

Intro to ECSE

Quiz 3

Fall 2023

1.	/13
2.	/27
3.	/8
Total	/48

Name Solutions

RIN _____

Section _____

Notes:

SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

Use the backs of pages if there is not enough room on the front.

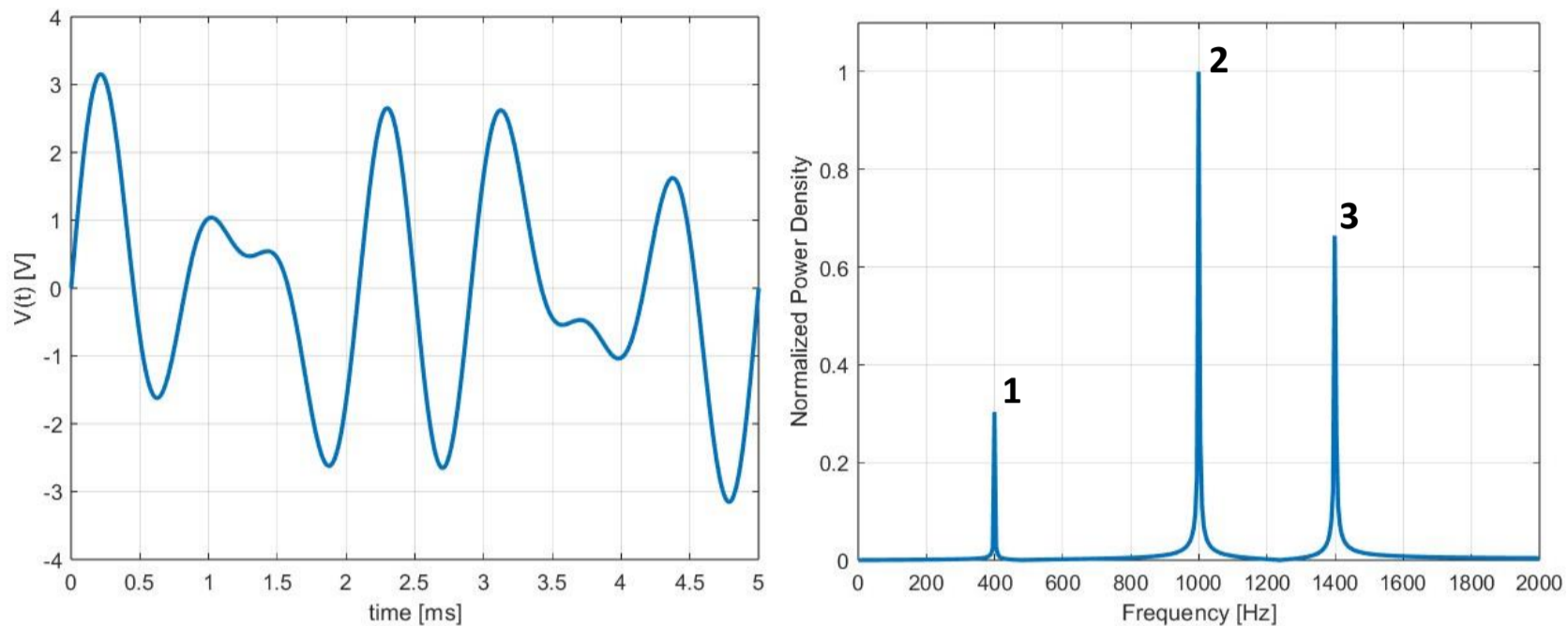
For partial credit on some questions, you may want to re-draw circuit diagrams as you simplify the circuits.

Many problems can be solved using more than one method. check your answers by using a second method.

At least skim through the entire quiz before you begin and then start with the problems you know best. The proctor will only answer clarification questions where wording is unclear or where there may be errors/typos. No other questions will be responded to.

Problem 1 [13 Points]: Time Domain vs. Frequency Domain

1.1 [10 pts] Answer the following questions about the signal below, whose time-domain and frequency-domain representations are shown.



- a. [3 pts] Write down the time-domain expression $f(t)$ for this signal in terms of sinusoids. You may use variables (such as A_1 , A_2 , etc.) for the amplitudes of individual signal components, but you must specify the frequency of each signal component numerically.

$$f(t) = A_1 \sin(2\pi \cdot 400t + \phi_1) + A_2 \sin(2\pi \cdot 1000t + \phi_2) + A_3 \sin(2\pi \cdot 1400t + \phi_3)$$

[+2] correct function (sum of sinusoids)

[+1] correct frequencies (in rad/s)

- b. [2 pts] Which of the peaks in the frequency-domain representation of the signal corresponds to the highest pitch? Explain.

Peak 3 corresponds to the highest pitch. Higher frequency equates to higher audible pitch and peak 3 has the highest frequency.

[+1] correct peak

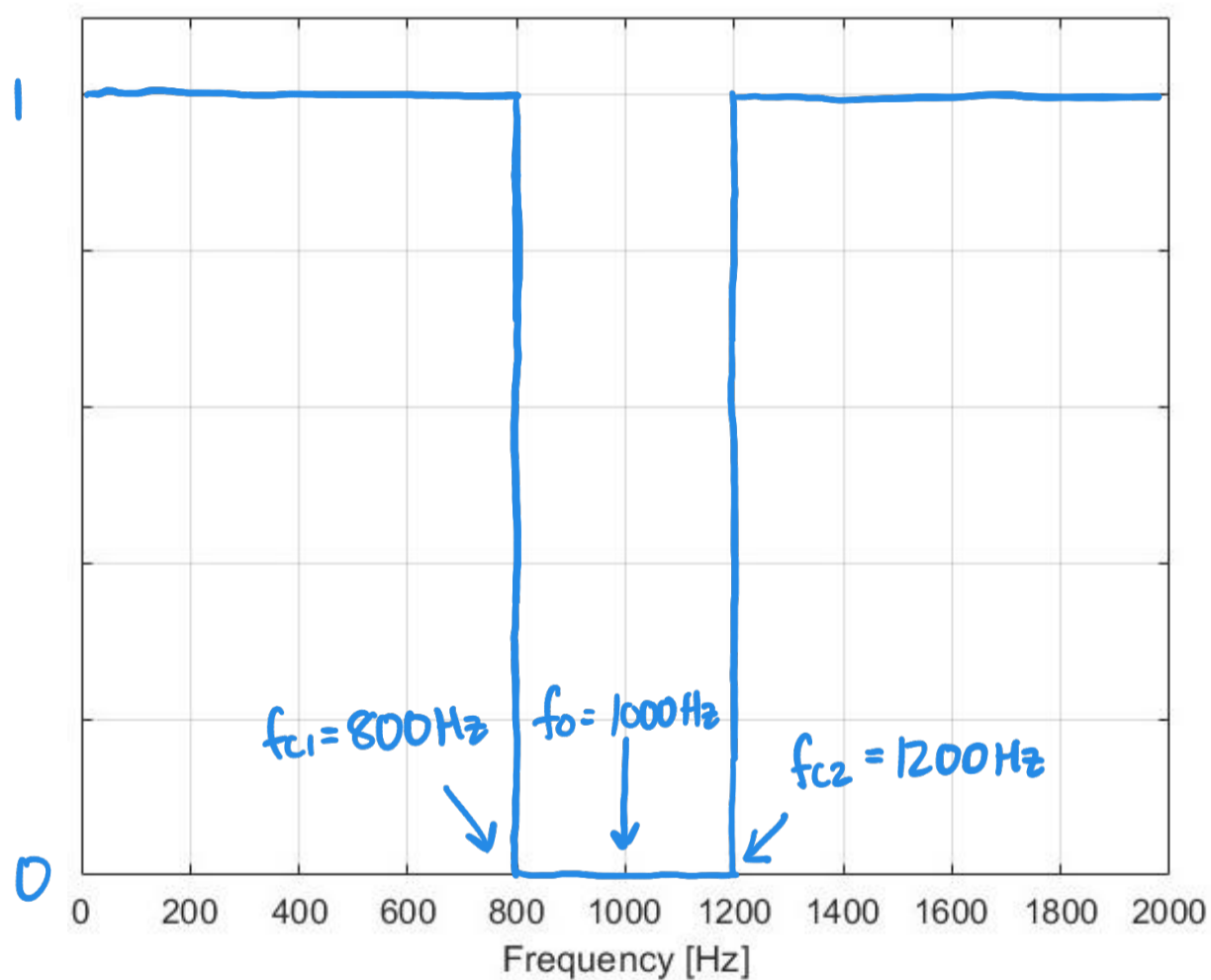
[+1] valid justification

- c. [2 pts] What kind of filter would you use to eliminate only the 2nd peak of the signal in the frequency domain (and not impact the others)?

band-stop filter

[+2] Correct filter type

- d. [3 pts] Draw the transfer function for the **ideal filter** you specified in part c. Label all relevant values for your transfer function on the y-axis and x-axis.

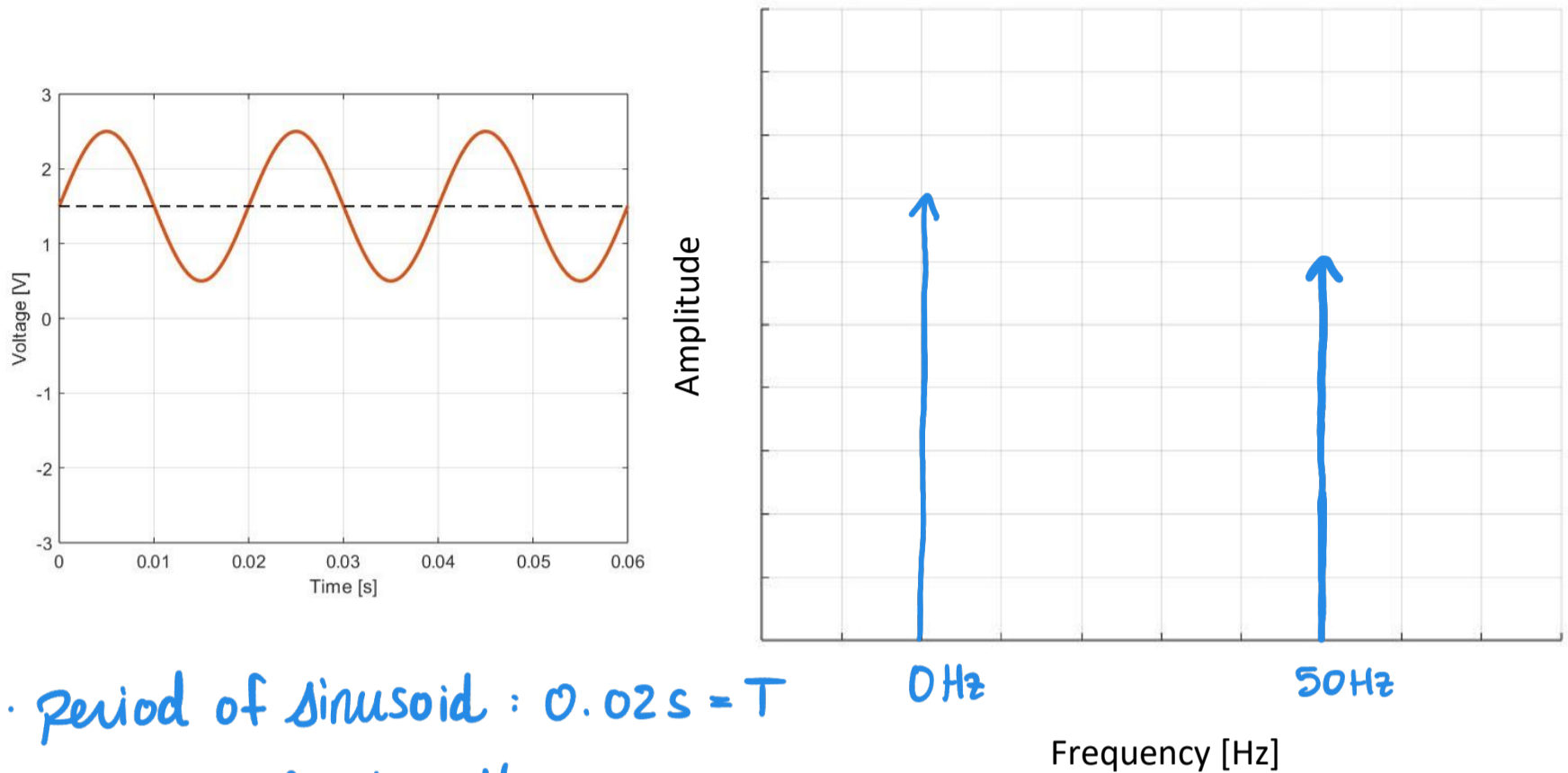


[+1] filter type matches part c

[+1] appropriate frequency range attenuated

[+1] max and min H values labeled

1.2 [3 pts] Sketch the frequency-domain representation of the time-domain signal shown below. Show all of your work. *Note:* the relative amplitudes of different signal components in the frequency domain will not be graded, only their locations on the frequency axis.



- Period of sinusoid : $0.02\text{ s} = T$
- frequency $f = 1/T = 1/0.02\text{ s} = 50\text{ Hz}$
- DC signal @ $f = 0\text{ Hz}$

[+1] shape: 2 peaks in f domain

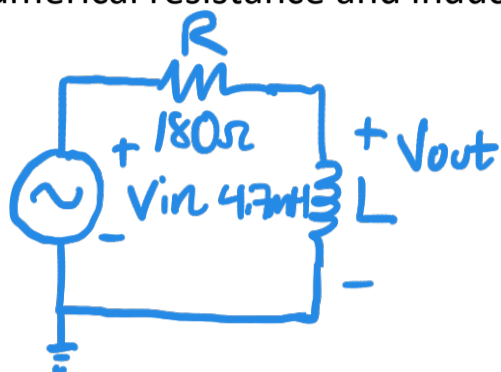
[+1] peak labeled at $f = 50\text{ Hz}$

[+1] peak labeled at $f = 0\text{ Hz}$ (DC offset)

Problem 2 [27 Points]: Filter Circuits and Transfer Functions

2.1 [12 pts] Design Problem: 1st Order High-Pass Filter

- a. [3 pts] Draw a circuit that implements a 1st order high-pass filter using a resistor and an inductor. You must specify the input voltage source and output voltage. You do not need to specify numerical resistance and inductance values (you will do that in part b).



[+1] Series RL circuit

[+1] V_{in} specified

[+1] V_{out} across L

- b. [3 pts] Choose values for your resistor and inductor to give a corner frequency of $f_c = 6 \times 10^3$ Hz, and label your circuit in part a with them. You have only the following resistor and inductor values available:

Resistance (Ω)	Inductance (H)
47	0.0010
180	0.0024
390	0.0047

$$f_c = \frac{\omega_c}{2\pi} = \frac{R}{2\pi L}$$

$$R = 2\pi f_c L = 37699 \cdot L$$

For $L = 0.0047$ H, $R = 177 \Omega$, so choose

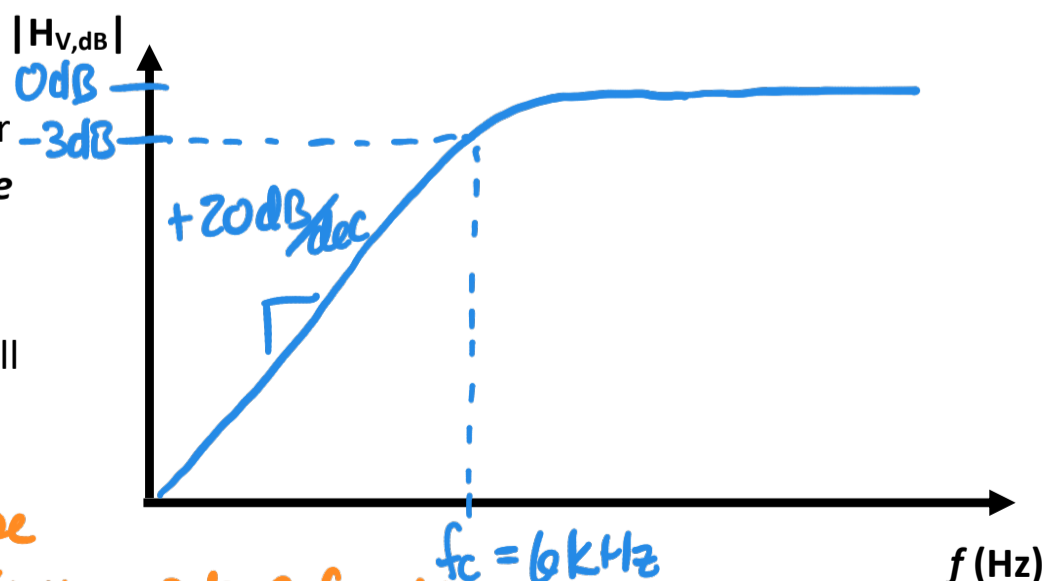
[+1] Correct expression for f_c, ω_c

[+1] Valid values for R, L selected

[+1] Values labeled on circuit

$$\begin{cases} L = 0.0047 \text{ H} \\ R = 180 \Omega \end{cases}$$

- c. [3 pts] Sketch the Bode plot of the magnitude of the transfer function for your high-pass filter **on a log-log scale** (decibels vs. frequency in Hz) to the right. Label the location of the corner frequency in dB and frequency, as well as the filter's roll-off in decibels per decade.



[+1] high-pass filter shape

[+1] Sufficient labeling ($|H| = -3\text{dB}$ @ $f_c = 6\text{kHz}$, $|H_{max}| = 0\text{dB}$)

[+1] correct roll-off

- d. [3 pts] Derive the transfer function $H(s)$ for your filter circuit in part *a* and put it in the form $H(s) = \frac{N(s)}{s+\alpha}$, where $N(s)$ is a function of s , and α is a constant. Express your transfer function in terms of R , L , and s .

$$V_{out} = V_{in} \frac{sL}{R + sL} = V_{in} \frac{s}{s + R/L}$$

$$H(s) = \frac{s}{s + R/L}$$

[+1] correct use of voltage divider to find $H(s)$

[+1] $H(s)$ given in specified form

[+1] no math error

- 2.2 [6 pts] *Transfer Functions and Complex Voltages*: Answer the following questions for the transfer function below. When referenced in this problem, the input voltage is $V_{in} = 8 \sin(5 \times 10^4 t)$.

$$H(j\omega) = \frac{10^4}{j\omega + 10^4}$$

- a. [2 pts] Find the expression for the magnitude of the transfer function $|H(j\omega)|$. Express your answer in terms of ω .

$$|H(j\omega)| = \sqrt{H(-j\omega) \cdot H(j\omega)} = \sqrt{\frac{10^4}{-j\omega + 10^4} \cdot \frac{10^4}{j\omega + 10^4}} = \frac{10^4}{\sqrt{\omega^2 + 10^8}}$$

[+1] valid approach to finding $|H|$

[+1] no math error

- b. [2 pts] Calculate the magnitude of the transfer function *in decibels* at the frequency of the input voltage V_{in} .

$$|H(j\omega)| = \frac{10^4}{\sqrt{\omega^2 + 10^8}} \quad \omega_{in} = 5 \times 10^4 \text{ rad/s}$$

$$|H(j\omega_{in})| = \frac{10^4}{\sqrt{(5 \times 10^4)^2 + 10^8}} = 0.196$$

$$|H_{v,dB}(j\omega_{in})| = 20 \log_{10}(0.196) = \underline{-14.15 \text{ dB}}$$

[+1] correct formula for $|H_{v,dB}|$

[+1] no math error

- c. [2 pts] Calculate the amplitude of the output voltage $|V_{out}|$ *in volts* for the given input voltage V_{in} .

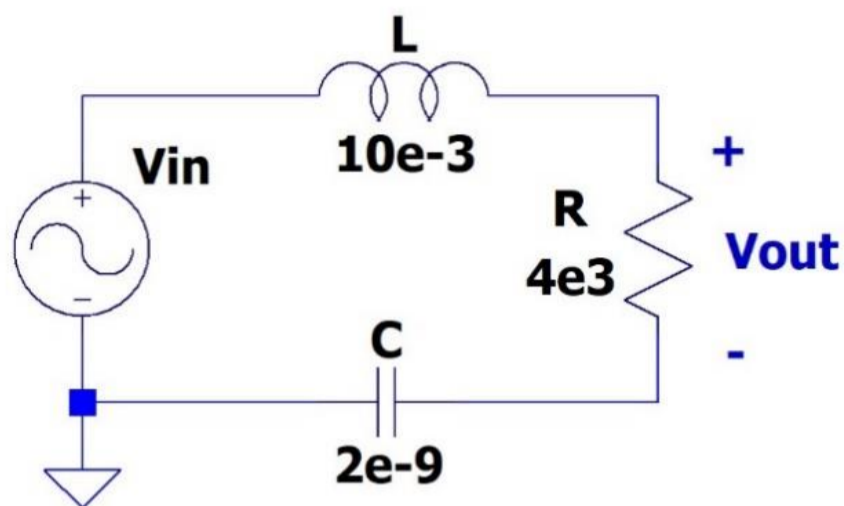
$$|V_{out}| = |H| \cdot |V_{in}| = 0.196 \cdot 8V = \underline{1.57V}$$

[+1] valid approach to finding $|V_{out}|$

[+1] no math error

2.3 [9 pts]: 2nd Order Filter Circuit Analysis

- a. [3 pts] Derive the transfer function for the circuit below in terms of L, R, C, and s. Express your transfer function in the form $H(s) = \frac{N(s)}{s^2 + \beta s + \alpha}$, where N(s) is a function of s, and β and α are constants.



$$\begin{aligned} V_{out} &= V_{in} \frac{Z_R}{Z_L + Z_R + Z_C} \\ &= V_{in} \frac{R}{sL + R + 1/sC} \\ &= V_{in} \frac{R/L \cdot s}{s^2 + R/L \cdot s + 1/LC} \end{aligned}$$

$$H(s) = \frac{R/L \cdot s}{s^2 + R/L \cdot s + 1/LC}$$

[+1] correct use of voltage divider to find H(s)

[+1] H(s) given in specified form

[+1] no math error

- b. [4 pts] Sketch a plot of $|H(j\omega)|$ **on a log-linear** scale. You must show your work in evaluating $H(j\omega)$ in the limit of very low frequencies and very high frequencies. Label all relevant filter frequency parameters (corner frequency/frequencies, center frequency, etc.) on the plot. *Note:* You do not need to give numerical frequencies for any corner frequencies, but you must give a numerical frequency for a center frequency (if applicable to your filter).

If you're not confident in your answer for part a, you may choose to sketch the alternate transfer function below instead for full credit. If you choose this option, you must clearly circle "I am sketching the alternate transfer function".

I am sketching the alternate transfer function

alternate $H(s) = \frac{s^2 + 10^8}{s^2 + 10^3 s + 10^8}$
 $H(s): R = 10 \Omega; L = 10 \times 10^{-3} \text{H}; C = 1 \times 10^{-6} \text{F}$

$$\begin{cases} s \rightarrow 0 : H(s) \rightarrow \frac{10^8}{10^8} = 1 \\ s \rightarrow \infty : H(s) \rightarrow \frac{s^2}{s^2} \rightarrow 1 \\ \text{at } \omega = \omega_0 : H(s) = 0 \end{cases}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{2\pi} \frac{1}{\sqrt{10\text{mH} \cdot 1\mu\text{F}}} = \underline{1.59 \text{ kHz}}$$

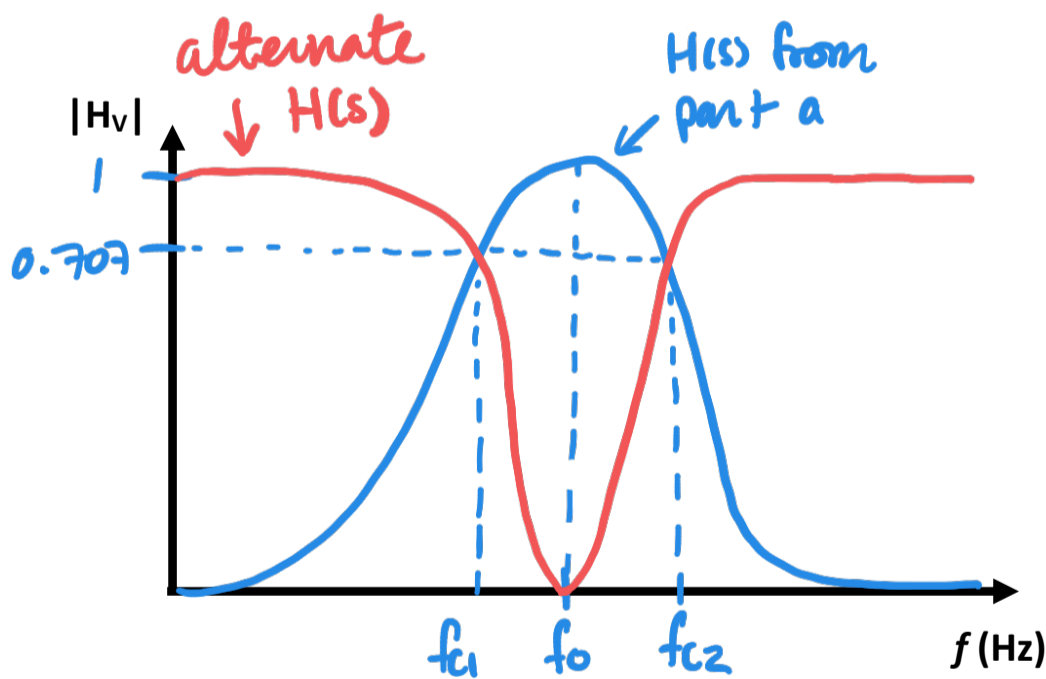
from a)

$$H(s) = \frac{\frac{R}{L} s}{s^2 + \frac{R}{L} s + \frac{1}{LC}}$$

$$\begin{cases} s \rightarrow 0 : H(s) \rightarrow 0 \\ s \rightarrow \infty : H(s) \rightarrow \frac{R/L}{s^2} \rightarrow 0 \\ \text{at } \omega = \omega_0 : |H(j\omega)| = 1 \end{cases}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} = \frac{1}{2\pi} \frac{1}{\sqrt{10\text{mH} \cdot 2\text{nF}}}$$

$$f_0 = \underline{35.6 \text{ kHz}}$$



- [+1] Correct $\omega \rightarrow 0$ limit
- [+1] Correct $\omega \rightarrow \infty$ limit
- [+1] $|H|$ at $f=f_0$ labeled correctly
- [+0.5] y-axis labeled sufficiently
- [+0.5] corner frequencies indicated

- c. [2 pts] What kind of filter does your transfer function sketch in part b represent?

from part a:
band-pass filter

alternate:
band-stop filter

[+2] answer agrees with part b

Problem 3 [10 Points]: Concepts and Short Answer

3.1 [2 pts] Explain how an RC circuit can serve as both a low-pass and high-pass filter. *Hint:* You should mention Kirchhoff's voltage law in justifying your response.

Since KVL states that the sum of the voltages around a loop is zero, in an RC circuit $V_{in} = V_R + V_C$. If the frequency of V_{in} is increased and its amplitude remains constant, when V_C decreases with frequency (LFF), V_R must increase with frequency (HPF) to keep V_{in} constant. [+2] valid response + justification

3.2 [2 pts] Short answer: Explain why using capacitors and inductors enables us to create filter circuits.

Capacitors and inductors have frequency dependent impedances, which allows us to control voltage and current as a function of frequency.

[+2] valid response and justification

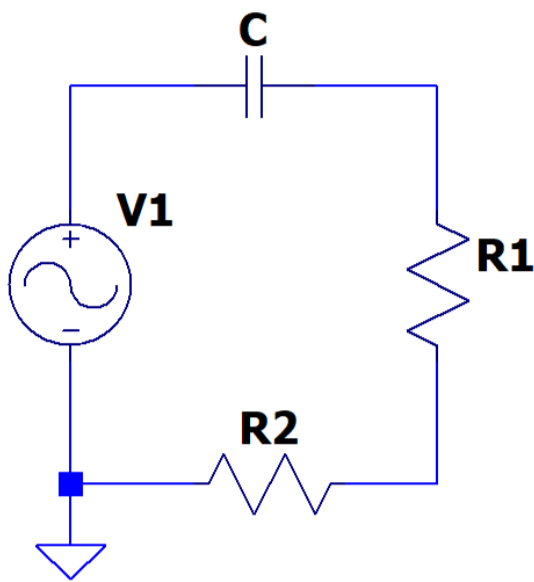
3.3 [2 pts] True or False: Only some signals can be represented as an infinite sum of sinusoidal waves with different frequencies and amplitudes. Circle one:

True

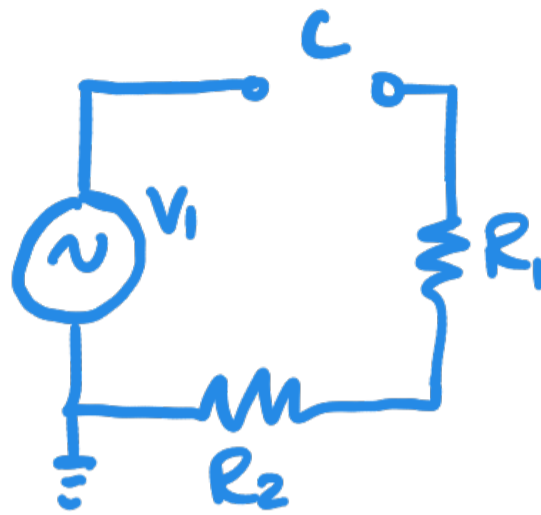
False

[+2] correct

3.4 [2 pts] For the circuit below, draw the equivalent circuit at both very low frequencies and very high frequencies.

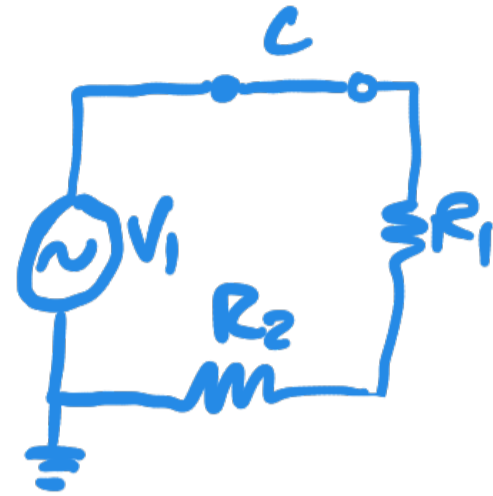


Low Frequency



at low ω : $\omega \rightarrow 0$
 $Z_c = \frac{1}{j\omega C} \rightarrow \infty$
 open-circuit

High Frequency



at high ω : $\omega \rightarrow \infty$
 $Z_c = \frac{1}{j\omega C} \rightarrow 0$
 short-circuit

[+1] correct low frequency circuit

[+1] correct high frequency circuit