by networks that use geographic routing to route packets. Directions of future research include extending our proposed information-theoretic framework to incorporate other types of network state.

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