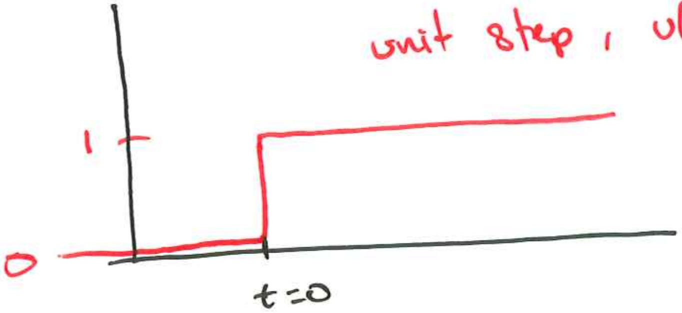
System Responses

→ System behavior to a given input.

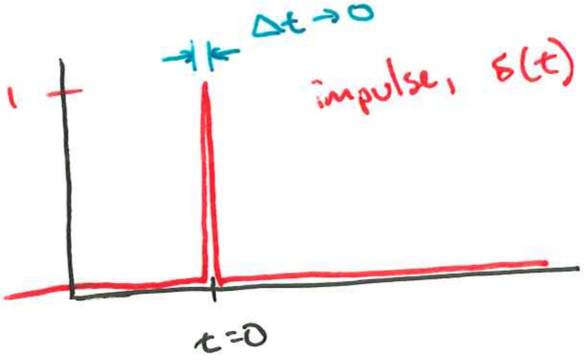
Types of input

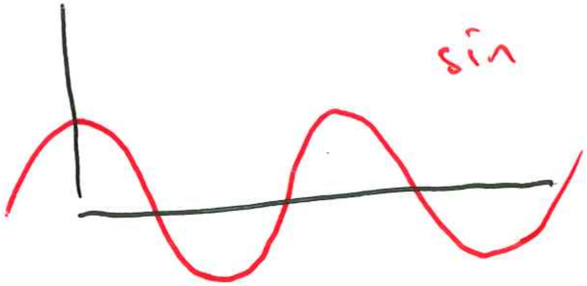
unit step, $u(t) = \begin{cases} 0 & t \le 0 \\ 1 & t > 0 \end{cases}$

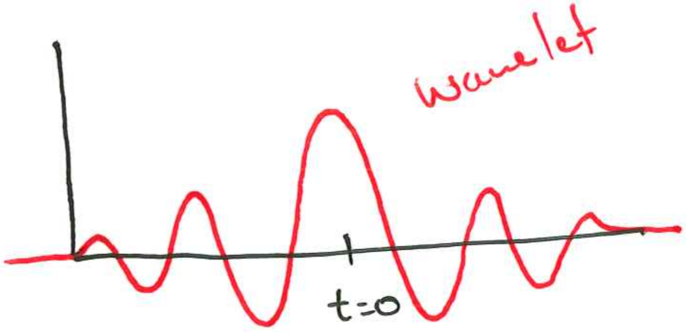
← we'll consider this one.



impulse, $\delta(t) = \begin{cases} 0 & t \neq 0 \\ 1 & t = 0 \end{cases}$



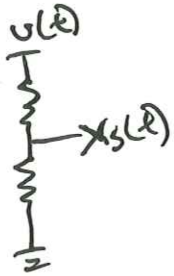


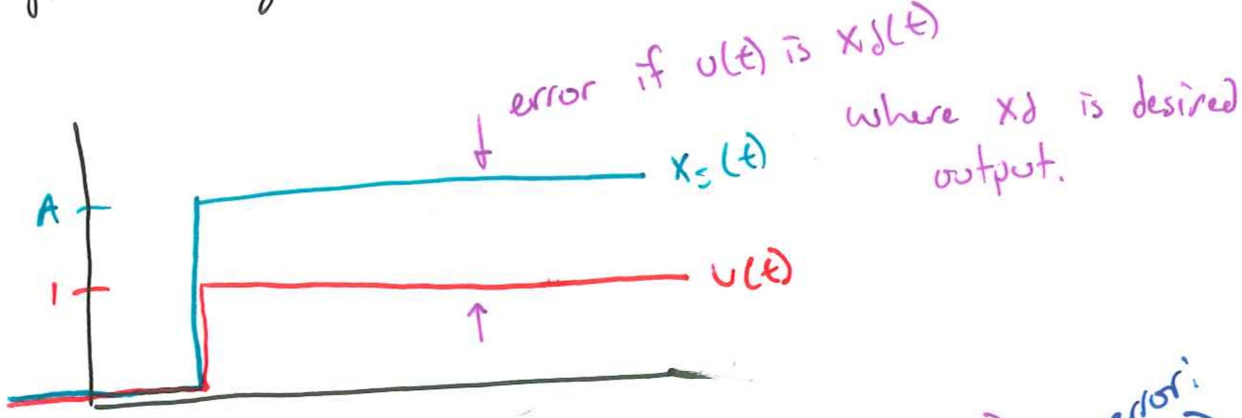


Basic linear System Response types using u(t)

0th Order Response

$x_s(t) = A u(t)$

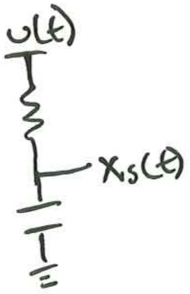




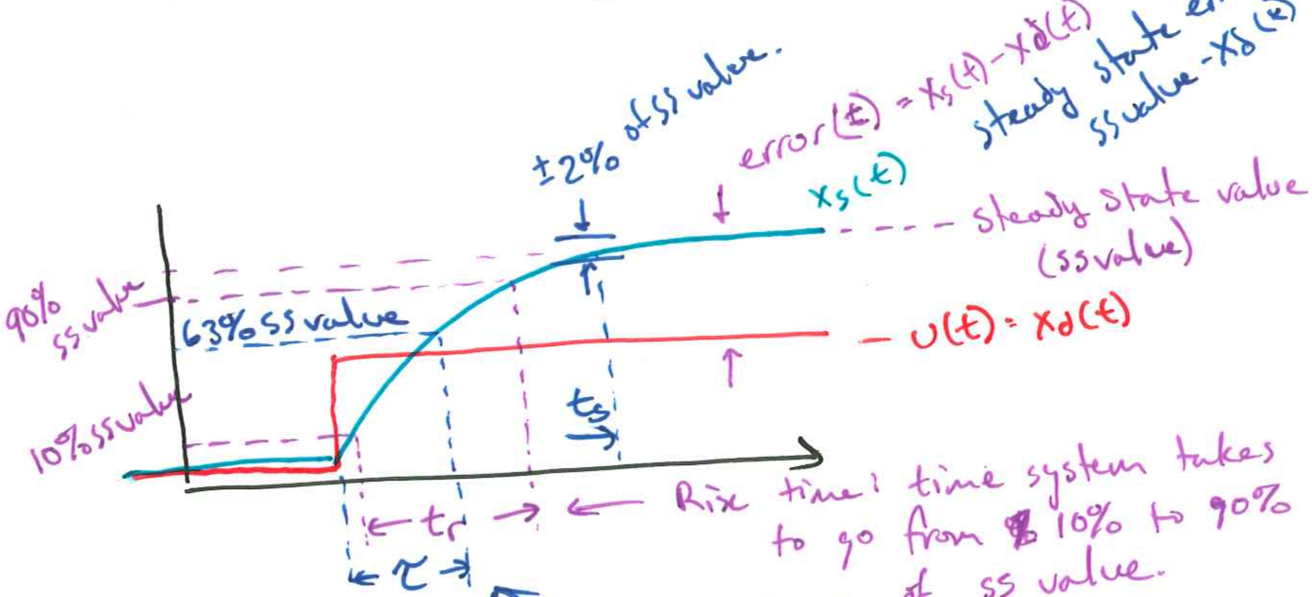
1st Order

$X_s(t) = A\underbrace{\left(1-e^{-t/\tau}\right)}u(t)$

starts at 0
approaches 1
as $t \to \infty$



$b\frac{dx_s}{dt} + Cx_s = u(t)$

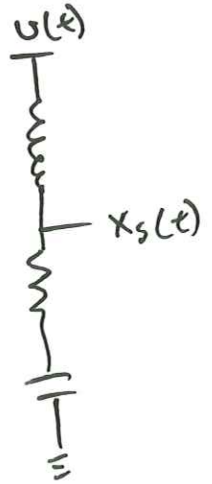


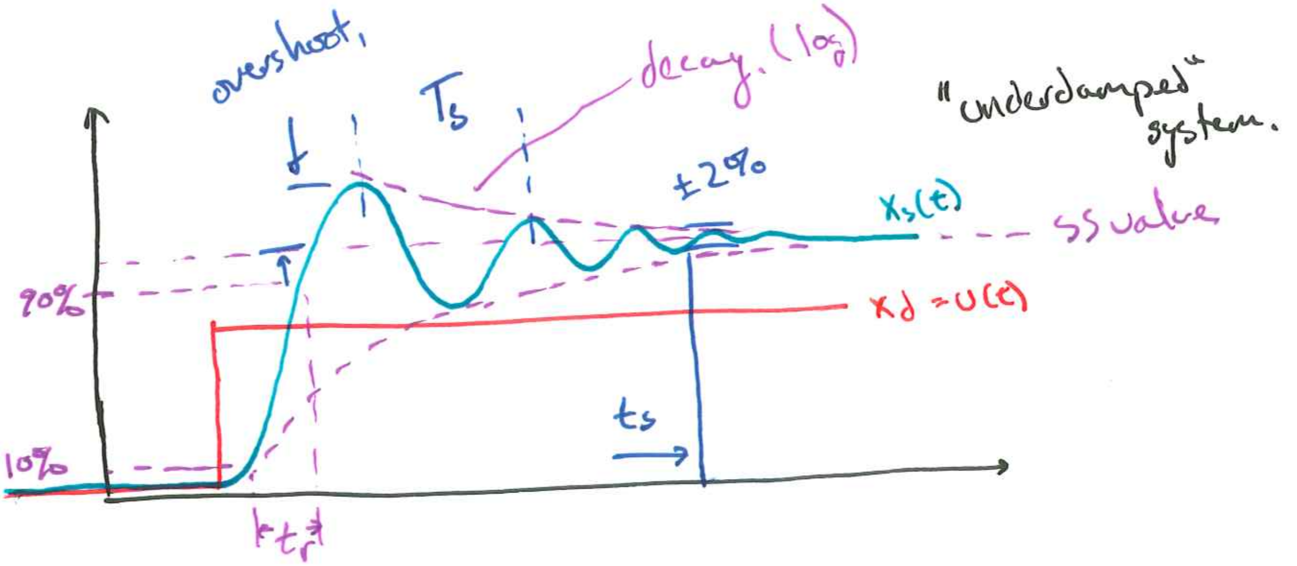
Rise time: time system takes to go from 10% to 90% of ss value.

time constant
time it takes to get from 0% to 63% ss value.

ts: settling time.
time it takes to get within (and stay within) ±2% of ss value.

2nd Order Response





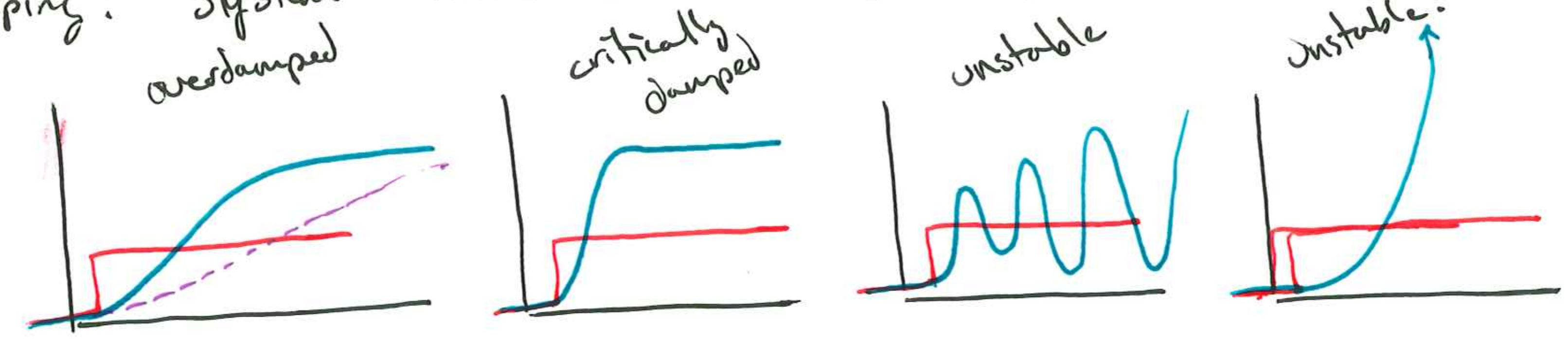
$$a\frac{d^2x_s}{dt}+b\frac{dx_s}{dt}+cx_s=u(t)$$

T_s : System Response period $\rightarrow \frac{1}{T_s}=f_s$ → response frequency.

f_s also known as: Natural frequency

Resonance frequency.

Damping: System resistance to change. (eg. inertia, friction)



PID

P- proportional
I - integral
D- derivative.

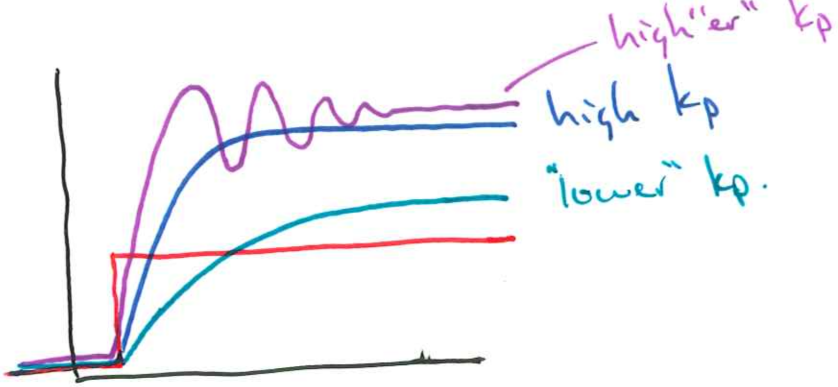
Operate on system error $e(t) = \underset{\text{desired}}{x_d(t)} - \underset{\text{actual.}}{x_s(t)}$

Proportional control - P

$$y_p(t) = k_p e(t)$$

y_p → forcing output.
k_p → prop. gain constant.

→ Act on instantaneous error → large error, large response.



→ higher kps may induce 2nd order behavior!

high kp will also amplify noise.

I - Integral Control

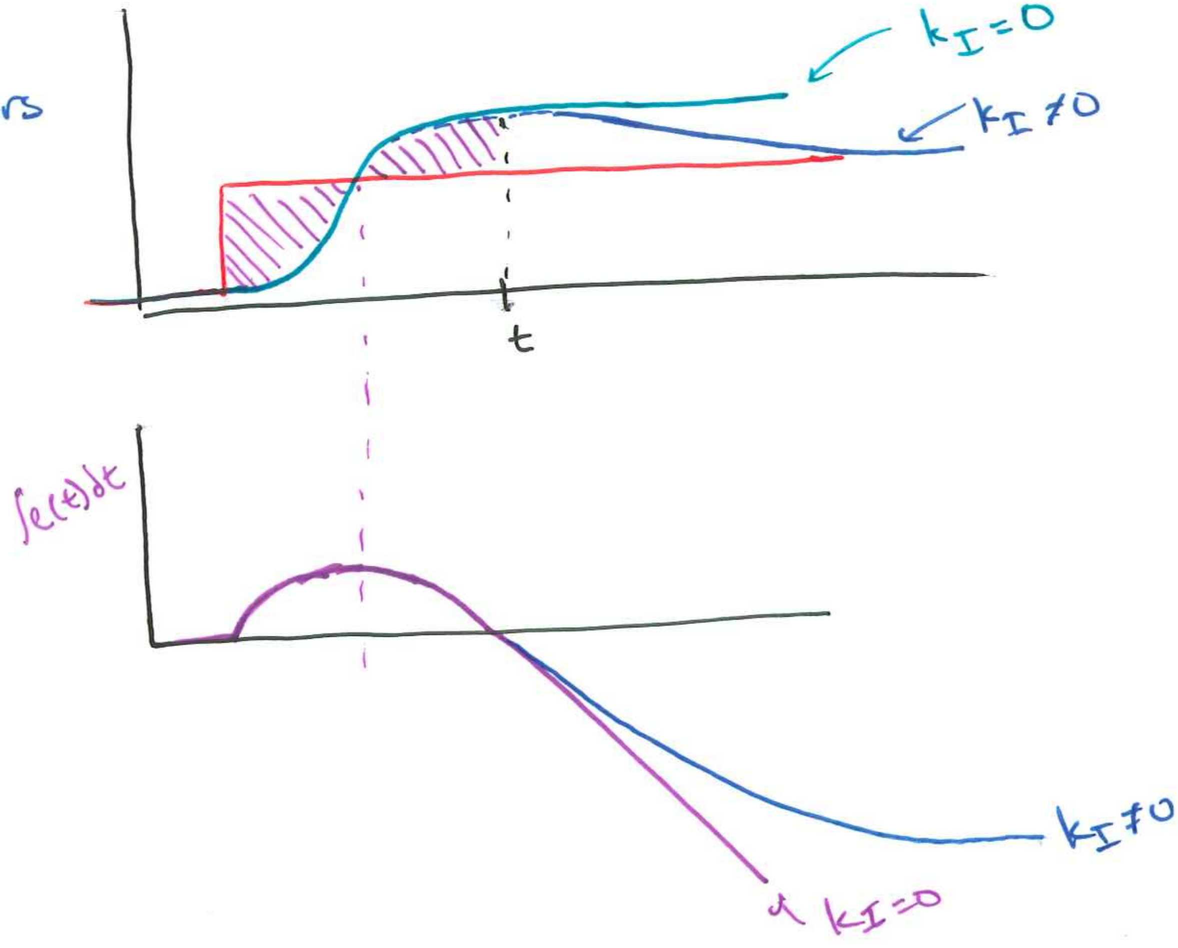
$$y_I(t) = k_I \int_0^t e(\tau)\,d\tau$$

y_I responds to system <u>history</u>

or <u>error history</u>

Acts on persistent errors

→ Remove Steady State errors



Derivative Control

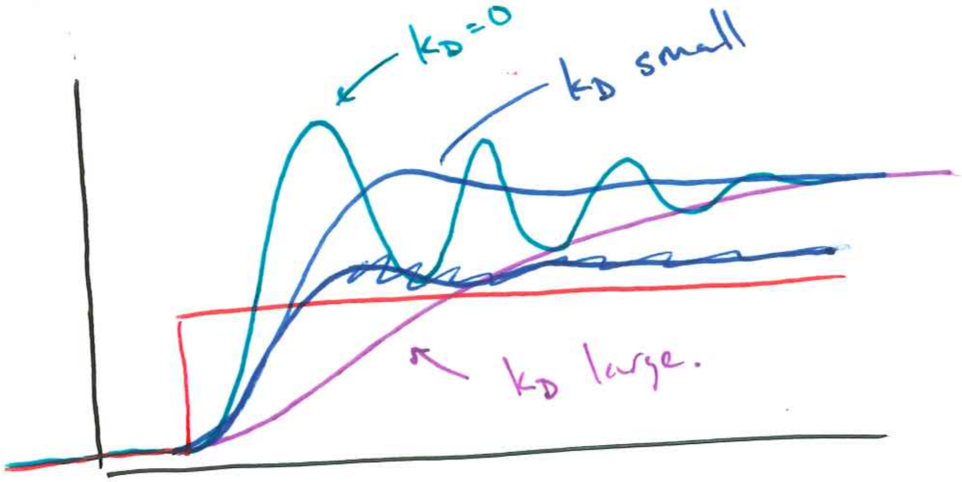
$$y_D(t) = k_D \frac{d\,e(t)}{dt}$$

$\frac{de(t)}{dt}$ → rate of change of error

Acts on system "velocity"

→ Injects system Damping.

slows down response time.



full PID:

$$y(t) = k_p e(t) + k_I \int_0^t e(\tau) d\tau + k_D \frac{d e(t)}{dt}$$

continuous equation!

$e(t) = x_d(t) - x_s(t)$

cannot implement in a μC.

Need Discetized form → evaluated at points in time

k = discrete time sample #

k = 0, 1, 2, ...

$e(k) = x_d(k) - x_s(k)$

$$y(k) = k_p e(k) + k_I \sum_{n=0}^{k} \Delta t e(n) + k_D \frac{(e(k) - e(k-1))}{\Delta t}$$

if Δt is constant, then simplify to:

$$y(k) = k_p e(k) + k_I' \sum_{n=0}^{k} e(n) + k_D' (e(k) - e(k-1))$$

$k_I' = k_I \Delta t$

$k_D' = k_D / \Delta t$

we will implement this

(and will assume $k_I = k_I'$

$k_D = k_D'$

Integral: