Intro to ECSE Class 08:
Circuit Reduction & KCL/KVL/Ohm's Law I

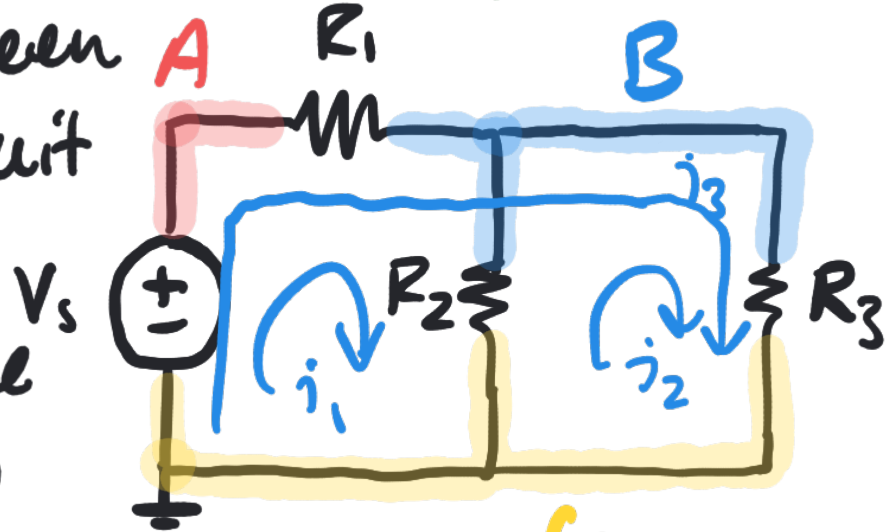
I | KCL & KVL: Nodes and Loops

- KCL: all currents flowing into or out of a node sum to zero
- KVL: all voltages around a loop sum to zero
- Ohm's Law: $V = IR$

1. What is a node?

how many nodes? 3

- any connection between any number of circuit components
- in circuits we assume ΔV across a wire is 0



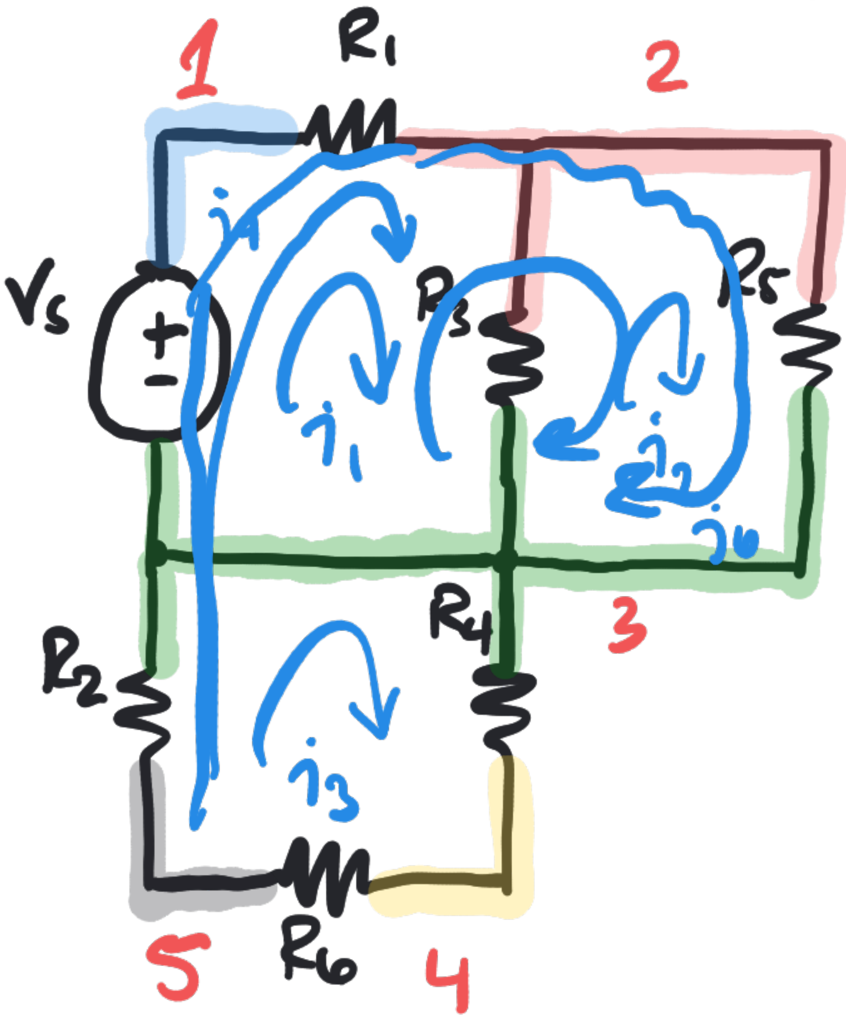
- All parts of a circuit connected to a node only by a wire have the same voltage as the node.

2. What is a loop?

how many loops? 3

- Any enclosed circle in a circuit

Example: How many nodes are in the circuit below? How many loops?



nodes: 5

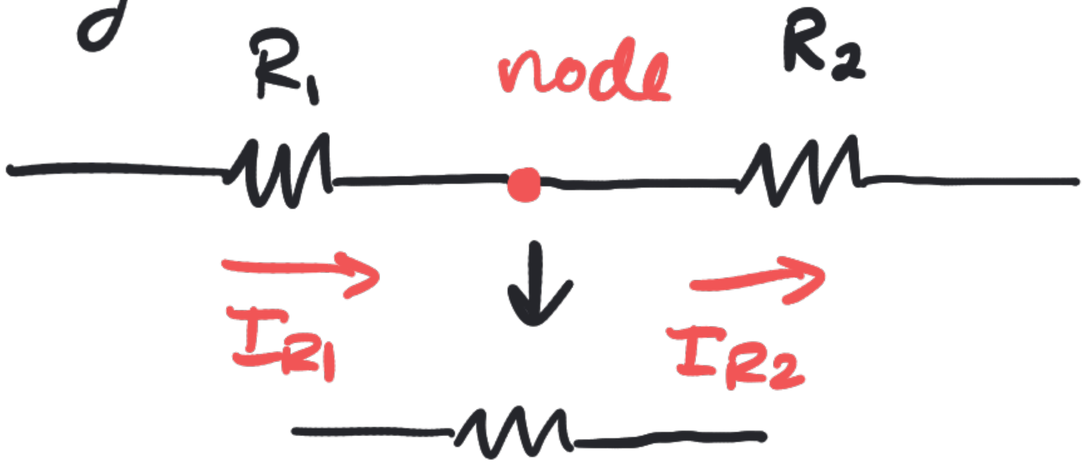
loops: 6

II| Circuit Analysis Method #1: Circuit Reduction

We can solve some circuits just by combining resistances

1. Combining resistors in series

- resistors in series have one common node with no other components sharing it



$$R_{eq} = R_1 + R_2$$

equivalent resistance

- in general, for N resistors $R_1, \ldots, R_i, \ldots, R_N$

$$R_{eq} = \sum_{j=1}^{N} R_i$$

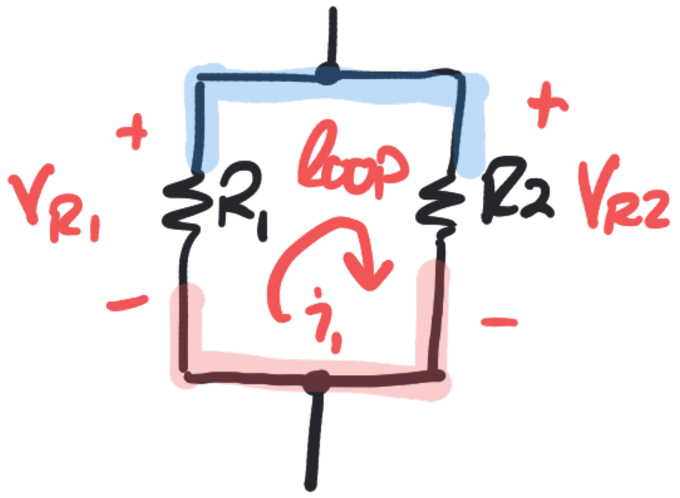
- if we apply KCL at the node between two resistors in series:

$$-I_{R1} + I_{R2} = 0 \rightarrow I_{R1} = I_{R2}$$

- resistors in series have the same current

2. Combining resistors in parallel

- resistors in parallel form a loop containing only two resistors



$\rightarrow$

$$R_{eq} = \left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

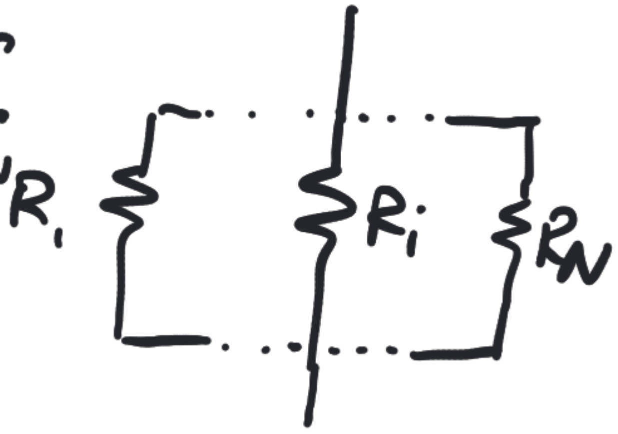
$$= \frac{R_1 R_2}{R_1+R_2}$$

for 2 R only

- in general, for N resistors in parallel $R_1, \dots, R_i, \dots, R_N$:

$$= \frac{1}{\frac{1}{R_1}+\dots+\frac{1}{R_i}+\dots\frac{1}{R_N}}$$

$$R_{eq} = \left(\sum_{j=1}^{N} 1/R_i\right)^{-1}$$



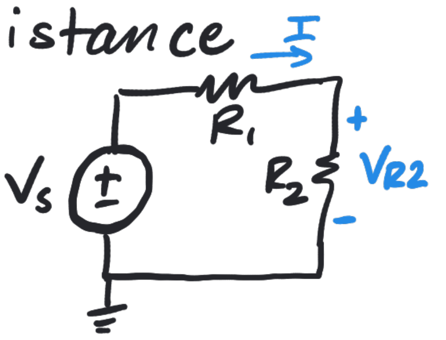
- if we apply KVL around the loop containing two resistors in parallel:

$$-V_{R1} + V_{R2} = 0 \rightarrow V_{R1} = V_{R2}$$

- resistors in parallel have the same voltage drop across them

3. The Voltage Divider

- In a series circuit with a voltage source V_s, the voltage gets divided up among the resistors proportional to the ratio of the individual resistance to the total series resistance



$$V_{R2} = V_s \frac{R_2}{\underbrace{R_1 + R_2}}$$

Resistor we want the voltage across

source

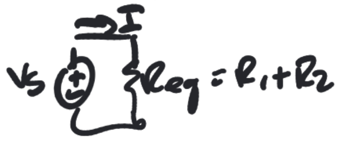
total series resistance

- where does this come from? Let's find an expression for the voltage across V_{R2}:

$$V_{R2} = I \cdot R_2 \qquad \text{Ohm's Law}$$

unknown

- What is I?



$$I = \frac{V_s}{R_{eq}} = \frac{V_s}{R_1 + R_2} \qquad \text{Ohm's Law}$$

- Calculate V_{R2}:

$$V_{R2} = I \cdot R_2 = \frac{V_s}{R_1+R_2} \cdot R_2 \rightarrow V_{R2} = V_s \frac{R_2}{R_1 + R_2}$$

we've derived the voltage divider eqn.

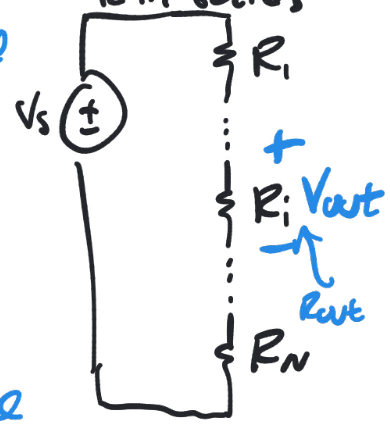
- General form:

$$V_{out} = V_s \frac{R_{out}}{\sum_{i=1}^{N} R_i}$$

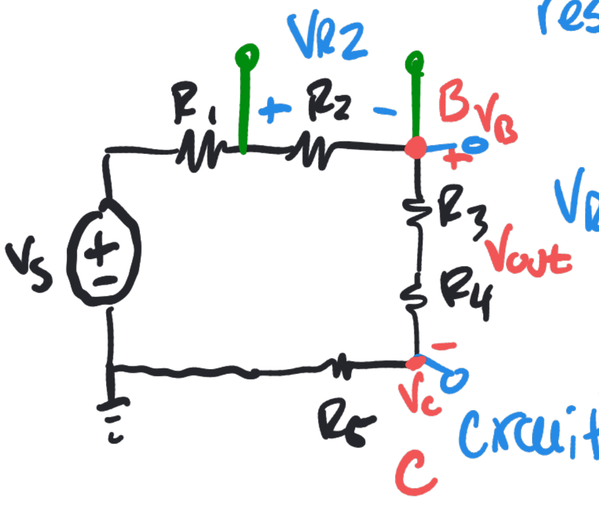
the resistor you want the voltage drop for

total series resistance

we have N R in series



Circuit 1



Circuit 2

$$V_{R2} = V_s \frac{R_2}{R_1 + R_2 + R_3 + R_4 + R_5}$$

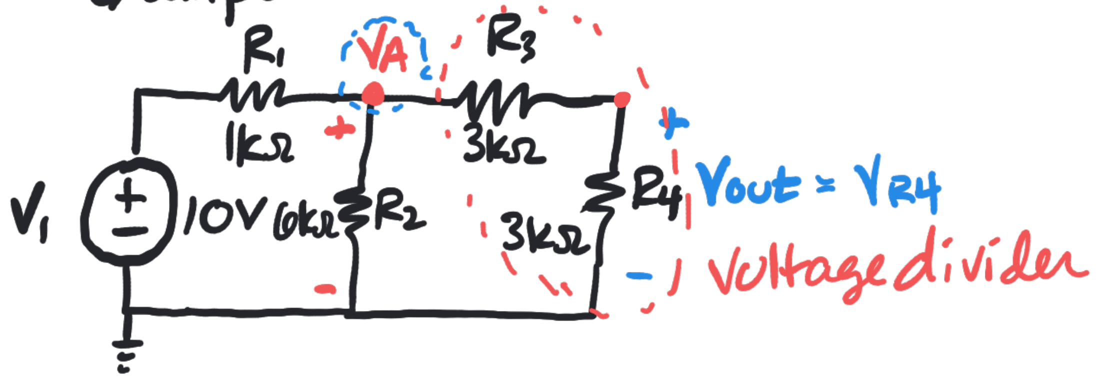
$$V_{out} = V_s \frac{R_3 + R_4}{R_1 + R_2 + R_3 + R_4 + R_5}$$

$$V_{out} = V_B - V_C = V_{R3} + V_{R4}$$

nodal voltages

4. Circuit Reduction Technique

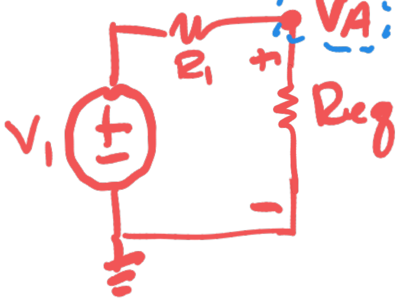
- Strategy: given a circuit with resistors + sources, combine the resistors to simplify the circuit, then use Ohm's Law and the voltage divider equation to solve for the desired quantity
- Example: Find $V_{out} = V_{R4}$



- We want to find V_A because

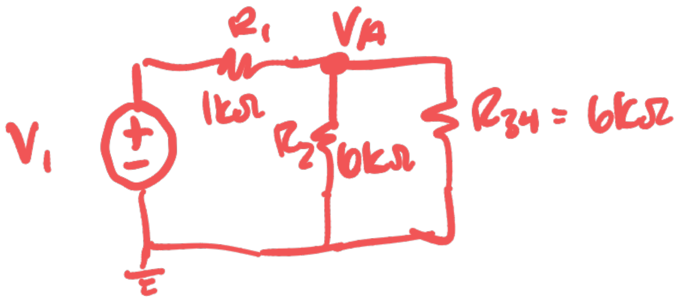
$$V_{out} = V_{R4} = V_A \frac{R_4}{R_3 + R_4} \quad (1)$$

- How do we find V_A? Reduce the circuit to



$$V_A = V_1 \frac{R_{eq}}{R_1 + R_{eq}}$$
(2)

- R_3 & R_4 are in series: $R_{34} = R_3 + R_4$
$= 3k\Omega + 3k\Omega = 6k\Omega$



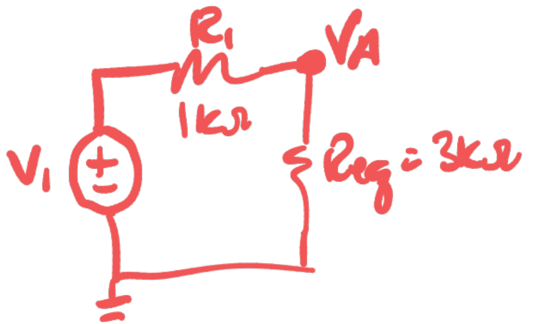
- R_2 and R_{34} are in parallel

in parallel ↓

$$R_{eq} = R_2 \| R_{34} = \frac{R_2 R_{34}}{R_2 + R_{34}} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_{34}}}$$

formal definition

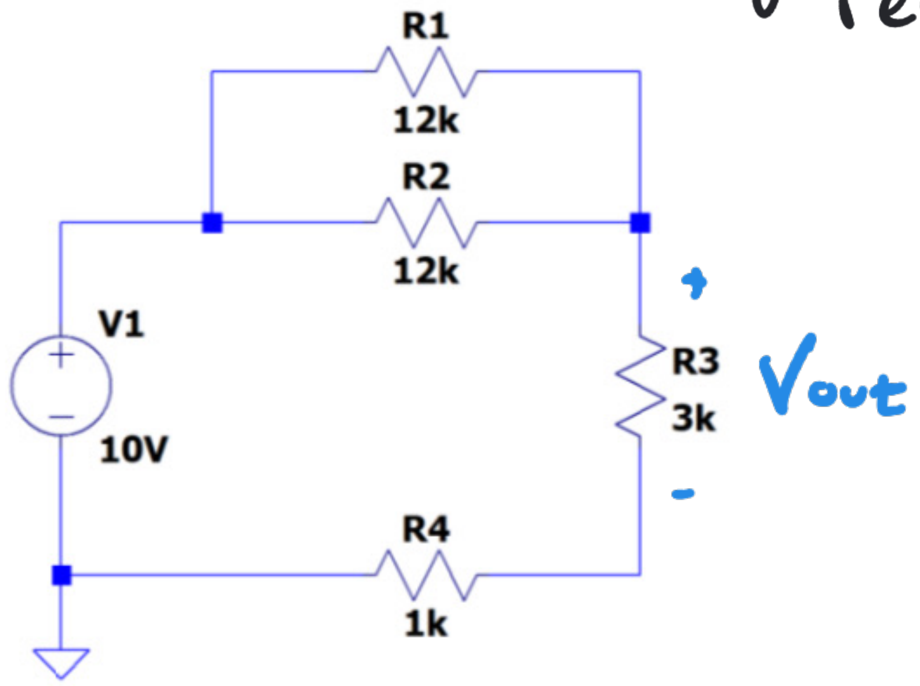
$$= \frac{6k \cdot 6k}{6k + 6k} = \underline{3k\Omega}$$



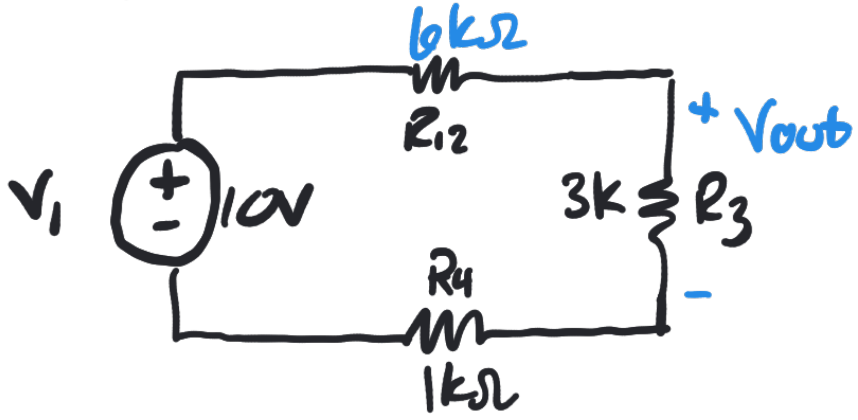
$$(2)\ V_A = V_1 \cdot \frac{R_{eq}}{R_1 + R_{eq}} = 10V \cdot \frac{3k\Omega}{1k\Omega + 3k\Omega} = 10V \cdot \frac{3}{4} = \underline{7.5V}$$

$$(1)\ V_{out} = V_A \cdot \frac{R_4}{R_3 + R_4} = 7.5V \frac{3k\Omega}{3k\Omega + 3k\Omega} = 7.5V \cdot \frac{1}{2} = \underline{3.75V}$$

Example #2: Find Vout using circuit reduction



- If we put the circuit in the following form, we can use a voltage divider



$$R_{12} = R_1 \| R_2 = \frac{1}{\frac{1}{12k\Omega} + \frac{1}{12k\Omega}} = 6k\Omega$$

- Using the voltage divider eqn.:

$$V_{out} = V_{R3} = V_1 \frac{R_3}{R_{12} + R_3 + R_4} = 10V \cdot \frac{3k\Omega}{6k\Omega + 3k\Omega + 1k\Omega}$$

$$= 10V \cdot \frac{3}{10} = \underline{3V}$$