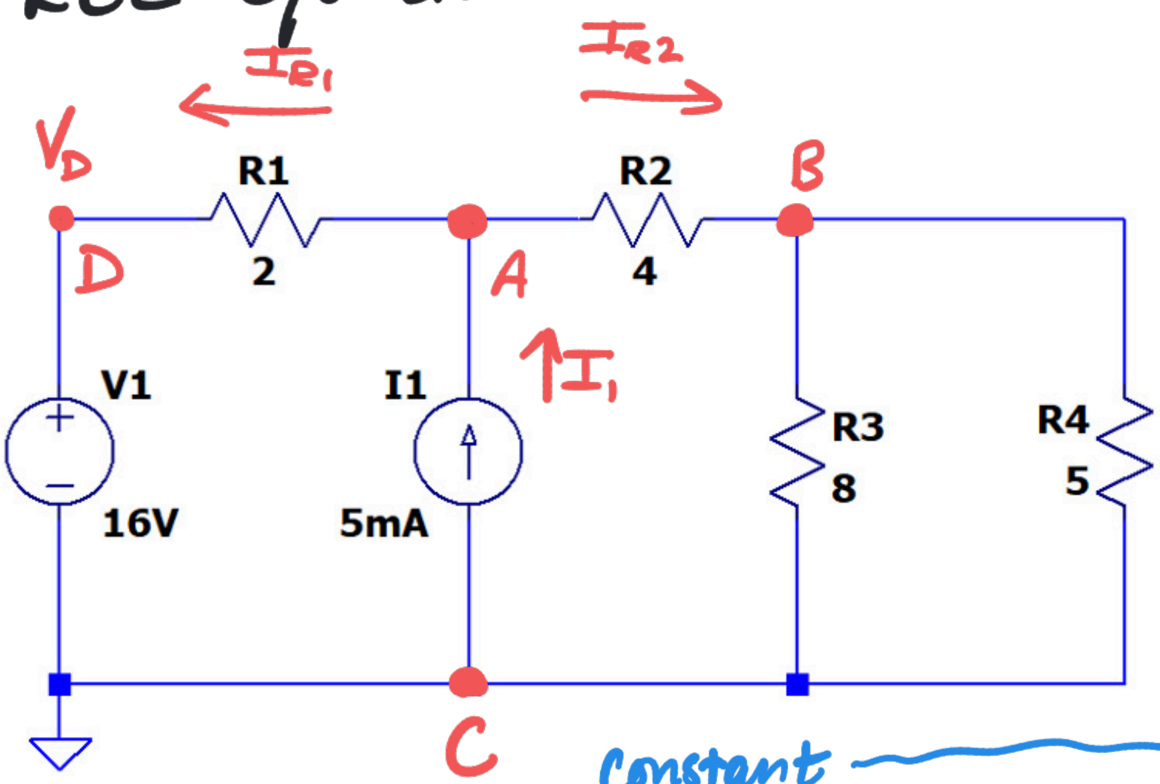
Intro to ECSE Class 13: Nodal Analysis Example Problems

I] Nodal Analysis with Current Sources

- If we have a current source, it will appear as a constant current term in KCL equations



constant   constant

KCL @ A: $\frac{V_A - V_D}{R_1} + \frac{V_A - V_B}{R_2} - I_1 = 0$

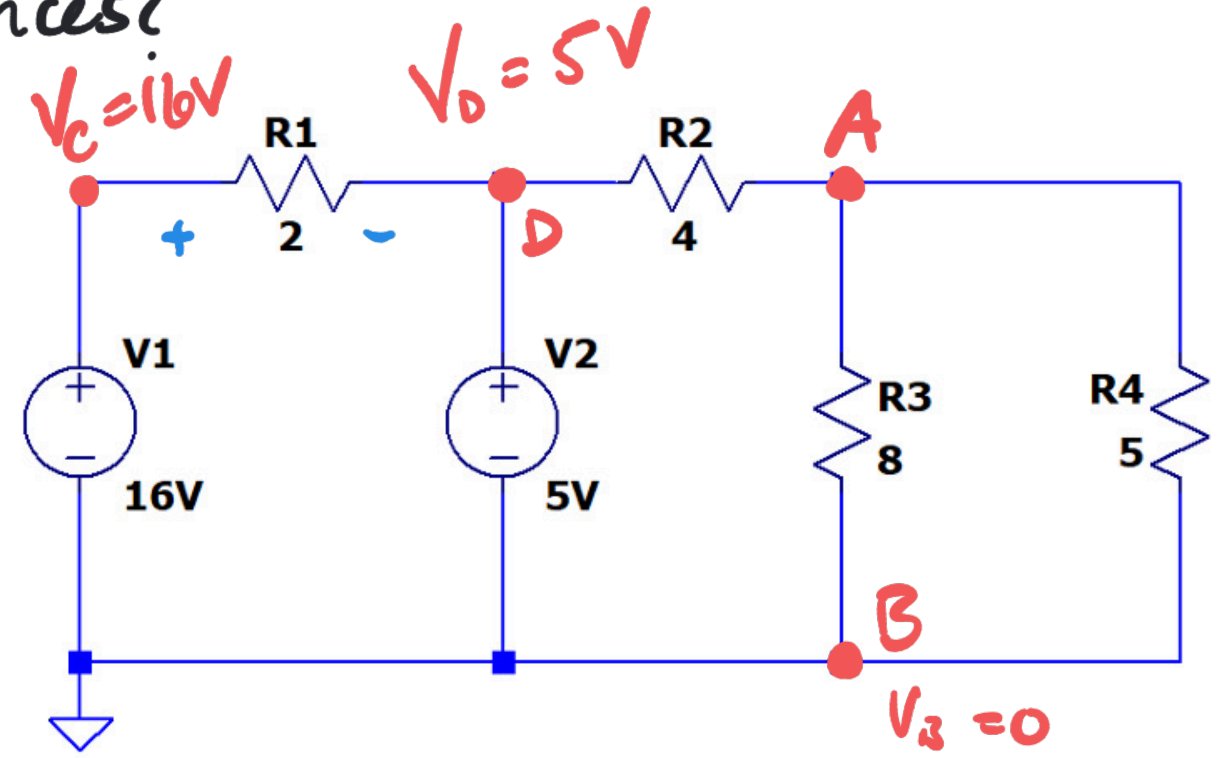
in standard form:

$$\left(1/R_1 + 1/R_2\right)V_A + \left(-1/R_2\right)V_B = I_1 + \frac{V_D}{R_1}$$

$$\left(1/2\Omega + 1/4\Omega\right)V_A + \left(-1/4\Omega\right)V_B = 5\times10^{-3}A + \frac{16V}{2\Omega}$$

II | Nodal Analysis with Multiple Voltage Sources

- What happens if we have multiple voltage sources?

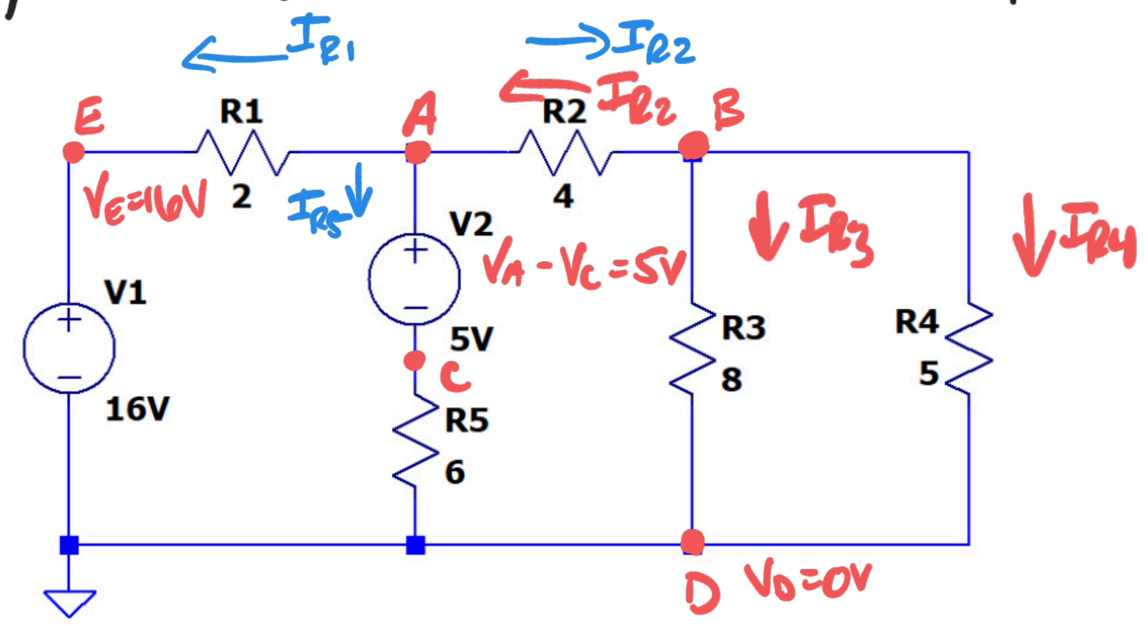


- #eqns = # nodes − # independent sources − 1

  4 − 2 − 1 = 1 equation

- effect: adding voltage sources to the circuit adds constraints to our system (we know the voltage at more nodes) and reduces the number of unknowns/equations

Example: What about this circuit?



1. How many eqns? 5 total nodes − 2 voltage sources − 1 = 2 equations

· Hint: How do we write I_{R5}? We know that the current flowing through the voltage source is the same as the current flowing through R_5 (they are in series)

equations: $V_A - V_C = 5V$ $\quad V_D = 0$

(1) B: $\frac{V_B - V_A}{R_2} + \frac{V_B}{R_3} + \frac{V_B}{R_4} = 0$

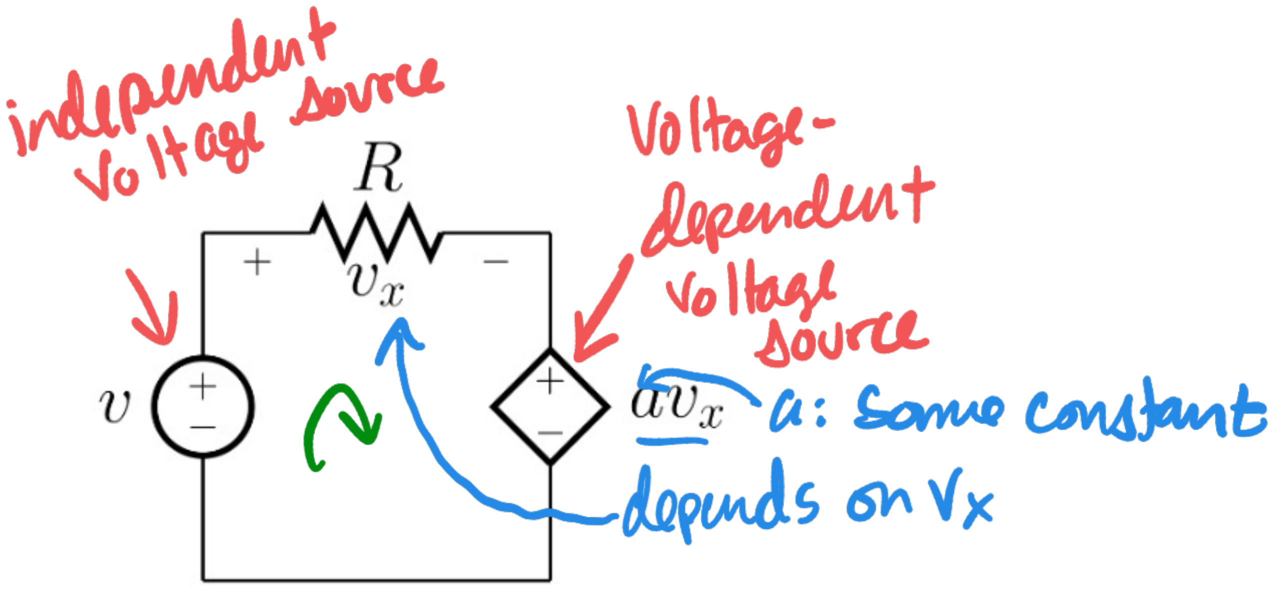
A: $\frac{V_A - V_E}{R_1} + \frac{V_A - V_B}{R_2} + \frac{V_C}{R_5} = 0$ $\quad (I_{R5})$

we know that $V_A - V_C = 5V \rightarrow V_C = V_A - 5V$

(2) $\frac{V_A - V_E}{R_1} + \frac{V_A - V_B}{R_2} + \frac{V_A - 5V}{R_5} = 0$

III | Dependent Sources

- Dependent Source: has a value that's dependent on some other value in the circuit.



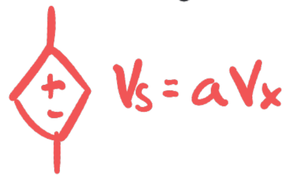
KVL: $-V + V_x + aV_x = 0$

- These aren't "real" sources like batteries - you can't buy one and put it in your circuit. They are used to create simple models of complicated circuits.

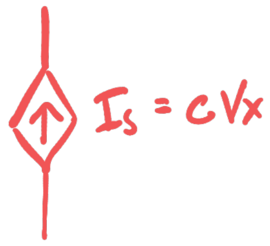
- Types of dependent source

Depends on \ Source Type	Voltage	Current
Voltage	$V_s = aV_x$	$I_s = bV_x$
Current	$V_s = cI_x$	$I_s = dI_x$

- Symbols
  - dependent voltage source



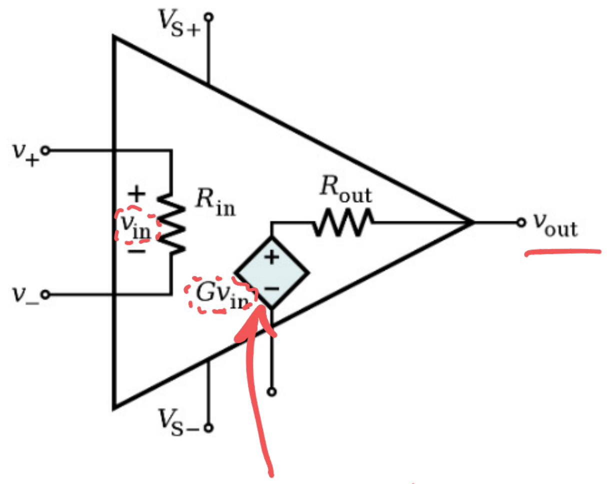
  - dependent current source



- Where do we use dependent sources?

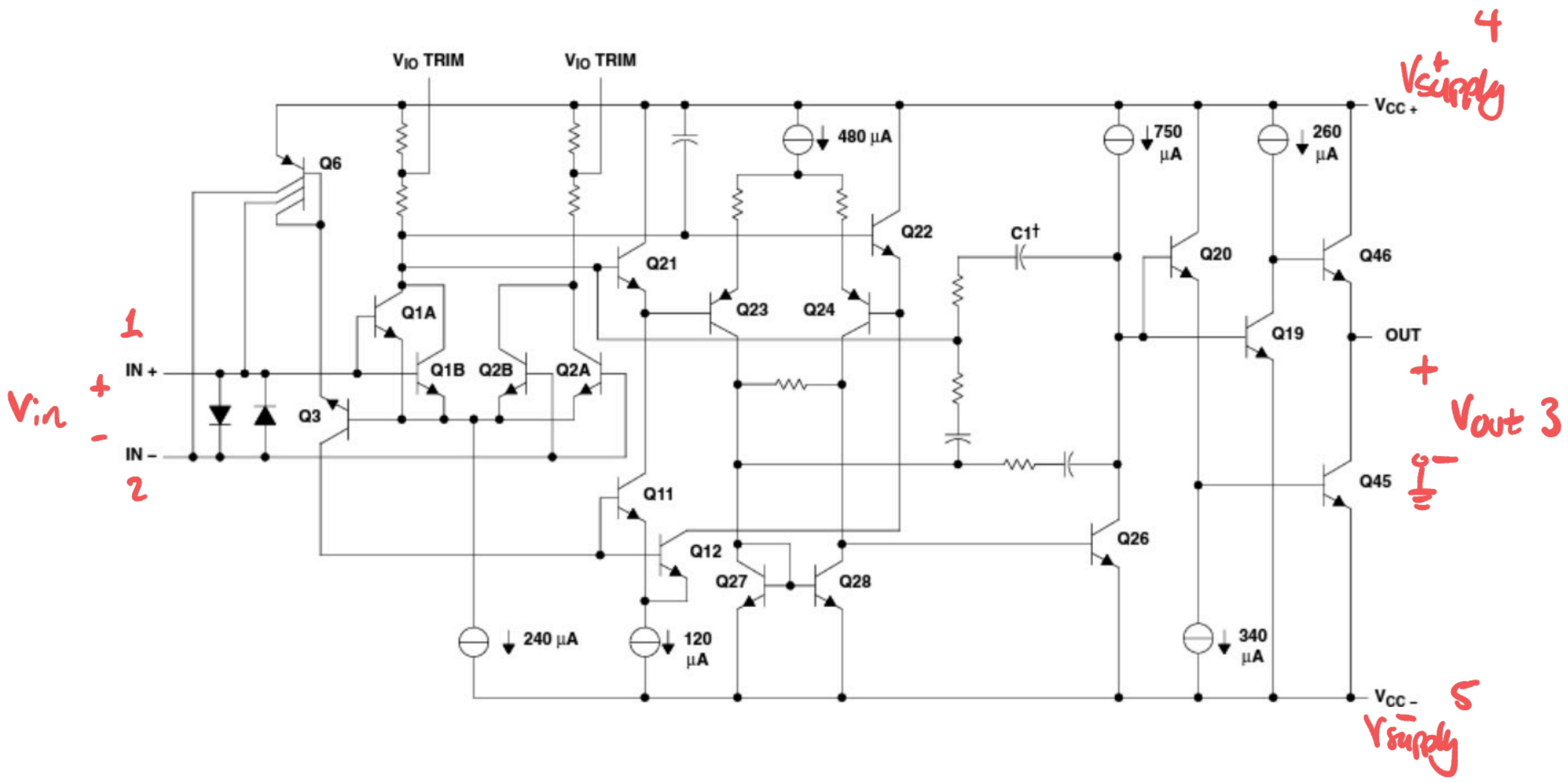
  models of electronic devices or systems

  operational amplifier model

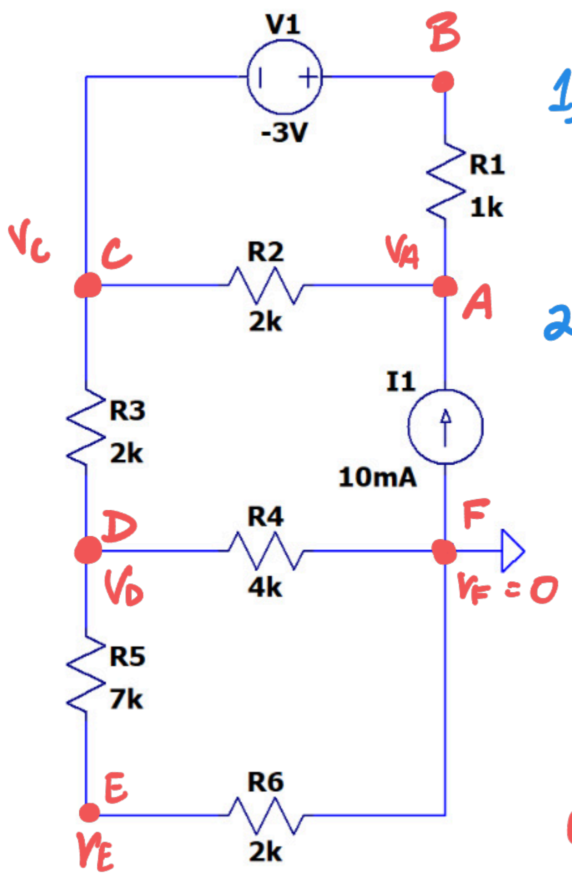


- model says: Vout depends on a constant (G) times Vin

- Why model an op-amp like this?
  - it describes the general behavior of an op-amp without having to analyze the whole circuit



IV | Extra Practice Problem



1) # nodes : 6

voltage sources : 1

unknowns : 6 − 1 − 1 = 4

2) KCL equations + $V_B - V_C = -3V$

<u>A</u>: $\dfrac{V_A - V_B}{R_1} + \dfrac{V_A - V_C}{R_2} - I_1 = 0$

(1) $\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_A + \left(-\frac{1}{R_1}\right)V_B + \left(-\frac{1}{R_2}\right)V_C = I_1$

<u>C</u>: $\dfrac{V_B - V_A}{R_1} + \dfrac{V_C - V_A}{R_2} + \dfrac{V_C - V_D}{R_3} = 0$

(2) $\left(-\frac{1}{R_1} - \frac{1}{R_2}\right)V_A + \left(\frac{1}{R_1}\right)V_B + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)V_C + \left(-\frac{1}{R_3}\right)V_D = 0$

<u>D</u>: $\dfrac{V_D - V_C}{R_3} + \dfrac{V_D}{R_4} + \dfrac{V_D - V_E}{R_5} = 0$

(3) $\left(-\frac{1}{R_3}\right)V_C + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_D + \left(-\frac{1}{R_5}\right)V_E = 0$

<u>E</u>: $\dfrac{V_E - V_D}{R_5} + \dfrac{V_E}{R_6} = 0$

(4) $\left(-\frac{1}{R_5}\right)V_D + \left(\frac{1}{R_5} + \frac{1}{R_6}\right)V_E = 0$

· we can either substitute in $V_B = -3V + V_A$ everywhere and solve 4 equations <u>OR</u> keep our 4 equations as they are (in terms of V_A, V_B, V_C, V_D, V_E) and add a 5th equation : $V_B - V_C = -3V$ (5)

$$\begin{array}{c}1\\2\\3\\4\\5\end{array}\begin{bmatrix} \left(\frac{1}{1000}+\frac{1}{2000}\right) & \left(-\frac{1}{1000}\right) & \left(-\frac{1}{2000}\right) & 0 & 0 \\ \left(-\frac{1}{1000}-\frac{1}{2000}\right) & \left(\frac{1}{1000}\right) & \left(\frac{1}{2000}+\frac{1}{2000}\right) & \left(-\frac{1}{2000}\right) & 0 \\ 0 & 0 & \left(-\frac{1}{2000}\right) & \left(\frac{1}{2000}+\frac{1}{4000}+\frac{1}{7000}\right) & \left(-\frac{1}{7000}\right) \\ 0 & 0 & 0 & \left(-\frac{1}{7000}\right) & \left(\frac{1}{7000}+\frac{1}{2000}\right) \\ 0 & 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \\ V_E \end{bmatrix} = \begin{bmatrix} 10\times10^{-3} \\ 0 \\ 0 \\ 0 \\ -3 \end{bmatrix} \rightarrow \begin{cases} V_A = 52.36V \\ V_B = 44.69V \\ V_C = 47.69V \\ V_D = 27.69V \\ V_E = 6.15V \end{cases}$$

Upcoming Assignment Due Dates

1. Problem Set #5 Late Due Date : 10/16
2. Mid-Semester Survey Due Date: 10/16
3. Lab 02 Parts A & B Proof of Concepts
   Due Date: 10/19
4. Problem Set #6
   Due Date: 10/19