

# Intro to ECSE

## Quiz 3 Solutions

Fall 2024

<b>1.</b>	<b>/10</b>
<b>2.</b>	<b>/15</b>
<b>3.</b>	<b>/10</b>
<b>Total</b>	<b>/35</b>

Name \_\_\_\_\_

RIN \_\_\_\_\_

Notes:

**SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS.** No credit will be given for numbers that appear without justification. Use the backs of pages if there is not enough room on the front.

For partial credit on some questions, you may want to re-draw circuit diagrams as you simplify the circuits.

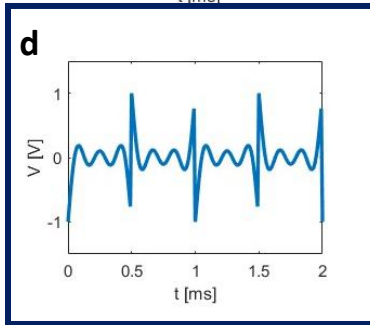
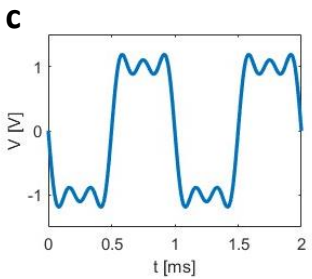
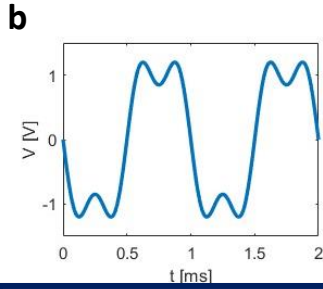
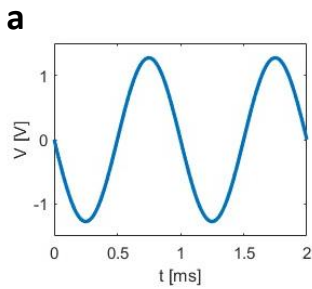
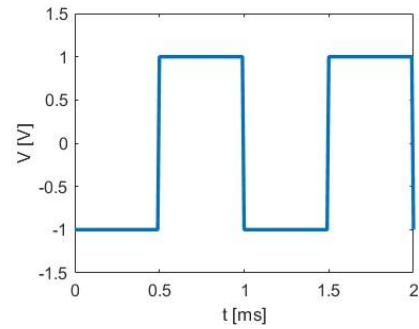
Many problems can be solved using more than one method. check your answers by using a second method.

At least skim through the entire quiz before you begin and then start with the problems you know best. The proctor will only answer clarification questions where wording is unclear or where there may be errors/typos. No other questions will be responded to.

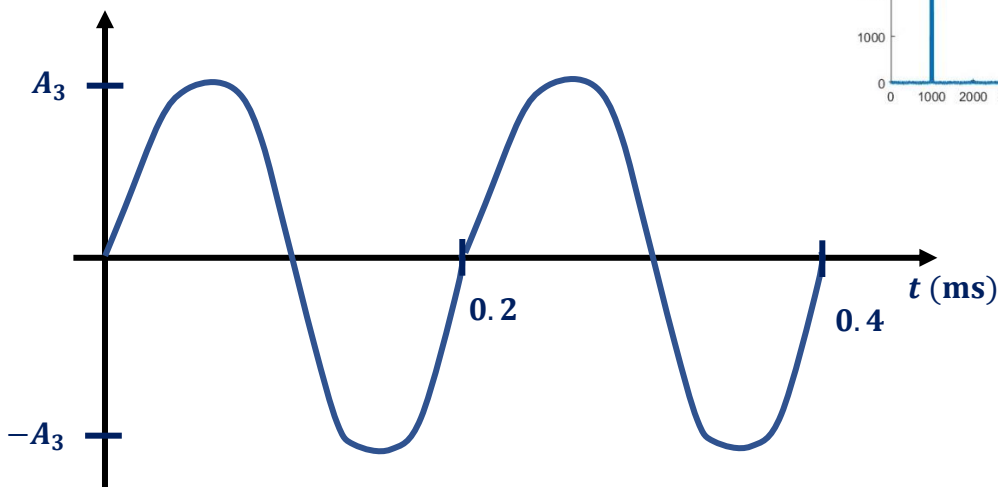
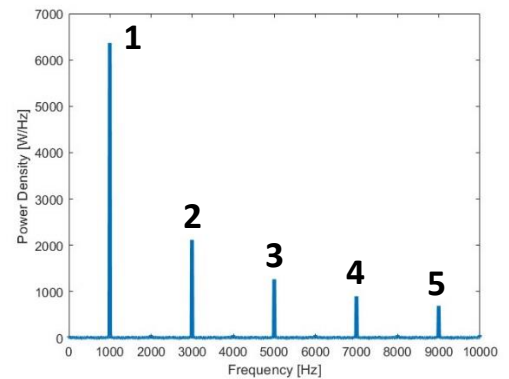
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**Problem 1 [10 Points]: Miscellaneous Concepts and Short Answer Questions**

**1.1** [2 pts] Shown to the right is a square wave in the time domain. Which of the time-domain signals below could result from applying a high-pass filter to the square wave? Circle one:

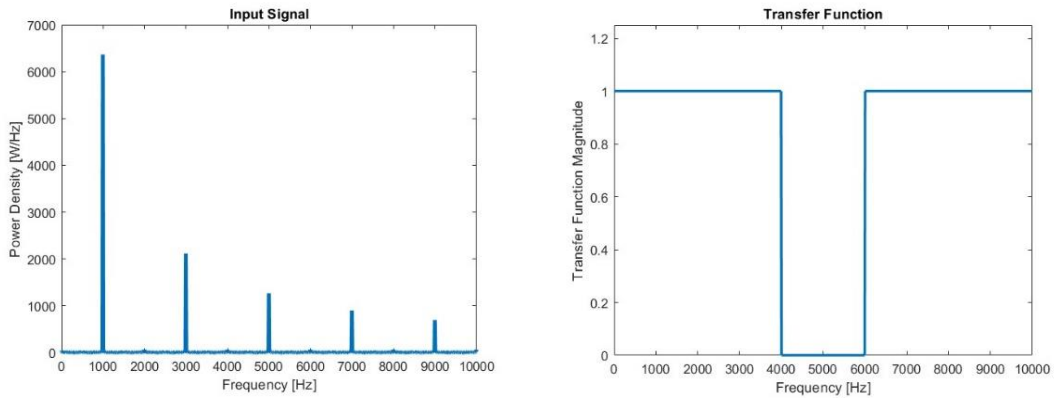


**1.2** [4 pts] The frequency spectrum of a signal is shown to the right. On the axes below, sketch two periods of the time domain waveform corresponding to peak 3 of the frequency spectrum. Label axes, all relevant numerical values and assume the amplitude of the wave is  $A_3$ .



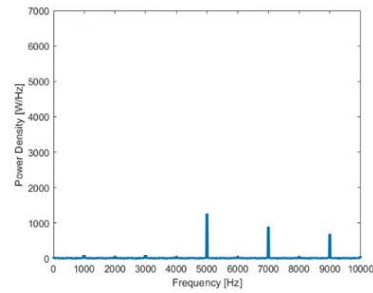
$$T = \frac{1}{f} = 0.2 \text{ ms}$$

1.3 [2 pts] Transfer functions and frequency spectra:

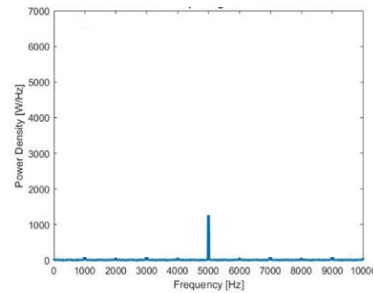


If the signal whose frequency spectrum is shown as “Input Signal” above (left) is passed through a system with the transfer function shown above (right), what would be the output signal’s frequency spectrum? Circle one:

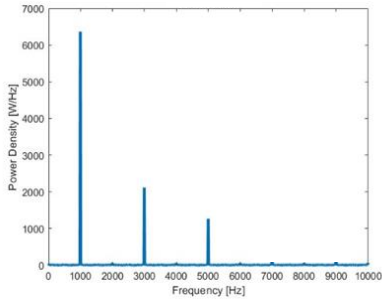
**a**



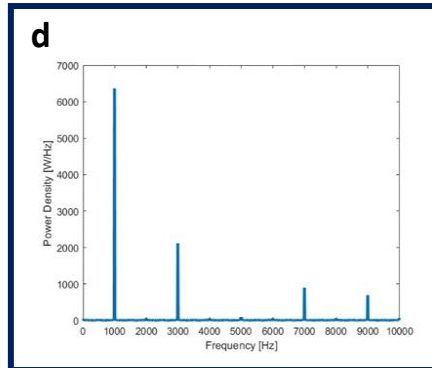
**b**



**c**



**d**



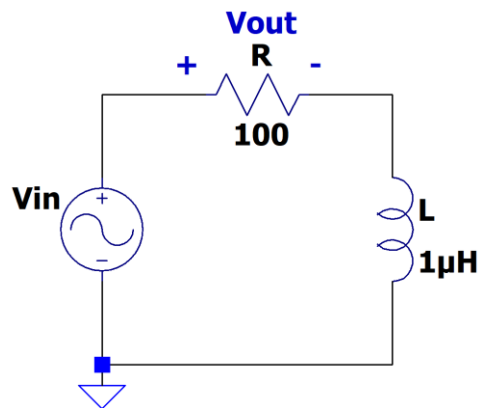
1.4 [2 pts] Convert the voltage transfer function magnitude  $|H_v| = 0.25$  to decibels.

$$|H_{v,dB}| = 20 \log_{10}(0.25) = -12.04 \text{ dB}$$

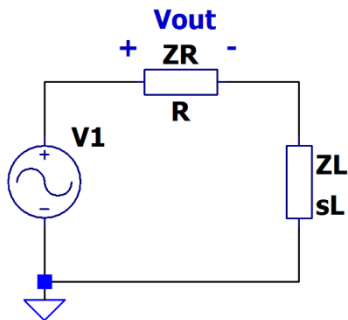
**$H_v$  in dB:**

**-12.04 dB**

**Problem 2 [15 Points]: First Order Filters**



- a. [2 pts] Redraw the circuit in terms of impedances as a function of  $s$ . You do not have to insert numerical values for the components.



- b. [3 pts] Find the transfer function for the circuit above and express it in the form  $H(s) = \frac{N(s)}{s+\alpha}$ , where  $N(s)$  is a function of  $s$  and  $\alpha$  is a constant. Express the transfer function in terms of  $R$ ,  $L$  and  $s$ .

$$H(s) = \frac{Z_R}{Z_R + Z_L} = \frac{R}{R + sL} \frac{1/L}{1/L} = \frac{R/L}{s + R/L}$$

$H(s) =$

$$\frac{R/L}{s + R/L}$$

c. [2 pts] Calculate the numerical value for the corner frequency for this circuit in Hz.

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi} \frac{R}{L} = \frac{1}{2\pi} \frac{100\Omega}{1 \times 10^{-6}\text{H}} = 1.59 \times 10^7 \text{ Hz}$$

**Corner Frequency:  $1.59 \times 10^7 \text{ Hz}$**

d. [2 pts] Find the magnitude of the transfer function  $|H(j\omega)|$ . Express the magnitude of the transfer function in terms of  $R$ ,  $L$  and  $\omega$ .

$$|H(j\omega)| = \sqrt{H(j\omega)H(-j\omega)} = \sqrt{\left(\frac{R/L}{j\omega + R/L}\right)\left(\frac{R/L}{-j\omega + R/L}\right)} = \frac{\left(\frac{R}{L}\right)}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}$$

$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}$$

e. [2 pts] Evaluate the magnitude of the transfer function at the frequency limits  $\omega \rightarrow 0$  and  $\omega \rightarrow \infty$ .

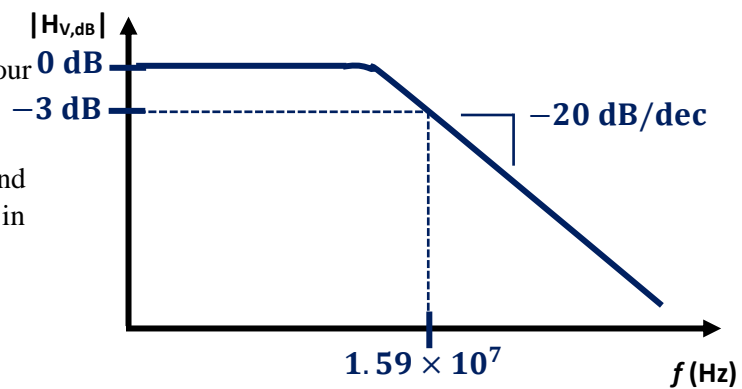
For  $\omega \rightarrow 0$ :  $|H(j\omega)| = \frac{R/L}{\sqrt{0^2 + \left(\frac{R}{L}\right)^2}} \rightarrow 1$

For  $\omega \rightarrow \infty$ :  $|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}} \approx \frac{R/L}{\omega} \rightarrow 0$

$$|H(\omega \rightarrow 0)| = 1$$

$$|H(\omega \rightarrow \infty)| = 0$$

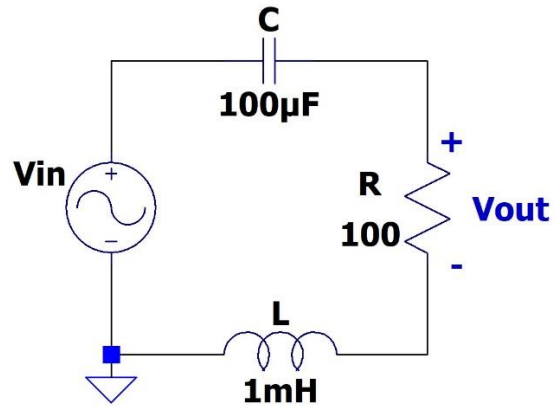
- f. [3 pts] Sketch the Bode plot of the magnitude of the transfer function for your circuit on a **log-log scale** (decibels vs. frequency in Hz) to the right. Label the location of the corner frequency in dB and frequency, as well as the filter's roll-off in decibels per decade.



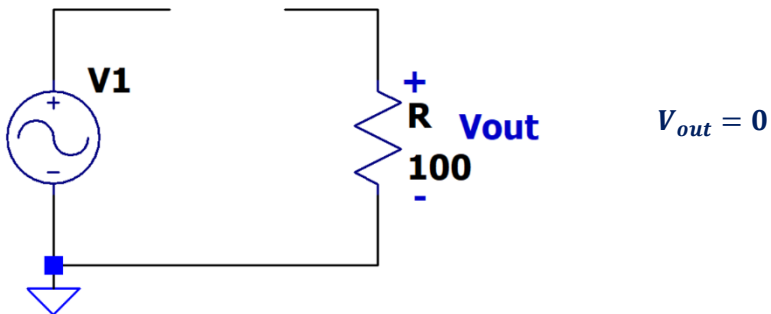
- g. [1 pt] What kind of filter is represented by the Bode plot you drew in part f?

**Filter Type:**  
**Low-pass**

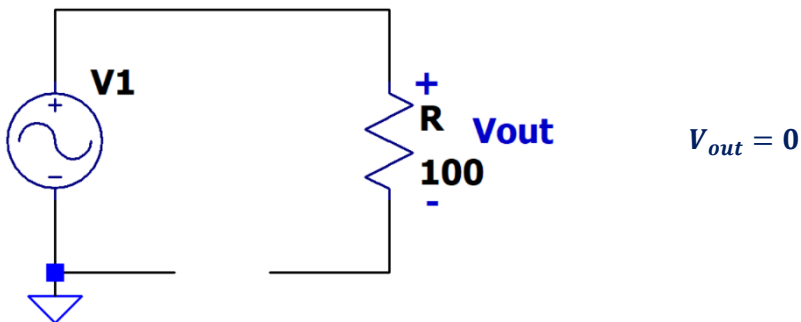
**Problem 3 [10 Points]: Second Order Filters**



- a. [2 pts] Redraw the equivalent circuit for the case  $s \rightarrow 0$  and determine  $V_{out}$  in terms of  $V_{in}$ .



- b. [2 pts] Redraw the equivalent circuit for the case  $s \rightarrow \infty$  and determine  $V_{out}$  in terms of  $V_{in}$ .

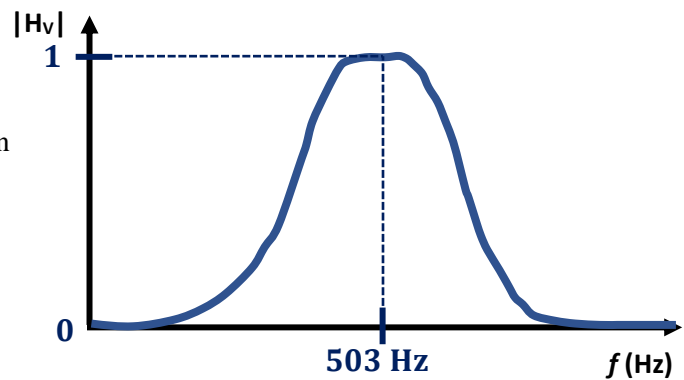


- c. [2 pts] Calculate the numerical value for the center frequency of this circuit in Hz.

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} = \frac{1}{2\pi \sqrt{1 \times 10^{-3} \text{H} \cdot 100 \times 10^{-6} \text{F}}} = 503 \text{ Hz}$$

Center  
Frequency: 503 Hz

- d. [3 pts] Sketch the Bode plot of the magnitude of the transfer function for your circuit on a **log-linear scale** to the right. Label the location of the center frequency in terms of  $|H|$  and frequency.



- e. [1 pt] What kind of filter is represented by the Bode plot you drew in part f?

Filter Type:

Band-pass