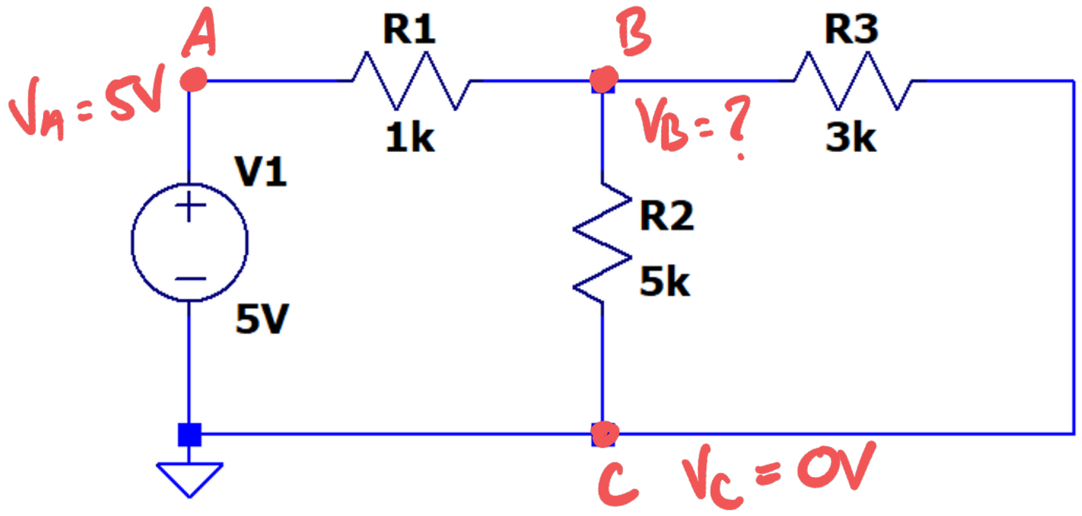
Intro to ECSE Class 12 :
Circuit Analysis Method #3: Nodal Analysis

I] How is it different from KCL/KVL/Ohm's Law?

1. It's more "efficient". No more "guessing" what the right equations are and usually fewer equations.

2. Covers more of the circuit: handles voltage and current sources more easily.

3. Unknowns are now nodal voltages, not resistor voltages/currents



- How many unknowns?
  - KCL/KVL/Ohm's Law method: 3 resistors = 3 unknowns
  - Nodal Analysis method: 1 unknown nodal voltage V_B

  ↳ there is an equation for this:

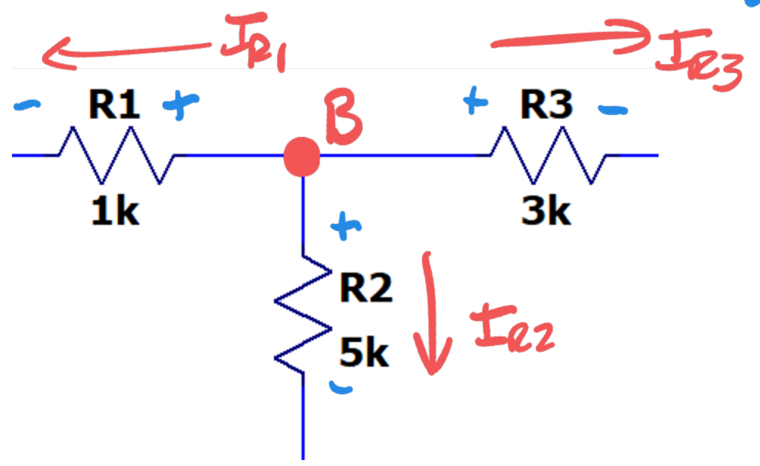
  \# unknowns = # nodes − # voltage sources − 1

  (# voltage sources: known nodal voltages; 1: ground)

- We only use KCL in nodal analysis: How many equations do we need?

  1 equation

4. Passive sign convention: we set the direction of current flow first, as opposed to how we measure voltage differences (the "+" and "-" signs)



. We must choose whether current entering or leaving a node positive or negative

→ Doesn't matter → just be consistent

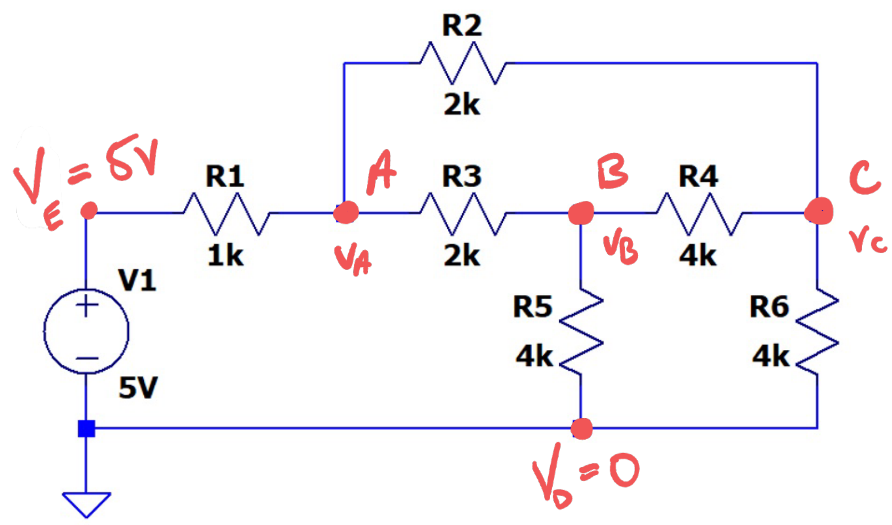
II| Nodal Analysis Method Steps

1. Determine the number of unknowns / how many equations we need

$$\#\, unknowns = \#\, nodes - \#\, voltage\ sources - 1$$

2. Label all nodes and the known nodal voltages (sources and ground)
3. Write a KCL equation for each node with an unknown voltage
   a. Write equations in terms of nodal voltages (V_A, V_B, V_C, etc.) → new
   b. Put equations in standard form (optional)
4. Put into matrix form and solve

III | Nodal Analysis Example Problem



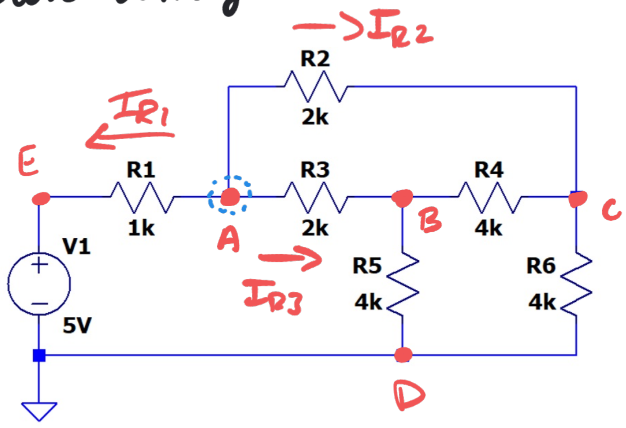
1. Determine the number of unknowns/equations required

$\#$ unknowns $= 5 - 1 - 1 = 3$

need 3 equations

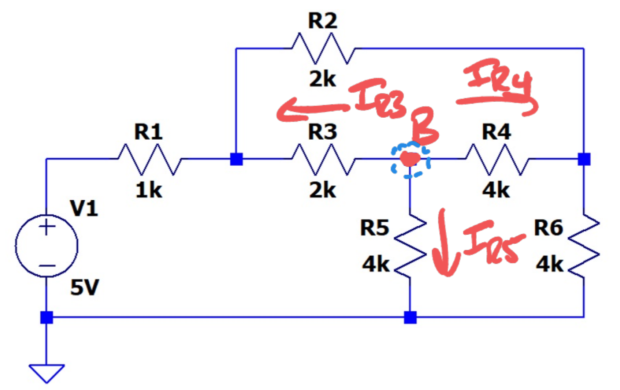
2. Label all nodes and known nodal voltages

3. Write a KCL equation for each node with an unknown voltage

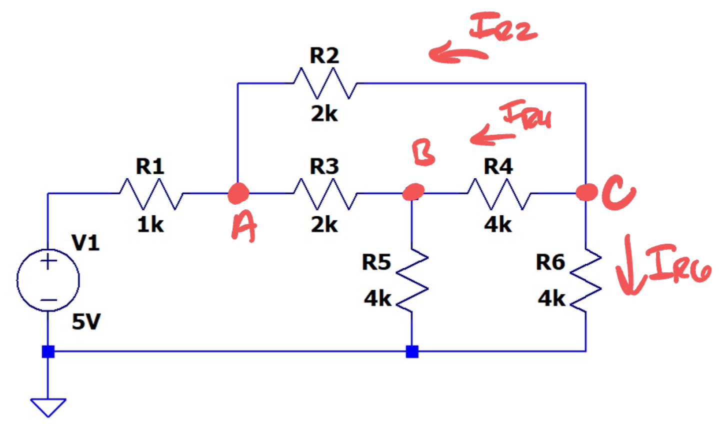


I've defined current exiting a node as positive

A: $I_{R1} + I_{R2} + I_{R3} = 0$



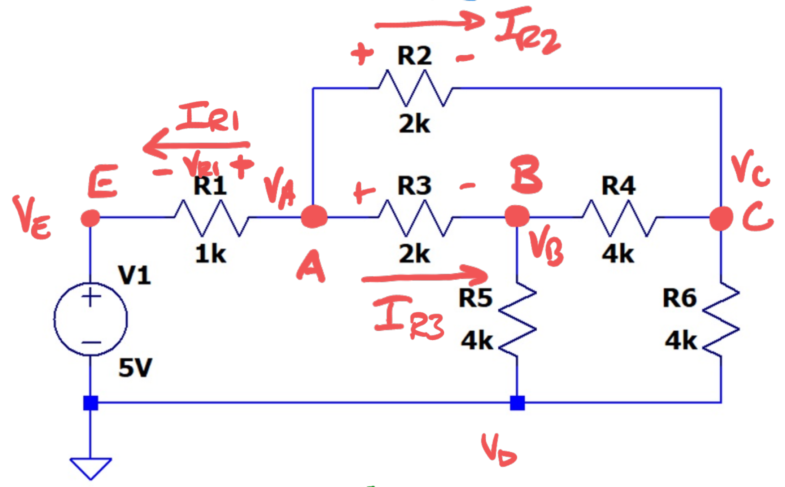
B: $I_{R3} + I_{R4} + I_{R5} = 0$



C: $I_{R2} + I_{R4} + I_{R6} = 0$

a) Write each equation in terms of nodal voltages. How?

A: $I_{R1} + I_{R2} + I_{R3} = 0$



node A: $I_{R1} = \dfrac{V_{R1}}{R_1} = \dfrac{V_A - V_E}{R_1}$

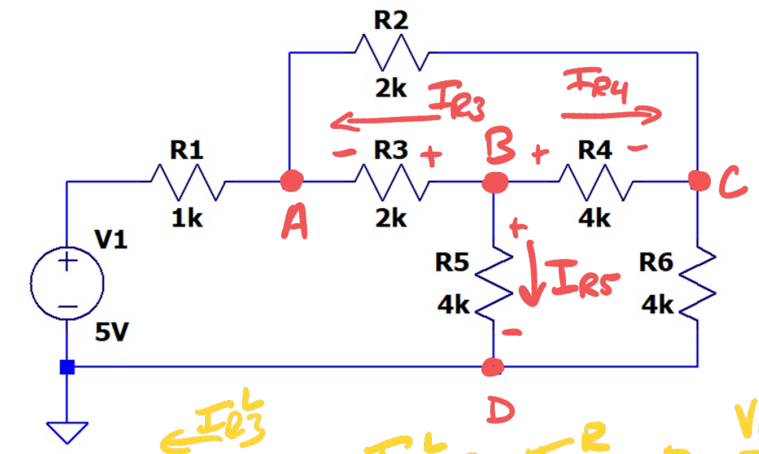
$I_{R2} = \dfrac{V_{R2}}{R_2} = \dfrac{V_A - V_C}{R_2}$

$I_{R3} = \dfrac{V_{R3}}{R_3} = \dfrac{V_A - V_B}{R_3}$

A: $\dfrac{V_A - V_E}{R_1} + \dfrac{V_A - V_C}{R_2} + \dfrac{V_A - V_B}{R_3} = 0$

I_{R3}^{R}

B: $I_{R3} + I_{R4} + I_{R5} = 0$

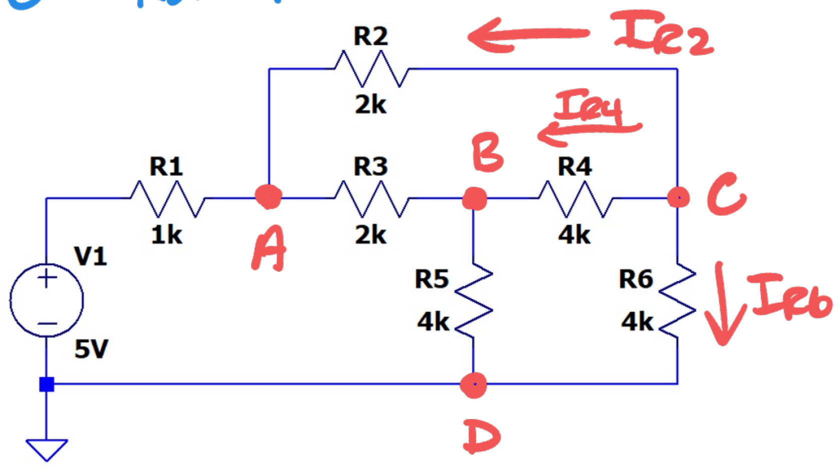


I_{R3}^{L}

$I_{R3}^{L} = -I_{R3}^{R} \Rightarrow \dfrac{V_B - V_A}{R_3} = -\left(\dfrac{V_A - V_B}{R_3}\right)$ ✓

B: $\dfrac{V_B - V_A}{R_3} + \dfrac{V_B - V_C}{R_4} + \dfrac{V_B - V_D}{R_5} = 0$

C: $I_{R2} + I_{R4} + I_{R6} = 0$



C: $\dfrac{V_C - V_A}{R_2} + \dfrac{V_C - V_B}{R_4} + \dfrac{V_C - V_D}{R_6} = 0$

b. Put equations in standard form (optional)

Standard form:

$$(\ldots)V_A + (\ldots)V_B + (\ldots)V_C = \text{Constants}$$

5V, const.

$$A: \frac{V_A - V_E}{R_1} + \frac{V_A - V_C}{R_2} + \frac{V_A - V_B}{R_3} = 0$$

$V_E = 5V$ constant

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_A + \left(-\frac{1}{R_3}\right)V_B + \left(-\frac{1}{R_2}\right)V_C = \frac{V_E}{R_1}$$

$$B: \frac{V_B - V_A}{R_3} + \frac{V_B - V_C}{R_4} + \frac{V_B - V_D}{R_5} = 0$$

$$\left(-\frac{1}{R_3}\right)V_A + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_B + \left(-\frac{1}{R_4}\right)V_C = \frac{V_D}{R_5}$$

$V_D = 0$ constant

$$C: \frac{V_C - V_A}{R_2} + \frac{V_C - V_B}{R_4} + \frac{V_C - V_D}{R_6} = 0$$

$$\left(-\frac{1}{R_2}\right)V_A + \left(-\frac{1}{R_4}\right)V_B + \left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_6}\right)V_C = \frac{V_D}{R_6}$$

constant

4. Put into matrix form & solve

$$A: \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_A + \left(-\frac{1}{R_3}\right)V_B + \left(-\frac{1}{R_2}\right)V_C = \frac{V_E}{R_1}$$

$$B: \left(-\frac{1}{R_3}\right)V_A + \left(1/R_3 + 1/R_4 + 1/R_5\right)V_B + \left(-1/R_4\right) = \frac{V_D}{R_5}$$

$$C: \left(-\frac{1}{R_2}\right)V_A + \left(-1/R_4\right)V_B + \left(1/R_2 + 1/R_4 + 1/R_6\right) = \frac{V_D}{R_6}$$

$$\begin{bmatrix} (1/R_1 + 1/R_2 + 1/R_3) & (-1/R_3) & (-1/R_2) \\ (-1/R_3) & (1/R_3 + 1/R_4 + 1/R_5) & (-1/R_4) \\ (-1/R_2) & (-1/R_4) & (1/R_2 + 1/R_4 + 1/R_6) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} V_E/R_1 \\ V_D/R_5 \\ V_D/R_6 \end{bmatrix}$$

↓ numerical values

$$\begin{bmatrix} (1/1000 + 1/2000 + 1/2000) & (-1/2000) & (-1/2000) \\ (-1/2000) & (1/2000 + 1/4000 + 1/4000) & (-1/4000) \\ (-1/2000) & (-1/4000) & (1/2000 + 1/4000 + 1/4000) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 5/1000 \\ 0 \\ 0 \end{bmatrix}$$

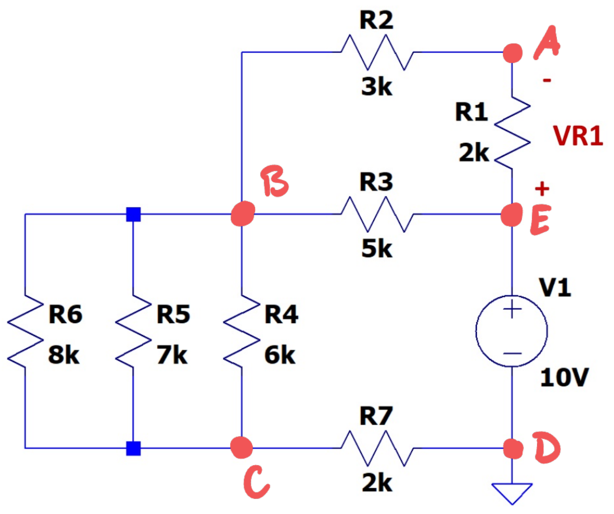
$V_A = 3.75V$

$V_B = 2.50V$

$V_C = 2.50V$

IV| Extra Practice Problem

Using nodal analysis, solve for V_{R1}



1) # equations: 5 nodes − 1 voltage source − 1
= 3 equations

2) Known nodal voltages: $V_D = 0V$; $V_E = 10V$

3) KCL equations:

node A: $\frac{V_A - V_B}{R_2} + \frac{V_A - V_E}{R_1} = 0$

$$(1/R_1 + 1/R_2)V_A + (-1/R_2)V_B = \frac{V_E}{R_1}$$

$$\rightarrow (1/2000 + 1/3000)V_A + (-1/3000)V_B = 10/2000$$

node B: $\frac{V_B - V_A}{R_2} + \frac{V_B - V_E}{R_3} + \frac{V_B - V_C}{R_4} + \frac{V_B - V_C}{R_5} + \frac{V_B - V_C}{R_6} = 0$

$$(-1/R_2)V_A + (1/R_2 + 1/R_3 + 1/R_4 + 1/R_5 + 1/R_6)V_B + (-1/R_4 - 1/R_5 - 1/R_6)V_C = \frac{V_E}{R_3}$$

$$\rightarrow (-1/3000)V_A + (1/3000 + 1/5000 + 1/6000 + 1/7000 + 1/8000)V_B + (-1/6000 - 1/7000 - 1/8000)V_C = \frac{10}{5000}$$

node C: $\frac{V_C - V_B}{R_4} + \frac{V_C - V_B}{R_5} + \frac{V_C - V_B}{R_6} + \frac{V_C - V_D}{R_7} = 0$

$$(-1/R_4 - 1/R_5 - 1/R_6)V_B + (1/R_4 + 1/R_5 + 1/R_6 + 1/R_7)V_C = \frac{V_D}{R_7}$$

$$\rightarrow (-1/6000 - 1/7000 - 1/8000)V_B + (1/6000 + 1/7000 + 1/8000 + 1/2000)V_C = 0$$

$$\begin{bmatrix} (1/2000 + 1/3000) & (-1/3000) & 0 \\ (-1/3000) & (1/3000 + 1/5000 + 1/6000 + 1/7000 + 1/8000) & (-1/6000 - 1/7000 - 1/8000) \\ 0 & (-1/6000 - 1/7000 - 1/8000) & (1/6000 + 1/7000 + 1/8000 + 1/2000) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 10/2000 \\ 10/5000 \\ 0 \end{bmatrix}$$

$V_A = 8.530V$

$V_B = 6.324V$

$V_C = 2.941V$

$V_{R1} = V_E - V_A = 10V - 8.530$
$= \underline{1.470V}$

Upcoming Assignments & Due Dates

1) Project Plan update (before Lab02) Due 10/10
   - Incorporate feedback from 1st project plan
   - Make any changes you want to
   - Add detail for lab02 and lab03

2) Metacognition journal entry on EE vs. CSE vs. CS Due 10/10

3) Problem Set #4 : Nodal Analysis
   Due 10/10

4) Proof of Skills Optimization #1
   (See instructions on course website under Course Resources/Proof of Skills) Due 10/12

5) Problem Set #5 : Nodal Analysis Practice
   Due 10/12

* Next Thursday 10/12 : Plan of study review!
- Complete your plan of study and bring a copy with you!